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著者	Nakamura Kohei
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Velocity of Long Gravity Waves in the Ocean

By Kôhei Nakamura

Geophysical Institute, Faculty of Science, Tôhoku University

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Abstract

Effects on the velocities of oceanic waves of compressibility of the sea water, elasticity of materials beneath the sea bed, the Corioli's force and curvature of the earth are examined particularly in case of long period waves as tsunamis. Computation of the dispersion curves for the phase and group velocities reveals that the usual velocities of oceanic waves whose wave lengths fall within the range 100 to 10000 km are reduced at most 1 percent owing to the above-mentioned effects.

1 Introduction

It is well known that the observed travel time of the initial disturbance of a tsunami is in most cases less than the one computed on the assumption that the tsunami is originated at the earthquake epicentre, and that the travel time is given by the formula $\int \frac{ds}{\sqrt{gh}}$, where g is the acceleration due to gravity and h is the depth of the sea, and the integration is to be carried along the orthogonal trajectory drawn in the refraction diagram. The discrepancy between the observed and computed travel times has been attributed to vast lateral extent of the deformed sea bed. The source area thus determined of the Sanriku tsunami of 1933 was in linear dimensions as large as about 600 km (N. Miyabe (1934)). Unless crustal deformations occurred in advance of the main shock, this figure seems to be incredibly large in spite of a large magnitude 8.0 of the shock, because there was no foreshock which could explain the earlier arrival of the tsunami.

Should the group velocity of long waves exceed $\sqrt{gh}$, the source area determined from the travel time differences could be diminished to some extent. It is not a little interesting to ascertain whether the group velocity increases or does not due to unconsidered effects on the progressive waves.

The factors which influence the velocity of oceanic waves with very long periods may be the curvature and rotation of the earth, the compressibility of the sea water, elasticity of the bottom materials, eddy viscosity of the sea water and the oceanic currents.

H. Nagaoka (1909) showed that the effect of the earth's curvature is to decrease the phase velocity. T. Terada (1921) examined the influence of elasticity of the earth's crust, and suggested that the group velocity of oceanic waves may exceed by about 10 percent the standard value computed on the assumption of a rigid sea bed.

In this paper the effect of elastic bottom and compressible water will be examined on a more reasonable model than Terada's. Also modifications due to the rotation and curvature of the earth, and eddy viscosity are investigated respectively. Then combined effects of these factors on the phase and group velocities will be exhibited in the form of dispersion curves.

2 Modification of Phase and Group Velocities due to Compressibility of Sea Water and Elasticity of Bottom Materials

We suppose that a plane sheet of water of infinite extent is underlain by a half space of an elastic solid. For simplicity's sake, the gravity terms in the equations for the elastic solid will be ignored. The velocity equation for the waves propagated in this system have been studied by R. STONLEY (1926), J.G. SCHOLTE (1943), and T. MATSUZAWA (1950). STONLEY considered the effect of the ocean on Rayleigh waves, SCHOLTE treated a problem of generation of microseisms, and MATSUZAWA's interest was in the modification of the velocity of tsunami.

The velocity equation contains two different types of dispersion, one is concerned with suboceanic Rayleigh waves, and the other is related to gravitational waves in water. H. MIYOSHI (1954) treated a problem of generation of a tsunami in compressible sea water, and showed that the phase velocity of gravity waves decreases by about 1 percent on account of the compressibility of the water.

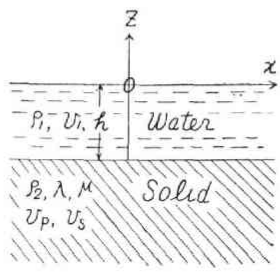

Fig. 1. Water layer over a half space of the elastic solid.

The coordinate system is shown in Fig. 1. The depth of the sea is h, the densities of water and solid are denoted by ρ_1 and ρ_2, respectively, and λ and μ designate the Lamè's constants. The velocity of sound in water and the velocities of dilatational and distorsional waves in the solid are denoted respectively by v_1; v_p and v_s. The time factor $e^{i\omega t}$ is assumed in this section.

Only plane waves are considered, and in the water part of the system, the linealized shallow water theory is used.

The motion of a water particle is governed by the equation for the velocity potential φ_1

$$\frac{\partial^2 \varphi_1}{\partial t^2} - v_1^2 \nabla^2 \varphi_1 + g \frac{\partial \varphi_1}{\partial z} = 0 . \tag{1}$$

The solution of (1) can be written

$$\varphi_1 = e^{\gamma z} (A \sinh \kappa z + B \cosh \kappa z)\, e^{i \xi x} , \tag{2}$$

where

$$\gamma = g/2 v_1^2 , \tag{3}$$

$$\kappa^2 = \xi^2 - \frac{\omega^2}{v_1^2} + \gamma^2 . \tag{4}$$

The pressure and the velocity of a particle along the z-direction are given respectively by

$$\rho_1 \dot{\varphi}_1 \quad \text{and} \quad -\frac{\partial \varphi_1}{\partial z} . \tag{5}$$

In the solid, the horizontal u and vertical w displacements are written in terms of the potentials φ and ψ,

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} . \tag{6}$$

The functions φ and ψ satisfy the wave equations

$$\nabla^2 \varphi = \frac{1}{v_p^2} \frac{\partial^2 \varphi}{\partial t^2}, \quad \nabla^2 \psi = \frac{1}{v_s^2} \frac{\partial^2 \psi}{\partial t^2} . \tag{7}$$

The solutions of (7) can be written

$$\varphi = P\, e^{\alpha z - i\xi x}, \qquad \psi = Q\, e^{\beta z - i\xi x}, \tag{8}$$

where

$$\alpha^2 = \xi^2 - h^2, \quad \beta^2 = \xi^2 - k^2, \quad h = \omega / v_p, \quad k = \omega / v_s , \tag{9}$$

and

$$\mathcal{R}e\,(\alpha, \beta) > 0 .$$

Substituting (8) into (6), we have

$$\left. \begin{aligned} u &= -\,(i\,\xi\, P e^{\alpha z} + \beta\, Q\, e^{\beta z})\, e^{-i\xi x}, \\ w &= (\alpha\, P e^{\alpha z} - i\,\xi\, Q\, e^{\beta z})\, e^{-i\xi x}. \end{aligned} \right\} \tag{10}$$

The stress components are

$$P_{zx} = \mu \left[2 \frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right], \quad P_{zz} = \lambda\, \nabla^2 \varphi + 2\,\mu \left[\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right]. \tag{11}$$

We assume hereafter that $\lambda = \mu$.

The boundary conditions are that the pressure at the sea surface is constant, and at the sea bed, the vertical velocity is continuous, the horizontal stress is zero, and the vertical stress is continuous. These conditions can be written as follows,

$$\left. \begin{aligned} &\frac{\partial^2 \varphi_1}{\partial z^2} + g \frac{\partial \varphi_1}{\partial z} = 0 && \text{at } z = 0 , \\ &-\frac{\partial \varphi_1}{\partial z} = \frac{\partial w}{\partial t} && \text{at } z = -h., \\ &P_{zx} = 0 && \text{at } z = -h., \\ &P_{zz} + \rho_1\, \dot{\varphi}_1 + g\,(\rho_2 - \rho_1)\, w = 0. && \text{at } z = -h. \end{aligned} \right\} \tag{12}$$

Substituting (2), (5), (10) and (11) in (12), the boundary conditions become

$$\left. \begin{aligned} &A\,[g\,\kappa] - B\,[\omega^2 - g\,\gamma] = 0 \\ &A\,[\kappa \cosh \kappa\, h - \gamma \sinh \kappa\, h] + B\,[\gamma \cosh \kappa h - \kappa \sinh \kappa\, h] \\ &\qquad + P\,[i\,\omega\,\alpha\, e^{h(\gamma - \alpha)}] + Q\,[-\omega\,\xi\, e^{h(\gamma - \beta)}] = 0 . \\ &P\,[2\,\xi\,\alpha\, e^{-\alpha h}] + Q\,[-i\,(2\,\xi^2 - k^2)\, e^{-\beta h}] = 0 \end{aligned} \right\} \tag{13}$$

$$\left.\begin{aligned}&A\left[-i\,\omega\,\rho_1\,e^{-\gamma h}\sinh\kappa h\right]+B\left[i\,\omega\,\rho_1\,e^{-\gamma h}\cosh\kappa h\right]+P\left[\{\mu\,(2\,\xi^2-k^2)\right.\\&\quad\left.+g\,(\rho_2-\rho_1)\}\,e^{-\alpha h}\right]+Q\left[-i\,\{2\,\mu\,\xi\,\beta+g\,(\rho_2-\rho_1)\,\xi\}\,e^{-\beta h}\right]=0\,.\end{aligned}\right\}$$

Eliminating A, B, P and Q from the simultaneous equations (13), we obtain the characteristic equation

$$\begin{aligned}&\left[\mu\,\{4\,\xi^2\,\alpha\,\beta-(2\,\xi^2-k^2)^2\}+g\,(\rho_2-\rho_1)\,\alpha\,k^2\right]\left[\gamma\,\{g\,\kappa\cosh\kappa\,h\right.\\&\quad\left.-(\omega^2-g\,\gamma)\sinh\kappa\,h\}+\kappa\,\{(\omega^2-g\,\gamma)\cosh\kappa\,h-g\,\kappa\sinh\kappa\,h\}\right]\\&\quad+\omega^2\,\rho_1\,k^2\,\alpha\left[g\,\kappa\cosh\kappa\,h-(\omega^2-g\,\gamma)\sinh\kappa\,h\right]=0\,.\end{aligned}$$

This can be rewritten in a convenient from

$$\left.\begin{aligned}&\left[\left(2-\frac{c^2}{v_s^2}\right)^2-4\sqrt{1-\frac{c^2}{v_p^2}}\sqrt{1-\frac{c^2}{v_s^2}}-\frac{v^2}{v_s^2}\,\frac{c^2}{v_2^s}\,\frac{1}{\eta}\sqrt{1-\frac{c^2}{v_p^2}}\right]\\&\quad+\frac{\tanh\kappa\,h}{\kappa\,h}\left[v^2\left(\frac{1}{2\,v^2}-\frac{1}{c^2}\right)\left\{\left(2-\frac{c^2}{v_s^2}\right)^2-4\sqrt{1-\frac{c^2}{v_p^2}}\sqrt{1-\frac{c^2}{v_s^2}}\right\}\right.\\&\quad\left.+\frac{c^2}{v_s^2}\sqrt{1-\frac{c^2}{v_p^2}}\,\eta\left\{\frac{\rho_1}{\rho_2}\,\frac{c^2}{v_s^2}-\frac{v^4}{v_s^2}\,\frac{1}{\eta^2}\left(\frac{\rho_1}{\rho_2}\,\frac{1}{c^2}-\frac{1}{c^2}+\frac{1}{2\,v_1^2}\right)\right\}\right]=0\,,\end{aligned}\right\}\tag{14}$$

where

$$\kappa\,h=\sqrt{\eta^2\left(1-\frac{c^2}{v_1^2}\right)+\left(\frac{v^2}{2\,v_1^2}\right)^2}\,,\qquad c=\frac{\omega}{\xi}\,,\quad v=\sqrt{g\,h}\,,$$
$$\eta=\xi\,h=2\,\pi\,h/\lambda\,.\tag{15}$$

In (15), c and λ represent the phase velocity and the wave length, respectively.

In computing the dispersion curves two models are used;

Model A : $h=4.5$ km, $\rho_2=3.47$, $\rho_1=1$, $v_s=4.60$ km/sec, $v_1=1.5$ km/sec,

Model B : $h=4.8$ km, $\rho_2=5.52$, $\rho_1=1$, $v_s=6.00$ km/sec, $v_1=1.5$ km/sec.

The model A represents the approximate natures of a portion of the upper few hundred kilometers of the mantle, and the model B represents the averaged natures of the

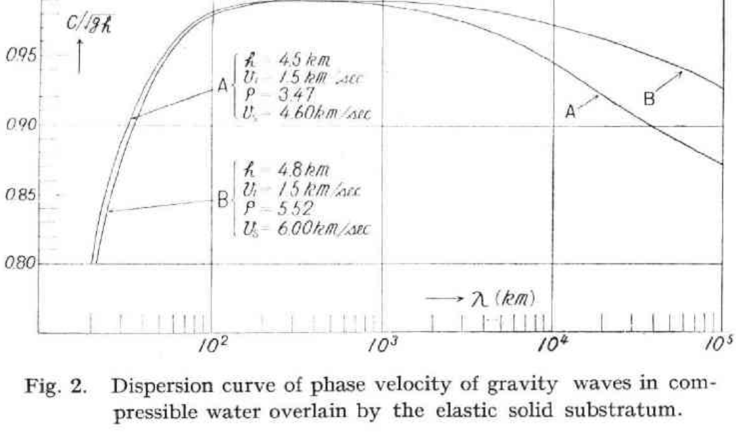

Fig. 2. Dispersion curve of phase velocity of gravity waves in compressible water overlain by the elastic solid substratum.

elastic part of the earth. In usual tsunamis which have small wave heights, the elastic yielding may not be transmitted far beneath the sea bed, so that the model A seems to be more realistic than the model B. Fig. 2 shows the dispersion curves for the phase velocity c, the abscissa indicating the wave length in kilometers.

The group velocity U is computed by the formula $U=c-\lambda\, dc/d\lambda$, through numerical differentiation of the dispersion curve for the phase velocity, and is shown in Fig. 3. In Figs. 2 and 3, A and B designate the two models assumed. We see that the maxima of both the phase and group velocities decrease at most by 1 percent.

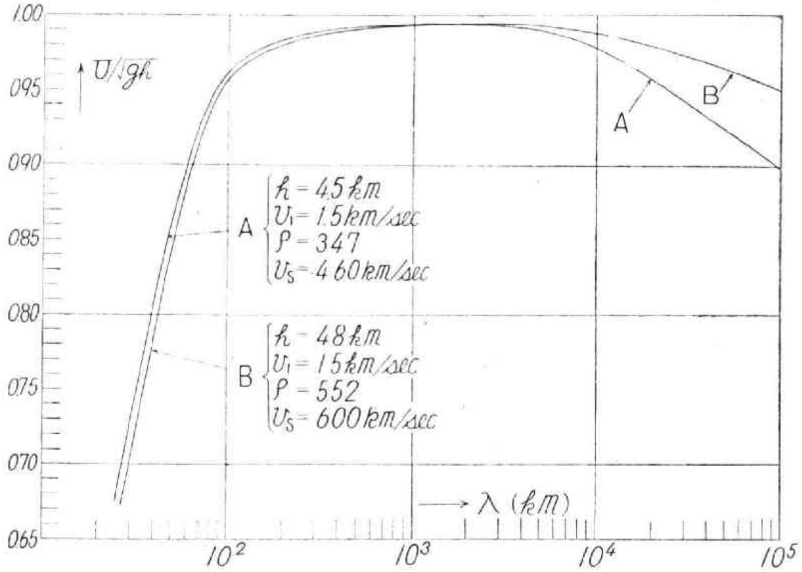

Fig. 3. Dispersion curve of group velocity of gravity waves in compressible water overlain by the elastic solid substratum.

3 Effect of Corioli Force

For long waves the effect of the rotation of the earth can be given by the equation (Ref. LAMB (1932)).

$$\nabla^2 \zeta + \frac{\sigma^2 - 4\,\omega^2 \sin^2 \varphi}{g\,h}\,\zeta = 0 \tag{16}$$

where ζ is the elevation, σ is the circular frequency, φ is the latitude, and ω is the angular velocity of the rotation of the earth. The phase velocity c is expressed by the relation

$$\frac{c}{\sqrt{g\,h}} = \left[1 - \frac{4\,\omega^2 \sin^2 \varphi\,\lambda^2}{\pi^2 g\,h}\right]^{1/2}, \tag{17}$$

and the group velocity becomes

$$\frac{U}{\sqrt{g\,h}} = \frac{c}{\sqrt{g\,h}} - \frac{\omega^2 \sin^2 \varphi}{\pi^2 g\,h}\,\frac{\lambda^2}{(c/\sqrt{g\,h})} \tag{18}$$

The dispersion curves for different latitudes of the phase and group velocities are shown in Figs. 4 and 5, respectively, assuming the depth of the sea to be 4.5 km. These figures show that the phase velocity increases, whereas the group velocity decreases with the wave length.

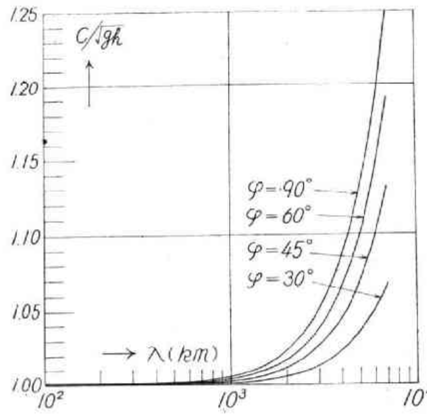

Fig. 4 Effect of Corioli's force on the phase velocity of oceanic waves. φ designates latitude, h=4.5km.

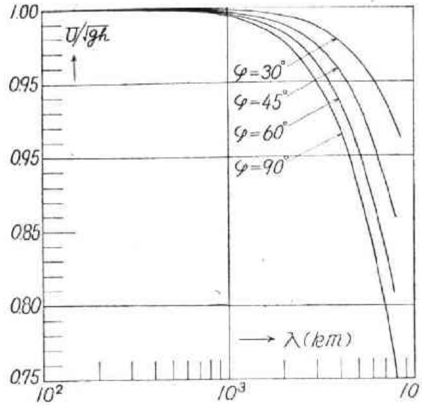

Fig. 5 Effect of Corioli's force on the group velocity of oceanic waves. φ designates latitude, h=4.5 km.

4 Effect of Curvature of the Earth

When the waves follow the zonal harmonic distribution and the wave length is not extremely long, the phase velocity of oceanic waves can be written by (cf. NAGAOKA (1909), LAMB (1932))

$$\frac{c}{\sqrt{g h}}=\left[1-\frac{3}{(2 n+1) \rho}\right]^{1/2} \tag{19}$$

where $n=2\pi a/\lambda$, a and ρ are the radius and the mean density of the earth, respectively.

The group velocity can be written

$$\frac{U}{\sqrt{g h}}=\frac{c}{\sqrt{g h}}+\frac{3 n}{(2 n+1)^2} \frac{1}{c/\sqrt{g h}} \tag{20}$$

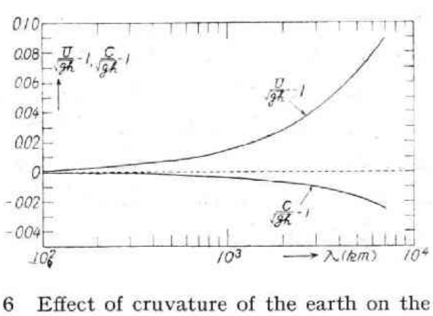

Fig. 6 Effect of cruvature of the earth on the phase and group velocities of oceanic waves.

Assuming $a=6370$ km and $\rho=5.52$, the effect of the curvature on the phase and group velocities is shown in Fig. 6. We see that the phase velocity decreases, whereas the group velocity increases with the wave length.

5 Effcct of Eddy Viscosity

When eddy viscosity is taken into account, the velocity $\sqrt{gh}$ of long waves decreases, the order of the decrease is given for example by NAKAMURA (1960) by the relation

$$\frac{\sqrt{gh}}{p}, \quad p=\frac{1}{\sqrt{2}}\sqrt{\frac{C}{A^2+B^2}}\sqrt{\sqrt{A^2+B^2}+A}, \tag{21}$$

where

$$A=2\kappa(\cosh 2\kappa+\cos 2\kappa)-(\sinh 2\kappa+\sin 2\kappa), \quad B=\sinh 2\kappa-\sin\kappa,$$

$$C=2\kappa(\cosh 2\kappa+\cos 2\kappa), \quad \kappa=h\sqrt{\sigma}/\sqrt{2\nu}$$

and $\sigma=2\pi/T$ is the circular frequency and ν is the coefficient of eddy viscosity. If we assume that $h=4.5$ km and $T=60$ min., it follows that

$$\frac{1}{p}=\begin{cases} 0.9998 & \text{for} \quad \nu=10^2\ \mathrm{cm^2/sec}, \\ 0.9981 & \text{for} \quad \nu=10^4\ \mathrm{cm^2/sec}, \\ 0.9941 & \text{for} \quad \nu=10^5\ \mathrm{cm^2/sec}. \end{cases} \tag{22}$$

The figures in (22) show that the effect of eddy viscosity is comparatively small, so that the factors considered in sections 2–4 will be most important ones.

6 Combined Effect on Phase and Group Velocicities of the Factors discussed in Sections 2-4

The effects on the velocity of the factors described in the sections 2–4 are superposed to give the total effects, and the dispersive property thus obtained is shown in Figs. 7 and 8 for the model A, and in Figs. 9 and 10 for the model B. Figs. 7 and 9 represent the dispersion for the phase velocity, and Figs. 8 and 10 show the dispersion for the group velocity. In these figures, the broken cruves indicate usual dispersion curves for the system of incompressible water over a rigid boundary when the roation and the curvature of the earth are ignored.

Fig. 8 which shows the dispersion of the group velocity for the model A indicates that the maximum group velocity decreases by about 0.7 percent for both latitudes $\varphi=30°$ and $\varphi=60°$. It may be certain that, even if the decrease by about several thousandths percents due to eddy viscosity is taken into account, the decrease in the groups velocity has the order of one percent. It is to be noticed that, in the case of model B, the group velocity increases by few percents.

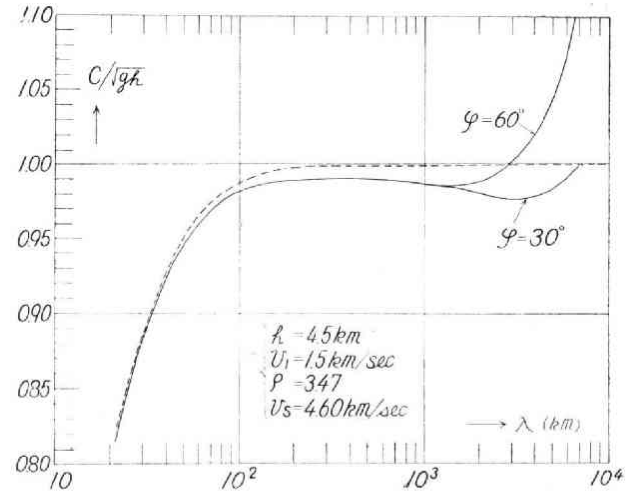

Fig. 7 Dispersion curve of the phase velocity of oceanic waves when effects of compressible water, elastic bottom, rotation and curvature of the earth are taken into account. Broken curve represents dispersion of usual gravity waves. (Model A)

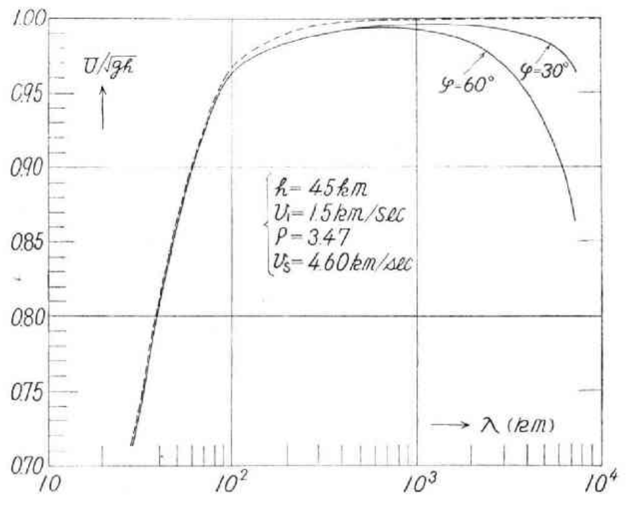

Fig. 8 Dispersion curve of the group velocity of oceanic waves when effects of compressible water, elastic bottom, rotation and curvature of the earth are taken into account. Broken curve represents dispersion of usual gravity waves. (Model A)

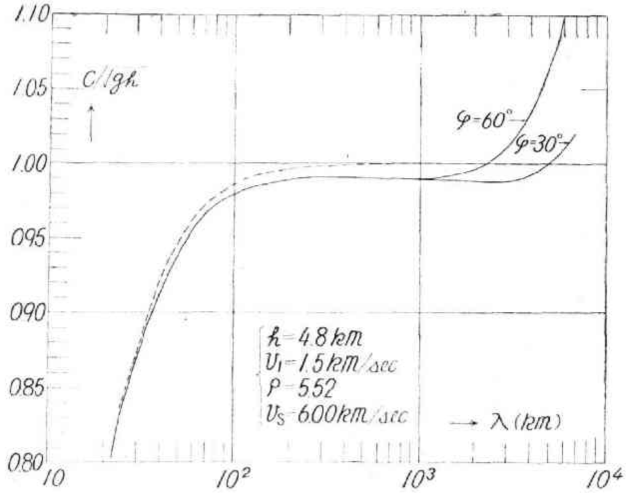

Fig. 9 Dispersion curve of the phase velocity of oceanic waves when effects of compressible water, elastic bottom, rotation and curvature of the earth are taken into account. Broken curve represents dispersion of usual gravity waves. (Model B)

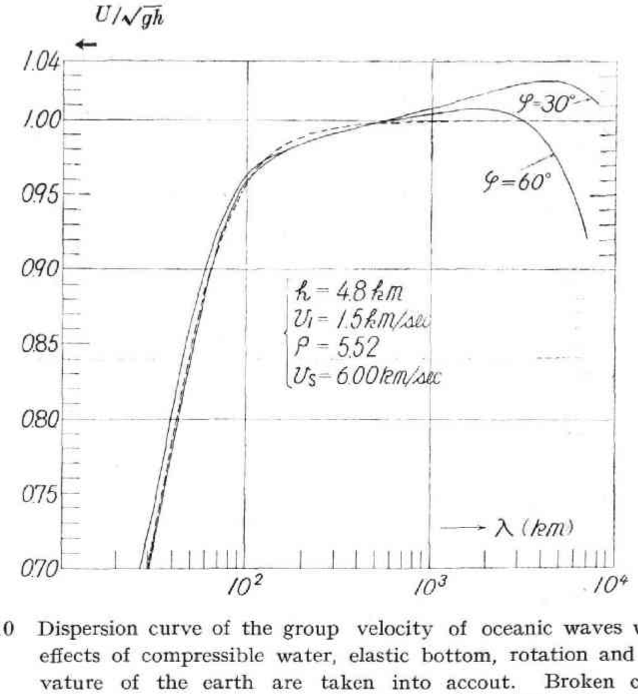

Fig. 10 Dispersion curve of the group velocity of oceanic waves when effects of compressible water, elastic bottom, rotation and curvature of the earth are taken into accout. Broken curve represents dispersion of usual gravity waves. (Model B)

7 Concluding Remarks

From the results obtained in the preceding sections, it is made clear that the initial distrubance of a tsunami is propagated with the velocity little less than $\sqrt{gh}$. However, the change in the group velocity seems to be too small to be considered, since the accuracy in the time keeping of the drums of mareographs is rather bad at least in the present stage of observation. This is probably true even if the effect of ocean currents is introduced in some way. However, in future, the increase in travel time amounting to about 15 min of a trans-Pacific tsunami such as the Chile tsunami of 1960, would become important in the travel-time analysis of a tsunami.

OMOTE (1948) reexamined the marigrams for the Sanriku tsunami of 1933 and determined the central area of the tsunami source. Excess travel time of a tsunami to that of the waves assumed to be radiated from the earthquake epicentre may certainly be ascribed in part to a finite area of the source, but does not seem to be a definite evidence that the source area has such a dimension as to account for the whole excess time. It is needed to study minute behaviour of the front of waves generated by finite deformations of the sea bed. Particularly, in tsunamis of nearby origin, the epicentre being located within a few hundred kilometers off the coast, it is important to take account of the mechanism of tsunami generation and also conspicuous change in water depth coastwards.

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