Elastic Waves Generated by a Directional Source (?)

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Elastic Waves generated by a Directional Source (I)

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Abstract

Generation of the transient elastic waves due to a surface linear force acting at an inclination, is investigated theoretically. The spatial distribution of relative amplitudes of the dilatational and distortional waves, and the effect of inclination of the force on the particle motion of the surface are elucidated for all possible values of Poisson's ratio.

1. Introduction

The control of generation of the elastic waves is a very important problem in seismic exploration, and has been the subject of several articles. To obtain a good signal-to-noise ratio, various methods are applied such as, for examples, pattern shooting or delayed detonation of the explosives. (PARR et. al., 1955, MUSGRAVE et. al., 1958). An attempt to determine the underground structure in terms of SH waves as well as P waves, has been made by the Seismic Exploration Group of Japan (KOBAYASHI, 1959).

It has been known that the energy radiated horizontally is the main source of the energy contributing to the background noise. The increase in the amount of energy emitted in the direction corresponding to the critical angle of incidence for a wave in a layered medium, may result in the improvement in signal-to-noise ratio.

In the present paper, the author intends to study primarily the surface linear force problem based on the exact solution attained by use of integral transform techniques, and elucidate the spatial distribution of relative amplitude of the dilatational and distortional waves generated in a semi-infinite elastic solid. The effect of the surface force on the generation of a seismic pulse with the major portion of its energy concentrated in a range of angle of emergence, and that on the particle motion of the surface, will be discussed. As the Poisson's ratio σ of more than 0.45 are occasionally encountered in seismic exploration, the results for all possible values of σ may be of use for the practical purposes.

2. Exact solution

Let us take x–y axes on the surface of a semi-infinite elastic solid, and z-axis vertically downward into the solid (Fig. 1). The impulsive force is supposed to act uniformly along a line coincident with the y-axis, at an inclination ϑ measured from a normal to the surface, the motion produced being independent of y. The strengths of the normal and tangential components of the force per unit length of the y-axis,

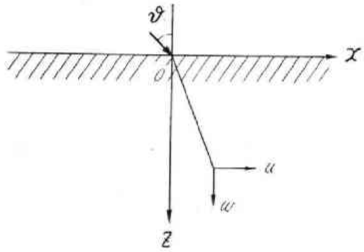

Fig. 1.

are assumed to be

$$\begin{Bmatrix} f_N(x,t) \\ f_T(x,t) \end{Bmatrix} = \delta(x)\,\delta(t) \begin{Bmatrix} \cos\vartheta \\ \sin\vartheta \end{Bmatrix} \tag{1}$$

respectively, where $\delta(x)$ is the Dirac's delta function, and t the time.

The displacement components u and w produced by this force system are given by the vector sum of the components due to the normal and tangential forces with the strength $\delta(x)\,\delta(t)$ per unit length of the y-axis, in such a way that

$$\left.\begin{aligned} u &= u_N \cos\vartheta + u_T \sin\vartheta \\ w &= w_N \cos\vartheta + w_T \sin\vartheta \end{aligned}\right\} \tag{2}$$

Subscripts N and T refer to the quantities for the normal and tangential forces respectively.

We define the Laplace and the complex Fourier transforms by

$$\bar{g}(x,z,p) = \int_0^\infty g(x,z,t)\, e^{-pt}\, dt \tag{3}$$

and

$$G(\xi,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x,z,t)\, e^{i\xi x}\, dx \tag{4}$$

respectively. When we consider the force varying as a simple harmonic function of the time $e^{i\omega t}$ instead of the delta function in (1), the derivation of the formal solutions for u_N, u_T, w_N and w_T is none other than that made in LAMB's (1904) paper. The frequency ω in the steady state solutions may be regarded as a parameter of the complex Fourier transformation with respect to t. And, ω is related to a parameter p of the Laplace transformation through the equation $\omega = ip$. Therefore, the solutions for the transient problem under consideration in the space of the Laplace transform with respect to t, are derived immediately by the substitutions

$$p \to ip,\ e^{ipt} \to 1,\ e^{i\xi x} \to e^{-i\xi x},\ 2\pi \to \sqrt{2\pi} \tag{5}$$

in LAMB's procedure.*)

*) The last two substitutions result from the LAMB's definition of the Fourier transform

$$G(\xi) = \int_{-\infty}^{\infty} g(x)\, e^{-i\xi x}\, dx$$

which is somewhat different from ours.

If we put

$$\xi = pv/\alpha \tag{6}$$

α being the velocity of the dilatational wave, we have

$$\left.\begin{aligned} \bar{u} &= -\frac{1}{\mu}\sqrt{\frac{2}{\pi}}\left[\cos\vartheta\, Im\int_0^\infty \bar{U}_N(v)\,dv + \sin\vartheta\, Re\int_0^\infty \bar{U}_T(v)\,dv\right] \\ \bar{w} &= \frac{1}{\mu}\sqrt{\frac{2}{\pi}}\left[\cos\vartheta\, Re\int_0^\infty \bar{W}_N(v)\,dv + \sin\vartheta\, Im\int_0^\infty \bar{W}_T(v)\,dv\right] \end{aligned}\right\} \tag{7}$$

where

$$\left.\begin{aligned} \bar{U}_N(v)\,F(v) &= v\,(2\,v^2+m^2)e^{-pt_1} - 2\,v\,a\,b\,e^{-pt_2} \\ \bar{U}_T(v)\,F(v) &= 2\,v^2\,b\,e^{-pt_1} - (2\,v^2+m^2)\,b\,e^{-pt_2} \\ \bar{W}_N(v)\,F(v) &= (2\,v^2+m^2)\,a\,e^{-pt_1} - 2\,v^2\,a\,e^{-pt_2} \\ \bar{W}_T(v)\,F(v) &= -2\,v\,a\,b\,e^{-pt_1} + v\,(2\,v^2+m^2)\,e^{-pt_2} \end{aligned}\right\} \tag{8}$$

$$F(v) = (2\,v^2+m^2)^2 - 4\,v^2\,a\,b$$

$$a = \sqrt{v^2+1}\,,\quad b = \sqrt{v^2+m^2}\,;\quad Re\,a > 0\,,\quad Re\,b > 0$$

$$\alpha\,t_1 = a\,z + i\,v\,x,\quad \alpha\,t_2 = b\,z + i\,v\,x$$

$$m = \alpha/\beta$$

the superimposed bar denotes the transformed quantity, μ the Lame's constant, and β the velocity of the distortional wave.

As the integration with respect to v and the Lapalce inverse transformation of the expressions in (7) can be performed by use of CAGNIARD's (1939) method, the final expressions of the exact solutions for our problem are given as follows:

$$\left.\begin{aligned} u &= -\frac{1}{\mu}\sqrt{\frac{2}{\pi}}\,(U_N\cos\vartheta + U_T\sin\vartheta) \\ w &= \frac{1}{\mu}\sqrt{\frac{2}{\pi}}\,(W_N\cos\vartheta + W_T\sin\vartheta) \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} U_N &= Im\left[v_1\,G_1(v_1)\,H(v_1) - b(v_1)\,G_2(v_2)\,H(v_2)\right] \\ U_T &= Re\left[v_1\,G_3(v_1)\,H(v_1) - b(v_2)\,G_1(v_2)\,H(v_2)\right] \\ W_N &= Re\left[a(v_1)\,G_1(v_1)\,H(v_1) - v_2\,G_2(v_2)\,H(v_2)\right] \\ W_T &= -Im\left[a(v_1)\,G_3(v_1)\,H(v_1) - v_2\,G_1(v_2)\,H(v_2)\right] \\ G_1(v)\,F(v) &= 2\,v^2+m^2 \\ G_2(v)\,F(v) &= 2\,v\,a \\ G_3(v)\,F(v) &= 2\,v\,b \end{aligned}\right\} \tag{10}$$

$$H(v_{1,2}) = \frac{\alpha}{R}\left[\cos\theta\,\frac{t}{\sqrt{t^2-R^2/\alpha^2,\beta^2}} - i\sin\theta\right] \tag{11}$$

$$v_{1,2} = \frac{a}{R}\left[\cos\theta\sqrt{t^2 - R^2/\alpha^2,\ \beta^2} - i\,t\sin\theta\right] \tag{12}$$

$$R = \sqrt{x^2+z^2}\,,\quad \theta = \tan^{-1} x/z\,.$$

If we put $\vartheta=0°$. (9) is the same as (1.46) in NAGUMO's (1960) paper.

3. Dilatational and Distortional Waves*)

The amplitudes of the dilatational (P) and distortional (S) waves due to the impulsive force prescribed for various values of ϑ, θ and σ, will be studied at first.

Substituting t by the arrival times of P or S waves in (12), we have

$$\tilde{v}_1 = -i\sin\theta\,,\quad \tilde{v}_2 = -i\,m\sin\theta\,. \tag{13}$$

It can be easily seen, refering to (10), that the second term in (11) is extraneous for $\tilde{v}_1$ or $\tilde{v}_2$. The first term becomes infinite for $\tilde{v}_1$ and $\tilde{v}_2$. Both infinities are of the order of $1/\sqrt{t}$, but the coefficient are not equal to each other. If in the first term in (11) we substitute $t=R/\alpha+\tau$, τ being positive and small, it takes the form

$$H(v_1) = \cos\theta\sqrt{\frac{\alpha}{2R\tau}}\left(1+\frac{\alpha\tau}{R}\right)\Big/\sqrt{1+\frac{\alpha\tau}{2R}} \tag{14}$$

In a similar manner, we get

$$H(v_2) = \cos\theta\sqrt{\frac{m\,\alpha}{2R\tau}}\left(1+\frac{\beta\tau}{R}\right)\Big/\sqrt{1+\frac{\beta\tau}{2R}} \tag{15}$$

for $t=R/\beta+\tau$. In the limiting case where τ tends to zero,

$$\lim_{\tau\to 0}\frac{H(v_2)}{H(v_1)} = \sqrt{m} \tag{16}$$

We should notice the fact that the infinity at $t=R/\beta$ is $\sqrt{m}$ times the infinity at $t=R/\alpha$. The amplitudes are proportional to $R^{-1/2}$.

Now let us define the amplitudes of P and S waves by those at the respective arrival times. For a detailed discussion on this definition, see Appendix. The amplitudes may be written as follows:

$$\left.\begin{aligned} u_p &= A\cos\theta\sin\theta\left[\cos\vartheta\,(m^2-2\sin^2\theta)+2\sin\vartheta\sin\theta\sqrt{m^2-\sin^2\theta}\right]/F_p(\theta) \\ u_s &= -A\cos^2\theta\left[2\cos\vartheta\sin\theta\sqrt{1/m^2-\sin^2\theta}-\sin\vartheta\sin 2\theta\right]/\sqrt{m}F_s(\theta) \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} w_p &= A\cos^2\theta\left[\cos\vartheta\,(m^2-2\sin^2\theta)-2\sin\vartheta\sin\theta\sqrt{m^2-\sin^2\theta}\right]/F_p(\theta) \\ w_s &= A\sin\theta\cos\theta\left[2\cos\vartheta\sin\vartheta\sqrt{1/m^2-\sin^2\theta}-\sin\vartheta\cos 2\theta\right]/\sqrt{m}F_s(\theta) \end{aligned}\right\} \tag{18}$$

where

$$\left.\begin{aligned} F_p(\theta) &= (m^2-2\sin^2\theta)^2+4\sin^2\theta\cos\theta\sqrt{m^2-\sin^2\theta} \\ F_s(\theta) &= \cos^2 2\theta+4\sin^2\theta\cos\theta\sqrt{1/m^2-\sin^2\theta} \end{aligned}\right\} \tag{19}$$

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*) Approximate solution for the normal force problem was studed by MILLER and PURSEY (1954) and HONDA, NAKAMURA and TAKAGI (1956). Three-dimensional problem for the case of normal as well as tangential forces, is solved approximately by HIRONO (1948, 49).

$$A=\frac{1}{\mu}\sqrt{\frac{2}{\pi}}\lim_{\tau\to 0}\left(1+\frac{\alpha,\beta\tau}{R}\right)\Big/\sqrt{\tau\left(1+\frac{\alpha,\beta\tau}{2R}\right)}. \tag{20}$$

When we denote the radial and transverse components of the displacements of P waves by δ_{pr} and $\delta_{p\theta}$, and those of S waves by δ_{sr} and $\delta_{s\theta}$ respectively (Fig. 2), we have

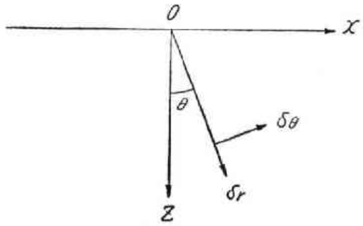

Fig. 2.

$$\delta_{p\theta}=\delta_{sr}=0$$

$$\delta_{pr}=A\left[\cos\vartheta\cos\theta\,(m^2-2\sin^2\theta)+\sin\vartheta\sin 2\theta\sqrt{m^2-\sin^2\theta}\right]/F_p(\theta) \tag{21}$$

$$\delta_{s\theta}=A\,Re\left[-\cos\vartheta\sin 2\theta\sqrt{1/m^2-\sin^2\theta}+\sin\vartheta\cos\theta\cos 2\theta\right]/\sqrt{m}\,F_s(\theta) \tag{22}$$

If the condition $Re\ a(v)>0$ in (8) is attended to, $\sqrt{1/m^2-\sin^2\theta}=-i\sqrt{\sin^2\theta-1/m^2}$. In the case $\sin\theta>1/m^2$ the S wave is subjected to a phase change

$$\epsilon=\tan^{-1}4\sin^2\theta\cos\theta\sqrt{\sin^2\theta-1/m^2}\,/\cos^2 2\theta$$

The phase shift results from the fact that in the region $\theta>\sin^{-1}1/m^2$, the diffraction

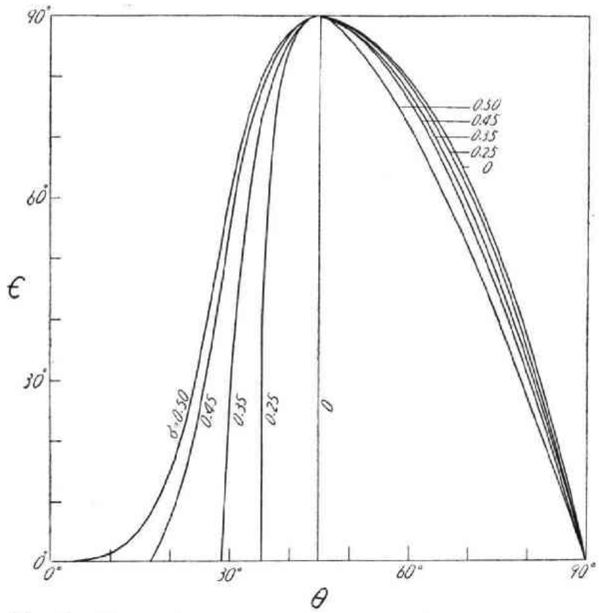

Fig. 3. Phase change ϵ for various angles of emergence.

wave PS travels ahead of the S waves (SHERWOOD, 1958). We define the amplitude of the S waves in such a case by

$$\delta'_{s\theta}=A\left[-\cos\vartheta\sin\epsilon\sin 2\theta\sqrt{\sin^2\theta-1/m^2}+\sin\vartheta\cos\epsilon\cos\theta\cos 2\theta\right]/\sqrt{m}f(\theta) \quad (22')$$

where

$$f(\theta)=\sqrt{\cos^4 2\theta+16\sin^4\theta\cos^2\theta\,(\sin^2\theta-1/m^2)}$$

In Fig. 3, is plotted the attendant phase change ϵ as a function of θ for some values of the Poisson's ratio σ, σ being related to m through the equation

$$m=\sqrt{2\,(1-\sigma)/(1-2\sigma)}\,.$$

As an unit of the amplitudes mentioned above, we take the amplitude of the P wave

$$\delta_{pN}\,(\theta=0)=A/m^2 \quad (23)$$

at point on the z-axis, generated by the normal force. And, we will consider hereafter in this section, the normalized amplitudes defined by

$$[D_P,\,D_S]=(m^2/A)\,[\delta_{pr},\,\delta_{s\theta}]\,. \quad (24)$$

3. 1 $\vartheta=0°$, $90°$

The relative amplitudes of P and S waves, D_{PN} and D_{SN}, generated by a normal force, are given by putting $\vartheta=0°$ in (21) and (22). D_{PT} and D_{ST} due to a tangential

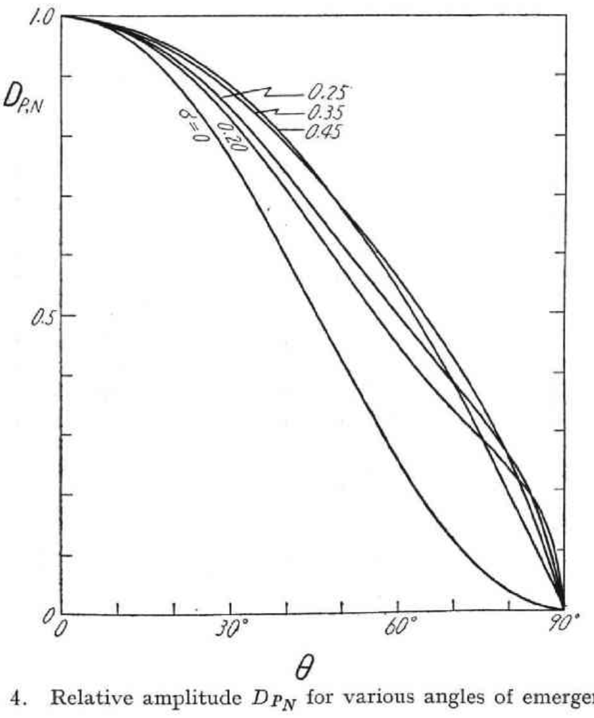

Fig. 4. Relative amplitude D_{PN} for various angles of emergence.

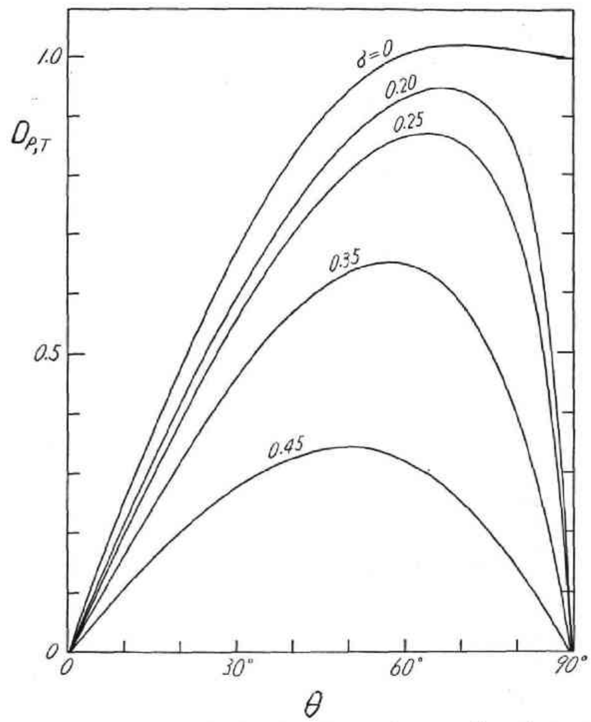

Fig. 5. Relative amplitude D_{P_N} for various angles of emergence.

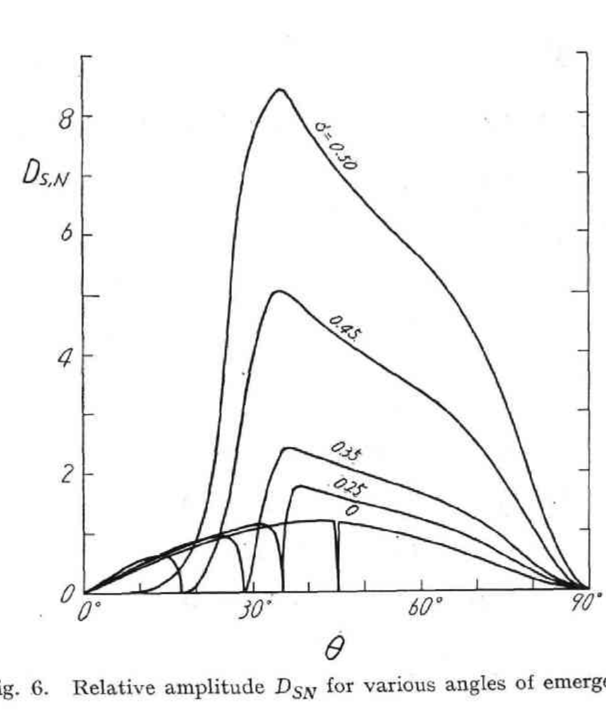

Fig. 6. Relative amplitude D_{S_N} for various angles of emergence.

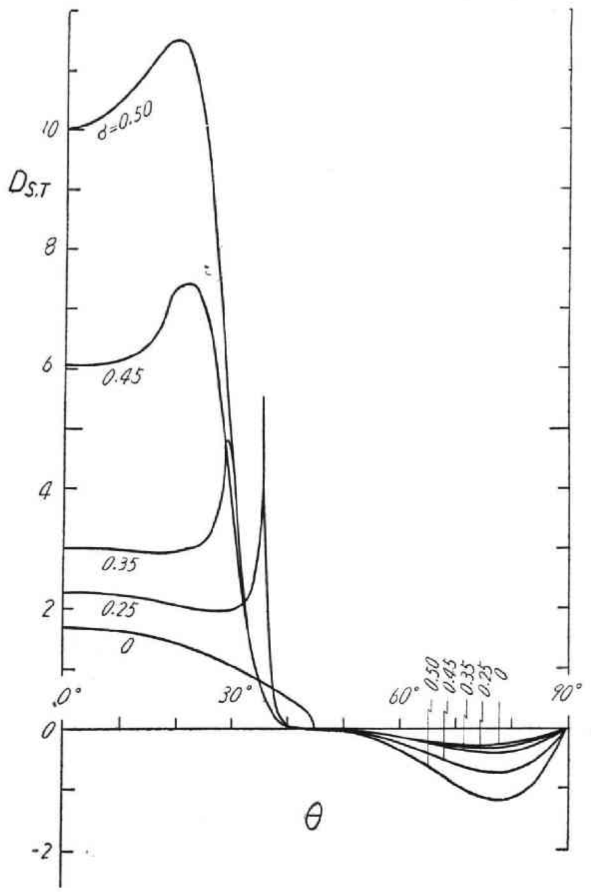

Fig. 7. Relative amplitude D_{ST} for various angles of emergence.

force are given by putting $\vartheta = 90°$. It is easily seen that D_{PN} and D_{ST} are the even functions of θ, and D_{PT} and D_{SN} the odd functions. We now consider the region of θ positive, or x positive.

In Figs. 4–7, are shown the variations of these relative amplitudes with θ, for some values of σ. In the limiting case where σ tends to 0.5, or m goes to infinity, we have the amplitude distributions as follows,

$$\left.\begin{aligned} D_{PN} &= \cos\theta \\ D_{PT} &= \sin 2\theta/m \\ D_{SN} &= 4\,m^{3/2}\sin^4\theta\cos\theta\sin 2\theta/g \\ D_{ST} &= m^{3/2}\cos\theta\cos^3 2\theta/g \end{aligned}\right\} \tag{25}$$

where $$g = \cos^4 2\theta + 16\sin^6\theta\cos^2\theta$$

The following conclusions are drawn from Figs. 4–7 and the expressions in (25).

D_{PN} : Pattern of the distribution is not very much different for different values of the

Poisson's ratio σ, at least in the range from 0.25 to 0.5. Hence D_{PN} may be considered as a function of θ only, for any value of σ that will be encountered in seismic exploration. The amplitude varies as $\cos\theta$ in the limiting case where σ tends to 0.5.

D_{PT} :, Pattern of the distribution is modified remarkably with the variation of σ. As σ tends to 0.5, the amplitude decreases as m^{-1} and varies with θ as $\sin 2\theta$. There is such a range of σ that D_{PT} exceeds unity. In Fig. 8 and 9, are shown the maximum amplitude D_{PT} (θ_{PT}) and the angle θ_{PT} respectively, as a function of σ.

D_{SN} : D_{SN} vanishes for $\theta_0=\sin^{-1} 1/m^2$, and is largest for θ_{SN} slightly larger than θ_0. The maximum amplitude $\bar{D}_{SN}$ (θ_{SN}) increases with increasing Poission's ratio. $\bar{D}_{SN}$ (θ_{SN}) and θ_{SN} are plotted in Fig. 8 and 9, respectively. The values of θ_{SN} range from 34°.2 to 45.°0. There is another maximum of D_{SN} at an angle, say θ'_{SN}, slightly less than θ_0. The amplitude of the subsidery maximum for θ'_{SN} slightly less than θ_0, decreases with σ.

D_{ST} : D_{ST} vanishes for $\theta=45°$ and the phase inversion takes place for the emergence beyond 45°. These behaviors of the S waves are independent of the amount of the Poisson's ratio, as will be seen from (22'). The amplitude becomes the maximum for an angle, say θ_{ST}. For the range of σ from zero to about 0.3, θ_{ST} is nearly equal to θ_0. The difference between the angles increases with σ, and in the limiting case where σ tends to 0.5, θ_{ST} is larger than θ_0 by 19.°7. The maximum amplitude $\bar{D}_{ST}$ (θ_{ST}) and θ_{ST} are also plotted in Fig. 8. and 9 respectively. The

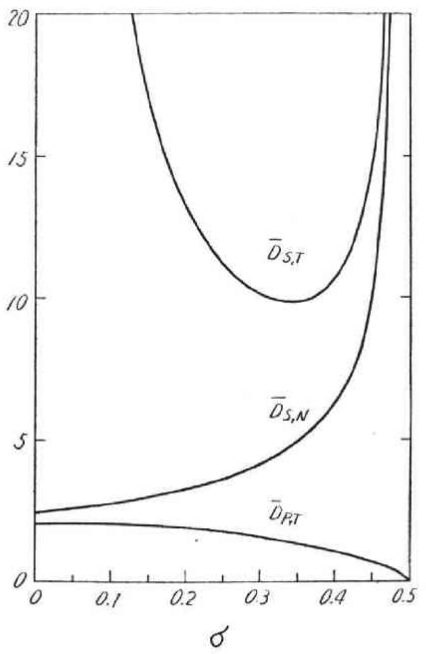

Fig. 8. Maximum amplitudes of D_{ST}, D_{PN} and D_{SN}.

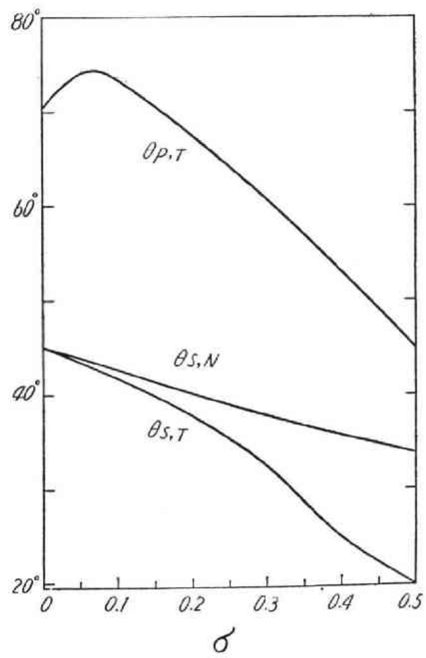

Fig. 9. Angles of emergence for which D_{ST}, D_{SN} and D_{PT} take maximum value.

energy radiated as the S waves into the region $\theta > 45°$ is negligible compared with one in the region $\theta > 45°$. Generally speaking, the energy radiated horizontally is the main source of the energy contributing to the background noise. Therefore the increase in the amount of energy directed downward in a layered medium, results in the improvement of signal-to-noise ratio.

Amplitude ratio of P to S waves for any of the force systems increases with σ in general tendency. Throughout the present paper, we take no account of energy dissipation which accompanies vibrations in solid media. However the tendency at

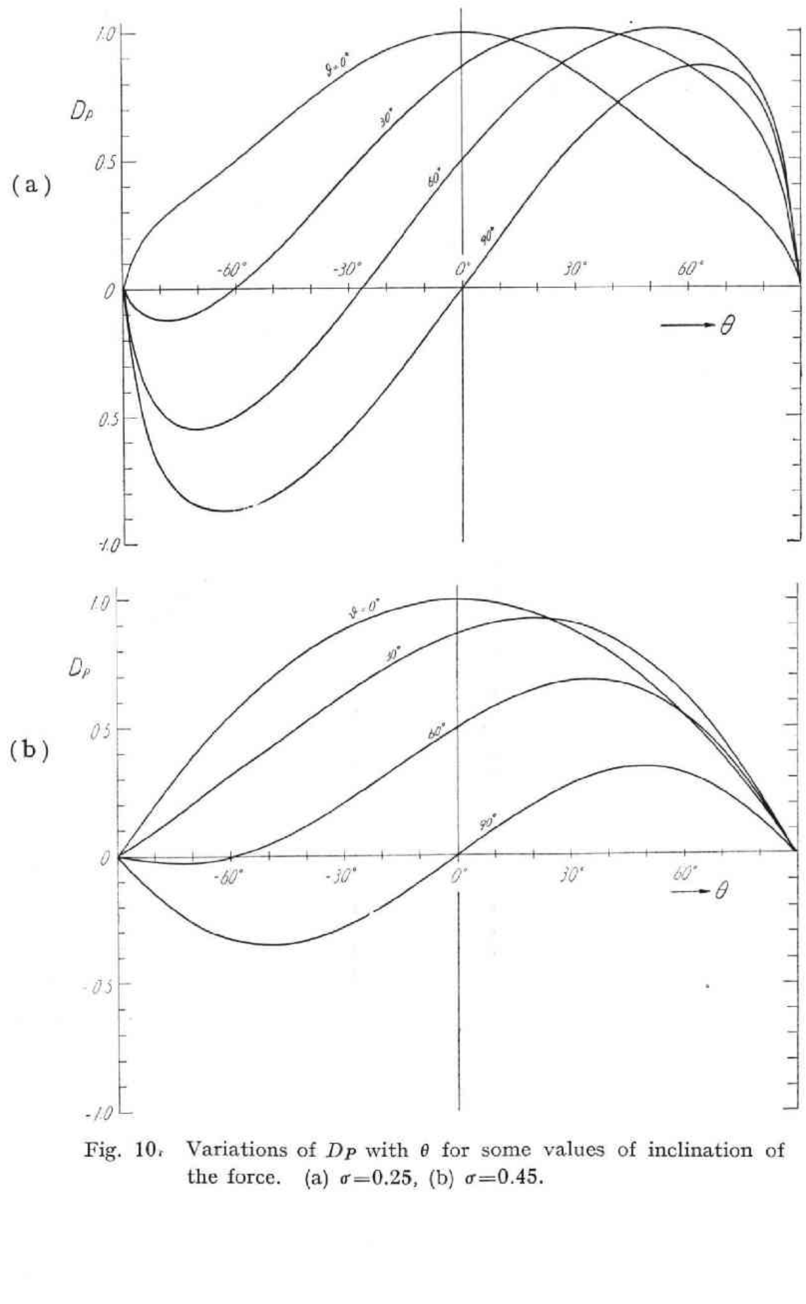

Fig. 10. Variations of D_P with θ for some values of inclination of the force. (a) $\sigma=0.25$, (b) $\sigma=0.45$.

least for σ 's of practical interest seems to be suggestive for the possibility of generation of the S waves with a considerable amount of the energy, even if σ tends to $1/2$.

3. 2 Arbitrary ϑ

We now proceed to study the variation of D_P and D_S with ϑ as well as θ. As the variation is now asymmetrical about to z-axis, we must be concerned with the range of θ from -90° to 90°. D_P and D_S are given by (21) and (22), taking into account

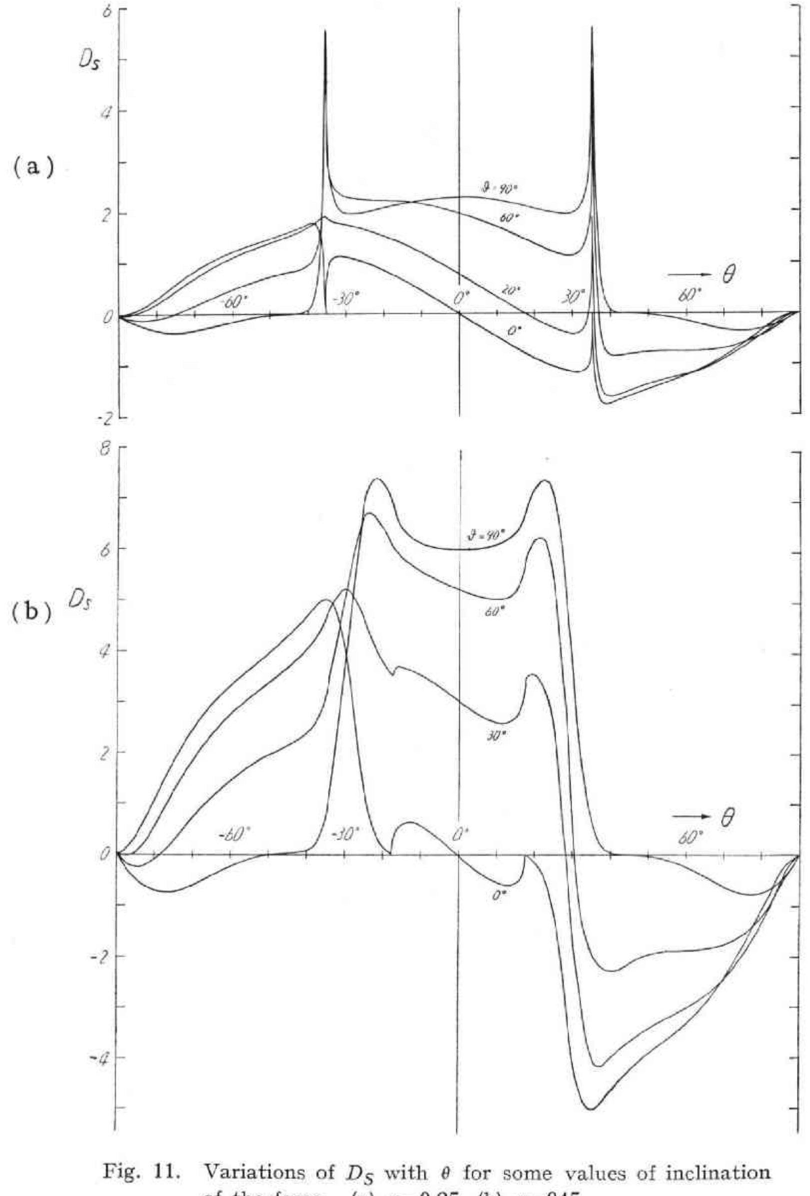

Fig. 11. Variations of D_S with θ for some values of inclination of the force. (a) σ=0.25, (b) σ=045.

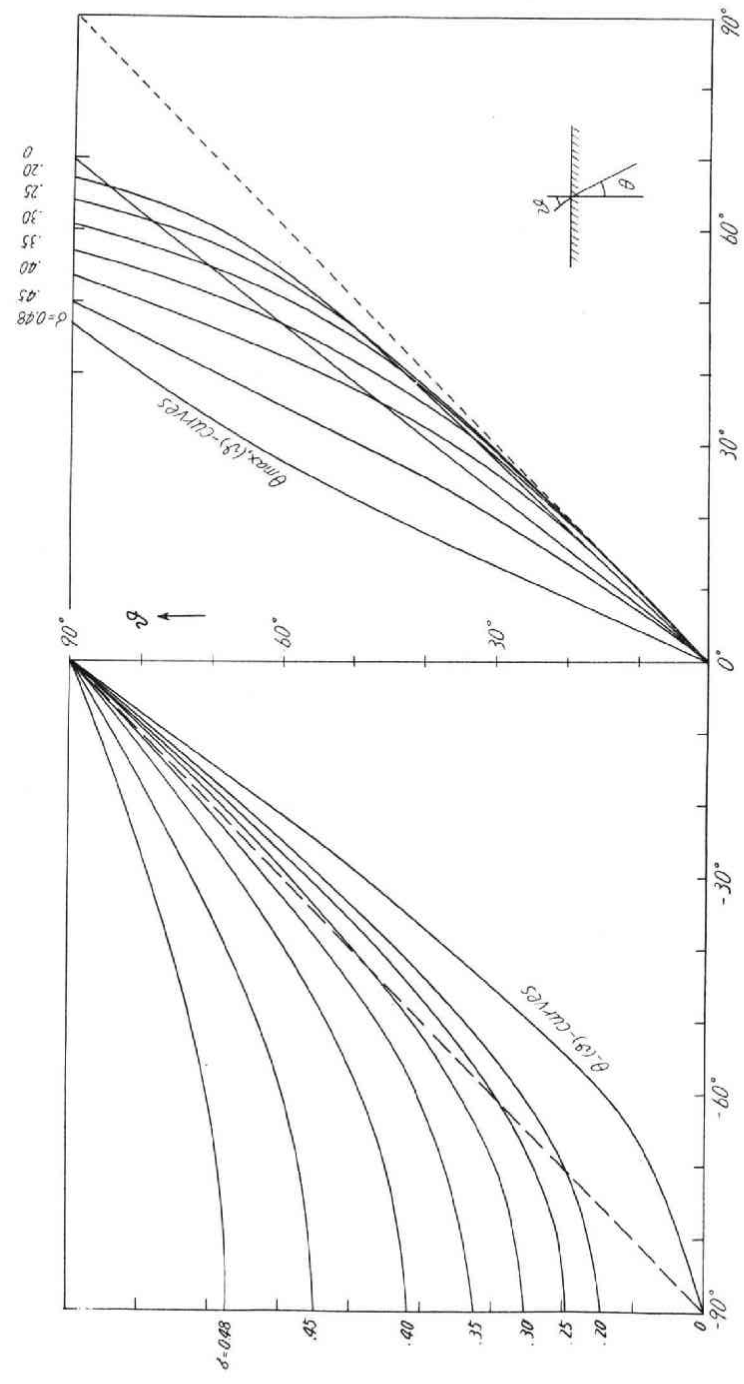

Fig 12. $\theta_{\max}(\theta)$ curves and θ–(ϑ) curves.

the normalization (24), and are shown in Figs. 10 and 11 respectively, for $\sigma=0.25$, 0.45, as a function of θ.

The general conclusions for all possible values of the Poisson's ratio σ are drawn as follows.

D_P: With increasing σ, θ_{max} for which the amplitude takes maximum increases, and the maximum amplitude decreases. θ_{max} is plotted as a function of ϑ in Fig. 12, for some values of σ. Dotted line shows the case in which θ_{max} is equal to ϑ. We may not expect, for the presence of a free surface in our medium that the direction of force acting agrees with θ_{max}. The deviation of θ_{max} (ϑ) curves from the dotted line increases with ϑ, and becomes more remarkably foı larger value of σ.

D_P vanishes for θ_- negative, and the phase inversion takes place for the emergence beyond θ_-. $\theta_-(\vartheta)$ curves are also shown in Fig. 12. The amplitude of the P waves in the absence of the free surface, vanishes for $\theta=\vartheta-90°$. (LAMB, p. 8). The broken line in Fig. 12 shows such a case. The deviation of θ-(ϑ) curves from the broken line is more sensitive to σ than that of the θ_{max} (ϑ) curves from the dotted line.

D_S: Pattern of the spatial distribution is rather sensitive to the variation of the inclination of the force as well as that of the Poisson's ratio. A large number of maximum and minimum in the amplitude variation makes the detection of the latter phase generated by such a S wave, somewhat infeasible as the case may be.

There are the values of inclination of the force, for which the amplitude of S waves are rather uniform. For the case $\sigma=0.25$, it takes about 20°. For these values of σ and ϑ, the amplitude of S waves is more than three times that of P waves, for all values of θ negative. Furthermore the amplitude of initial motion of surface becomes minimum, as will be seen in the next section.

The uniformity of amplitude variation of P or S waves and the larger amplitude of a wave than that of another, facilitate the interpretation of the latter phases which are due to the incidence of the initial P or S waves on the interface in a layered medium. (c. f. Appendix).

4. Initial Motion of the Surface of the Elastic Solid.*)

For the practical purposes, it may be worth while, in some cases, to reduce the amplitude of an initial motion propagated along the surface of an elastic solid. We now study the effect of the inclination of the force considered in the preceding sections, on the initial motion of the surface of the solid.

Putting $z=0$, or $\theta=90°$ in (9), we have the exact solutions for the surface motions as follows,

*) If we perform the numerical calculation of the exact solutions in (10), we will see that the sudden commencement of displacement for $z\neq0$, or $\theta\neq 90°$, is followed by a gradual recovery, or by an ocsillatory motion as the case may be. In the limiting case where z tends to zero, the infinite jerk at the onset vanishes, and the motion begins with finite displacement corresponding to the ocsillatory motion stated above.

$$\left.\begin{aligned} u_0 &= \sqrt{\frac{2}{\pi}}\,\frac{a}{\mu x}\left[\cos\vartheta\, Re\left(v\,(2v^2+m^2)-2ab\right)/F(v)+m^2\sin\vartheta\, Im\, b/F(v)\right] \\ w_0 &= \sqrt{\frac{2}{\pi}}\cdot\frac{a}{\mu x}\left[m^2\cos\vartheta\, Im\, a/F(v)-\sin\vartheta\, Re\, v\,(2v^2+m^2-2ab)/F(v)\right] \end{aligned}\right\} \quad (26)$$

where $v=-iat/x,\ t>x/a$.

The horizontal and the vertical components of the displacements for x positive are denoted by u_0^+ and w_0^+, and those for x negative by u_0^- and w_0^- respectively. These are shown for times before the arrival of the phase propagated with the velocity of S wave, in Fig. 13, the displacements being measured in units of $\sqrt{2/\pi}\,a/\mu x$. Increase in ϑ results in the increase in the maximum amplitude of initial motions for x positive,

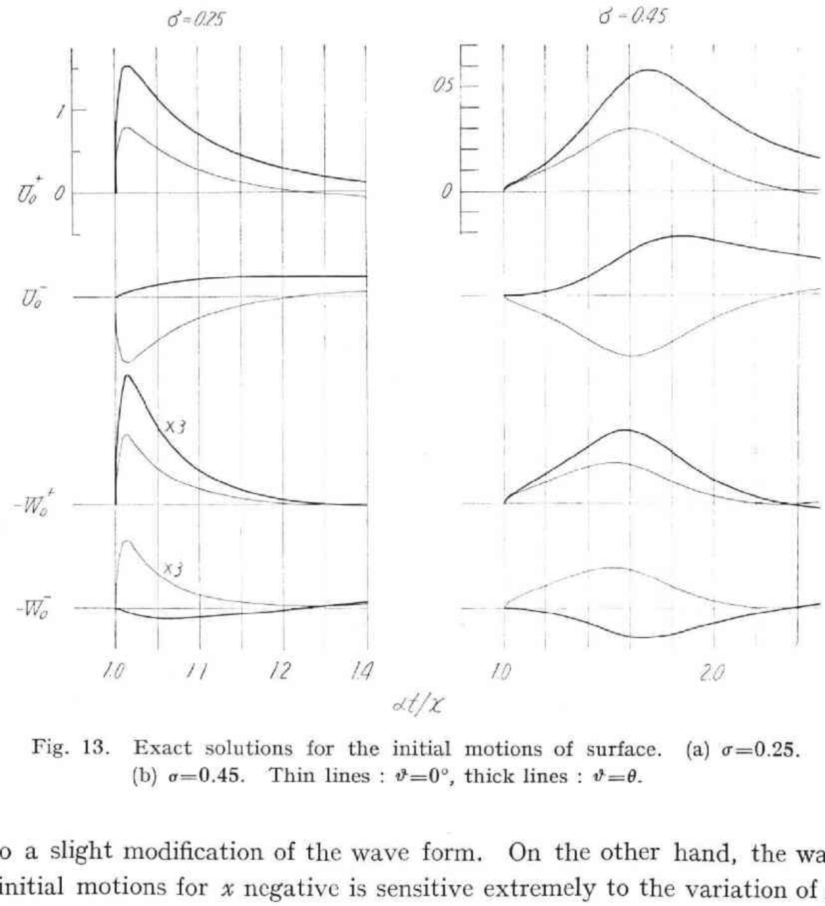

Fig. 13. Exact solutions for the initial motions of surface. (a) $\sigma=0.25$. (b) $\sigma=0.45$. Thin lines : $\vartheta=0°$, thick lines : $\vartheta=\theta$.

and also a slight modification of the wave form. On the other hand, the wave form of the initial motions for x negative is sensitive extremely to the variation of ϑ. The period of the motions increase apparently with increasing Possion's ratio, as would be expected.

There is an inclination of the force, say ϑ_-, beyond which the application of the force yields the inversion of the sense of a component of the initial motion. In other words, the particle velocity of the component of motion at the onset, vanishes for ϑ_-.

Substituting $t=x/a$, or $v=1$, and $\vartheta=0°$ and $90°$, we have the ratios of vertical to horizontal amplitude of the initial motions due to the normal and tangential forces, as follows,

$$R_p=\left(\frac{w_{0N}}{u_{0N}}\right)_{t=x/\alpha}=\left(\frac{w_{0T}}{u_{0T}}\right)_{t=x/\alpha}=\frac{\sigma}{\sqrt{1-2\sigma}}. \qquad (27)^{*)}$$

As the expressions for u_{0N} is equivalent to that for $-w_{0T}$, we have

$$w_{0\,N}^{-}=R_p\,u_{0\,N}^{-}=-R_p\,w_{0\,T}^{-}=-R_p^2\,u_{0\,T}^{-} \qquad \text{for } t=|x|/\alpha \qquad (28)$$

Considering at first the horizontal amplitude, the equation

$$u_{0\,N}^{-}\cos\vartheta_{-}+u_{0\,N}^{-}\sin\vartheta_{-}=0 \qquad (29)$$

which must be satisfied by ϑ_-, is rewritten as

$$R_p\cos\vartheta_{-}-\sin\vartheta_{-}=0 \qquad (30)$$

by use of (28). The latter equation can be applied also for the vertical amplitude, as is easily seen from (28). We understand therefore that the particle velocities of both components of the motion at $t=x/\alpha$ vanishes for ϑ_-.

Now ϑ_- is given by the relation

$$\vartheta_{-}=\cos^{-1}\frac{1}{\sqrt{1+R_p^2}} \qquad (31)$$

and shown in Fig. 14 together with R_p as a function of σ. For $\sigma=0.25$, $\vartheta_-=19.°5$ and $R_p=0.3535$.

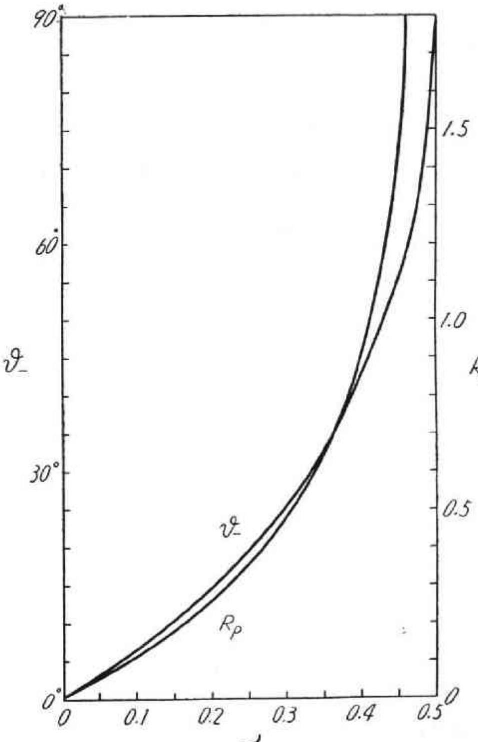

Fig. 14. ϑ_- and R_p.

5. Rayleigh Wave

We will study the Rayleigh waves propagated along the surface of an elastic solid, which are generated by the force system similar to that provided in section 2, except the time variation of it. In the integrals for $z=0$ in (7), the zero of $F(v)=0$ corresponding to the velocity of the Rayleigh waves, is on the distorted path of integra-

*) The ratio may be defined also in terms of the maximum amplitudes of the components of the initial motion. Generally speaking, (Max. of w_{0N}/Max. of U_{0N}), for the case is smaller than R_p defined above, e.g. by about 5% for $\sigma=0.25$. In seismic exploration, the geophones used are of the velocity type for the most part. It seems to be significant, therefore, to consider the ratio at the onset of the motion, such as defined by (27).

tion, and the integrands become infinite at the zero. For the problem under consideration, it seems to be suitable to concern with a residue at the pole.

Now, we suppose that the force such as stated above, varies with time as

$$f(t) = \frac{cA}{t^2+c^2}\,;\, c, A > 0 \tag{32}$$

In order to obtain the displacement solutions for Rayleigh waves generated by such a force, we perform the operation

$$\frac{1}{\pi}\, Re \int_0^\infty d\,\omega \int_{-\infty}^\infty f(\lambda)\, e^{-i\omega\lambda}\, d\,\lambda \tag{33}$$

to the displacement solutions for steady state propagation of the waves, the latter being given as

$$\left.\begin{aligned} u_{NR} &= -\frac{1}{\mu} H_N\, e^{i\omega(t-x/C_R)}, \qquad & w_{NR} &= -i\,\frac{1}{\mu} K_N\, e^{i\omega(t-x/C_R)} \\ u_{TR} &= -i\,\frac{1}{\mu} H_T\, e^{i\omega(t-x/C_R)}, \qquad & w_{TR} &= \frac{1}{\mu} K_T e^{i\omega(t-x/C_R)} \end{aligned}\right\} \tag{34}$$

where

$$\left.\begin{aligned} & H_N = (2R^2-1)^3/R\,D(R)\,, \quad K_N = 2\,(2R^2-1)^2\sqrt{R^2-1/m^2}/D(R) \\ & H_T = 2\,(2R^2-1)^2\sqrt{R^2-1}\,/D(R)\,, \qquad K_T = H_N \\ & \quad D(R) = 16\,R\,\{1-(6-4/m^2)\,R^2+6\,(1-1/m^2)\,R^4\} \\ & \quad R = \beta/C_R \end{aligned}\right\} \tag{35}$$

subscripts N and T refer to the quantities for the normal and tangential forces respectively, ω is the frequency, and c_R the velocity of Rayleigh wave. (cf. LAMB, 1904).*)

Performation of the operation (33) yields the transient solutions for our problem, as follows.

$$\left.\begin{aligned} U_R &= -A\,[H_N \cos\vartheta\cos(\tan^{-1}\tau) - H_T \sin\vartheta\sin(\tan^{-1}\tau)]/c\,\mu\sqrt{1+v^2} \\ W_R &= A\,[H_T \cos\vartheta\sin(\tan^{-1}\tau) + K_T \sin\vartheta\cos(\tan^{-1}\tau)]/c\,\mu\sqrt{1+v^2} \end{aligned}\right\}$$

$$\tau = \frac{t-x/C_R}{c}$$

where

In Fig. 15, are shown the wave form and particle motion for the wave for x positive, $\sigma=0.25$ and some values of ϑ, the displacements being measured in units of $AH_N/c\mu$.

Increase in ϑ results the reduction of the amplitude and also the modification of the wave form, as if the wave undergoes a phase change. Whereas the particle motion is elliptical retrograde, being regardless of the variation of ϑ. As cos $(\tan^{-1}\tau)$ and sin $(\tan^{-1}\tau)$ in (36) are even and odd functions of x respectively, the locus of the particle

*) For the convenience of numerical computation, the transfer has been made in some expressions.

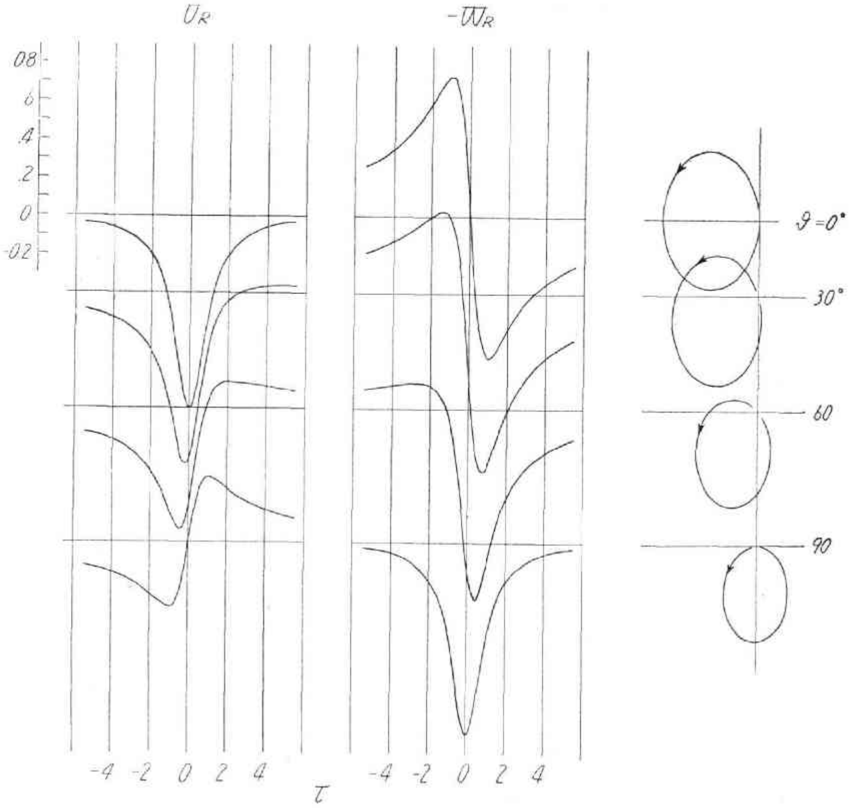

Fig. 15. Particle motions for Rayleigh waves and the loci, at the surface.

motion for x negative must be symmetrical about the origin. The variation of ϑ yields the displacement of center of the ellipse and no change in the ratio of the major to the minor axes.

It is easily seen from (26) that the variation of the Poisson's ratio affects the ratios H_T/H_N and K_N/H_N alone, and not the wave form or the locus of the particle motion. Therefore the results stated above for $\sigma=0.25$ are applicable altogether to any case of the value of σ. In Fig. 16, are shown the ratios H_T/H_N and K_N/H_N as well as H_N, H_T and K_N as a function of σ, for the benefit of reference.

6. Conclusions

The transient elastic waves in a semi-infinite elastic solid, generated by a surface linear force acting at an inclination, are investigated theoretically for all possible values of the Poisson's ratio. The results are related to a practical problem, that of the improvement in signal-to-noise ratio in seismic exploration.

The general conclusions from the present investigation are drawn as follows :

1. Increase in the inclination ϑ of the force results the increase in the angle θ of emergence, for which the amplitude of P waves has the maximum value, and results also

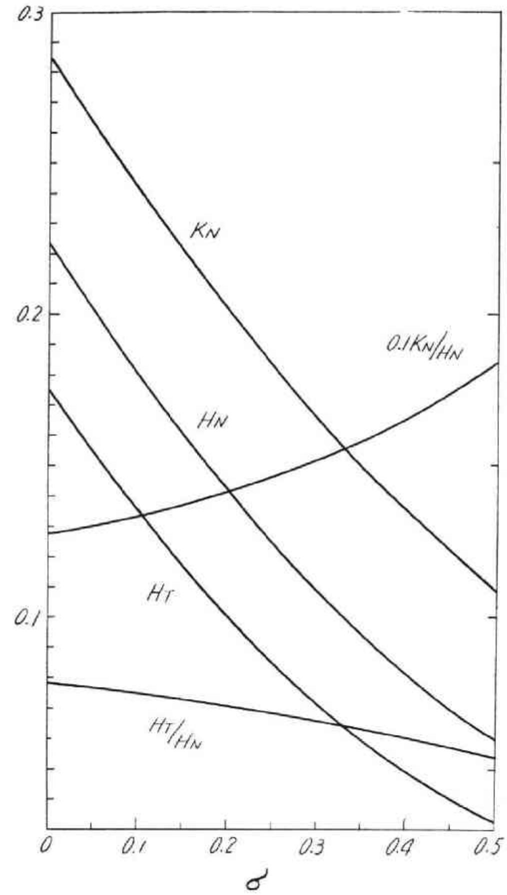

Fig. 16. Ratios H_N, H_T, K_N, K_N/H_N H_T/H_N as functions of Poisson's ratio.

in the decrease in the maximum amplitude. The angle θ is smaller than the inclination of force for any value of the Poisson's ratio σ. The difference increases with increasing σ,

2. Amplitude of P waves passes through zero at an angle of emergence, and the phase inversion takes place for the emergence beyond this angle.

3. Pattern of the amplitude variation for S waves is rather sensitive to the variation of the inclination of force as well as that of the Poisson's ratio. A large number of maximum and minimum in the amplitude variation makes the detection of the latter phases on a seismogram, somewhat infeaseble as the case may be. There are the range of inclination of the force, for which the amplitudes of S waves are rather uniform, e.g. about 20 degrees for the case $\sigma=0.25$.

4. Ratio of the amplitudes of P to S waves for any of the force systems, increases with increasing Poisson's ratio. Throughout the present paper we take no account of energy dissipation. However the tendency at least for σ's of practical interest seems to be suggestive for the possibility of generation of the S waves with a considerable

amount of energy, even if σ tends to $1/2$.

5. The inclination of the force, for which the particle velocity of the surface motion vanishes at the onset, increases with σ, It takes about 20 degrees for $\sigma=0.25$.

6. Particle motion for Rayleigh waves at the surface is elliptical retrograde regardless of the values of the inclination as well as of the Poisson's ratio. Increase in the inclination yeilds, to the displacement of the center of the locus and no change in the ratio of the major to the minor axes of the ellipse.

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Appendix

The reason why we define the ampltidues of P and S waves by (21), (22) and (22'), will be elucidated as follows :

According to the exact solution for the problem on the two-dimentional propagation of elastic waves in two semi-infinite media in contact, the displacement of refracted waves due to an initial pulse, the time variaiton in the pulse being similar to $H(v)$ in (14), must be given by

$$A = D(\theta) f(\theta, \theta_0; R)\, Im\, G(\theta), \quad \theta > \theta_0 \tag{1}$$

where $D(\theta)$ expresses the amplitude of the initial pulse radiated in the direction which makes an angle θ with a normal to the interface, and is none other than that defined by (24). $f(\theta, \theta_0; R)$ denotes the distance factor, θ_0 the critical angle of incidence, and $G(\theta)$ the coefficient of reflection (EMURA, 1960). For the initial pulse without the

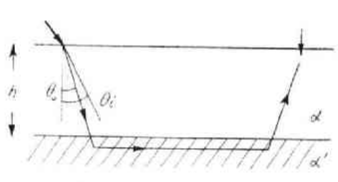

Fig. A. Refracted wave $P_1P_2P_1$

directive properties, $D(\theta)$ must be unity. If we consider, for example, a refracted wave $P_1P_2P_1$ as is shown in Fig. A, the angle θ is connected with time by the relation

$$t = \frac{h+z}{a} \cos\theta + \frac{x}{a'} \tag{2}$$

Basing on the ray theoretical interpretation, it is seen that the amplitude of $P_1P_2P_1$ at the time t_i after onset depends on the amplitude of initial pulse at the time $t=h/a \cos\theta_i$ when the pulse strikes the interface with an angle θ_i, and not on the amplitude of the residual displacement following the sudden jerk of the pulse. In other words, the wave form of a refracted wave must be determined by the initial amplitude of the pulses striking the interface.

If $D(\theta)$ is assumed to be unity, the wave form of a refracted wave will be given, in general, by $Im\ G(\theta)$ alone. When we consider the directive property in the initial pulse, the wave form of the refracted or critically reflected waves must be rather complicated, as the case may be. For example, the amplitude of S waves due to a surface tangential force passes through zero for $\theta=45°$, and the phase inversion takes place for the emergence beyond 45°. If the critical angle of incidence for such a initial S wave is about 45°, the amplitude of the refracted waves generated may be so small that it is impossible to identify the waves on a seismogram. And, the detection of the reflected waves may be troublesome, on account of the inversion of sense of the motion as well as the complicated variation of the wave form with distance along the interface.

These are the interesting problem encountered in seismic exploration and will be the subjects of subsequent publications.

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