Maximum Water Height at Bay Head in Case of Tsunami Invasion

著者	Nakamura Kohei, Emura Kinya
雑誌名	Science reports of the Tohoku University. Ser. 5, Geophysics
巻	13
号	1
ページ	32-42
発行年	1961-04
URL	http://hdl.handle.net/10097/44625

Maximum Water Height at Bay Head in Case of Tsunami Invasion

KÔHEI NAKAMURA and KINYA EMURA

Geophysical Institute, Faculty of Science, Tôhoku University

(Received March 31, 1961)

Abstract

For three bays situated along the Sanriku Coast, Japan, comparison of the water heights attained by the Chile tsunami of 1960 with those by the Sanriku tsunami of 1933 is made, and a remarkable difference is found between them. In the 1960 tsunami, the water height at the head was 2–3 times large the height at the mouth, whereas in the 1933 tsunami, the water height decreased with the distance from the mouth.

The increase in height of the 1960 tsunami is ascribed to the seiche developed to a considerable extent, and the decrease in height of the 1933 tsunami can be explained by assuming turbulent motion of the bay water.

In the case of the invasion of a tsunami into bays with varied seiche periods, general features about the water height at the head are described on the data for the 1933 and 1960 tsunamis.

1 Introduction

The bays situated along the eastern coast of Japan have been frequently attacked by tsunamis originated in distant as well as near earthquake zones. Several tsunamis of South American origin had struck Japan earlier, some without causing any damage. The Chile tsunami of May 24, 1960 was the first of South American origin that caused disaster on the shores of Japan. Some bays situated along the Sanriku Coast, the northeastern coast of Japan, were severely damaged. In almost of these bays the behaviours of bay water in case of the tsunami invasion were quite different from those for the Sanriku tsunami of 1933. Especially, the height of water at the bay head was much higher than at the bay mouth in the 1960 tsunami, and vice versa in the 1933 tsunami. Consequently, the bay head districts which have been considered to be safe from the attack of near tsunamis, have suffered unexpected damage. This may be due to different ways in which the bays response to distant or near tsunamis, and our interest is centered in the elucidation of this point.

We have made a field observation in Ofunato and Hirota bays of which the location and topography are shown in Figs. 1, 2. The bay head districts of these bays received tremendous damage principally by innundation. The results of water height measurements in these bays indicate that the height of the incident tsunami was amplified about 2 to 3 times at the bay head. Whereas, in the 1933 tsunami, it was reported that in many bays the wave decreased in height with the distance from the mouth.

This paper first deals with the characteristic behaviours of the 1933 and

1960 tsunamis in Ofunato, Hirota and Miyako bays. Then the general features of transient response of bays to tsunami are studied, using the data of these tsunamis. In the course of the analysis, we need a marigram which can afford some indications as to a seiche damping. However, as both the tide gauges installed in Ofunato and Hirota bays were washed out, use is made of the marigram made in Miyako bay whose size and seiche period are comparable with those of the two bays. In addition, the water height distributions in this bay in the 1933 and 1960 tsunamis are also used, since rather complete wave height measurements were made for these tsunamis. The periods of seiche in Ofunato, Hirota and Miyako bays are 40, 48, 45 minutes, respectively.

2 Observed Water Heights in Ofunato and Hirota Bays

The height of water traced by the Chile tsumani of 1960 was measured at the points indicated in Figs. 2 a, b. The measured water heights were reduced to the values above the mean sea level in Tokyo bay. The distribution of the water height thus corrected is shown in Figs. 3 a, b, where the abscissa is the distance from the head measured along the median line of the bay, and the ordinate shows the corrected water height. Similar plots, for the 1933 tsunami are made as shown in Figs. 4 a, b, basing upon the data given in the report of the Earthquake Research Institute[1]. In Figs. 3, 4 circles and triangles indicate the values on the eastern and western coasts of the bay, respectively. It is clearly shown that, in the 1960 tsunami, the water height decreases, whereas, in the 1933 tsunami, it increases with the distance from the head. In the 1960 tsunami, the ratio of the water height at the head to that at the mouth is about 1.9 in Ofunatio bay, 2.9 in Hirota bay, and 2.9 in Miyako bay, the last numeral being the result of other observation parties. In the 1933 tsunami, the ratio becomes about 0.29, 0.55, and 0.50, in the order above described. This is a remarkable fact about which we are to study.

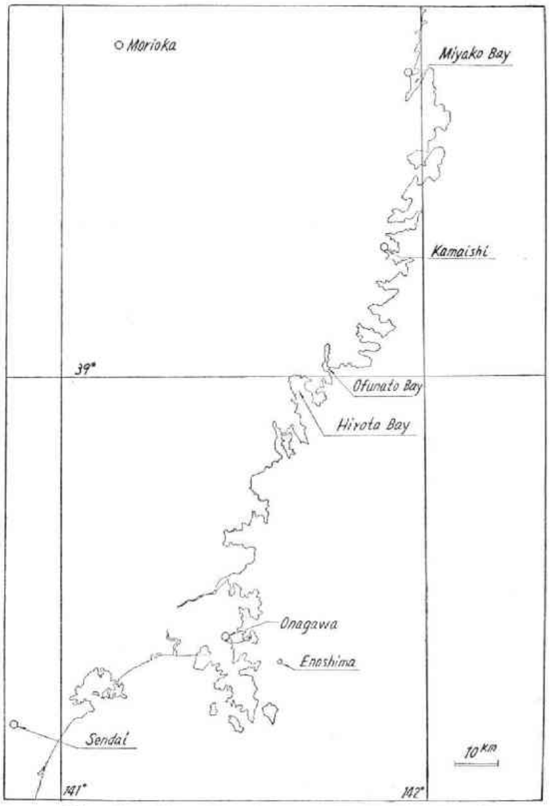
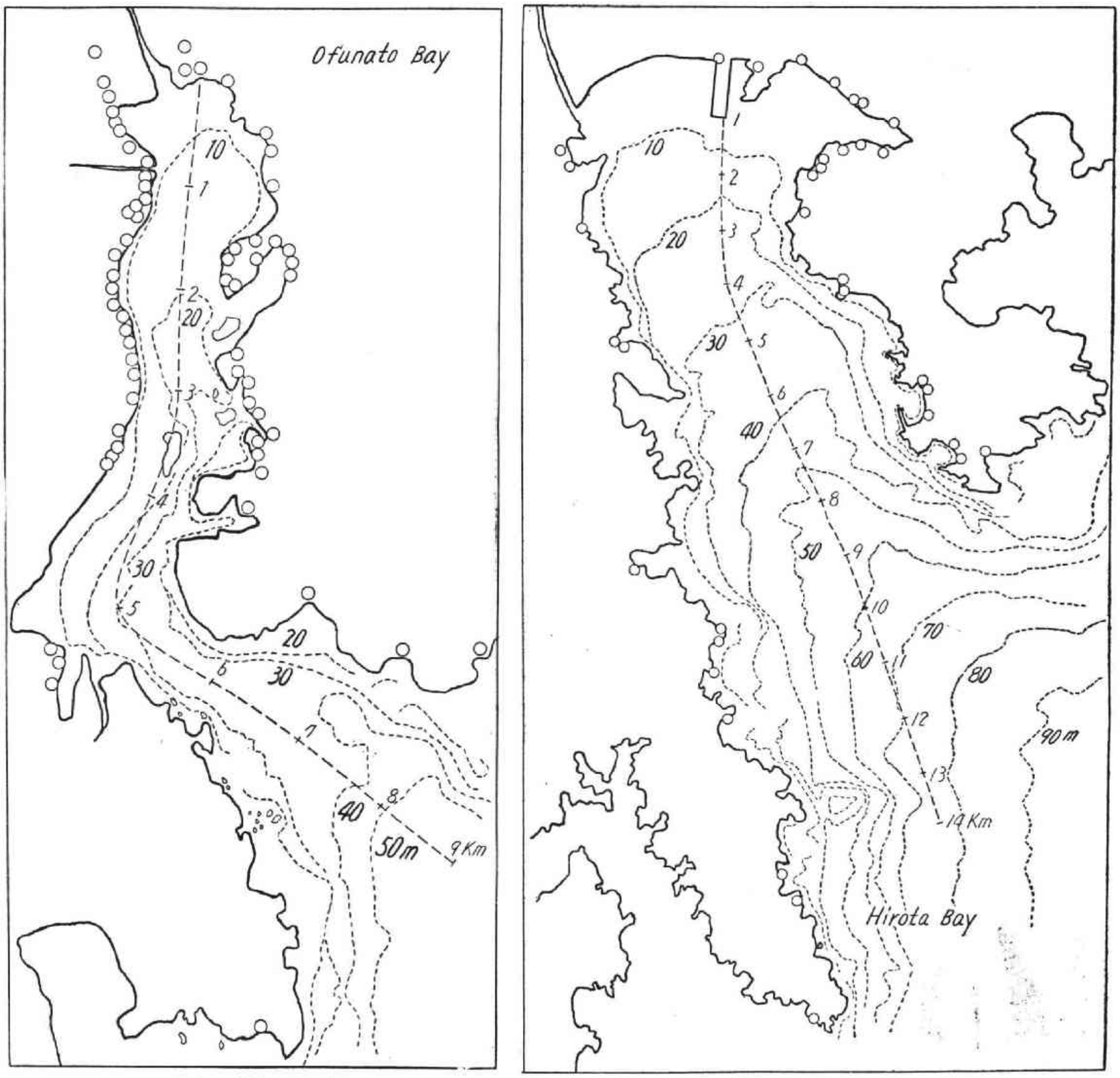


Fig. 1. Map showing part of the Sanriku Coast, Japan.

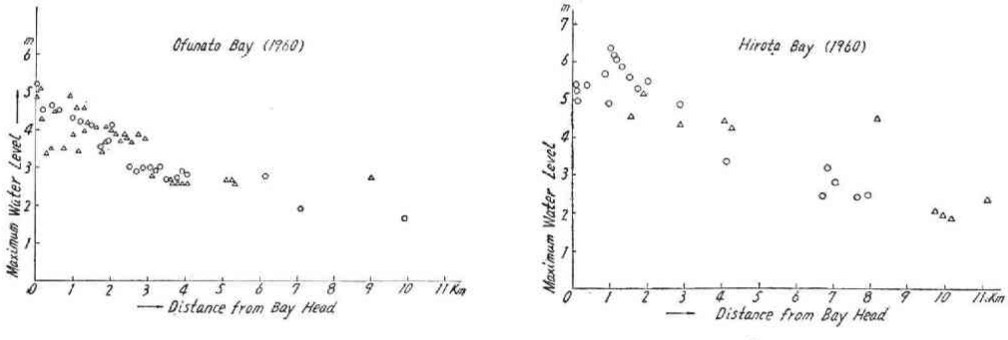


(a) : Ofunato bay (b) : Hirota bay

Fig. 2. a, b. Water height was measured at points indicated by circles.

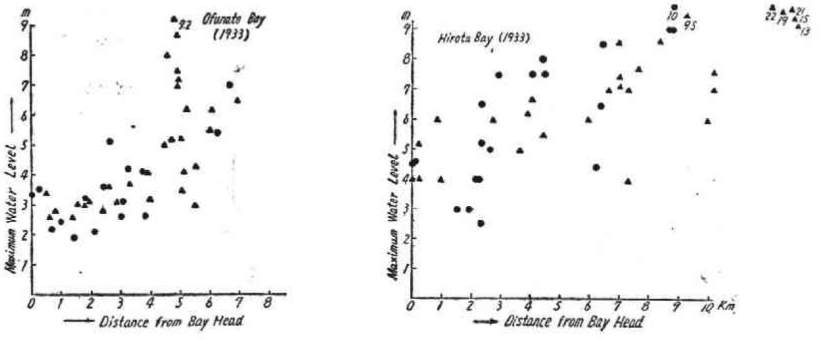
3 Heights of Water in Ofunato, Hirota and Miyako Bays in Cases of the 1960 and 1933 Tsunamis

Near and distant tsunamis differ in several features; among these, period and the number of crests and troughs of the incident waves are most easily recognized. These factors evidently influence the maximum amplitude of the forced oscillation of bay water due to the incident waves. In a previous paper [3], the wave height at the head as influenced by these has been obtained for the case of a rectangular bay with uniform depth. Since many bays are more or less of rectangular shape, it may be permissible, at



a b

Fig. 3. a, b. Distribution of water height for the Chilean tsunami of 1960. Circles and triangles show values on the eastern and western coasts, respectively.



a b

Fig. 4, a, b. Distribution of water height for the Sanriku tsunami of 1933. Circles and triangles show values on the eastern and western coasts, respectively.

least in the first approximation, to compare the actual wave height with the theoretical one obtained on the assumption of rectangular shape. In this comparison, discrepancies, if any, may be attributed to other elements such as the bottom topography, the bay shape, and eddy viscosity of the bay water. Magnitude of the third element may be supposed to vary with the first two, and also with the period and the direction of approach of the tsunami.

A previous result [3] above referred to is shown in Fig. 5, which represents the ratio η/h of the water height at the head to the amplitude of the incident waves, as a function of the ratio T/T_0, taking m as a parameter, where, T and T_0 are the period of the tsunami and that of the first-mode free oscillation of the bay, and m represents the number of crests and troughs contained in a wave packet incident upon the bay mouth.

Inspection of marigrams taken at many tide stations along the Pacific Coast, especially that of the record made at Enoshima, which is shown in Fig. 1, make it possible to determine the period of the incident waves of the 1960 tsunami to be about 60 minutes. To infer the rate of damping of free oscillation of bay water due to eddy

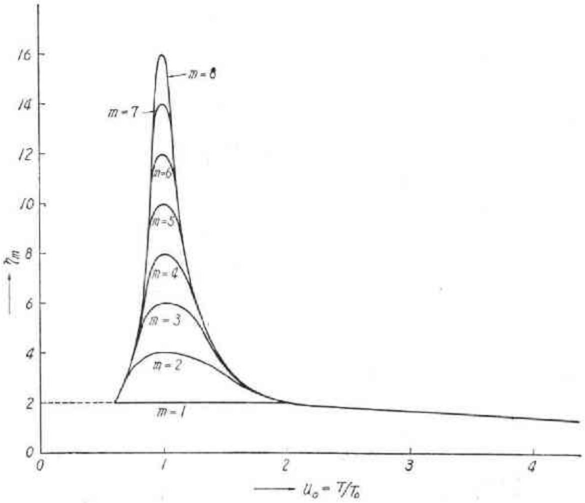


Fig. 5. Response curves.

T : period of incident waves.
T_0 : seiche period.
m : number of crests and troughs contained in the incident wave packet.
$\eta_m = \eta/h$, η : water height at bay head.
h : amplitude of incident waves.

viscosity, it is necessary to examine the marigram of the bay concerned. However, as stated in the preceding section, being unable to get the marigrams at Ofunato and Hirota, we have inspected only the record at Miyako. It shows that the undulation of the bay water lasts considerably longer as compared with the cases of other tsunamis. This suggests a small value of eddy viscosity and small amount of energy dissipation in the form of diverging waves from the bay mouth.

Ignoring eddy viscosity, and assuming tentatively $T = 60$ min, the ratio η/η_0 of the water levels between head and mouth is plotted in Fig. 6, for Ofunato, Hirota and Miyako bays. The curves in the figure are the same as those in Fig. 5, except that the scale of the ordinate is diminished to 1/2. This change of scale results from the assumed condition of coast-lines near the mouth, that is, the coast, similarly as the head, is composed of a vertical cliff.

The period of the incident waves of the 1933 tsunami can be determined to be about 16 minutes from the aspect of resonance shown in Fig. 7, which represents the relationship for some bays between the value η/η_0 above stated and the seiche period. In this figure the plot for the case of the 1960 tsunami is also shown, the result of which does not conflict with the assumed value $T = 60$ min.

For comparison's sake, the plot of η/η_0 against T/T_0 for the three bays is also made for the tsunami of 1933, assuming $T = 16$ min as shown in Fig. 6.

It can be seen from Fig. 6 that, in the Chile tsunami, the number of crests and troughs contained in the incident wave packet is more than 4, and that this value becomes larger, the longer we assume the period of the tsunami. This is in accord with the fact that the marigram at Enoshima indicates a period a little longer than 60

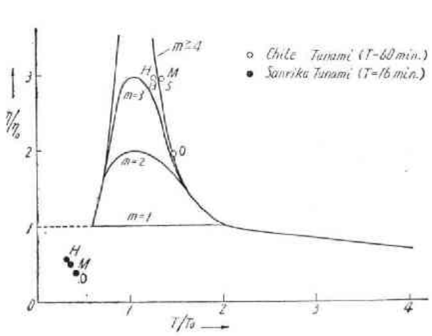


Fig. 6.

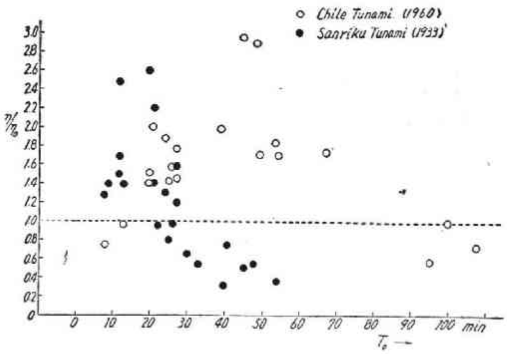


Fig. 7.

Fig. 6. Responses of three bays to Chile and Sanriku tsunamis.

η : water height at bay head.
η_0 : water height at bay mouth.
T_0 : seiche period
T : period of tsunami

T is assumed to be 16 min. for Sanriku tsunami and 60 min. for Chile tsunami. Letters O, H and M correspond to Ofunato, Hirota and Miyako bays. Curves indicate the response curves shown in Fig. 5.

Fig. 7. Ratio η/η_0 for some bays of water heights between head and mouth against seiche period T_0.

minutes, and about 8 for the m-value. It may be also believed that eddy viscosity in the earlier stage of the oscillation of water, or the forced oscillation due to the tsunami, would not be large, since there is no appreciable decrease of the observed value for η/η_0 from the theoretical ones obtained for non-viscous water. We will estimate the order of the coefficient of eddy viscosity in the later stage of the marigram where only the free oscillation of the first mode predominates.

When a tsunami is incident upon a rectangular bay with uniform depth, the water level η at the head is given by two different expressions; the normal mode solution and the ray solution [3]. From the former, the logarithmic decrement ρ in the later stage of the bay water oscillation can be written

$$\rho = \pi\left(\frac{\eta^2-\xi^2}{2\,\xi\,\eta}\right). \tag{1}$$

In this expression, $w=\xi+i\eta$ is the root of the equation

$$\beta\, w^4 + 1 - \frac{\tanh w}{w} = 0\,,$$

and eddy viscosity ν is connected with β by the relation $\beta=\nu^2/\omega^2 H^4$, $\omega=2\pi/T_0$, where T_0 is the period of the first mode free oscillation, and H is the depth of the bay.

The factor that represents the decrease in height of the first wave of a tsunami is given by

$$e^{-\frac{\pi q}{2 u_0}}, \tag{2}$$

where

$$u_0 = T/T_0 \,, \qquad q = \frac{1}{\sqrt{2}} \sqrt{\frac{C^2}{A^2+B^2}} \sqrt{\sqrt{A^2+B^2}-A} \,,$$

$$A = 2\kappa\,(\cosh 2\kappa + \cos 2\kappa) - (\sinh 2\kappa + \sin 2\kappa) \,,$$

$$B = \sinh 2\kappa - \sin 2\kappa \,,$$

$$C = 2\kappa\,(\cosh 2\kappa + \cos 2\kappa) \,,$$

$$\kappa = \frac{H\sqrt{\omega}}{\sqrt{2\nu}} \,, \qquad \omega = \frac{2\pi}{T} \,,$$

and T is the period of the tsunami. In deriving (1) and (2), it is assumed that the bay head is composed of a rigid wall, there is no horizontal flow of water at the bottom, and no energy is dissipated from the bay mouth.

For Miyako bay, the coefficient of eddy viscosity is given in Table 1, assuming that $T=60$ min, $T_0=48$ min, $H=23$ m, for various values of logarithmic decrement ρ of the first mode free oscillation. The value $\rho=0.01754$ indicates the one for Miyako bay in the case of the 1960 tsunami. This is obtained from a portion of the marigram at Miyako where noticeable phases of reflected waves are no more recorded. As Ofunato and Hirota bays are comparable in size and seiche period with Miyako bay, the coefficient of eddy viscosity relevant to the bay water oscillation in the three bays may be estimated to be of the order of 3 cm²/sec.

Table1.

ρ	β	ν (cm²/sec)
0.01125	1.026×10^{-8}	1.40
0.01317	1.897×10^{-8}	1.91
0.01579	3.921×10^{-8}	2.74
0.01754	5.962×10^{-8}	3.38
0.01973	9.523×10^{-8}	4.27
0.02638	2.985×10^{-7}	7.57
0.03980	4.616×10^{-6}	29.8
0.05317	1.486×10^{-6}	16.9
0.08065	2.258×10^{-5}	65.8
0.1172	8.991×10^{-5}	131
0.1830	4.000×10^{-4}	277
0.2379	9.766×10^{-4}	433

The plot for the 1933 tsunami, as shown in Fig. 6, indicates considerable discrepancy between the calculated and observed values. The only possibility that accounts for the decrease of water height at the head, is to assume eddy viscosity of the bay water. We have seen in a previous paper [3] that the decrease of wave height at the head is caused only by damping of the first wave. This was understandable from the procedure of constructing the theoretical marigrams. In Table 2 the third line shows the

Table 2.

2κ	30.51	9.658	4.000	2.632	1.316
q	0.0172	0.1047	0.1926	0.3893	1.113
ν(cm²/sec)	7.500	1792	4328	10^4	4×10^4
r (u_0=0.4)	0.935	0.662	0.530	0.217	0.0127
r (u_0=0.3)	0.914	0.578	0.430	0.130	0.003

coefficients of eddy viscosity ν which can reduce the amplitude of the tsunami in various degrees. The fourth and fifth lines indicate the ratios of the amplitude of the first wave at the mouth to that at the head. The coefficient $\nu = 10^4$ cm²/sec given in the fifth column may account for the decrease of wave height in the three bays in the case of the 1933 tsunami.

This considerable difference in eddy viscosity between the water motions in the 1933 and 1960 tsunamis seems natural, since the eyewitnesses which happened to observe the invasion of both tsunamis, told that, in 1933, the tsunami rushed into the bay with much turbidity, while in 1960, it came very quietely. It may be reasonable to suppose that the large value in the 1933 tsunami is due to the comparatively short period of the invaded wave and its oblique incidence and the small value in the 1960 tsunami is principally due to long period of the invaded waves. This inference is in accord with the result of R. TAKAHASHI [2] who accounted for the decrease of wave height in Ofunato bay in the 1933 tsunami by comparing the marigrams due to swell and tsunami.

It is to be noticed that the magnitude of eddy viscosity will differ according as the water motion is in the transient stage or in the stationary stage. The order of magunitude 10^4 cm²/sec above obtained, of course, coresponds to the initial stage of motion. If we continue to assume this value in the later stage, the period of free oscillation is considerably lengthened, but, actually, no appreciable change in the seiche period was observed.

4 General Features of Water Height at Bay Head

So far we have treated only three bays, but there are many other bays, large or small, inspected by other survey groups. Basing upon their results, we will investigate the general features of bay response to tsunami. Figs. 8 a, b show the plots of the ratio η/η_0 against T/T_0 in the cases of 1960 and 1933 tsunamis, respectively. The period of tsunami is again assumed to be 60 min in 1960, and 16 min in 1933. The

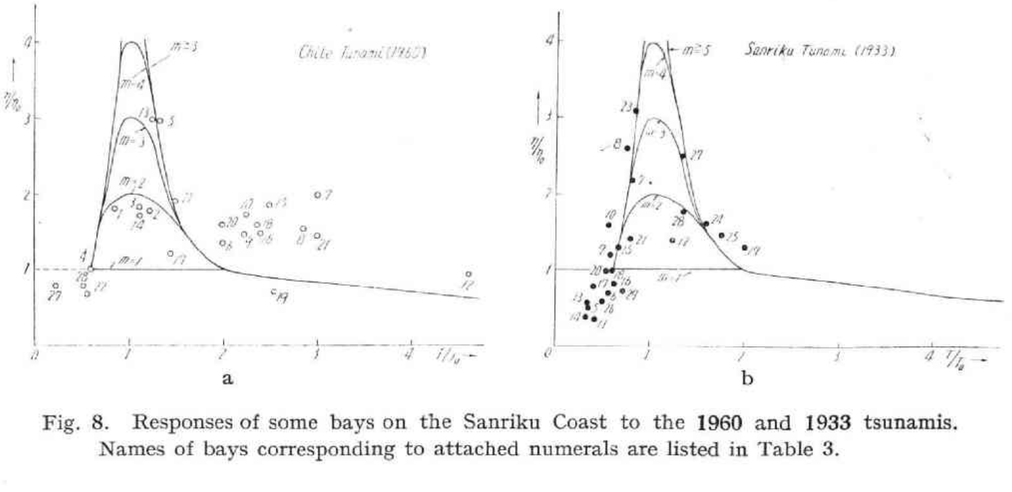


Fig. 8. Responses of some bays on the Sanriku Coast to the 1960 and 1933 tsunamis. Names of bays corresponding to attached numerals are listed in Table 3.

Table 3

Bay	Seiche Period (min.)	Chile Tsunami (1960)		Sanriku Tsunami (1933)	
		T/T_0	η/η_0	T/T_0	η/η_0
1. Akkeshi	67	0.90	1.75		
2. Hamanaka	49	1.22	1.71		
3. Usu	53	1.13	1.84		
4. Aomori	100	0.60	1.00		
5. Miyako	45	1.33	2.90	0.36	0.50
6. Oduti	30	2.0	1.34	0.53	0.65
7. Ryoishi	20	3.00	2.00	0.80	2.20
8. Yoshihama	21	2.86	1.52	0.76	2.60
9. Toni	27	2.22	1.45	0.59	1.20
10. Okkirai	27	2.22	1.80	0.59	1.60
11. Ofunato	40	1.50	1.90	0.40	0.29
12. Kadonohama	13	4.61	0.98	1.23	1.40
13. Hirota	48	1.25	2.90	0.33	0.55
14. Kesennuma	54	1.11	1.70	0.30	0.38
15. Koizumi	24	2.50	1.89	0.67	1.30
16. Oppa	25	2.40	1.43	0.64	0.80
17. Shizugawa	41	1.46	1.18	0.39	0.77
18. Okati	26	2.31	1.58	0.62	0.99
19. Ayukawa	8	2.50	0.75	2.00	1.30
20. Onagawa	30	2.00	1.59	0.53	0.96
21. Samenoura	20	3.00	1.41	0.80	1.40
22. Matushima	95	0.63	0.59		
23. Ryori	19			0.34	3.10
24. Ryoriko	10			1.60	1.50
25. Attari	9			1.78	1.40
26. Yamada	33			0.48	0.57
27. Taro	12			1.33	2.50
28. Funakoshi	12			1.33	1.70
29. Kamaishi	22			0.73	0.72
30. Tokyo	220	0.27	0.77		
31. Kagoshima	107	0.56	0.73		

seiche period, and the values for T/T_0 and η/η_0 for some bays are listed in Table 3. The curves in these figures are the same as those shown in Fig. 5.

Figs. 8 a, b indicate several features about the water height at the bay head.

(1) Remarkably large heights of water are observed only in the bays, T/T_0-values of which lie in the range $0.7 < T/T_0 < 2$, so that the condition of resonance seems to be an essential factor for the incident waves to become very large at the head.

(2) Even if the resonance condition is nearly satisfied, the tsunami decreases in height when the depth of the bay is small, the shallow bottom being the main cause of highly turbulent motion of the bay water. Examples for this case are Kessennuma and Hamanaka (Hokkaido) bays in the 1960 tsunami.

(3) When the T/T_0-value is less than about 0.7, to speak in plain language, when a bay has a small depth or a large length, the height of water at the head is lower than at the mouth. The decease of the wave height is caused, as stated in previous sections, by the turbulent motion of the bay water.

(4) When the T/T_0-value is larger than about 2, in other words, when a bay has a comparatively large depth or a small length, the height of water at the head becomes larger than at the mouth. Most predominant factors which influence the height at the

head are the shallowing bottom and the decreasing bay-width toward the head.

(5) The high water level occurred in Ofunato or Hirota bay in the 1960 tsunami may be attributed to the well developed bay seiche, while abnormally high water levels experienced in Ryôri and other bays in the 1933 tsunami may be caused, as has been reported, by the swelling of water due to their V-shaped configuration.

(6) If we take out of account the plots for V-shaped bays, we can find a distinctive nature of the 1933 tsunami, that is, the incident wave of the tsunami is compsoed of relatively few crests and troughs ($m=2$). This nature seems to be common in tsunamis of nearby origin.

5 Remarks

To protect from a tsunami a bay head where often are situated industrial and business districts, it is necessary to know the characteristic behaviours of the tsunami, near and distant, in the bay. Damage to buildings and other structures are caused partly by streaming or running up of water upon the land, and partly by innundation. The wave height at the bay head is, of course, the most essential factor that influence these phenomena, since it determines the maximum values of both the water level and the flow velocity.

The response curves in Figs. 6 and 8 have been obtained on the assumption that the bay is rectangular in shape and uniform in depth. This assumption is a simplification that can not be applied rigorously to account for the plots in detail. However, it is certain that the response curves indicate the effect due only to period, and the deviation of the plots from the curves may be attributed to other effects such as due to the peculiar configuration and conspicuous depth change of the bay.

It may be generally accepted that the magnitude of eddy viscosity is large if the bay is shallow, its configuration is complicated, the period of the tsunami is short, and the incident angle of the tsunami is large. The last two cases are most usually realized in the case of a near tsunami.

6 Summary

(1) As a typical example of different ways in which a bay responses to near and distant tsunamis, the maximum water heights at the heads of Ofunato, Hirota and Miyako bays in the cases of the Sanriku (1933) and Chile (1960) tsunamis are investigated. From the field observation it is found that the maximum water height in the Chile tsunami occurred at the bay head, whereas, in the Sanriku tsunami it occurred at the bay mouth according to the report published by the Earthquake Research Institute.

It can be shown that the decrease of water height toward the head in the 1933 tsunami may be caused by damping of the invaded waves due to the turbulent motion of the bay water. The coefficient of horizontal eddy viscosity that makes the tsunami damp in Ofunato and other two bays can be estimated to be of the order of 10^4 cm^2/sec.

It is to be noticed that the eddy viscosity specified by such a value meaures the turbidity of the bay water motion caused mainly by bore-like invasion, and reflection and refraction of waves arising from the irregular shape of the bay.

In the Chile tsunami, owing to its long period waves, the effect of the eddy viscosity was negligibly small. The ν-value in the later stages of the bay water disturbance is determined to be of the order of 1 cm^2/sec from the damping of the seiche in Miyako bay.

(2) By comparing the response curves obtained in a previous paper [3] with the plots of water height ratio for many bays along the Sanriku Coast, it is found that, generally, the water height at the bay head is lower or higher than that at the bay mouth, according as the seiche period is somewhat shorter or longer than the period of the tsunami. The decrease of water height at the head may be, as stated above, caused by the turbulent motion of the bay water. While the incresase of the water level at the head may arise form the swelling of water due to the configuration and bottom topography of the bay.

That the period of a tsunami is practically coincident with that of the bay seiche is a necessary condition for the incident waves to become considerably increased in height at the head. Even if the approximate condition for resonance is satisfied, the water height decreases toward the head when the bay is very shallow. On the contrary, if the bay has a large depth and a V-shaped form, the tsunami can increase abnormally in height at the bay head.

It is suggested that shorter period tsunamis are likely to produce abnormally heigh water levels through the swelling due to V-shaped bays, rather than through a bay seiche, whereas, very high water levels caused by longer period tsunamis may be attributed to well developed seiches.

Acknowledgement We thank all those persons in the visited area for their help in collecting data. We also acknowlege the financial aid of the Ministry of Education.

References

1. Papers and Reports on the Tsunami of 1933 on the Sanriku Coast, Japan. *Bull. Earthq. Res. Inst. Tokyo. Suppl.* Vol. 1, 1934.
2. TAKAHASHI, R.: Seiches and Surface Waves in Ofunato Bay and two other Bays. *Bull. Earthq. Res, Inst., Tokyo, Suppl.*, Vol. 1, 198–217, 1934.
3. NAKAMURA, K.: The Motion of Water due to Long Waves in a Rectangular Bay of Uniform Depth. *Sci. Rep. Tôhoku Univ., Fifth Ser., Geophy.*, **12**, 191-213, 1960.