Studieson Geomagnetic Pulsation

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Studies on Geomagnetic Pulsation, Pc. *

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Abstract

It is well known that the quasi-stationary pulsation of the geomagnetic field occurs principally in the day time, at the time of magnetic disturbance. Geomagnetic pulsations of this type are called Pc's.

Observational evidences obtained show that the geomagnetic pulsation would be caused by the hydro-magnetic oscillation of the earth's outeratmosphere expanding beyond the ionosphere and Pc's correspond to the poloidal type of oscillation. The radius of the outeratmosphere is guessed to be of the order of serveral earth's radii, assuming the shape of the outeratmosphere to be spherical. It is very interesting that the dimension of the outeratmosphere is of the order of that of the Chapman-Ferraro's cavity or of the forbidden region. There is a observational fact suggesting that the outeratmosphere would be contracted at the magnetic disturbance.

Besides, a preliminary study on the agent of excitation of the outeratmospheric oscillation predicts that such oscillations would be caused by the turbulent motion of the solar corpuscular stream.

1 Introduction

We recognize morphologically the three types of geomagnetic pulsations : Pc's, Pt's and giant pulsations.

Pc's appear as continued trains of pulsations lasting, sometimes, for several hours. They occur mostly in the daytime, but, on disturbed days, appear even in the night time. Their periods lie commonly between ten seconds and a minute, and their amplitudes are of the order of 1/10 gamma.

Pt's occur principally in the night time, and are often accompanied by bay-disturbances. Thier amplitudes amount sometimes to a few tenths of gamma.

Various names are given to Pc's and Pt's by bifferent authors. Pc's are called the day-time pulsation (E. R. R. HOLMBERG [1]), or the a-group pulsation (G. ANGENHEISTER [2]). Beautiful and sinusoidal pulsations, which seem to belong to Pc's, occur sometimes and are called regular pulasations by T. TERADA [3]. On the other hand, Pt's are named irregular pulsations (by S. UTASHIRO [4]), the night time pulsations (by HOLMBERG [1]), or the b-group pulsations (by ANGENHEISTER [1]).

The 10 th Committee of IAGA [5] has determined to tentatively take up the names 'Pc' and 'Pt'. It is, therefore, advisable that these names should be used commonly by the researchers in this field all over the world.

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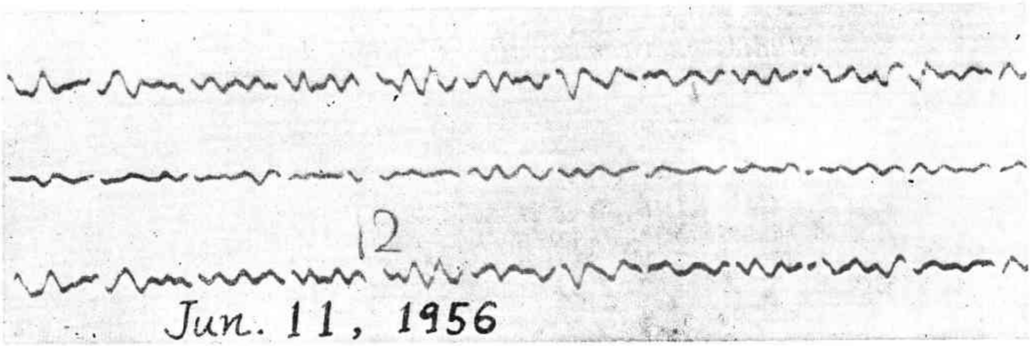
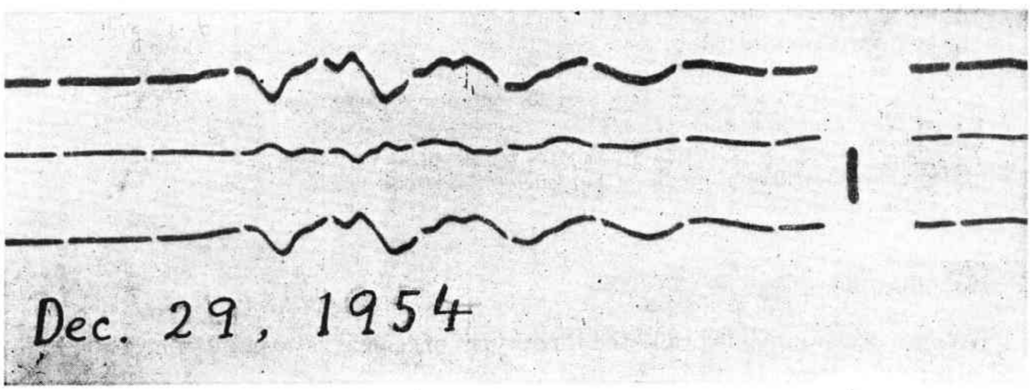


12 : 00 12 : 05

Fig. 1 a *Pc* type pulsation
From above to down : dZ/dt, dD/dt and dH/dt components. Date : about 12 h 135° E L.M.T., Jun. 11, 1956.



00 : 55 01 : 00

Fig. 1 b *Pt* type pulsation
From above to down : dZ/dt, dD/dt and dH/dt components. Date : about 01 h 135° E L.M.T., Dec. 29, 1954.

Besides Pc's and Pt's, there is another type of pulsations, which appear only in the auroral region and have very large amplitudes amounting sometimes to a few ten gammas. By this reason, they are named the giant micropulsations by B. ROLF [6] or more shortly the giant pulsations by E. SUCKSDORFF [7].

Theoretical studies has been made too by several authors : TERADA [3], H. HATAKEYAMA [8], [9], C. STÖRMER [10], Y. KATO et al. [11], [12] and J. W. DUNGEY [13]. Their proposals are divided into two groups : the intra-ionospheric origin theory and the extra-ionospheric origin theory. The former lays the origin of the geomagnetic pulsation in the ionosphere, whereas the latter lays it outside the ionosphere.

TERADA [3] proposed that the geomagnetic pulsations would be caused by fluctuaton of the S_q-current system. According to him, the idea was originated from BIRKELAND. H. HATAKEYAMA [9] cosidered that they were caused not by fluctuation of the S_q-current system but by fluctuation of the S_D-current system.

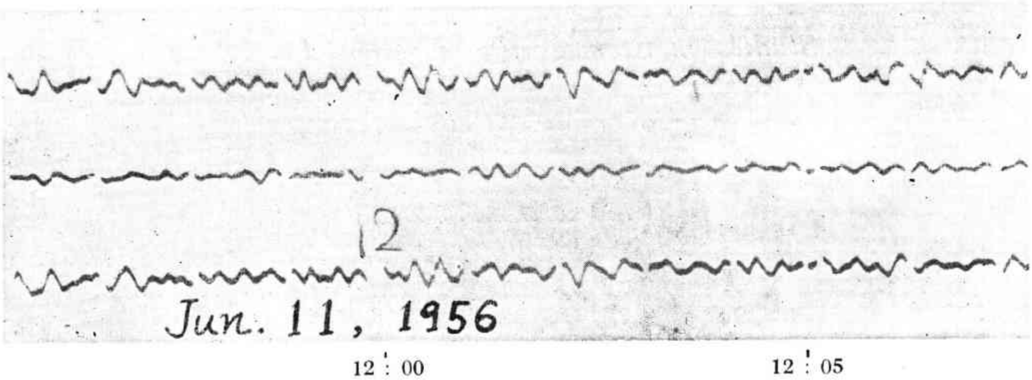


Fig. 1 a *Pc* type pulsation
From above to down : dZ/dt, dD/dt and dH/dt components. Date : about 12h 135° E L.M.T., Jun. 11, 1956.

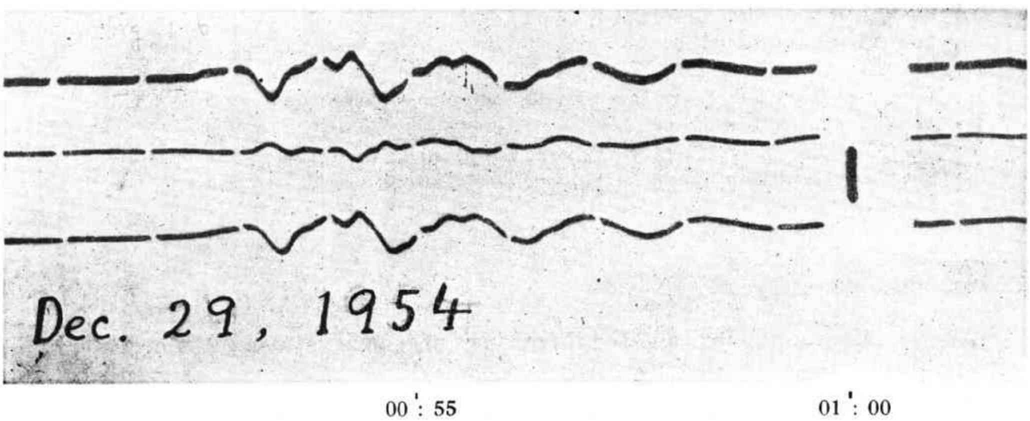


Fig. 1 b *Pt* type pulsation
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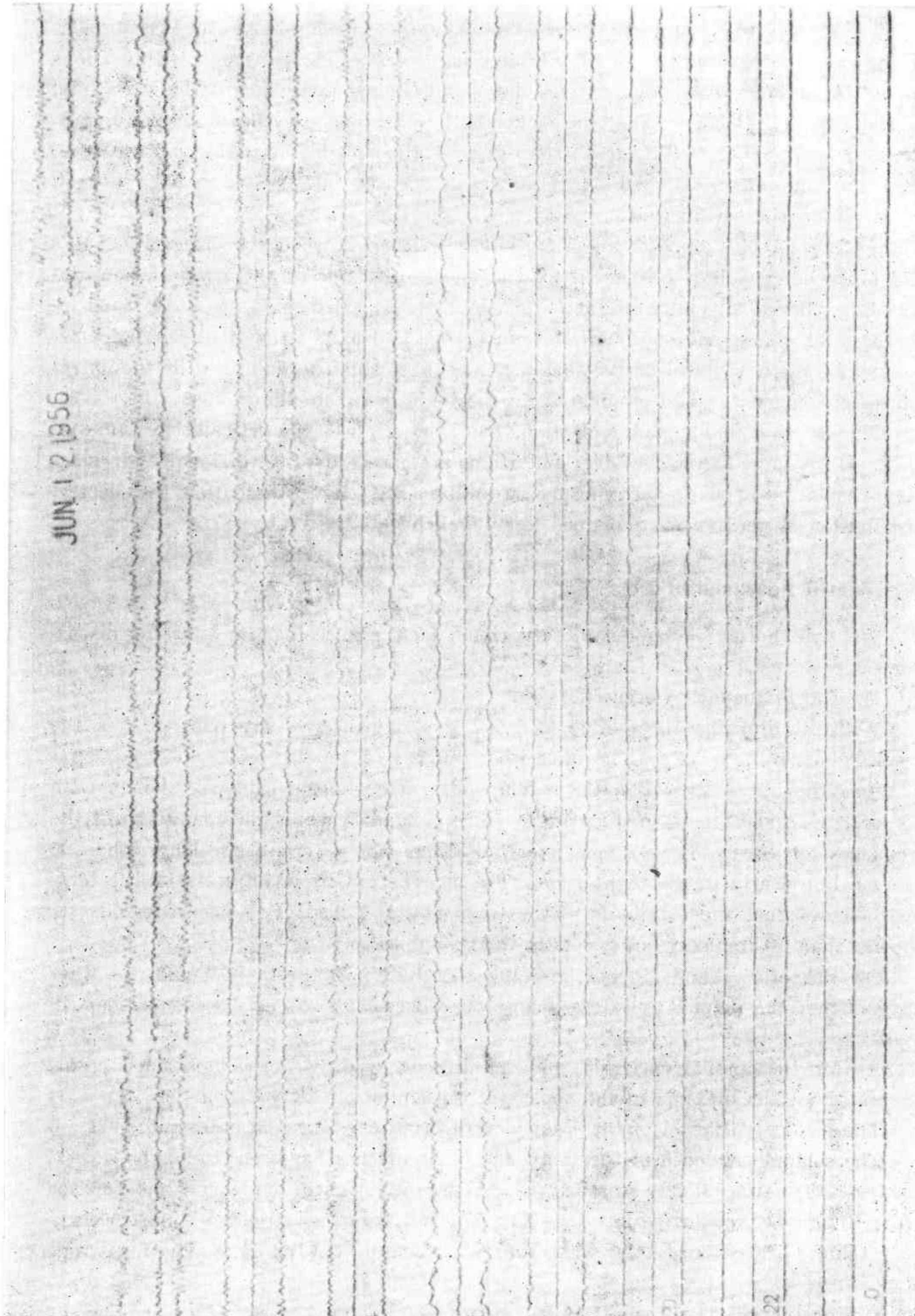


Fig. 2 Examples of Pc's in the moderately disturbed time interval

C. STÖRMER [10] found that the periodical orbits of the solar charged corpuscules might exist in the earth's magnetic field, and has proposed that geomagnetic pulsations should be caused by the solar corpuscular clouds flying along such periodical orbits.

DUNGEY [13] claimed that the geomagnetic pulsations are caused by the hydromagnetic oscillation of the earth's ionized outeratmosphere expanding outwards beyond the ionosphere, of which existence was guessed by L.R.O. STOREY [14] through his study on the whisting atmospherics.

KATO [11], [12] opposed the intra-ionospheric origin theory in his study of Pt's and proposed that Pt's are caused by the effect of beams of the charged particles springing periodically into the earth. In our study [15] of Pt's, we have stood by the side of the extra-ionospheric origin theory and have found that the daily behaviour of the horizontal perturbing vector of Pt's could be described as the extra-ionospheric magnetic field modulated by the ionospheric shielding effect.

On the other hand, giant pulsations has been studied theoretically by DUNGEY [13]. He proposed that the giant pulsations are caused by the torsional hydromagnetic oscillation of the outeratmosphere, and we [25] have found that the spectral distribution of periods are explained fairly well by his theory.

2 General Features of Pc's

We will shortly review the observational facts concerning the following discussions.

1) The frequency of occurence

With the difficulty in practice to define the frequence of occurrence of Pc's, the results given by several authors have been differed in detail.

According to TERADA [3], the regular pulsations whose periods are shorter than 70 sec. ca. occurr most frequently in the forenoon, whereas the maximum of the frequency of occurrence of the regular pulsations whose periods are longer than 90 sec. ca. lies clearly in the night time. The recent study by ANGENHEISTER [2], however, shows that the maximum of the frequency of general Pc's whose periods are shorter than 10 min. ca. lies generally in the day time.

On the other hand, it has been reported by Y. BEAUFILS [17] and the other authors that the regular pulsations whose periods are near 20 sec. occur particularly at dawn.

As for the annual variation, any definite conclusion is not yet obtained. But, it seems that the maxima might appear in the equinoxes.

The secular variation of the frequency of occurrence has not been studied at all.

The solar dependence of the frequency of occurrence has been studied by several authors. TERADA [3] has searched the 27 day-reccurrence tendency, but has not reached to any conclusive result. The 27 day-reccurrence tendency has recently been found with certainty by KATO and S. AKASOFU [18], or by KATO, J. OSSAKA et al. [19].

During the year 1953, the UM region had continued to appear in the solar surface (H.W. BABCOCK and H.D. BABCOCK [20]), and a clear correspondence of the occur-

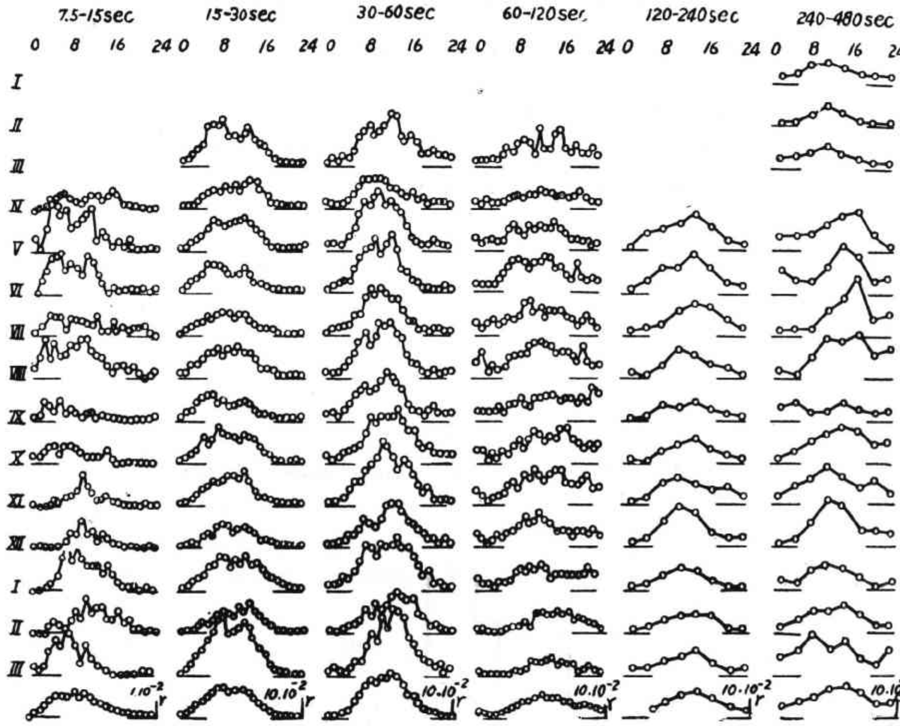


Fig. 3 Frequency of occurrence of Pc's (after G. Angenheister)

rence of Pc's and the appearence of the UM regions have been pointed out by KATO and AKASOFU.

The 27 day-reccurrence tendency during the same period has been also found by KATO, OSSAKA et al. It has been pointed out by K. BURKHART [21] too.

ANGENHEISTER [2] has found a clear correlation between K_p and the frequency of occurrence.

2) The relative frequency of occurrence.

The problem has been studied by TERADA [3] and HOLMBERG [1]. The both results differ in some respects. The frequency of occurrence of lower periods is smaller in TERADA's results, whereas the occurrence frequency in the longer period region is greater in it. The difference might be due to the difference of the instruments used. TERADA has used the usual magnetomer of high sensitivity and HOLMBERG the induction loop. The shorter period is apt to be cut off more eaily in the usual magnetometer and, on the other hand, is likely to be overtoned more easily in the induction magnetometer.

3) Amplitudes

Fig. 5 which has been obtained by ANGENHEISTER [2] shows the relation between the mean double amplitude and the period. At the periods near a few ten sec. which we observe most frequently, the double amplitude becomes nearly of the order of 1 gamma.

4) World wide character of occurrence

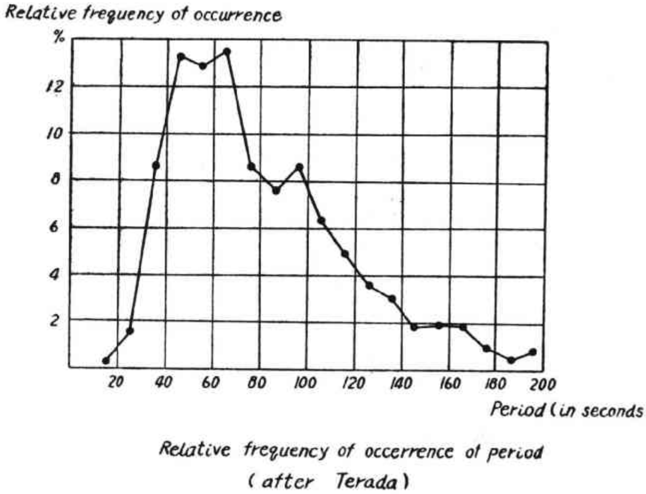


Fig. 4 a Relative frequency of occurrence of period

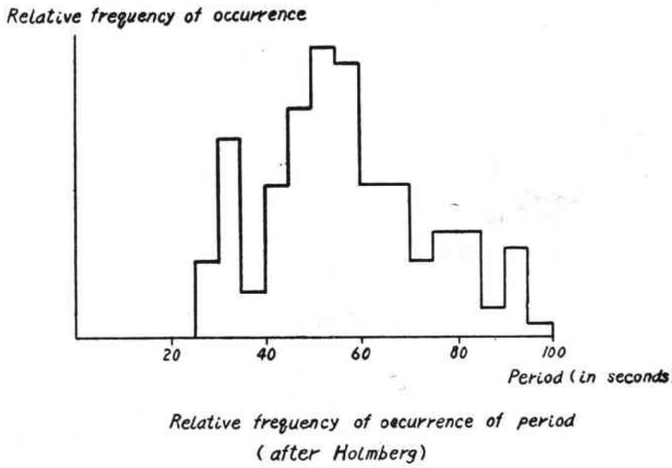


Fig. 4 b Relative frequency of occurrence of period

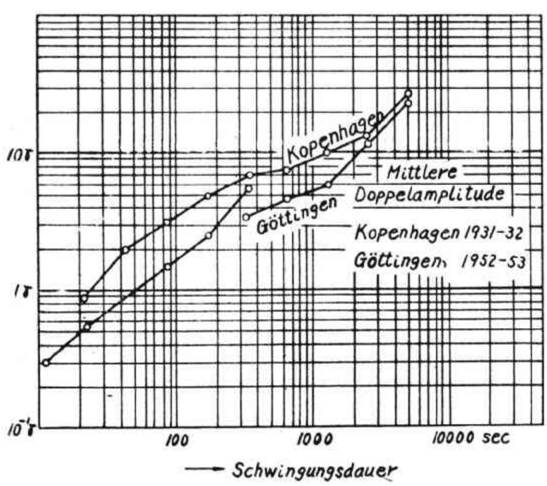


Fig. 5 Relation between the amplitude and the period (after G. Angenheistor)

It may be noticed that Pc's occur simultaneously in the considerably wide region. It has been pointed out by KATO and M. OKUDA [22] that the very similar Pc type pulsations were observed at the same time in stations as far as apart as Memambetsu, Hokkaidô, Japan (geograpfic longitude=208°4, geograpfic latitude=+34°0) and Paradeniya, Ceylon (geograpfic longitude=149°8, geograpfic latitude=−2°7). The phase differences were nearly zero, and the amplitudes were slightly greater in Memambetsu.

A similar fact was observed by M. SCHLUMBERGER and G. KUNETZ [23]. They reported a good correlation of the variation of the north component at the both places, through their cooperative observations at Madagascar and France. KUNETZ [24] made also the cooperative observations of the earth currents at U.S.A., Venezuela, Gabon (Africa), Sahara (Africa) and Scilie, where he had found a considerably good correlations in some cases.

3 Fundamental Formulation

In order to study Pc according to DUNGEY's theory outlined in the section 1, we shall first formulate the equations of the hydromagetic oscillation of the outer-atmosphere, which were given as follows by DUNGEY (in Gaussian units) :

$$\left[4\pi\rho H^{-2}\frac{\partial^2}{\partial t^2}-r^{-2}\sin\theta\frac{\partial}{\partial\theta}\sin^{-1}\theta\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial r^2}\right](r\sin\theta\, E_\phi)$$
$$=c^{-1}\sin\theta\left(H_r\frac{\partial}{\partial\theta}-H_\theta r\frac{\partial}{\partial r}\right)\left((r\sin\theta)^{-1}\frac{\partial u_\phi}{\partial\phi}\right), \qquad (1)$$

$$\left[4\pi\rho\frac{\partial^2}{\partial t^2}-(r\sin\theta)^{-1}\left((\boldsymbol{H}\cdot\nabla)(r\sin\theta)^2(\boldsymbol{H}\cdot\nabla)+H^2\frac{\partial^2}{\partial\phi^2}\right)\right]\left(\frac{u_\phi}{r\sin\theta}\right)$$
$$=c(r\sin\theta)^{-3}\left(H_r r^{-1}\frac{\partial}{\partial\theta}-H_\theta\frac{\partial}{\partial r}\right)\left(r\sin\theta\frac{\partial E_\phi}{\partial\phi}\right). \qquad (2)$$

We use the system of the spherical polar cordinates r, θ and ϕ. r represents the distance of a representative point from the magnetic dipole situated at the earth's centre, which coincides with the origin of the polar axis, and its sense is coincided with the negative direction of the polar axis ($\theta=\pi$). The magetic field (H_r, H_θ, 0) produced by the dipole represents the earth's main magnetic field and its intensity H is expressed by

$$H=\frac{H_0 a^3}{r^3}\sqrt{1+3\cos^2\theta}, \qquad H_0=0.3\,\Gamma,$$

where a represents the length of the earth's radius. ρ in eqs. (1) and (2) represents the mass density of the earth's outeratmosphere.

The general equations of oscillation are too difficult to be solved analytically. But, if we assume that the field quantities are independent of the azimuth ϕ, the above equations are reduced to the following equations :

$$\left[4\pi\rho H^{-2}\frac{\partial^2}{\partial t^2}-r^{-2}\sin\theta\frac{\partial}{\partial\theta}\sin^{-1}\theta\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial r^2}\right](r\sin\theta E_\phi)=0, \qquad (3)$$

and

$$\left[4\pi\rho\frac{\partial^2}{\partial t^2}-(r\sin\theta)^{-2}(\boldsymbol{H}\cdot\nabla)(r\sin\theta)^2(\boldsymbol{H}\cdot\nabla)\right]\left(\frac{u_\phi}{r\sin\theta}\right)=0, \qquad (4)$$

which are rather simpler to be solved.

In this case, the following two sets of quantities are governed by the different equations independenly.

$$\boldsymbol{u}\ (u_r,\ u_\theta,\ 0) \qquad \boldsymbol{h}\ (h_r,\ h_\theta,\ 0) \qquad \boldsymbol{E}\ (0,\ 0,\ E_\phi)$$

$$\boldsymbol{u}\ (0,\ 0,\ u_\phi) \qquad \boldsymbol{h}\ (0,\ 0,\ h_\phi) \qquad \boldsymbol{E}\ (E_r,\ E_\theta,\ 0)$$

The above set of quantities are governed by the equation (3), which is called the equation of the poloidal oscillation. On the other hand, the lower set of qnantities are given by the equation (4), which is named the equation of torsional oscillation by DUNGEY [13].

Since the only differential operator occurring in the epuation (4) is $(\boldsymbol{H}\cdot\nabla)$, this equation means, as noticed by DUNGEY, that each surface of revolution of a line of force oscillates independently. Accordingly, the eigenperiods of oscillation differ for each surface of revolution. The eigenperiod is largely. dependent of the latitude, at which the magnetic line of force intersects the earth's surface, namely the latiude λ_0 of the observing station. By our analysis [25], the periods depen on λ_0 through the factor $\sec^2\lambda_0 \sin\lambda_0\ F\ (\sin^2\lambda_0)$. $F(\sin^2\lambda_0)$ is of the order 0.5 for $\lambda_0=45°\sim65°$. The feature is not consistent with the observational fact stated in the section **2** : it has been noticed, as above stated, that the very similar Pc-type pulsations are observed practically at the same time in stations as far apart as Memambetsu, Hokkaido, Japan and Paradeniya, Ceylon or in stations as far apart as France, and Madagascar.

In fact, the torsional oscillation gives not the field of Pc pulstaion, but that of the giant pulsation (see [13], [25]). On the other hand, the poloidal oscillation is favoured as the origin of Pc's, because eigenperiods are independent of the latitude of a observing station as shown later, and the assumption that the field quantities are independent to the azimuthal angle seems roughly satisfiable by the above mentioned fact.

4 Mathematical Analysis of the Equation of the Poloidal Oscillation

The equation of the poloidal oscillation is studied by one of the authors (Y.K.) and S. AKASOFU [26]. Now, we shall treat it under somwhat different conditions.

(i) The fundamental equation

Instead of r, we use R, which is the distance measured in units of the earth's radius. Then, the equation of the poloidal oscillation is written as follows :

$$\left(4\pi a^2\rho H^{-2}\frac{\partial^2}{\partial t^2}-\frac{1}{R^2}\sin\theta\frac{\partial}{\partial\theta}\sin^{-1}\theta\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial R^2}\right)(R\sin\theta E_\phi)=0\,. \tag{5}$$

Putting

$$R\sin\theta E_\phi=\chi, \qquad \mu=\cos\theta\,, \tag{6}$$

and assuming that the field quantities depend on t through the factor $e^{i\omega t}$:

$$E_\phi \propto e^{i\omega t}$$

we have

$$\left(\frac{\partial^2}{\partial R^2}+\frac{1-\mu^2}{R^2}\frac{\partial^2}{\partial\mu^2}+\frac{4\pi\rho a^2\omega^2}{H^2}\right)\chi=0. \tag{7}$$

According to Dungey [13], ρ depends on R as follows : $\rho \propto e^{2.5/R}$, then the value of ρ at $R=1$, $R=10$ are approximately in ratio 10 : 1.

On the other hand, $1/H^2$ is given as follows :

$$\frac{1}{H^2} = \frac{1}{H_0^2} \cdot \frac{R^6}{(1+3\cos^2\theta)} . \tag{8}$$

Because $1/H^2$ depends on θ through the factor $1/(1+3\cos^2\theta)$, the values of $1/H^2$ at $\theta=0$ and $\theta=\pi/2$ are in ratio $1:4$. But, $1/H^2$ depends on R far more sensitively, and so, the behavior of the last term in the bracket of the equation (7) may be governed largely by the variable R only. Then, we have approximately :

$$\frac{4\pi\rho a^2\omega^2}{H^2} \rightarrow \frac{4\pi\rho a^2\omega^2}{\varepsilon^2 H_0^2} R^6 ,$$

where ρ may be assumed constant, and ε^2 lies between 1 and 4.

Putting

$$\frac{1}{V_0} = \frac{\sqrt{4\pi\rho}}{H_0} , \tag{9}$$

and

$$\frac{a\omega}{V_0} = 2b , \tag{10}$$

the equation (7) is written as follows :

$$\left(\frac{\sigma^2}{\partial R^2} + \frac{1-\mu^2}{R^2} \frac{\partial^2}{\partial\mu^2} + 4b^2R^6 \right) \chi = 0 . \tag{11}$$

We take this equation as the fundamental equation of the poloidal oscillation.

(ii) Solution of the equation

Putting

$$\chi(R, \theta) = F(R)\,\Theta(\theta) ,$$

the above equation is seperated into the following equations :

$$\frac{d^2F}{dR^2} + \left(4b^2R^6 - \frac{n(n+1)}{R^2} \right) F = 0 , \tag{12}$$

$$(1-\mu^2) \frac{d^2\Theta}{d\mu^2} + n(n+1)\,\Theta = 0 , \tag{13}$$

where $n(n+1)$ is a constant of seperation. In order to make Θ to be finite and univalent, n must be integers, and so Θ is given as follows :

$$\Theta = (1-\mu^2)^{1/2} P_n^1(\mu) . \tag{14}$$

n may be determined, if we know the latitudinal distribution of the magnetic field from observations. But, the observational results are too scanty to draw any definite conclusion. It may, however, be possible that $n=1$, because the cooperative observation made in France and Madagascar [23] shows us that the observed magnetic field is wonderfully similar in both places : when the magnetic field is directed northwards in one place, it is directed not sowthwards but northwards in the other place too. This tendency is observed too in the cooperative observation in

Memambetsu (Hokkaidô, Japan), and Paradeniya, Ceylon [22]. It is, fortunately, not necessary to know the value of n in order to evaluate the eigenperiods of oscillation approximately as shown later. But, it is needed for the calculation of the field quantities. If we take $n=1$, Θ is given as follows :

$$\Theta = \sqrt{1-\mu^2}\, P_n^1(\mu) = \sin^2\theta\,, \tag{15}$$

and the equation (12) is reduced to

$$\frac{d^2F}{dR^2} + \left(4b^2R^2 - \frac{2}{R^2}\right)F = 0\,, \tag{16}$$

whose solution is given as follows :

$$F = \sqrt{R}\left(AJ_{\frac{3}{8}}(bR^4) + BN_{\frac{3}{8}}(bR^4)\right), \tag{17}$$

where A and B are constants of integration.

(iii) Condition at the lower boundary

According to DUNGEY [27], we may put $E_\phi = 0$ at the lower boundary of the outeratmosphere (viz. at the upper boundary of the ionosphere). In our case, it may be permitted to assume that we take the earth's surface ($R=1$) as the lower boundary. Then the boundary condition at the lower boundary is given :

$$F = 0\,, \qquad \text{at} \quad R = 1\,.$$

Accordingly, we have

$$F = E_0\sqrt{R}\left(N_{\frac{3}{8}}(b)J_{\frac{3}{8}}(bR^4) - J_{\frac{3}{8}}(b)N_{\frac{3}{8}}(bR^4)\right), \tag{18}$$

and

$$E_\phi = E_0\frac{1}{\sqrt{R}}\left(N_{\frac{3}{8}}(b)J_{\frac{3}{8}}(bR^4) - J_{\frac{3}{8}}(b)N_{\frac{3}{8}}(bR^4)\right)\sin\theta\,. \tag{19}$$

(iv) Field quantities

The field quantities $\boldsymbol{h}$ and $\boldsymbol{u}$ are calculated by the following equations :

$$\frac{\partial \boldsymbol{h}}{\partial t} = -c\,\mathrm{curl}\,\boldsymbol{E}\,,$$

$$\rho\,\frac{\partial \boldsymbol{u}}{\partial t} = \frac{1}{4\pi}\,(\mathrm{curl}\,\boldsymbol{h}) \wedge \boldsymbol{H}.$$

Finally we have,

$$\left.\begin{aligned} u_r &= -c\,\frac{H_\theta}{H^2}E_\phi \\ u_\theta &= \;\;\,c\,\frac{Hr}{H^2}E_\phi \end{aligned}\right\} \tag{20}$$

and

$$\left.\begin{aligned} h_R &= \frac{ic}{a\omega R}\left(\frac{\partial E_\phi}{\partial\theta} + \cot\theta\,E_\phi\right) \\ &= 3E_0\,\frac{ic}{a\omega}\,R^{-3/2}\left[N_{\frac{3}{8}}(b)J_{\frac{3}{8}}(bR^4) - J_{\frac{3}{8}}(b)N_{\frac{3}{8}}(bR^4)\right]\cos\theta \\ h\theta &= -\frac{ic}{a\omega R}\,\frac{\partial}{\partial R}(RE_\phi) \\ &= -4E_0\,\frac{ic}{a\omega}\,R^{-3/2}\left[\frac{5}{8}\left\{N_{\frac{3}{8}}(b)J_{\frac{3}{8}}(bR^4) - J_{\frac{3}{8}}(b)N_{\frac{3}{8}}(bR^4)\right\}\right. \\ &\quad \left. - bR^4\left\{N_{\frac{3}{8}}(b)J_{\frac{11}{8}}(bR^4) - J_{\frac{3}{8}}(b)N_{\frac{11}{8}}(bR^4)\right\}\right]\sin\theta \end{aligned}\right\} \tag{21}$$

(v) Condition at the outer boundary

The shape of the outerboundary of the outeratmosphere is not yet cleared as stated by DUNGEY [13]. For the sake of simplicity, however, we assume that the shape of the outer boundary as a spherical surface, whose radius is aR_1. The assumption is completely conventional, and so 'the radius of the outeratmosphere' means merely one of meaures of the dimension of the outeratmosphere.

What condition must be imposed at the boundary, under the above stated assumption ?

If it is vacum ontside the outeratmosphere, there hold the Maxwell equations. The field quantitions are propagated with the light velocity there, whereas in the outeratmosphere they are propagated with the Alfvén-wave velocity, which would be considerably smaller than the light velocity. At the boundary of two media, whose velocities of propagation are decidedly different, the energy of wave is genearlly reflected almost completely. Then, the component of the Poynting vector normal to the outerboundary is nearly zero. $h_\theta E_\phi$ can, therefore, be put equal into zero with enough certainty : one of two quantities h_θ and E_ϕ is, at least, equal to zero.

If the outerboundary is fixed rigidly, so $\boldsymbol{u}$ must be zero at the outerboundary : the equation (20) teaches us E_ϕ must be zero at the outerboundary. It is, however, doubtful that the outerboundary is fixed rigidly.

If we postulate that quantities h_θ and E_ϕ must be continuous at the boundary, we find that h_θ almost vanishes. In this case, however, E_ϕ is not zero, and so $\boldsymbol{u}$ at the outerboundary is not zero, viz., the outerboundary is moved. If its motion is small enough, it may be permitted to assume that the outerboundary is fixed rigidly as treated by KATO and S. AKASOFU [26].

Even with the above considerations, the boundary conditions at the outerboundary still remain unclear. In the first place, the space outside the outerboundary is not vacum. There is interplanetary matter, whose density is not smaller than that of the outeratmosphere. Next, there may be interplanetary magnetic field, and so it is possible that the field quantities are governed not by the Maxwell equations but by the hydromagnetic equations.

In spite of the above-mentioned difficulties, it is fortunately possible that the eigenperiods are calculated with enough approximation as stated in the next section.

(vi) Eigenperiods of oscillation

In the boundary value problems, there is a helpful theorem, which is stated as follows [28] :

Under any of the bounary conditions considered, the number n of eigenvalues less than a fixed λ of the differential equation,

$$\frac{d}{dx}\left(p\,\frac{du}{dx}\right) - qu + \lambda\rho u = 0\,, \qquad (0 \leqq x \leqq \pi) \tag{22}$$

is asymptotically equal to $\sqrt{\lambda}/\pi\int_0^\pi\sqrt{\frac{\rho}{p}}\,dx$; in other words,

$$\lim_{n\to\infty}\frac{u}{\sqrt{\lambda_n}} = \frac{1}{\pi}\int_0^\pi\sqrt{\frac{\rho}{p}}\,dx\,. \tag{23}$$

By this theorem, we may evaluate the eigenperiods without any knowlegde of the boundary conditions. Remembering that the considered differential equation (12) is defined in the interval $[1, R_1]$, we evaluate the eigenvalues b_n as follows :

$$b_n \sim \frac{n\pi}{2} \frac{1}{(R_1-1) F\,(R_1-1)}, \tag{24}$$

where

$$F(\alpha) = 1 + \frac{3}{2}\alpha + \alpha^2 + \frac{1}{4}\alpha^3. \tag{25}$$

If T_n means the n-th eigenperiod, we obtain

$$b_n = \frac{\pi a}{\varepsilon V_0} \frac{1}{T_n},$$

from the equation (10), and then

$$T_n \sim T_1/n, \tag{26}$$

where

$$T_1 = \frac{2a(R_1-1)\,F\,(R_1-1)}{\varepsilon V_0}.$$

If $R_1 \gg 1$,

$$T_1 \simeq \frac{1}{2\varepsilon} \frac{aR_1^4}{V_0}, \tag{27}$$

or,

$$T_1 \simeq \frac{1}{\varepsilon} \frac{\frac{aR_1}{2}}{V_1}, \tag{28}$$

where

$$V_1 = V_0/R_1^3.$$

εV_1 is the Alfvén-wave velocity at the outerboundary in the equatorial plane when $\varepsilon = 1$, or in the polewards direction when $\varepsilon = 2$.

The relation (28) is very useful. Because aR_1 is the radius of the outeratmosphere, T_1 (say the fundamental period) is obtained from the half of the radius of the outeratmosphere divided by the Alfvén-wave velocity at the boundary.

Equation (28) teaches us that

(a) T_1 depends on R_1 very sensitively : $T_1 \propto R_1^4$

(b) and on ρ through the factor $\sqrt{\rho}$, because $V_0 \propto 1/\sqrt{\rho}$

To evaluate T_1, it is necessary to know the value of ρ. Under the observation of the zodiacal light, the electron density is approximately 600/cc (Siedentopf [29]). Under the observation of whistlers, the particle density lies between 10^2/cc and 10^3/cc. If we assume that the constituents of the outeratmosphere is H^+ ion and electron, the density of the outeratmosphere is given as follows :

$\rho = 5.0 \times 10^{-22}$ gm/cc		300 protons/cc
$= 1.0 \times 10^{-21}$ gm/cc	approximately corresponding to	600 protons/cc
$= 2.0 \times 10^{-21}$ gm/cc		1200 protons/cc

The following tables give the values of T_1 for the various values of R_1's, ρ's, and ε's.

Table 1
The Fundamental Period T_1 (in sec)

ρ(gm/cc) / R_1	$5.0 \cdot 10^{-22}$		$1.0 \cdot 10^{-21}$		$2.0 \cdot 10^{-21}$	
	$\varepsilon=1$	$\varepsilon=2$	$\varepsilon=1$	$\varepsilon=2$	$\varepsilon=1$	$\varepsilon=2$
1	0	0	0	0	0	0
2	1	1	2	1	3	2
3	7	4	10	5	14	7
4	24	12	34	17	48	24
5	53	27	75	38	106	53
6	112	56	158	79	224	112
7	206	103	293	147	415	208
8	344	172	488	244	693	347
9	553	252	783	392	1110	555
10	844	422	1190	595	1680	840

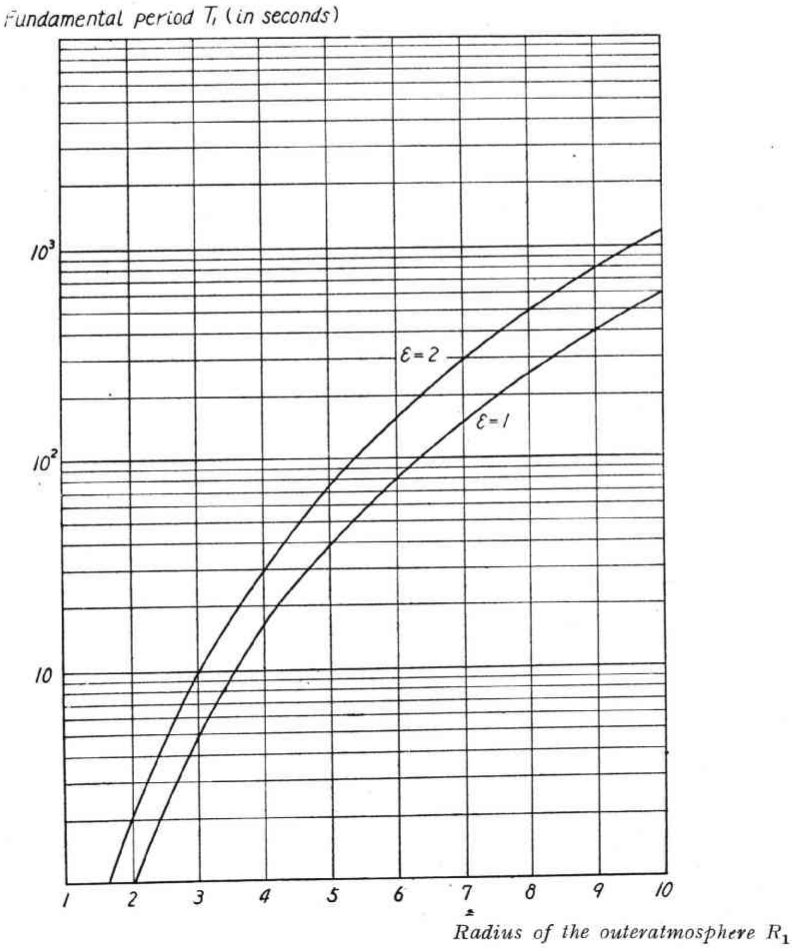


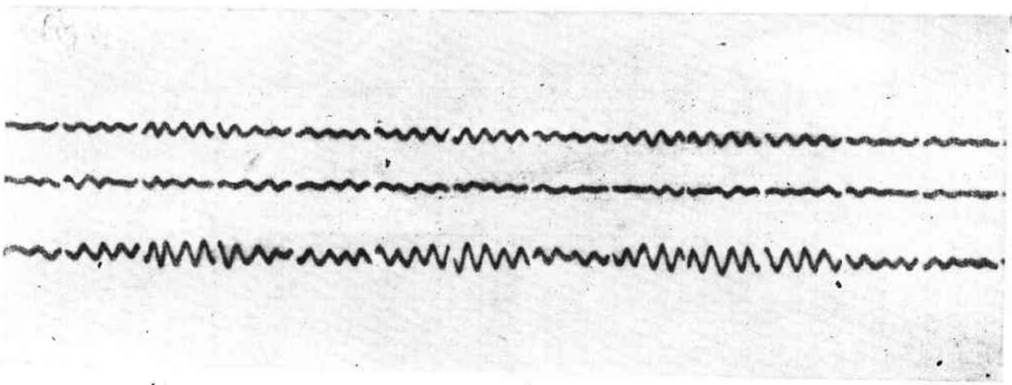
Fig. 6 Relation between the fundamental period T_1 and the radius of the outeratmosphere R_1, when the number density of the outeratmosphere is assumed to be 600 protons/cc.

5 Law of Spectral Distribution

The law of specrum of the eigenperiods (26) is not found cleary in Pc's, although such a spetrum is found fairly clearly in case of the giant pulsations [25]. This is perhaps due to the reason that the outerboundary is not formed definitely in the geometrical meaning. On the other hand, the boundary condition in case of the giant pulsations is determined only at the surface of the earth (see [13] or [25]).

We can find, however, a exprimental evidence showing that Pc's would be caused by the outeratmospheric poloidal oscillation having eigenperiods as shown in the equation (26).

We find sometimes a beatiful pulsation on the papid-run magnetogram, whose typical case is shown in Fig. 7. Beating shows us that such an oscillation is produced



10 : 25 10 : 30 10 : 35

Fig. 7 Typical 'Beating Pc'

From above to down : dZ/dt, dD/dt and dH/dt components. Date : about 11 h 135° E L.M.T., Jul. 26, 1956.

by superposing of oscillations whose eigenperiods are approached. If one oscillation is exicted with eigenperiod T_1/n, the eigenperiod of the other oscillation, most probably, $T_1/n+1$ or $T_1/n-1$. If such two oscillations are superposed, the period of beating becomes just to be T_1 as shown by simple calculation.

We have picked up approximately 100 beats from the records obtained by the induction-magnetometer at the Onagawa Geomagnetic Observatory near Sendai, Japan. When we have chosen them from the magnetogram, we have not taken any isolated beat but the beats repeated, at least, twice. Finally, we have chosen approximately 30 wave trains, from which we have picked up approximalely 100 beats.

The mean value of the periods of the beats is 180 sec. and the root mean square deviation is 50 sec : approximately the period of beat lies between 2 min. and 4 min.

It is considered that the period of beat indicates the fundamental period T_1 of the poloidal oscillation. Comparing the theoretical calculation shown in **4**, we find that the radius of the outeratmosphere is several times of the earth's radius. It is interesting that the dimension of the outeratmosphere is of the order of the forbidden reigion or of the CHAPMAN-FERRARO's cavity ([30], [31], [32] and [34]). It is, however, rather dangerous to overtone the quantitative agreement, because our model is too simplified.

6 Relation between the Period of Pc's and the Fundamental Period

Fig. 8 shows that the relation between the periods of beats and the mean periods of pulsation. The mean period is determined in dividing the time of interval of beating by the number of pulses.

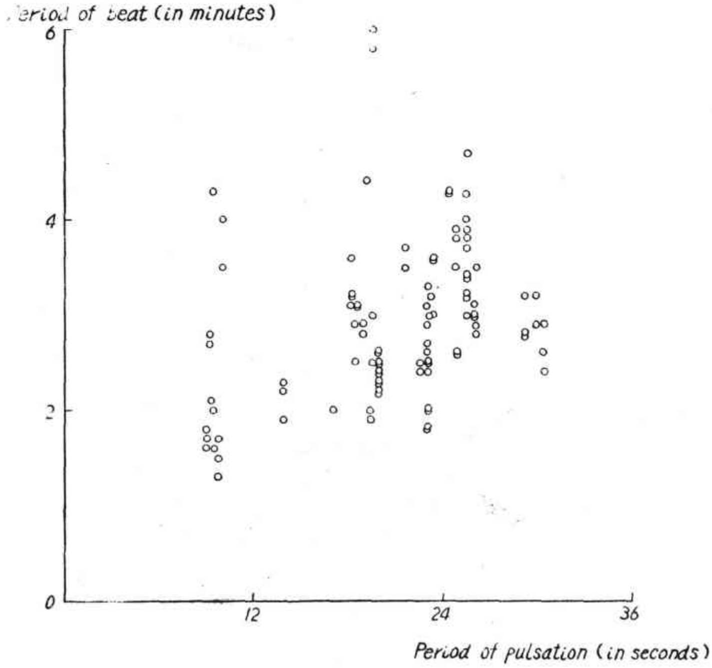


Fig. 8 Relation between the period of pulsation and the beat length

It may be noticed that the longer is the mean period of pulsation the longer is the period of beat.

7 Relation between the Disturbance Grade and the Fundamental Period

Looking at the rapid-run magnetogram, we notice the tendency that the heavier is the magnetic disturbance the shorter period pulsation is apt to occur. The shorter period pulsation like to appear, particulary, during magntic storms. To examine this tendency more carefully, we contrusted the relation between the periods of regular pulsations and the degree of magnetic disturbance K_p. We have picked up regular pulsations wich appeared in the interval from 12h to 15h L.M.T. during two year 1954 and 1955. The result is shown in Fig. 9, and it ascertains the expected tendency.

Combining it with the above stated fact that the longer is the period the longer is the beat period (viz., the fumdamental period T_1), we know that the heavier is magnetic disturbance the shorter is the beat period (viz., the fundamental period T_1). Since the eigenperiod depends on the radius of the outeratmosphere very sensitively, this fact may be explained by assuming that the more severely outeratmosphere is contracted when the stronger is the magnetic disturbance.

8 Agent of Pc's

We shall consider the mechanism by which the outeratmospheric oscillation is excited.

It is no doubt that the ultimate origin of the geomagnetic pulsation lies in the sun as shown by the solar dependence of occurrence stated in the section 2. As the possible agents of the excitation of the outeratmospheric oscillation, we can enumerate (i) the solar ultraviolet radiation and (ii) the solar corpuscular radiation. Besides these, there may be any disturbance propagated from the sun to the earth through the interplanetary medium.

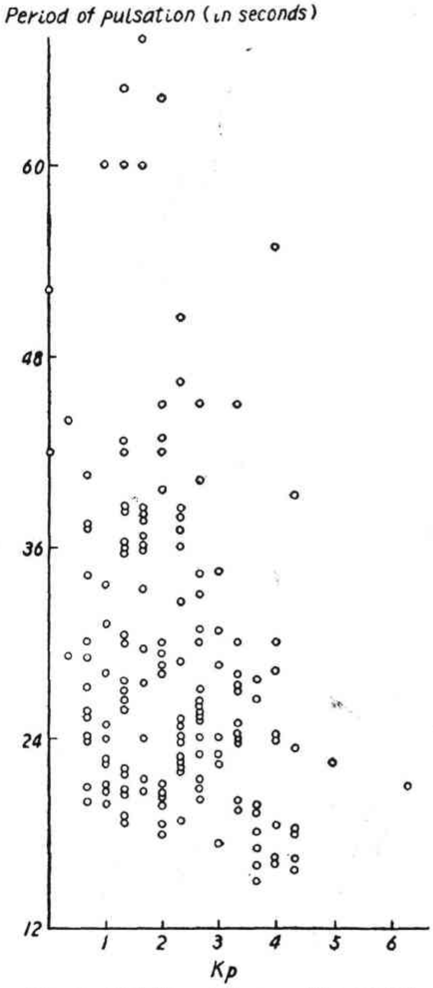


Fig. 9 Relation between the period of pulsation and the grade of disturbance

It fails at first for the ultraviolet radiation to stand for the agent of the geomagnetic pulsation, because we can not find any pulsation at solar flares*.

On the other hand, the solar corpuscular emission seems more hopeful, because the other magnetic disturbances are generally accepted to be due to the solar corpuscular radiation and further the clear correlation is proved between the pulsational disturbance and the general magetic disturbance (Angenheister [2]).

It is clear that the outeratmosphere is a dissipative system *a prioi* or by the observational fact that Pt's show damped oscillations. On the other hand, Pc's execute continued oscillations, and this fact suggests that any supplement of energy is needed for the outeratmospheric oscillation. We shall roughly estimate the required supplement of energy.

If we take account of the dissipative effects (due to Joule's heat loss and viscosity), the solution of the equation of the outeratmospheric oscillation would be given as follows for a particular mode of osillation :

$$\boldsymbol{u}(\boldsymbol{r}, t) = \boldsymbol{u}(\boldsymbol{r}) e^{-\lambda t + i\omega t}$$

This equation means that the volume element $d\tau$ of the outeratmosphere would oscillate with the period $2\pi/\omega$ around the equilibrium point and it would damp with the damping time $1/\lambda$. If we regard the outeratmosphere as an ensemble of mass elements $\rho d\tau$, which are bounded around their equilibrium points, the dissipation function F

* It is a very remarkable fact, which was confirmed by one of our colleagues, Mr. T. Kazazawa.

is calculated from the well known theorem [34] in the dynamics of the vibrating system :

$$F = \lambda \int \rho u^2 \, d\tau .$$

The integration is extended over the whole space of the outeratmosphere. If it is permitted to assume that the kinetic energy and the magnetic field energy do not differ greatly ;

$$\int \frac{1}{2} \rho u^2 d\tau \sim \int \frac{h^2}{8\pi} d\tau ,$$

the dissipation function F is reduced to

$$F = \frac{\lambda}{4\pi} \int h^2 \, d\tau .$$

Since the rate of dissipation of energy is twice of F,

$$\text{The rate of the energy dissipation} = \frac{\lambda}{\pi} \int h^2 d\tau ,$$

where we take account of the dissipation of both the kinetic energy and the magnetic field energy.

Assuming that the mean intensity of the magnetic field as h_m, we have :

$$\text{The above mentioned rate} = \frac{4}{3} \lambda (aR_1)^3 h_m{}^2 .$$

In order to take the time-mean value, we replace h_m by the effective value $h_0/\sqrt{2}$, and then, we obtain :

$$\text{The rate of the energy dissipation} = \frac{2}{3} \lambda (aR_1)^3 h_0{}^2 .$$

The damping time $1/\lambda$ is expected to be of the order of 100 sec. from the observation of Pt's. On the other hand, the magnetic field in the outeratmosphere may be of the orber of $10\gamma \sim 100\gamma$, in order that magnetic field observed at the earth's surface may be of the order of a tenth of 1 gamma, taking the ionospheric shielding effect to be approximately 99% [35].

Table 2

The rate of the energy dissipation (erg/sec)

$h(\gamma)$ \ R_1	5	7	10
10	$2.2 \cdot 10^{18}$	$5.9 \cdot 10^{18}$	$1.7 \cdot 10^{19}$
100	$2.2 \cdot 10^{20}$	$5.9 \cdot 10^{21}$	$1.7 \cdot 10^{21}$

It is, therefore, demanded, that the energy of the above mentioned order must be supplied to the outeratmosphere, in order that the geomagnetic pulsation can be held steady as Pc.

We shall compare the quantities with the energy of the corpuscular stream springing into the outeratmosphere in a unit time. The kinetic energy of the solar corpuscules colliding with the cross section of the outeratmosphere $\pi(aR_1)^2$ in a unit time is $1/2\ \pi\rho(aR_1)^2 V^3$, where ρ is the mass density of the solar corpuscular stream, and V

is the mean velocity of the stream. If the solar corpuscular stream is composed of protons and electrons, ρ is given as follows ; $\rho = N m_H$, where m_H is the mass of proton and N is the number density of protons. The following table shows the values of $1/2\,\pi\rho(aR_1)^2V^3$, where the number density N of protons is taken as equal to 1.

Table 3
The energy of the incident corpuscular stream
when the particle density=1 particle/cc

R_1 / V(cm/sec)	5	7	10
$3\cdot10^7$	$6.9\cdot10^{17}$	$1.4\cdot10^{18}$	$2.8\cdot10^{18}$
$2\cdot10^8$	$2.0\cdot10^{20}$	$4.0\cdot10^{20}$	$8.2\cdot10^{21}$

The ratio of the two quantities κ, the energy to be supplied in a unit time to maintain the steady pulsation and the kinetic energy of the solar corpuscular stream springing into the outeratmosphere in a unit time, is given as follows :

$$\kappa = \frac{\frac{2}{3}\lambda\,(aR_1)^3 h_0^2}{\frac{1}{2}\pi m_H N\,(aR_1)^2\,V^3} = \frac{4\lambda}{3\pi}\,\frac{aR_1 h_0^2}{m_H N V^3}. \tag{29}$$

The value of κ are given in various cases. The radius of the outeratmosphere is taken as 7 times of the earth's radius. Even with the number density $N=1\sim100$/cc, which is expected from the Chapman-Ferraro's theory for the geomagnetic storm, it is possible to make κ to be of the order of unity. If N is more greater as suggested by L. BIERMANN [35], viz., 10^5 particles/cc, the above ratio κ becomes very small.

Table 4
The ratio of the two quantities,
the rate of the energy dissipation and the
energy of the incident corpuscular stream
(N is the number density of corpuscular stream)

V(cm/sec) / $h(\gamma)$	$3\cdot10^7$	$2\cdot10^8$
10	$4.4/N$	$1.5/N\cdot10^{-2}$
100	$4.4/N\cdot10^2$	$1.5/N$

It may, therfore, be possible that Pc's are caused by the following mechanism that the solar corpuscular stream holds the outeratmospheric osillation steady delivering a part of its kinetic energy to the outeratmosphere, even though the process is not yet clear by which the kinetic energy of the solar corpuscular stream might be delivered to the outeratmosphere. It might be considered that the turbulent energy of the solar corpuscular stream would excite the outatmospheric oscillation.*

It is said [36] that the turbulent velocity in the low March flow is the order of several percent (8 % on the average) of the mean flow velocity V. On the other hand, the solar corpuscular stream is a extremely high Mach flow : the velocity of

* Mr. S. Akasofu considers also that Pc s are caused by the outeratmospheric oscillation forced by the turbulent motion of the solar corpuscular stream (see the following paper).

sound propagation may be 10^6cm/sec. and the velocity of the stream itself is of the order of $10^7 \sim 10^8$cm/sec. If the above stated fact holds for the high Mach flow, the turbulent velocity v_t is given approximately $v_t = 0.08V$. The turbulent energy contained in the stream springing into the outeratmospheric cross section in a unit time, therefore, is given as follows :

$$\frac{1}{2} m_H v_t^2 N \pi (aR_1)^2 V = (0.08)^2 \times \left(\frac{1}{2} m_H N \pi (aR_1)^2 V^3\right)$$

Then, the ratio of the energy dissipation and above stated turbulent energy κ' is given by

$$\kappa' = \frac{1}{(0.08)^2} \kappa$$

where κ is the same as (29).

If the outeratmospheric oscillation is excited by the turbulent motion of the solar orpuscular stream, κ must not exceed 1 :

$$\frac{1}{(0.08)^2} \frac{4\lambda}{3\pi} \frac{aR_1 h_0^2}{m_H N V^3} \leqq 1$$

The equality holds, only when all the turbulent energy is effectively used to excite the outeratmospheric oscillation. In the other words, N is unable to be smaller than N_0, where N_0 is given as follews :

$$\frac{1}{(0.08)^2} \frac{4\lambda}{3\pi} \frac{aR_1 h_0^2}{m_H V^3} = N_0$$

Table 5 shows the values of N_0 in various cases.

Table 5
The value of N_0 (particles/cc), the smallest value of the particle density required to excite the outeratmospheric oscillation

$h(\gamma)$ \ V(cm/sec)	$3 \cdot 10^7$	$2 \cdot 10^8$
10	$6.9 \cdot 10^3$	$2.3 \cdot 10$
100	$6.9 \cdot 10^5$	$2.3 \cdot 10^3$

The above discussions are of course incomplete. The problem needs to be studied more carefully.

9 Formation of the Outeratmosphere

The space around the earth or between the earth and the sun is fullfilled with the highly ionozed gas, which is called the interplanetary matter. It was found that such a ionized gas is considerably dense : the particle density is thought of the order of $10^2 \sim 10^3$ particles / cc or sometimes, of the order of 10^5 particles / cc as already stated.

DUNGEY [13] has considered that the relative motion between the interplanetary matter and the earth due to the earth's revolution would produce a region, in which the earth's magneic field would survive and outside which it would be died away.

Such a region is namely the DUNGEY's outeratmosphere. The mechanism by which the outeratmosphere is formed is quite analogous to that proposed by S. CHAPMAN and V.C.A. FERRARO [30] concerning the formation of the forbidden region in their theory of the geomagnetic storm.

If such a region were not formed, the earth's magnetic field would suivie at the farest point from the earth, and so we must take the radius of the earth's outeratmosphere very large. Because the fundamental period T_1 depends on R_1 very sensitively : $T_1 \propto R_1^4$, T_1 becmes very large and so the outeratmosphere, in the extreme case, would not have the finite and discrete eigenperiods. It is inconsistent of our observation. We find often that the beautiful and sinusoidal pulsations continue steadily for a considerably long period (sometimes for several hours and rarely for the whole day), and this fact suggests that the outeratmosphere has, at least, one definite eigen period : it is very difficult to think that such a steady oscillation will be caused by the dynamical system without any definite and discrete eigenperiod.

Is the outeratmorphere, then, bounded to all the directions, or open to some directions ? In order to answer this problem, it is necessary to solve the following question : under what shape of the boundary of the outatmosphere has the equation of the poloidal oscillation (11), or more generally have, the equations of the general oscillation (1) and (2) the definite and discrete eigenperiods ? That is very difficult probrom to be solved mathematically. We will reserve our answer in the present stage of our study.

There may be two ways how the outeratmosphere is formed :

(1) The relative motion between the interplanetary matter and the earth due to the earth's revolution forms the outeratmosphere, as proposed by DUNGY [13].

(2) The outeratmosphere is formed by the relative motion between the solar corpuscular stream and the earth's magnetic field. In this case, the outeratmosphere is namely the forbidden region or the cavity proposed by CHAPMAN and FERRARO [31].

In the case (1), at the dawn side of the earth, there is the top of the outeratmosphere, which is approximately ten earth radii distant from the earth' center (DUNGEY [13]). In the case (2), the top lies in the day side, and it is several earth's radii distant, as it is shown by the CHAPMAN–FERRARO's theory (CHAPMAN and FERRARO [30], FERRARO [32], D.F.MARTYN [31], T. NAGATA [33]).

It is uncertain which mechainism is more favoured. It might be, however, considered that the mechanism (2) would be more favoured, taking into account of the following facts.

(a) The maximum of the frequency of occurrence lies generally in the day side as shown by ANGENHEISTER [2].

(b) The peirod of beat shows us that the radius of the outeratmosphere would be of several earth's radii as stated in the section 5, although it is somewhat dangerous to overtone the quantitative agreement because of roughness of our model.

(c) The more severe is the magnetic disturbance, the smaller becomes the radius of the outeratmosphere (see section 7). It is understood quite naturally by the CHAPMAN–FERRARO's theory.

10 Concluding Remarks

By the above discussions, the mechanism is made clear, by which the geomagnetic pulsation would be excited : the turbulent motion of the solar corpuscular stream would excite the outeratmospheric poloidal oscillation, which gives the field of the geomagnetic pulsation.

On the other hand, there are some points to be examined. At first, it is unclear, as stated in the section 9, whether the outeratmosphere is closed up or not. It is quite a conventional assumption that the shape of outeratmosphere is to be spherical. The world wide character of occurrence must also be examined more carefully. Pc's are likly to appear in the day time and this fact is in the apparent contradiction with the world wide character. It may be explained, if we notice that the excitation of the outeratmospheric oscillation would be done at the limb on the sunlit side of the outeratmosphere and the outeratmosphere is, ***a priori***, dissipative. Our consideration is perfectly speculative on the agent of excitation of the outeratmospheric oscillation, which assume the turbulent motion of the solar corpuscular stream. We shall discuss it elsewhere in the near future.

Acknowlegdement

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