Corrected Paper on the Phase Difference of Earth Current Induced by the Charges of the Earth's Magnetic Field.

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Corrected Paper on the Phase Difference of Earth Current Induced by the Changes of the Earth's Magnetic Field

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Abstract

The phase difference between the earth's magnetic field and the induced earth's current is discussed, when the electrical conductivity of the earth varies as

$$\sigma(z) = \sigma_0\left(1 + \frac{z}{a}\right)^{-\beta}.$$

Present report is the corrected one of the former published.

It is well-known fact that the phase or amplitude of the variation of the earth current induced by the changes of the earth's magnetic field varies according to the period of the earth's magnetic variation or to the electrical property of the earth's interior. Also we can find the electrical property of the earth's interior approximately, by the analysis of above mentioned two phenomena.

In the previous papers, one of present writers and T. KIKUCHI have treated the phase difference of above two phenomena under the assumption that the earth's crust has two layers of the electrical conductivity σ_1, and σ_2 respectively [1], or that its conductivity varies with the depth from the surface as

$$\sigma(z) = \sigma_0\left(1 + \frac{z}{a}\right)^{-\beta}, \ [2],$$

where σ_0 is the electrical conductivity at the surface,
a is constant which has the length's dimension, and
β is constant larger than 0.

In the former case, the phase difference of above mentioned two phenomena is $\frac{\pi}{4}$ when $T = 0$ and it tends to a minimum, as the period becomes longer, while after it takes a minimum, increases again. In the latter case the result obtained differs from the former. It decreases from $\frac{\pi}{4}$ to 0 as the period becomes longer.

Recently Dr. J. G. J. SCHOLTE, Kon. Ned. Meteo. Institute, Holland, pointed out a mistake in our calculation of the latter case. Present report is a corrected paper according to his advice.

Here we repeat the fundamental equations briefly.

$$\text{rot } \mathbf{H} = 4\pi \mathbf{i} \tag{1}$$

$$\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\mathbf{i} = \sigma(z)\ \mathbf{E} \tag{3}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{4}$$

$$\operatorname{div} \mathbf{B} = 0 \tag{5}$$

$$\sigma(z) = \sigma_0 \left(1 + \frac{z}{a}\right)^{-\beta} \tag{6}$$

If we eliminate $\mathbf{E}$ from above equations and put $p = \partial/\partial t$, then H_x (x-component of $\mathbf{H}$) satisfies the following differential equation.

$$\left(1 + \frac{z}{a}\right)^{\beta} \frac{d^2 H_x}{dx^2} + \frac{\beta}{a}\left(1 + \frac{z}{a}\right)^{\beta-1} \frac{dH_x}{dz} - 4\pi\sigma_0 \mu p H_x = 0 . \tag{7}$$

Through the transformation such as $\zeta = \frac{1}{2-\beta}\left(1 + \frac{z}{a}\right)^{1-\frac{\beta}{2}}$, (6) is transformed as

$$\frac{d^2 H_x}{d\zeta^2} + \frac{\beta}{2-\beta}\frac{1}{\zeta}\frac{dH_x}{d\zeta} - 16\pi\sigma_0 \mu a^2 p H_x = 0 . \tag{8}$$

In the former report we have missed the second term of (8) by a factor 2 (in excess). After the transformation such as $H_x = e^{-2k\zeta}u(\zeta)$, we get the solution of (8) as follows.

$$H_x = H_0(t) \frac{\left(1+\frac{z}{a}\right)^{\frac{1-\beta}{2}} H_{1,-\frac{1-\beta}{2-\beta}}\left\{\frac{i2k}{2-\beta}\left(1+\frac{z}{a}\right)^{1-\frac{\beta}{2}}\right\}}{H_{1,-\frac{1-\beta}{2-\beta}}\left\{\frac{i2k}{2-\beta}\right\}} , \tag{9}$$

where $k^2 = 4\pi\sigma_0 \mu p a^2$.

Therefore from (2) we find E_y easily,

$$E_y = \frac{H_0(t)}{4\pi\sigma_0 a}\left[(1-\beta)\left(1+\frac{z}{a}\right)^{-\frac{1-\beta}{2}} H_{1,-\frac{1-\beta}{2-\beta}}\left\{\frac{i2k}{2-\beta}\left(1+\frac{z}{a}\right)^{1-\frac{\beta}{2}}\right\}\right.$$
$$\left. + ik\left(1+\frac{z}{a}\right)^{\frac{1}{2}} H_{1,-\frac{3-2\beta}{2-\beta}}\left\{\frac{i2k}{2-\beta}\left(1+\frac{z}{a}\right)^{1-\frac{\beta}{2}}\right\}\right] \Big/ H_{1,-\frac{1-\beta}{2-\beta}}\left\{\frac{i2k}{2-\beta}\right\} \tag{10}$$

Then the final result is obtained by solving the next DUHAMEL's and BROMIWICH's integral, that is

$$E_y = \frac{1}{4\pi\sigma_0 a}\frac{d}{dt}\int_0^t \gamma(\xi) H_0(t-\xi)\, d\xi, \tag{11}$$

where

$$\gamma(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[(1-\beta)\left(1+\frac{z}{a}\right)^{-\frac{1-\beta}{2}} H_{1,-\frac{1-\beta}{2-\beta}}\left\{\frac{i2a\sqrt{4\pi\sigma_0\mu\lambda}}{2-\beta}\left(1+\frac{z}{a}\right)^{1-\frac{\beta}{2}}\right\} +\right.$$
$$\left. ai\sqrt{4\pi\sigma_0\lambda}\left(1+\frac{z}{a}\right)^{\frac{1}{2}} H_{1,-\frac{3-2\beta}{2-\beta}}\left\{\frac{i2a\sqrt{4\pi\sigma_0\mu\lambda}}{2-\beta}\left(1+\frac{z}{a}\right)^{1-\frac{\beta}{2}}\right\}\right] e^{\lambda t} d\lambda \Big/ \lambda H_{1,-\frac{1-\beta}{2-\beta}}\left(\frac{i2a\sqrt{4\pi\sigma_0\mu\lambda}}{2-\beta}\right) .$$

In the numerical calculation we assumed μ (earth's magnetic permeability) = 1, σ_0 (earth's electrical conductivity at the surface) = 10^{-13} e. m. u., and constant a = 5×10^6cm, 10^7cm respectively, and used the HANKEL's asymptotic expansion formula. The result obtained are as follows.

a) In the case of $\beta = 0$.

The result of this case is just same as the former report. The phase difference is constant ($= \pi/4$) for all period, and the amplitude ratio E_y/H_x becomes smaller as the period becomes longer.

b) In the case of $\beta = 1$.

In this case E_y is as follows

$$E_y = \frac{H_0(t)}{4\pi\sigma_0 a}\left[ik\left(1+\frac{z}{a}\right)^{\frac{1}{2}} H_{1,-1}\left\{i2k\left(1+\frac{z}{a}\right)^{\frac{1}{2}}\right\}\right] \Big/ H_{1,0}(i2k)\,. \tag{12}$$

Therefore the phase difference φ is calaulated, that is

$$\varphi = \tan^{-1}\frac{B}{A}, \tag{13}$$

where

$$\begin{cases} A = \frac{512^2}{\sqrt{2}}\delta^5\left(\frac{2\pi}{T}\right)^{\frac{5}{2}} + 96\cdot512\ \delta^4\left(\frac{2\pi}{T}\right)^2 - \left(\frac{24\cdot512}{\sqrt{2}} + 16\cdot96\cdot\sqrt{2}\right)\delta^3\left(\frac{2\pi}{T}\right)^{\frac{3}{2}} \\ \qquad + 30\cdot16\ \delta^2\frac{2\pi}{T} - \frac{9\cdot15}{\sqrt{2}}\delta\left(\frac{2\pi}{T}\right)^{\frac{1}{2}} \\ B = \frac{512^2}{\sqrt{2}}\delta^5\left(\frac{2\pi}{T}\right)^{\frac{5}{2}} - 32\cdot512\ \delta^4\left(\frac{2\pi}{T}\right)^2 + \left(\frac{24\cdot512}{\sqrt{2}} - 16\cdot96\cdot\sqrt{2}\ \delta^3\left(\frac{2\pi}{T}\right)^{\frac{3}{2}}\right. \\ \qquad + 9\cdot96\cdot\ \delta^2\frac{2\pi}{T} - \frac{9\cdot15}{\sqrt{2}}\delta\left(\frac{2\pi}{T}\right)^{\frac{1}{2}} \\ \delta^2 = 4\pi\sigma_0\mu a^2. \end{cases}$$

These numerical calculations at the earth's surface ($z = 0$) are shown in Fig. 1 and Fig. 2.

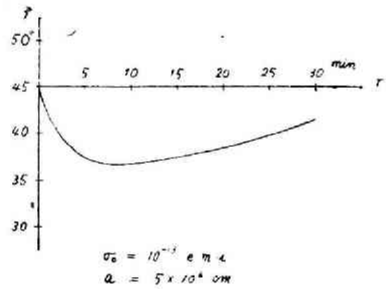

Fig. 1

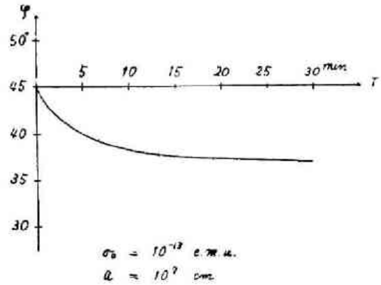

Fig. 2

The amplitude ratio E_y/H_x can be calculated, too, as follows.

$$\frac{E_y}{H_x} = \frac{1}{4\pi\sigma_0 a}\cdot\frac{\sqrt{A^2+B^2}}{D}, \tag{14}$$

where

$$D = (9-16\sqrt{2})^2 + 512^2\ \delta^4\frac{4\pi^2}{T^2} + 2\cdot16^2\cdot\ \delta^2\frac{2\pi}{T} - 2\sqrt{2}\cdot16\cdot512\cdot\ \delta^3\left(\frac{2\pi}{T}\right)^{\frac{3}{2}}.$$

As the eq. (13), and the Fig. 1, 2 show the phase difference is $\pi/4$ when $T = 0$ and it tends to a minimum as the period becomes longer.

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