A Remark on the Amplitude of the Initial Motion of Very Shallow Earthquake

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A Remark on the Amplitude of the Initial Motion of Very Shallow Earthquake

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1. Introduction

The amplitude of the initial motion of the longitudinal waves of very shallow earthquake observed at a station near the epicenter, whose epicentral distance and azimuth being Δ and φ respectively, was shown to be approximately proportional to $\Delta^{-2} \sin 2\varphi$, in some cases by H. HONDA (1931, 1932). HONDA explained the relation by the result of H. NAKANO's (1930) theoretical research on the motion of the surface of a semi-infinite elastic solid subjected to the tractions on the surface near the origin. H. KAWASUMI (1934) took the effect of the heterogeneous crustal structure or the rapid increase of the velocity of the seismic waves with the depth from the earth's surface into account, and he also assumed that the amplitude of the initial motion to be proportional to $\sin^2\theta \sin 2\varphi$, where θ being the angle subtended by the seismic ray of the waves emitted into the solid and the vertical line at the hypocenter on the earth's surface.

Recently T. HIRONO (1948, 1949) investigated theoretically the elastic waves emitted into the solid, when the tractions of any form are applied on the surface of a semi-infinite elastic solid. In the present paper the authors intend to calculate the amplitude of the initial motion to be observed at the earth's surface, after the result of HIRONO's theoretical investigation, taking the heterogeneous crustal structure into account, and compare them with the results of the observations.

2. Theoretical Consideration

We deal with a semi-infinite elastic solid, and take the cylindrical coordinates (r, φ, z), so that $z = 0$ should coincide with the surface of the solid in its natural state, and the positive sense of the z-axis is directed towards the interior of the solid. Let us assume that the radial traction $\Pi_2(r) \cdot \sin 2\varphi \cdot e^{ipt}$ to act on the surface within a small circle of radius r_0 constructed around the origin, corresponding to the mechanism of typical very shallow earthquakes. The displacement ϑ_R, $(R = \sqrt{r^2+z^2})$, of the motion of the longitudinal waves in the direction θ, $(\tan\theta = r/z)$, and φ from the origin, was obtained by HIRONO as follows;

$$\vartheta_R = -2A_2 \frac{\cos\theta \sin^2\theta \sqrt{\varepsilon^2-\sin^2\theta}}{D(\cos\theta)} \sin 2\varphi \frac{e^{-ihR+ipt}}{hR}$$

$$A_2 = -\frac{h^2}{\mu} \frac{i}{4} \int_0^{r_0} \Pi_2(r) r^2 dr, \quad h = \sqrt{\frac{\rho}{\lambda+2\mu}}\, p, \quad \varepsilon = \sqrt{\frac{\lambda+2\mu}{\mu}}$$

$$D(\cos\theta) = (\varepsilon^2 - 2\sin^2\theta)^2 + 4\cos\theta \sin^2\theta \sqrt{\varepsilon^2-\sin^2\theta},$$

ρ; density, λ, μ; Lame's constants.

Putting $\lambda = \mu$, the amplitude of the longitudinal waves is seen to be proportional to $F(\theta) = \cos\theta \sin^2\theta \sqrt{3 - \sin^2\theta}/D(\cos\theta)$ with respect to θ.

The amplitude of the waves arriving at a station whose epicentral distance is Δ, not very far from the epicenter, is f times that of the original amplitude at the origin, owing to the effect of the heterogeneous crustal structure, instead of $1/R$ in the homogeneous medium, where f being

$$f = \sqrt{\frac{\sin\theta}{\Delta \sin e_0}\left|\frac{d\theta}{d\Delta}\right|} = \sqrt{\frac{\tan\theta}{\Delta}\left|\frac{d\theta}{d\Delta}\right|}\,.$$

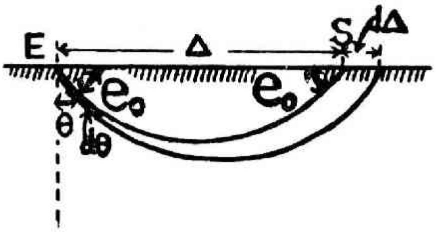

Figure. 1

The amplitude a of the resultant of the horizontal component and the vertical one of the motion of the earth's surface is two times that of the incident longitudinal waves.

In this way, we can suppose that $a/\sin 2\varphi$ to be proportional to $f \cdot F(\theta)$. The values of f, $F(\theta)$ and $f \cdot F(\theta)$ are given in the table 1. for Δ less than 1,000 km.

Table 1. f, $F(\theta)$, and $f \cdot F(\theta)$.

Δ km	f 10^{-2}km^{-1}	$F(\theta)$ 10^{-1}	$f \cdot F(\theta)$ 10^{-3}km^{-1}
40	1.02	0.77	0.79
60	0.66	0.63	0.42
80	0.47	0.50	0.24
120	0.19	0.40	0.076
160	0.098	0.37	0.036
200	0.068	0.36	0.024
240	0.052	0.34	0.018
280	0.043	0.34	0.015
320	0.040	0.33	0.013
360	0.037	0.33	0.012
440	0.032	0.31	0.0099
480	0.026	0.31	0.0081
560	0.012	0.30	0.0036
640	0.0083	0.30	0.0025
720	0.0078	0.30	0.0023
800	0.0074	0.30	0.0022
880	0.0070	0.30	0.0021

3. Comparison with the Results of the Observations

The North Idu Earthquake* occurred on Nov. 25, 1930 and the Fukui Earthquake** occurred on June 28, 1948 are both the typical very shallow earthquakes. The dilatations and compressions of the initial motions of the longitudinal waves are distributed according to the expression $\sin 2\varphi$, as are shown *e.g.* by HONDA (1952). The amplitude a of the initial motion observed at the stations in this country and 2φ of these stations are given in the table 2 (a), (b). The observed values of $\log(a/\sin 2\varphi)$ and the theoretical values of $\log(f \cdot F(\theta))$ are shown (by continuous line) for $\log\Delta$, in the figure 2 (a), (b).

Table 2 (a). The North Idu Earthquake

Station	Epicentral Distance Δ	Initial Motion a	2φ	$a/\sin 2\varphi$	Station	Epicentral Distance Δ	Initial Motion a	2φ	$a/\sin 2\varphi$
Mishima	10$^{\text{km}}$	18310$^{\mu}$	3°	(349800)$^{\mu}$	Kyôto	290$^{\text{km}}$	11$^{\mu}$	15°	43$^{\mu}$
Numazu	14	8127	14	(33600)	Ôsaka	322	6.8	14	28
Yokohama	73	298	78	305	Hukushima	323	7.6	32	14
Tôkyô	99	473	80	480	Wazima	317	18	84	18
Kumagaya	126	73	19	224	Kôbe	345	9.4	26	21
Chôshi	180	15	60	17	Shiomisaki	342	5.5	75	5.7
Kakioka	171	43	67	47	Wakayama	356	3.8	42	5.7
Nagoya	187	24	7	197	Sumoto	377	4.2	36	7.1
Nagano	191	106	58	125	Morioka	543	2.4	54	3.0
Gifu	204	6.6	7	54	Matsuyama	577	12	37	20
Hatidyôzima	226	12	36	20	Miyazaki	788	7.6	63	8.5
Hikone	247	2.9	3	5.5	Unzendake	850	18	45	25

Table 2 (b). The Fukui Earthquake

Station	Epicentral Distance Δ	Initial Motion a	2φ	$a/\sin 2\varphi$	Station	Epicentral Distance Δ	Initial Motion a	2φ	$a/\sin 2\varphi$
Turuga	52^{km}	728^{μ}	64	810^{μ}	Yokohama	318^{km}	14^{μ}	50	18^{μ}
Gifu	92	296	36	504	Ôshima	321	23	78	24
Toyama	108	97	50	127	Utsunomiya	330	26	4	372
Hikone	91	315	26	719	Kôchi	373	2.5	72	2.6
Nagoya	122	242	36	412	Hiroshima	393	15	32	28
Toyooka	142	360	30	720	Matsuyama	403	10.2	50	13
Kyôto	127	317	64	353	Onahama	428	18	4	258
Wazima	155	191	70	203	Sendai	479	7.1	38	12
Nagano	188	48	16	174	Hachinohe	676	2.5	76	2.6
Ôsaka	181	562	72	591	Morioka	589	1.8	68	1.9
Kôbe	180	331	88	331	Fukuoka	600	2.5	28	5.3
Owashi	220	407	28	867	Kumamoto	622	8.1	45	11
Sumoto	227	61	90	61	Miyazaki	638	13	66	14
Funatsu	239	89	76	92	Miyako	639	5.2	58	6.1
Kumagaya	283	14	20	41	Aomori	657	17	88	17
Shiomisaki	297	49	42	73	Kagoshima	725	19	61	22

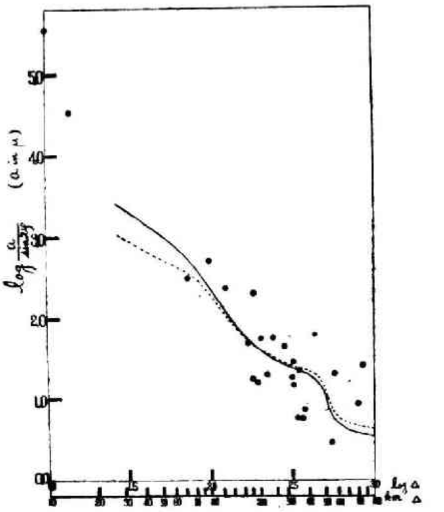

Figure 2 (a). The North Idu Earthquake.

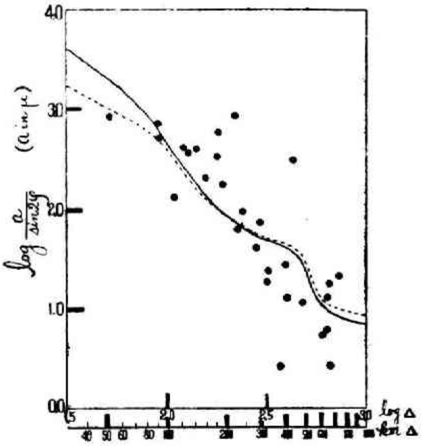

Figure 2 (b). The Fukui Earthquake

The curve of $\log (f \cdot F(\theta))$ is displaced uniformly by a proper amount along the axis of the ordinates for each earthquake respectively. One can see in these figures, that the observed values of $\log (a/\sin 2\varphi)$ are represented fairly well by the theoretical curve (continuous curve) of $\log (f \cdot F(\theta))$.

Assuming provisionally $F(\theta)$ to be independent of θ, the values of $\log f$ for $\log \Delta$ are shown by the broken lines in the same figures. The broken line is not very different from the continuous line, but they are nearly parallel each other in the region $100\,\mathrm{km} < \Delta < 600\,\mathrm{km}$. The fact shows that the effect of the heterogeneous crustal structure manifest itself very conspicuously compared with the effect of the variation of the amplitude with θ, within the domain considered at present. The theoretical variation of $F(\theta)$ for θ and corresponding Δ is shown in the figure 3. To ascertain the variation of $F(\theta)$ for θ more clearly, it may be hoped that the amplitude of the initial motion at the satations lying far beyond $\varDelta = 1{,}000$ km may be treated for the very shallow earthquakes of large magnitudes.

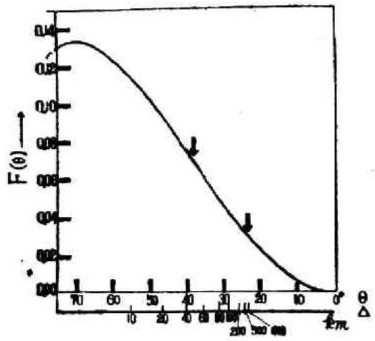

Figure 3. $F(\theta)$

4. Summary

The variation of the amplitude of the initial motion of the longitudinal waves of very shallow earthquakes with the epicentral distance and the azimuth of the station, can he explained pretty well, the results of HIRONO's theoretical investigation on the waves generated in the semi-infinite elastic body subjected to the radial traction on the surface near the origin being used, and the marked effect of the heterogeneous crustal structure being taken into account.

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