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Influence of Stability on Turbulent Transfer Near the Ground

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1. Introduction

It will be almost certain that in adiabatic conditions the vertical profiles of wind speed and absolute humidity near the ground follow the logarithmic law, while in non-adiabatic conditions the effect of buoyancy will modify the profiles from the logarithmic ones. ROSSBY and MONTOGOMERY [5] have investigated the problem as early as 1935 and have proposed the following generalized velocity profile for stable atmospheric conditions :

$$\frac{du}{dz}=\frac{u_*}{kz}\sqrt{1+\sigma R_i} \qquad (1)$$

where u = wind velocity at the height z

$u_*=\sqrt{\tau_0/\rho}$, called the friction velocity

τ_0 = surface value of the horizontal shear stress

ρ = density of the air

k = von Karman's constant = 0.40

$$R_i=\frac{g\dfrac{d\theta}{dz}}{\theta\left(\dfrac{du}{dz}\right)^2} = \text{Richardson's number}$$

σ = universal constant

θ = potential temperature

g = acceleration due to gravity.

SVERDRUP [7] found that observations over snow (mainly stable atmospheric conditions) indicated σ to be approximately 11. Recently, SHEPPARD [6], DEACON [1] and PASQUILL [3] have carried out extensive series of observations of vertical profiles of wind speed, temperature and humidity near the ground and have found that S-$\log z$ curve is concave upwards in unstable conditions and convex upwards in stable conditions, where S is either wind speed or absolute humidity. In addition DEACON found that values of σ calculated from his observations vary from about 2 for very unstable conditions to more than 20 for conditions of marked stability, and according to him a modified formula, proposed by HOLZMAN [2] entirely on empirical grounds, seems to provide a better representation. HOLZMAN's formula is as follows :

$$\frac{du}{dz}=\frac{u_*}{kz}\frac{1}{\sqrt{1-\sigma_1 R_i}} \quad (\sigma_1=\text{constant}), \qquad (2)$$

which it will be seen is identical with equation (1) when the Richardson number is small. Mean value of σ evaluated by DEACON from his observations is 7.1.

PASQUILL has also compared equations (1) and (2) with his experimental results and has found that the reasonable agreement with the results in stable conditions is provided by the ROSSBY-MONTGOMERY equation with σ equal to 12, while in unstable conditions the HOLZMAN formula gives coresponding agreement using the same value for σ_1. From the result he considered that neither (1) nor (2) can satisfactorily describe the whole range of experimental results.

Such is the brief summary of the present status of the problem. Now in the present paper HOLZMAN's empirical formula will be derived theoretically and it will also be shown that the constant σ_1 in HOLZMAN's formula takes different values in both unstable, and stable conditions; larger in unstable conditions than in stable conditions.

2. Derivation of the Formulae

Recently PRIESTLEY and SWINBANK [4]

have demonstrated that the fluctuations in temperature of eddies play an important rôle in the turbulent transfer of heat. It is because of buoyancy which acts on eddies having excess temperatures over the surrounding air. They showed that even in stable conditions, if the fluctuations in temperature are sufficiently large, warm eddy may move upwards and cold eddy, downwards, and the resultant transfer of heat may be upwards in discordance with the older theory of turbulence, but in accordance with obserrvations at such conditions. In their theory, however, the effect of buoyancy only was emphasized and the effect of mechanical turbulence was disregarded. In the present theory we will consider these two effects simultaneously.

We consider an eddy having an excess potential temperafure θ' from the surrounding air at datum level and having moved upwards a distance z from the level by the mechanical turbulent action. The eddy will, then, have an acceleration whcih was already given by PRIESTLEY and SWINBANK, *i.e.*,

$$\frac{dw'}{dt} = w'\frac{dw'}{dz} = \frac{g}{\theta}\left[\theta' - z\frac{\partial\theta}{\partial z}\right], \qquad (3)$$

where w' is the vertical velocity of the eddy. It is at present not clear that the direction of initial movement of an eddy is determined whether by the mechanical turbulent action or by the buoyancy due to the excess temperature. If the latter case is more probale the theory will become as PRIESTLEY and SWINBANK have derived. In the case of large temperature fluctuations, the matters will be probably as such. However, if the fluctuations in temperature are not so large, we can assume that the eddy will be moved initally by the mechanical shearing force. Then the excess potential temperature θ' of the eddy can take either positive or negative values. As was assumed by PRIESTLEY and SWINBANK from dimensional considerations, we can assume

$$\theta' = \pm L'\frac{\partial\theta}{\partial z},$$

where L' is the constant of dimension of length. At first we will consider in unstable conditions in which $\frac{\partial\theta}{\partial z} < 0$. For the eddy of positive excess temperature, we have

$$\theta' = -L'\frac{\partial\theta}{\partial z}. \qquad (4)$$

Inserting (4) in (2), and assuming θ and $\frac{\partial\theta}{\partial z}$ to be constant for the short path L_1, which will probably connected with mixing, we have

$$w'^2 = w_0'^2 - \frac{g}{\theta}\frac{\partial\theta}{\partial z}\left[2L'L_1 + L_1^2\right], \qquad (5)$$

where w_0' is the initial velocity due to mechanical turbulence, which will be given by

$$w_0' = l\frac{\partial u}{\partial z},$$

where l is the length constant, independent of stability, and so, equal to that in adiabatic conditions.

Next, we consider the eddy of negative excess temperature. In this case we have

$$\theta' = L'\frac{\partial\theta}{\partial z}. \qquad (6)$$

Again' inserting (6) in (3), we can integrate (3). In this case we assume that integration should be performed over the same time as the case of the positive excess eddy, because the time is concerned with mixing process. Then in the case of negative excess eddy the integration path L_2 should be smaller than L_1 of the case of positive excess eddy, because the acceleration and accordingly the mean velocity will be smaller in the former case than in the latter case, Thus we have

$$w'^2 = w_0'^2 - \frac{g}{\theta}\frac{\partial\theta}{\partial z}\left[-2L'L_2 + L_2^2\right]. \qquad (7)$$

If we put that

$$\frac{1}{2}\left[2L'(L_1 - L_2) + L_1^2 + L_2^2\right] = L^2, \qquad (8)$$

we have from (5), (7) and (8) following formula as the average eddy velocity:

$$w'^2 = l^2\left(\frac{\partial u}{\partial z}\right)^2 - L^2\frac{g}{\theta}\frac{\partial \theta}{\partial z}$$
$$= l^2\left(\frac{\partial u}{\partial z}\right)^2 (1 - \sigma_1 R_i), \qquad (9)$$

where $R_i = \dfrac{\frac{g}{\theta}\frac{\partial \theta}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2}$ = Richardson's number

and $\sigma_1 = \dfrac{L^2}{l^2}$.

Hence,

$$w' = l\left(\frac{\partial u}{\partial z}\right)\sqrt{1-\sigma_1 R_i}. \qquad (10)$$

If we write that

$$w' = l'\left(\frac{\partial u}{\partial z}\right), \qquad (11)$$

then l' is the mixing length in the case of unstable conditions and it is given by

$$l' = l\sqrt{1-\sigma_1 R_i}. \qquad (12)$$

And we have

$$\frac{\tau_0}{\rho} = l'w'\left(\frac{\partial u}{\partial z}\right) = l^2(1-\sigma_1 R_i)\left(\frac{\partial u}{\partial z}\right)^2 \qquad (13)$$

Introducting the PRANTL's relation : $l = kz$ ($k = 0.4$ = von Karman constant), we have

$$\frac{\tau_0}{\rho} = u_* = k^2z^2(1-\sigma_1 R_i)\left(\frac{\partial u}{\partial z}\right)^2. \qquad (14)$$

This is the required HOIZMAN's equation in unstable couditions.

In stable conditions the derivation of the formula is formally quite similar to above. However there is some difference concering the range of integration of equation (3). In stable conditions in which $\frac{\partial \theta}{\partial z}>0$, the acceleration given by (3) is smaller than that in unstable conditions for the same value of $\left|\frac{\partial \theta}{\partial z}\right|$. Hence in stable conditions, if we denote the integration path for the eddy of positive excess temperature by L_3 and for the eddy of negative excess temperature by L_4, then we have $L_3 < L_1$ and $L_4 < L_2$ by the same reason as was described concerning the relation between L_1 and L_2. So that we have in stable conditions

$$\frac{\tau_0}{\rho} = u_*^2 = k^2z^2(1-\sigma_2 R_i)\left(\frac{\partial u}{\partial z}\right)^2, \qquad (15)$$

where $\sigma_2 < \sigma_1$.

The transfer of heat and of water vapour can be treated similarly. Let the flux of heat be F and the rate of evaporation be E, then we have

$$-\frac{F}{\rho c_p} = l'w'\frac{\partial \theta}{\partial z}$$
$$= k^2z^2(1-\sigma_1(\text{or}\,\sigma_2)R_i)\frac{\partial u}{\partial z}\frac{\partial \theta}{\partial z}, \qquad (16)$$

$$-E = l'w'\frac{\partial c}{\partial z}$$
$$= k^2z^2(1-\sigma_1(\text{or}\,\sigma_2)R_i)\frac{\partial u}{\partial z}\frac{\partial c}{\partial z}, \qquad (17)$$

where c is the vapour concentration. By this theory the eddy viscosity, the eddy conduction and the eddy diffusion are all given by

$$K = l'w' = k^2z^2\frac{\partial u}{\partial z}(1-\sigma_1(\text{or}\,\sigma_2)R_i). \qquad (18)$$

According to PASQUILL's analysis of his observations, there is a striking agreement between the eddy viscosity and the eddy diffusion in unstable conditions, while with the increase ot atmospheric stability a considerable disparity appears between them; the eddy diffusivity larger than the eddy visvosity. Further according to him, a quite remarkable identity is shown between the eddy conduction and the eddy diffusivity in stable conditions, while with the increase of stability the former coefficient is systematically greater of the two. So that it may be necessary to modify the theory to explain these facts. At the same time further experimental investigations on these coefficients are hoped, because it is very difficult to determ'ne them accurately by experiments.

3. Profiles of Wind Speed and Absolute Humidity

As the Richardson number includes the temperature and the tempeature gradient, profiles of wind speed and absolute humidity are dependent upon the profile of temperature.

At the same time the profile of temperature is related not only to the profile of wind speed, but also to the profile of humidity, because the radiative transfer and evaporation, which are important factors in determining temperature profile, are concerned with the profile of humidity. Thus the exact solutions of the transport phenomena can only be obtained by solving simultaneously the transport equations of momentum, (14) or (15), and of humidity, (17), and a more general equation of heat transfer than (16). These processes are impossible at present, so that, here, we will assume that the profile of temperature is given, and try to solve (14) or (15) and (17).

According to DEACON the profile of potential temperature is generally given by

$$\frac{d\theta}{dz} = b\,z^{-\delta} \tag{19}$$

where, in unstable conditions $b < 0$ and $\delta > 1$: in adiabatic conditions $b = 0$; and in stable conditions $b > 0$ and $\delta < 1$.

If we assume that the temperature profile is given by (19) and introduce it to (14) and further replace θ to θ_0 (mean temperature) in (14), we have in unstable conditions

$$\frac{du}{dz} = \frac{u_*}{kz}\sqrt{1-az^m}, \tag{20}$$

where

$$a = -\frac{k^2\sigma_1 gb}{u_*^2\theta_0} > 0, \tag{21}$$

$$m = 2 - \delta > 0. \tag{22}$$

As is expected, the $u-\log z$ relation given by (20) is concave upward (or concave with regard to $u-$axis), because $\frac{d^2u}{d(\log z)^2}$ is negative. The solution of equation (20) with the boundary condition that $u = 0$ at $z = z_0$ is given by

$$u = \frac{u_*}{km}\left[\log\frac{(1-\sqrt{1-az^m})(1+\sqrt{1-az_0^m})}{(1-\sqrt{1-az_0^m})(1+\sqrt{1-az^m})} + 2\sqrt{1-az^m} - 2\sqrt{1-az_0^m}\right]. \tag{23}$$

In stable conditions, we have from (15), (19),

$$\frac{du}{dz} = \frac{u_*}{kz}\sqrt{1+a'z^m}, \tag{24}$$

where

$$a' = \frac{k^2\sigma_2 gb}{u_*^2\theta_0} > 0. \tag{25}$$

The u-log z relation given by (24) is convex upward (or convex with regard to u-axis) because in this case $\frac{d^2u}{d(\log z)^2}$ is positive. The solution of (24) is

$$u = \frac{u_*}{km}\left[\log\frac{(\sqrt{1+a'z^m}-1)(\sqrt{1+a'z_0^m}+1)}{(\sqrt{1+a'z_0^m}-1)(\sqrt{1+a'z^m}+1)} + 2\sqrt{1+a'z^m} - 2\sqrt{1+a'z_0^m}\right]. \tag{26}$$

As these equations, (23) and (26), are the generalized formulae of usual logarithmic law, it is necessary that they are transformed to $u = \frac{u_*}{k}\log(z/z_0)$ when a or a' tends to zero. That this condition is satisfied both by (23) and (26) is easily seen.

Next we will seek for the solution of equation (17). By (14) and (17) we have

$$-E\frac{du}{dz} = u_*^2\frac{dc}{dz} \tag{27}$$

and

$$Eu = u_*^2(c_0 - c), \tag{28}$$

where c_0 is the vapour concentration at the surface. Or if we want to express the formula using the values of wind speed and vapour concentration at heights z_1 and z_2, *i.e.*, u_1, c_1 and u_2, c_2, we have

$$E(u_2 - u_1) = u_*^2(c_1 - c_2) \tag{29}$$

The friction velocity u_* in equation (29) is given by (23) in unstable conditions and by (26) in stable conditions. It is, however, not easy to obtain u_* from (23) or (26). If some approximation on u_* is pemitted, then from (23) and (29) we have

$$E = \frac{k^2m^2(u_2-u_1)(c_1-c_2)}{\left[\log\frac{(1-\sqrt{1-az_2^m})(1+\sqrt{1-az_1^m})}{(1-\sqrt{1-az_1^m})(1+\sqrt{1-az_2^m})} + 2\sqrt{1-az_2^m} - 2\sqrt{1-az_1^m}\right]^2} \tag{30}$$

for unstable coonditions, and from (26) and (29) we have

$$E = \frac{k^2 m^2 (u_2-u_1)\ (c_1-c_2)}{\left[\log \frac{(\sqrt{1+a'z_2^m}-1)\ (\sqrt{1+a'z_1^m}+1)}{(\sqrt{1+a'z_1^m}-1)\ (\sqrt{1+a'z_2^m}+1)} + 2\sqrt{1+a'z_2^m} - 2\sqrt{1+a'z_1^m}\right]^2} \qquad (31)$$

for stable conditions. Here it must be remarked that a and a' contain u_*, so that in evaluating the rate of evaporation u_* must be determented in any way.

Equations (30) and (31) are the generalization of the well-known THORNTHWAITE-HOLZMAN's eqnation [8].

4. Comparison with PASQUILL's Observations

PASQUILL has compnted from his observations the values of $\frac{-E}{z^2 \frac{\partial u}{\partial z} \frac{\partial c}{\partial z}}$ and R_i and has shown the relation between them in fig. 4 of his paper [3]. The dots in fig. 1 were taken from PASQUILL's paper. Although he draw a curved line to represent the average relation between $\frac{-E}{z^2 \frac{\partial u}{\partial z} \frac{\partial c}{\partial z}}$ and R_i, according to the present theory, there must be a linear relationship between them in the range of $R_i < 0$ and another linear relation in the range of $R_i > 0$. The straight lines in the figure were drawn from such a point of view. From these straight lines we have the value of σ_1 to be 12 which is in agreement with PASQUILL's estimation of σ_1, while the value of σ_2 is estimated to be about one half of σ_1, that is, $\sigma_2 = 5.7$.

The experimental check of the theory can also be possible by using the values of $\frac{\partial u}{\partial z}$ and R_i, which were computed by PASQUILL at three different heights, *i.e.*, 37.5, 75 and 150 cm, and were listed in table 3 of his paper. In this case the values of the surface drag are not known, so that we will examine whether or not the values ot $z^2\left(\frac{\partial u}{\partial z}\right)^2 \{1 - \sigma_1 \text{ (or } \sigma_2) R_i\}$ are independent of height by assuming that $\sigma_1 = 12$ and $\sigma_2 = 5.7$. The results of cmputation of the values are listed in table 1. The obtained values of $z^2\left(\frac{\partial u}{\partial z}\right)^2 \{1-\sigma_1(\text{or } \sigma_2)R_i\}$ are nearly independent of heitht at each case, althotugh the accuracy is not so good. Thus, within some error, equations (14) and (15) will be considered to hold.

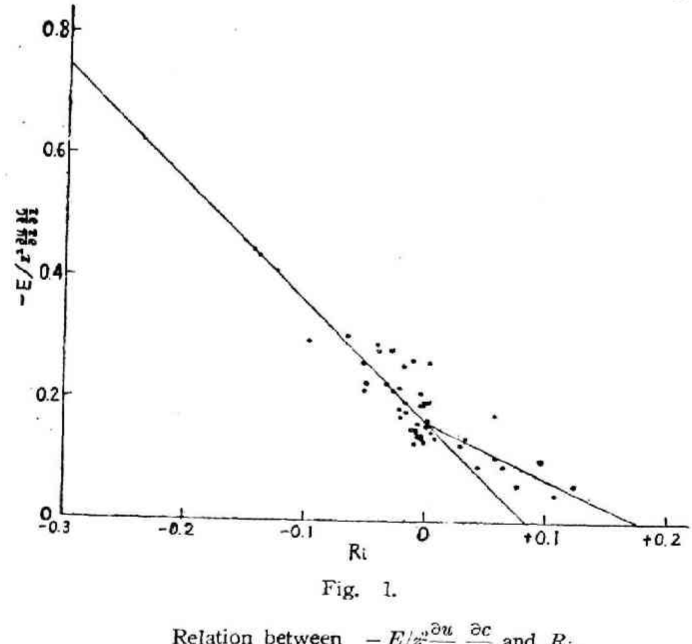

Fig. 1.

Relation between $-E/z^2 \frac{\partial u}{\partial z} \frac{\partial c}{\partial z}$ and R_i.

Table 1 Computed values of $\left(z\frac{\partial u}{\partial z}\right)^2\left\{1-\sigma_1\ (\text{or}\ \sigma_2)\ Ri\right\}$

	z cm	$\frac{\partial u}{\partial z}$ cm sec^{-1}	Ri	$\left(z\frac{\partial u}{\partial z}\right)^2\left\{1-\sigma_1\ (\text{or}\ \sigma_2)\ Ri\right\}$
Obs. No. 15	37.5	1.15	−0.022	2350
	75	0.44	−0.066	1954
	150	0.19	−0.144	2215
No. 16	37.5	1.09	−0.033	2335
	75	0.40	−0·125	2250
	150	0.19	−0.245	3200
No. 17	37.5	1.28	−0.022	2910
	75	0.56	−0.052	2894
	150	0.24	−0.140	3472
No. 19	37.5	1.17	+0.029	1607
	75	0.65	+0.058	1593
	150	0.38	+0.095	1490
No. 20	37.5	1.42	−0.021	3542
	75	0.57	−0.051	2953
	150	0.28	−00.97	3818
No. 22	37.5	2.09	−0·001	6220
	75	0.99	−0.004	5772
	150	0.53	−0.008	6928
No· 23	37.5	1.25	+0.044	1648
	75	0.72	+0.076	1653
	150	0.45	+0.107	1776
No. 26	37.5	2.23	−0.004	7323
	75	1.10	−0.008	7460
	150	0.59	−0.012	8980
No. 29	37.5	3.13	−0.002	14120
	75	1.43	−0.005	12080
	150	0.71	−0.011	12740

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