A Discussion of Slice Method for Determining the Vertical Stability of Air

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A Discussion of Slice Method for Determining the Vertical Stability of Air

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I. Introduction

There are two methods by which the vertical stability or instability conditions of the atmosphere are determined, *i.e.* parcel method and slice method. The atmosphere is in a stable state of equilibrium if a parcel of air which is moved a small distance upward or downward and then brought to rest has a tendency to return to its original level. If such a parcel has a tendency to move farther away from the original level, the air is in an unstable state of equilibrium. The parcel method is based on the assumption that an isolated parcel of air can move through an undisturbed environment, thus the stability criteria are determined only by the initial distribution of temperature.

Slice method was first discussed by J. BJERKNES (1) and then extended by S. PETTERSSEN (2). A few years ago S. SHYÔNO (3) put foward a uniqued theory of parcel and slice method. The slice method by them took into consideration not only the initial distribution of temperature but also the change of the environment of the ascending air and the nature of the perturbation that started the motion.

In studying the problem J. BJERKNES and S. PETTERSSEN considered that the excess of heating (or cooling) of the ascending air relative to the environment was the measure for the solenoid-producing energy. In this case they treated the descending air as the environment, and estimated the excess of heating (or cooling) of the ascending air relative to the descending air. However, when a part of air ascends (or descends) the other part will inevitably descends (or ascends) if the horizontal motion does not maintain net inflow or outflow as was assumed by them. Therefore it is not reasonable to distinguish the descending air from the ascending one and treat the former as the environment.

In the dynamical treatment by S. SHYÔNO, which yielded the same results as those of J. BJERKNES and S. PETTERSSEN, he assumed hydrostatic condition for the descending air, whereas for the ascending air vertical acceleration has been taken into consideration.

The present author discussed the problem by using the dynamical method and obtained a new result for the slice method.

II. Discussion of BJERKNES-PETTERSSEN Slice Method

If the air were originally at rest and an impulse affecting the mass M_1 to ascend is applied, then the surrounding air M_2 will descend, where M_1 and M_2 denote the mass of unit thickness of the respective air. The heat gained per second within the slice is expressed by

$$\frac{\partial Q}{\partial t} = C_p\left(M_1\frac{\partial T_1}{\partial t}+M_2\frac{\partial T_2}{\partial t}\right)=C_p(M_1+M_2)\frac{\partial T_2}{\partial t} + C_pM_1\left(\frac{\partial T_1}{\partial t}-\frac{\partial T_2}{\partial t}\right), \quad (1)$$

where T_1 and T_2 are the temperature of the ascending and the descending air respectively, and C_p is the specific heat of air at constant pressure.

The first term on the right-hand side of (1) represents a uniform heating (or cooling) of the entire slice at the rate $\partial T_2/\partial t$ equal to the heating (or cooling) of the descending air. The last term represents the excess heating (or cooling) of the ascending air relative to the environment, and only this term contributes to the solenoid-producing energy. BJERKNES-PETTERSSEN slice method starts from the above consideration. However, if one part of air ascends without any net inflow or outflow due to the horizontal motion, then the other part of the air will descend. Similarly, if one part of the air descends, ascension of the other part will occur at the same time. In other words the descension of the air as well as the ascension is nothing but a part of one phenomenon or convection, therefore we cannot decide which of the ascending or the descending air to be the environment.

Now we shall consider the case in which an impulse affecting the mass M_2 to descend is applied and the mass M_1 of the environment ascends. In this case the heat gained in the entire slice will be expressed by

$$\frac{\partial Q}{\partial t} = C_p(M_1+M_2)\frac{\partial T_1}{\partial t} + C_pM_2\left(\frac{\partial T_2}{\partial t} - \frac{\partial T_1}{\partial t}\right). \quad (2)$$

The first term of the right-hand side expresses a uniform heating (or cooling) of the entire slice at the rate equal to the heating (or cooling) of the environment (now the ascending air) and the last term expresses the excess of heating (or cooling) of the descending air relative to the environment. The character of the convction which was established by applying an impulse affecting M_1 to ascend is quite same as that established by applying an impulse affecting M_2 to descend. Therefore, if the solenoid-producing energy of the convection were expressed by the last term of the right-hand side of (1), it would be also expressed by the corresponding term of (2). In the second case, however, the available energy should be given by $-C_pM_2\left(\frac{\partial T_2}{\partial t} - \frac{\partial T_1}{\partial t}\right)$, because the excess of heating of the descending air leads the air to retard in the direction of the impulse. It is clear that these two expressions have not a same value except when $M_1 = M_2$. In other words, the solenoid-producing energy cannot be expressed by the last term of (1). As stated above, BJERKNES-PETTERSSEN slice method is based on (1), therefore the results obtained by them whould not be considered to be correct.

III. Dynamical Treatment of Slice Method

Instead of the thermodynamical treatment of the problem as was done by J. BJERKNES and S. PETTERSSEN, the author studied it dynamically.

Similarly as S. PETTERSSEN we shall make the following assumptions.

(1) The horizontal motion does not at any level maintain any net inflow to, or outflow from, the region under consideration.

(2) At the initial moment the conditions are barotropic.

(3) The temperature changes adiabatically.

In addition to (1), we shall assume that the motion of the air occurs only vertically. This will not be inconsistent with the assumptions made by J. BJERKNES and S. PETTERSSEN, as they have not taken the horizontal motion into consideration. In these circumstances, the pressure of the ascending air will at any level be equal to that of the descending air.

Let us consider that the mass M_1 which was originally ($t=0$) at rest at a small distance Δz_1 below the standard level z ascends to z-level in a short time interval Δt and that the mass M_2 at a distance Δz_2 above the z-level descends to the z-level in the same time interval. The pressure, density and temperature of the air at the time when it was at rest and when it arrived at z-level are shown in Fig. 1 (a) and (b) respectively.

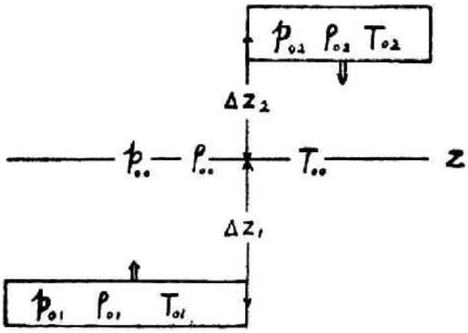

Fig. 1 (a) Pressure, Density and Temperature at the Initial Moment, $t=0$.

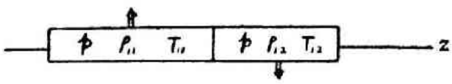

Fig. 1 (b) Pressure, Density and Temperature at $t = \Delta t$.

When the upward and downward velocities of the air are denoted by w_1 and w_2 (<0) respectively, then the equations of the respective air are expressed by

$$\frac{dw_1}{dt} = -g - \frac{1}{\rho_1}\frac{dp_1}{dz}, \qquad (3)$$

and

$$\frac{dw_2}{dt} = -g - \frac{1}{\rho_2}\frac{dp_2}{dz}. \qquad (4)$$

From the assumption mentioned above we have at z-level

$$\frac{dp_1}{dz} = \frac{dp_2}{dz} \equiv \frac{dp}{dz}, \text{ and } M_1 w_1 = -M_2 w_2$$

$$\text{or } M_1 \frac{\Delta z_1}{\Delta t} = M_2 \frac{\Delta z_2}{\Delta t}, \qquad (5)$$

which is reduced to

$$M_1 \Delta z_1 = M_2 \Delta z_2. \qquad (6)$$

Thus the left-hand sides of (3) and (4) are expressed by

$$M_1 \frac{dw_1}{dt} = M_1 \frac{\Delta z_1}{\Delta t \Delta t} = M_2 \frac{\Delta z_2}{\Delta t \Delta t} = -M_2 \frac{dw_2}{dt}.$$

The force F_1 realized in the ascending air by the displacement Δz_1 is given by

$$F_1 = M_1\left(\frac{dw_1}{dt}\right)_{t=\Delta t} = -M_1 g - \frac{M_1}{\rho_{11}}\frac{dp}{dz}.$$

Similarly, the force F_2 realized in the descending air is

$$F_2 = M_2\left(\frac{dw_2}{dt}\right)_{t=\Delta t} = -M_2 g - \frac{M_2}{\rho_{12}}\frac{dp}{dz}.$$

At the standard level, therefore, we have

$$F_1 = -F_2.$$

Combining the above three equations we get

$$\frac{dp}{dz} = -(M_1 + M_2) g \bigg/ \left(\frac{M_1}{\rho_{11}} + \frac{M_2}{\rho_{12}}\right), \quad \text{and}$$

$$F_1 = -F_2 = M_1 M_2 \frac{\rho_{12} - \rho_{11}}{\rho_{12} M_1 + \rho_{11} M_2} g$$

$$= M_1 M_2 \frac{1 - \frac{\rho_{11}}{\rho_{12}}}{M_1 + \rho_{11}/\rho_{12} \cdot M_2} g. \qquad (7)$$

Above results show that the force acting on the ascending air is at the standard level equal in magnitude and opposite in direction to that on the descending air, which corresponds to the "Principle of Action and Reaction of Force" in dynamics. Also from the above result it will be clear that the ascending and descending air should be treated equally.

In the paper by S. SHYÔNO cited afore he used the equation (3) for the ascending air, while for the descending air the acceleration term was dropped from the equation (4). This corresponds to the treatment by J. BJERKNES and S. PETTERSSEN, thus it need not be surprised that they have reached to the same conclusion.

Now, criteria of stability and instability of the air will be given by

(1) $F_1 < 0$ *or* $F_2 > 0$ stable equilibrium

(2) $F_1 = 0$ *or* $F_2 = 0$ indifferent equilibrium

(3) $F_1 > 0$ *or* $F_2 < 0$ unstable equilibrium.

As the densities ρ_{11} and ρ_{12} involved in (7) are those at $t = \Delta t$, it is necessary to express them by quantities at the initial moment. From the assumption (3) the temperature change is adiabatic, but for the convenience of calculation it was assumed as polytropic. Wet adiabatic change is not perfectly polytropic, the departure from which is, however, negligible in our case.

(1) Non-saturated air and saturated air

First, we shall consider the case in which both the ascending and the descending air are non-saturated or saturated.

In the case of the dry adiabatic change the rank k of the polytropic change is related with the dry adiabatic lapse rete γ_d by the expression

$$\frac{k-1}{k}\frac{g}{R} = \gamma_d, \tag{8}$$

where R is the gas constant of dry air. Similarly, when the air is saturated the relation between k and the wet adiabatic lapse rate γ_w is given by

$$\frac{k-1}{k}\frac{g}{R} = \gamma_w. \tag{8'}$$

From the assumption of polytropic change we have

$$p_{01}/\rho_{01}^k = p/\rho_{11}^k \quad \text{and} \quad p_{02}/\rho_{02}^k = p/\rho_{12}^k \tag{9}$$

for the ascending and the descending air respectively. Eliminating p from the above equations we have

$$\frac{\rho_{11}}{\rho_{12}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{1}{k}}\frac{\rho_{01}}{\rho_{02}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{1}{k}-1}\frac{T_{02}}{T_{01}}.$$

The right-hand side of the equation can be expanded with respect to z, which leads to

$$\left(\frac{p_{02}}{p_{01}}\right)^{\frac{1}{k}-1}\frac{T_{02}}{T_{01}} \fallingdotseq \left\{1-\left(\frac{1}{k}-1\right)\frac{g}{RT_{00}}(\Delta z_1+\Delta z_2)\right\}$$

$$\times\left\{1-\frac{\gamma}{T_{00}}(\Delta z_1+\Delta z_2)\right\}$$

$$\fallingdotseq 1-\frac{\Delta z_1+\Delta z_2}{T_{00}}\left\{\gamma+\left(\frac{1}{k}-1\right)\frac{g}{R}\right\},$$

where γ is the lapse rate of the air temperature at the initial moment. The final expression of ρ_{11}/ρ_{12} is then given by

$$\frac{\rho_{11}}{\rho_{12}} = 1-\frac{\gamma-\gamma_d}{T_{00}}(\Delta z_1+\Delta z_2) \tag{10}$$

for non-saturated air,

and

$$\frac{\rho_{11}}{\rho_{12}} = 1-\frac{\gamma-\gamma_w}{T_{00}}(\Delta z_1+\Delta z_2) \tag{10'}$$

for saturated air.

Putting these expressions into (7) and neglecting the small quantities we get

$$F_1 = \frac{M_1 g}{T_{00}}(\gamma-\gamma_d)\Delta z_1 \tag{11}$$

for non-saturated air,

and

$$F_1 = \frac{M_1 g}{T_{00}}(\gamma-\gamma_w)\Delta z_1 \tag{11'}$$

for saturated air.

Criteria of stability and instability of the air are then given by

(A) Non-saturated air

$\gamma < \gamma_d$ stable, $\gamma = \gamma_d$ indifferent, $\gamma > \gamma_d$ unstable.

(B) Saturated air

$\gamma < \gamma_w$ stable, $\gamma = \gamma_w$ indifferent, $\gamma > \gamma_w$ unstable.

The above results show that the criteria and the available energy are quite same as those obtained by parcel method.

(2) Saturated ascent through a dry-adiabatically descending air

In this case the rank of the polytropic change for the ascending air is different from that of the descending one. Denoting them by k_1 and k_2 respectively, we have

$$\frac{k_1-1}{k_1}\frac{g}{R} = \gamma_w, \quad \text{and} \quad \frac{k_2-1}{k_2}\frac{g}{R} = \gamma_d.$$

Two relations corresponding to (9) are then given by

$$p_{01}/\rho_{01}^{k_1} = p/\rho_{11}^{k_1} \quad \text{and} \quad p_{02}/\rho_{02}^{k_2} = p/\rho_{12}^{k_2} \tag{12}$$

respectively. Now, elimination of p from the above two equations does not give the expression of $\rho_{11}/_{12}$ by the quantities at the initial moment. Therefore we shall consider the continuity conditions for the ascending and the descending air. They are given by

$$\frac{\partial\rho_1}{\partial t} + w_1\frac{\partial\rho_1}{\partial z} + \rho_1\frac{\partial w_1}{\partial z} = 0$$

and

$$\frac{\partial\rho_2}{\partial t} + w_2\frac{\partial\rho_2}{\partial z} + \rho_2\frac{\partial w_2}{\partial z} = 0 \tag{13}$$

respectively. Neglecting the small quantities (13) will be written as

$$\frac{\partial\rho_1}{\partial t} + w_1\frac{\partial\rho_{00}}{\partial z} + \rho_{00}\frac{\partial w_1}{\partial z} = 0$$

and

$$\frac{\partial\rho_2}{\partial t} + w_2\frac{\partial\rho_{00}}{\partial z} + \rho_{00}\frac{\partial w_2}{\partial z} = 0. \tag{13'}$$

Multiplying M_1 and M_2 by both sides of the respective equation and adding them we have at z-level,

$$M_1\frac{\partial\rho_1}{\partial t}+M_2\frac{\partial\rho_2}{\partial t}+\frac{\partial\rho_{00}}{\partial z}(M_1w_1+M_2w_2)+\rho_{00}\Big(M_1\frac{\partial w_1}{\partial z}+M_2\frac{\partial w_2}{\partial z}\Big)$$

$$=M_1\frac{\partial\rho_1}{\partial t}+M_2\frac{\partial\rho_2}{\partial t}+\rho_{00}\frac{\partial}{\partial z}(M_1w_1+M_2w_2)-\rho_{00}\Big(w_1\frac{\partial M_1}{\partial z}+w_2\frac{\partial M_2}{\partial z}\Big)=0.$$

If we assume that M_1/M_2 is independent of z as J. BJERKNES has assumed, then we have

$$M_1\frac{\partial M_2}{\partial z}-M_2\frac{\partial M_1}{\partial z}=0, \qquad \text{which leads to}$$

$$w_1\frac{\partial M_1}{\partial z}+w_2\frac{\partial M_2}{\partial z}=\frac{1}{M_1}\frac{\partial M_1}{\partial z}(M_1w_1+M_2w_2)=0.$$

Thus we have
$$M_1\frac{\partial\rho_1}{\partial t}+M_2\frac{\partial\rho_2}{\partial t}=0. \tag{14}$$

As $\frac{\partial\rho_1}{\partial t}$ and $\frac{\partial\rho_2}{\partial t}$ express the change of the density at the standard level they are also expressed by $\frac{\partial\rho_1}{\partial t}=\frac{\rho_{11}-\rho_{00}}{\Delta t}$ and $\frac{\partial\rho_2}{\partial t}=\frac{\rho_{12}-\rho_{00}}{\Delta t}$ respectively, from which the following expression connecting ρ_{11} with ρ_{12} is obtained.

$$M_1(\rho_{11}-\rho_{00})+M_2(\rho_{12}-\rho_{00})=0. \tag{15}$$

Now, from (12) we have

$$p=p_{01}\Big(\frac{\rho_{11}}{\rho_{00}}\Big)^{k_1}=p_{01}\Big(\frac{\rho_{11}}{\rho_{00}}\frac{\rho_{00}}{\rho_{01}}\Big)^{k_1}=p_{01}\Big(\frac{\rho_{00}}{\rho_{01}}\Big)^{k_1}\Big(1+\frac{\rho_{11}-\rho_{00}}{\rho_{00}}\Big)^{k_1}\fallingdotseq p_{01}\Big(\frac{\rho_{00}}{\rho_{01}}\Big)^{k_1}\Big(1+k_1\frac{\rho_{11}-\rho_{00}}{\rho_{00}}\Big),$$

and also
$$p\fallingdotseq p_{02}\Big(\frac{\rho_{00}}{\rho_{02}}\Big)^{k_2}\Big(1+k_2\frac{\rho_{12}-\rho_{00}}{\rho_{00}}\Big).$$

Eliminating p from the above two equations and using (15) we have

$$\frac{\rho_{12}-\rho_{00}}{\rho_{00}}=\frac{1-\frac{p_{02}}{p_{01}}\Big(\frac{\rho_{01}}{\rho_{00}}\Big)^{k_1}\Big(\frac{\rho_{00}}{\rho_{02}}\Big)^{k_2}}{k_2+\frac{p_{02}}{p_{01}}\Big(\frac{\rho_{01}}{\rho_{00}}\Big)^{k_1}\Big(\frac{\rho_{00}}{\rho_{02}}\Big)^{k_2}k_1\frac{M_2}{M_1}},$$

in which the second term of the numerator of the right-hand side is expressed by

$$\frac{p_{02}}{p_{01}}\Big(\frac{\rho_{01}}{\rho_{00}}\Big)^{k_1}\Big(\frac{\rho_{00}}{\rho_{02}}\Big)^{k_1}=\frac{p_{02}}{p_{01}}\Big(\frac{p_{01}}{p_{00}}\frac{T_{00}}{T_{01}}\Big)^{k_1}\Big(\frac{p_{00}}{p_{02}}\frac{T_{02}}{T_{00}}\Big)^{k_1}\fallingdotseq\Big\{1-\frac{g}{RT_{00}}(\Delta z_1+\Delta z_2)\Big\}\Big(1+\frac{k_1g}{RT_{00}}\Delta z_1\Big)$$
$$\times\Big(1-\frac{k_1\gamma}{T_{00}}\Delta z_1\Big)\Big(1+\frac{k_2g\,\Delta z_2}{RT_{00}}\Big)\Big(1-\frac{k_2\gamma\,\Delta z_2}{T_{00}}\Big)\fallingdotseq 1-\frac{k_1}{T_{00}}(\gamma-\gamma_w)\Delta z_1-\frac{k_2}{T_{00}}(\gamma-\gamma_d)\Delta z_2.$$

Therefore we have

$$\frac{\rho_{12}-\rho_{00}}{\rho_{00}}\fallingdotseq\frac{k_1(\gamma-\gamma_w)\Delta z_1+k_2(\gamma-\gamma_d)\Delta z_2}{T_{00}\Big(k_2+k_1\frac{M_2}{M_1}\Big)}.$$

On the other hand, from (15) we get

$$\frac{\rho_{11}}{\rho_{12}}=\frac{\rho_{00}+(\rho_{11}-\rho_{00})}{\rho_{00}+(\rho_{12}-\rho_{00})}\fallingdotseq 1+\frac{(\rho_{11}-\rho_{00})-(\rho_{12}-\rho_{00})}{\rho_{00}}=1-\frac{\rho_{12}-\rho_{00}}{\rho_{00}}\Big(1+\frac{M_2}{M_1}\Big). \tag{16}$$

Combining the above two equations we have the final expression of ρ_{11}/ρ_{12},

$$\frac{\rho_{11}}{\rho_{12}} = 1 - \frac{1+\dfrac{M_2}{M_1}}{T_{00}\left(k_2+k_1\dfrac{M_2}{M_1}\right)}\left\{k_1(\gamma-\gamma_w)\Delta z_1 + k_2(\gamma-\gamma_d)\Delta z_2\right\}. \tag{17}$$

The force realized in the ascending air is then given by

$$F_1 = M_1M_2g\frac{1-\dfrac{\rho_{11}}{\rho_{12}}}{M_1+\dfrac{\rho_{11}}{\rho_{12}}M_2} \doteqdot \frac{M_1g\Delta z}{T_{00}}\,\frac{1}{1+\dfrac{k_2}{k_1}\,\dfrac{M_1}{M_2}}\left\{(\gamma-\gamma_w)+\frac{k_2}{k_1}\,\frac{M_1}{M_2}\,(\gamma-\gamma_d)\right\}. \tag{18}$$

If we put $k_1=k_2$, or $\gamma_d=\gamma_w$ in the above expression, then we can obtain the results for the non-saturated and the saturated air, which were given by (10) and (11) respectively.
Putting $\gamma_d-\gamma=\alpha$ and $\gamma-\gamma_w=\beta$,
(18) may be written as

$$F_1 = \frac{M_1g\Delta z}{T_{00}}\,\frac{1}{1+\dfrac{k_2}{k_1}\,\dfrac{M_1}{M_2}}\left(\beta-\alpha\frac{k_2}{k_1}\,\frac{M_1}{M_2}\right). \tag{18$'$}$$

Thus the air is stable, indifferent or unstable state of equilibrium according to whether

$$\beta \lesseqgtr \alpha\frac{k_2}{k_1}\,\frac{M_1}{M_2}, \qquad \text{or} \quad \frac{M_1}{k_1}\Big/\frac{M_2}{k_2} \gtreqless \frac{\beta}{\alpha}. \tag{19}$$

IV. Comparison with Parcel Method and BJERKNES-PETTERSSEN Slice Method

To compare the results obtained above with those by parcel method and BJERKNES-PETTERSSEN slice method, the force realized in the ascending air was tabulated below.

	(a) Parcel method	(b) Slice method by BJERKNES etc.	(c) Slice method by OGIWARA
(1) Non-saturated air	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_d)$	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_d)\left(1+\dfrac{M_1}{M_2}\right)$	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_d)$
(2) Saturated air	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_w)$	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_w)\left(1+\dfrac{M_1}{M_2}\right)$	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}(\gamma-\gamma_w)$
(3) Saturated ascent, dry adiabatic descent	—	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}\left(\beta-\alpha\dfrac{M_1}{M_2}\right)$	$F_1=\dfrac{M_1g\Delta z_1}{T_{00}}\,\dfrac{\beta-\alpha\dfrac{k_2}{k_1}\dfrac{M_1}{M_2}}{1+\dfrac{k_2}{k_1}\dfrac{M_1}{M_2}}$

It is clear from the table that when both the ascending and the descending currents are non-saturated or saturated the criteria of stability and instability deduced on the assumption of an undisturbed environment (the parcel method) hold also for both slice methods.

In the case of (3), *i.e.* saturated ascent through a dry-adiabatically descending air, the air is stable for $\gamma \leqq \gamma_w$ and unstable for $\gamma \geqq \gamma_d$. When $\gamma_w < \gamma < \gamma_d$ the air is stable, indifferent, or unstable according to whether $M_1/M_2 \gtreqless \beta/\alpha$ from BJERKNES-PETTERSSEN slice method and $\dfrac{M_1}{k_1}\Big/\dfrac{M_2}{k_2} \gtreqless \dfrac{\beta}{\alpha}$ from our method. As is obvious from the definition of k_1 and k_2, k_2/k_1 is greater than unity, therefore it is concluded from both slice methods that the air is stable or unstable according to whether $\beta < \alpha\,\dfrac{M_1}{M_2}$ or $\beta < \alpha\dfrac{k_2}{k_1}\,\dfrac{M_1}{M_2}$. When $\alpha\dfrac{k_2}{k_1}\,\dfrac{M_1}{M_2} > \beta > \alpha\dfrac{M_1}{M_2}$, it is stable according to our method and is unstable according to BJERKNES-PETTERSSEN slice method. As the numerical value of γ_w depends on the pressure and the temperature of the air, the numerical value of k_2/k_1 varies also according to the conditions of the air. For instance when

$\gamma_w = 0.5°/100$ m and $0.7°/100$ m,

$k_1 = 1.16$ and 1.25, thus $k_2/k_1 = 1.20$ and 1.12 respectively.

Next we shall examine the available energy or the force realized in the ascending air by the displacement. In the case of (1) and (2) our results agree with that obtained by the parcel method, whereas the slice method by BJERKNES gives an available energy which is $\left(1+\frac{M_1}{M_2}\right)$ times as large as that was obtained by the parcel method and also our slice method.

In the case of (3) we had an available energy which was not numerically equal to that obtained by BJERKNES etc. When the air is conditionally unstable ($\gamma_w < \gamma < \gamma_d$), however, the air is selectively unstable also in our case and there is also a tendency for the impulse that causes the ratio M_1/M_2 to be a minimum to gain over other neighboring impulses. Furtheremore it is clear from the table above that BJERKNES-PETTERSSEN slice method overestimates the available energy both when $\beta < a\frac{M_1}{M_2}$ (stable) and $\beta > a\frac{k_2}{k_1}\frac{M_1}{M_2}$ (unstable).

V. Conclusions and Acknowledgment

The author discussed the slice method and obtained some new results. The reductions of the results, however, are based on the assumptions which are not strictly fulfilled in the atmosphere. Especially the assumption which is expressed by (14) seems rather artificial.

These difficulties will be overcome by the hydrodynamical treatment of the problem taking into consideration the horizontal distribution of the temperature and the velocity of the convective currents.

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