Studies on the Twilight by the Scattering.

STUDIES ON THE TWILIGHT BY THE SCATTERING

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1 Introduction

In olden times the observation of twilight was executed by FESSENKOFF (1), SMART (2), HULBURT (3). The latest one was perhaps given by LJUNGHALL (4), who introduced us much historical process and many useful references in this paper.

As far as the theoretical investigation is concerned, HULBURT computed by considering only first scattering, so that the error mounted up to 30% and he confessed that the rigorous calculation would be impossible with the sun's zenith distance greater than 98° owing to the far predominancy of secondary scattering. Hence a satisfactory explanation of twilight has not been completed until now.

Giving a paper on the scattering of the sun in the day time (5) the author has now researched the twilight as the extention of the paper by assuming also the earth's atmosphere to be composed of numberless homogeneous thin layers bounded by concentric spheres, and considering the extinction of the sun's ray and further discussing strictly the secondary scattering with respect to each wave-length, and computed the radiation of each portion of the sky dome and that falling on a horizontal plane on the earth's surface in the case of the sun's zenith distances 96°, 96°.404, 97°; but as the above papers (1), (2), (3) and that introduced in (4) are giving the observation of particular portion on the sky, and (4) is concerned to the luminosity on a vertical plane, so we have some unreliance and inconvenience in the representation of solid angle and the method of measurement.

On the contrary, the observation of Mr. ŌSAWA of Tokyo Astro. Obs. (6) can not only give full reliance but is concerned to the horizontal plane on the earth's surface with the unit of Lux, so his measurement is most convenient to be compared with the author's computation. This comparison has been made with good identity. Eventurely the explanation of twilight which has not been hitherto researched has became possible up to about 100° of the sun's zenith distance if we consider up to secondary scattering on the practical point of view. For the greater zenith distance, the consideration of the scattering of higher grade is necessary, but we can assert that we must conquer much difficulty. As already mentioned, this paper is a extention of the above paper of the author. Please the reader refer to it.

Let us take two points T and E in the atmosphere. Define any arbitrarily chosen set of axes of rectangular coordinates X_1 Y_1 Z_1 with its center at T, with the X_1 axis drawn towards the sun. Now, let i be the direct insolation reaching T, $ET=r$, φ the angle between X_1 axis and r, so the intensitv of primary scattering generated by unit volume becomes

$$\frac{3}{16}\frac{k_{\lambda T}}{\pi r^2}\ (1+\cos^2\ \varphi)\ i$$

in which we can substitute $k\rho_T$ for $k_{\lambda T}$, k being the function of wave-length λ, ρ_T the atmospheric density at T. The wave generated at E by a plane polarized light coming from X_1

direction and oscillating in Z_1 direction is also a plane polarized and advances in r direction and oscillates in a definite direction normal to it.

Now take the origin at E, X_2 axis in r, that is, TE direction, Z_2 axis in the direction of oscillation, Y_2 axis normal to X_2 Z_2.

Now take any point O′ and notate EO′= R and let ω and ω' be the angle between r and Z_1, and Y_1, Ω the angle between Z_2 and EO′.

The plane polarized light coming from X_1 and oscillating in Y_1 direction produces at E a plane polarized light which advances in the direction r and oscillates in a definite direction normal to it. Draw from E the axis X_2' and Z_2' in the direction of r and oscillation respectively and Y_2' normal to them, and let Ω' be the angle between EO′ and Z_2'. Hence, when we take unit volume of atmosphere at T and E respectively, T being exposed by solar ray, emits primarily scattered light to E and again E, receiving this light, emits secondarily scattered light to O′. The intensity of this final scattered light at O′ becomes

$$\frac{i}{2}\left(\frac{3}{8\pi}\right)^2 \frac{1}{R^2r^2} (\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega')\, k_{\lambda T}\, k_{\lambda E},$$

in which $k_{\lambda E}$ is the extinction coefficient at E. If we consider the effect of absorption on the optical path, we must only multiply the absorption term.

2 First scattering

§ 1 General aspect

Take a point E in the atmosphere seen from O′ point on the earth's surface. Let us define a set of axes of coordinates X, Y, Z, with its origin at the earth's center O, taking O O′ as Z axis, X axis being normal to Z axis in the plane determined by this and the sun's centre, and on the sun-side, and Y axis being normal to the other two.

Let the coordinates of E referred to XYZ system be

$$X = e \sin\gamma \cos A, \qquad Y = e \sin\gamma \sin A,$$
$$Z = e \cos\gamma,$$

where
$$0 \leqq \gamma \leqq \frac{\pi}{2}, \quad 0 \leqq A \leqq 2\pi.$$

Let the transformed system of XYZ obtained by moving the origin to O′ be $X'\,Y'\,Z'$ system, and define the polar angle θ and A by

$$X' = R\cos\theta\cos A, \quad Y' = R\cos\theta\sin A,$$
$$Z' = R\sin\theta,$$

and denote the coordinates of E referred to this new system by $X'\,Y'\,Z'$, then we get

$$X = X', \quad Y = Y', \quad Z = Z' + a_0,$$

where a_0 is the earth's radius, now, let H' be the height of a point from the earth's surface of a point distant from E by s on a solar ray passing through E, and d be the sun's dip (*i. e.* negative altitude) so is

$$(a_0+H')^2 = (X+s\cos d)^2 + Y^2 + (Z-s\sin d)^2$$
$$= s^2 + 2s\ (X\cos d - Z\sin d) + e^2,$$

from this H' can be expressed by a function of s. Consider a new axis of x_1 drawn at the angle d with X axis. Then the relation between the new system $x_1\,y_1\,z_1$ and the original are shown below

	x_1	y_1	z_1
X	$\cos d$	0	$\sin d$
Y	0	1	0
Z	$-\sin d$	0	$\cos d$

Here, needless to say, x_1 is directed towards the sun. If the $x_1\,y_1\,z_1$ coordinates of E be denoted by a, b, c, we get

$$a = X\cos d - Z\sin d, \quad b = Y,$$
$$c = X\sin d + Z\cos d.$$

Hence the optical path along the solar ray in the atmosphere between E and the upper limit, is

$$\delta_1 = \sqrt{(a_0+l)^2 - b^2 - c^2} - a$$

where l is the height of upper limit. Thus, δ_1

can be expressed by a function of XYZ, the coordinate of E. So becomes the direct solar intensity at E

$$\frac{I_0}{D^2}\exp\left(-\int_0^{\delta_1} k\rho(H')ds\right).$$

Now, the light scattered by this direct ray at E will be of course similarly affected by the atmospheric absorption on the way from E to O′.

The height of a point, H, distant from O′ by s on the line O′E is evidently

$$H=\sqrt{a_0^2+s^2+2a_0 s\sin\theta}-a_0,$$

and the absorption effect between O′E is

$$\exp\left(-\int_0^{R} k\rho\ (H)\ ds\right)$$

and the angle φ, between O′E line and solar ray passing through E is

$$\begin{aligned}\cos\varphi &= \frac{X}{R}\cos d-\frac{Z-a_0}{R}\sin d\\ &= \cos\theta\cos A\cos d-\sin d\sin\theta.\end{aligned}$$

Thus the horizontal intensity dS_1 at O′ due to primary scattering produced by 1 cm³ with its centre at E in a domain in the atmosphere radiated by direct sun on the horizon at O′ (this will be called hereafter the bright zone) is

$$\begin{aligned}dS_1=&\frac{3k\rho_E}{16\pi R^2}(1+\cos^2\varphi)\sin\theta\frac{I_0}{D^2}\\ &\times\exp\left(-\int_0^{\delta_1}k\rho(H')\,ds-\int_0^{R}k\rho(H)ds\right)\end{aligned}$$

where ρ_E is the air density at E.

dS_1 can be expressed by a function of R, θ, A by the above explanation. Now letting R_1 be the distance from O′ to a point of intersection between the upper limit and a line passing through O′, so

$$R_1=\frac{\cos(\theta+\alpha)}{\cos\theta}(a_0+l),$$

where $$\sin\alpha=\frac{a_0}{a_0+l}\cos\theta.$$

The domain in the atmosphere besides the bright zone is not contributory to lst scattering since it is never radiated by the direct sun. Therefore, the total intensity is obtainable by integrating dS_1 all over the bright zone. Since this zone is a fraction of the total sky above horizone at O′ outside a cylinder generated by the sun's ray touching the earth, it is convenient to express this zone by $X'Y'Z'$ system. The equation of cylinder referred to XYZ system is

$$Y^2+(X\sin d+Z\cos d)^2=a_0^2,$$

converting this to $X'Y'Z'$ system we get

$$Y'^2+(X'\sin d+(Z'+a_0)\cos d)^2=a_0^2.$$

Next consider a new system $X_1'Y_1'Z_1'$ which is made from $X'Y'Z'$ system by rotating it by the angle A around Z' axis, then we get evidently the next relation between both systems.

	X_1'	Y_1'	Z_1'
X'	$\cos A$	$-\sin A$	0
Y'	$\sin A$	$\cos A$	0
Z'	0	0	1

From this the equation will become

$$\begin{aligned}(X_1'\sin A+Y_1'\cos A)^2+\{(Z_1'+a_0)\cos d\\ +(X_1'\cos A-Y_1'\sin A)\sin d\}^2=a_0^2.\end{aligned}$$

Substituting $Y_1'=0$ in this, we get the section of cylinder by plane $O'X_1'Z_1'$, *i. e.*

$$\begin{aligned}(X_1'\sin A)^2+\{(Z_1'+a_0)\cos d\\ +X_1'\cos A\sin d\}^2=a_0^2. \qquad \cdots (a)\end{aligned}$$

Secendly, the equation of sphere representing the atmospheric upper limit is

$$\begin{aligned}X^2+Y^2+Z^2&=(a_0+l)^2,\\ X'^2+Y'^2+(Z'+a_0)^2&=(a_0+l)^2,\\ (X_1'\cos A-Y_1'\sin A)^2+(X_1'\sin A+Y_1'\cos A)^2&\\ +(Z_1'+a_0)^2&=(a_0+l)^2.\end{aligned}$$

The line of intersection produced by this and $Y_1'=0$ is

$$X_1'^2+(Z_1'+a_0)^2=(a_0+l)^2. \qquad \cdots (b)$$

Solving X_1' and Z_1' by the combination of (a) and (b) which are curves on $Y_1'=0$ plane, and getting $\Theta=\text{tg}^{-1}\frac{Z_1'}{X_1'}$, then Θ is obviously the upper limit of integration with respect to θ for one given A. Next, since the limiting value with respect to A must of course satisfy $\Theta=0$, *i.e.* $Z_1'=0$, we must only to eliminate X_1' from (a), (b) and $Z_1'=0$ to get it, A', consequently from

$$(X_1'\sin A)^2+(a_0\cos d+X_1'\cos A\sin d)^2=a_0^2$$
$$X_1'^2 + a_0^2 = (a_0+l)^2.$$

Substituting $X'=R\cos\theta\cos A$,

$$Y'=R\cos\theta\sin A,\qquad Z'=R\sin\theta,$$

in the equation of cylinder, we get

$$R^2\cos^2\theta\sin^2 A+\{(R\sin\theta+a_0)\cos d + R\cos\theta\cos A\sin d\}^2=a_0^2.$$

The solution R_1 of this equation for given θ and A is naturally one limit of integration with respect to R. Still the integral formula representing the bright zone will take different expression according as whether the intersected point of Z axis by the upper atmospheric limit (hereafter denoted by G$'$ point) is in or outside the cylinder.

§ 2 The case when $0\leqq d\leqq d_1=\cos^{-1}\frac{a_0}{a_0+l}$

In this case, the G$'$ point is within the bright zone, and yet the intensity of radiation is symmetry as to $Y'=0$ plane, so that the total horizontal intensity due to 1st scattering is

$$D_1=2\left\{\int_0^{A'}dA\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta\int_{R_0}^{R_1}R^2dR + \int_{A'}^{\pi}dA\int_{\Theta}^{\frac{\pi}{2}}\cos\theta\,d\theta\int_{R_0}^{R_1}R^2dR\right\}dS_1.$$

§ 3 The case $d_1\leqq d\leqq 2\,d_1$

In this case, the G$'$ point is within the dark zone. Using the same notations in the same meanings as in § 1, yet Θ being applied in $0<A<A'$, we get as the total intensity

$$D_1'=2\left\{\int_0^{A'}dA\int_0^{\Theta}\cos\theta\,d\theta\int_{R_0}^{R_1}R^2\,dR\right\}dS_1.$$

However, when it becomes $d\leqq 2d_1$, the 1st scattering will entirely disappear since there exists no bright zone, reducing the total intensity to zero.

3. Secondary scattering

§ 4 General aspect

Now consider a cylindrical coordinate system given by

$$x_1=x_1,\quad y_1=r_1\cos\alpha,\quad z_1=r_1\sin\alpha.$$

Consider a point T exposed by direct sun in the atmosphere visible from a point E in the sky dome. Let $x_1\,y_1\,z_1$ be the coordinates of T referred to $x_1\,y_1\,z_1$ system, then that of a point distant by s from T on a solar ray passing through T are x_1+s, y_1, z_1 and its height H_2 is given by a function of s as follows

$$(a_0+H_2)^2=(x_1+s)^2+y_1^2+z_1^2.$$

The optical path L traversed by the sun's ray until it reaches T is

$$L=\sqrt{(a_0+l)^2-y_1^2-z_1^2}-x_1,$$

thus the intensity of direct sun at T is given by a function of $x_1\,y_1\,z_1$ as follows

$$\frac{I_0}{D^2}\exp\left(-\int_0^L k\,\rho\,(H_2)ds\right).$$

Since the coordinates of E referred to $x_1\,y_1\,z_1$ system are a, b, c, so is the distance of a line ET becomes

$$r=\sqrt{(x_1-a)^2+(y_1-b)^2+(z_1-c)^2}$$

and its direction cosines are

$$\lambda=\frac{x_1-a}{r},\ \mu=\frac{y_1-b}{r},\ \nu=\frac{z_1-c}{r}$$

and the height H_1 of a point distant by s from E on it becomes

$$a_0+H_1=\sqrt{(a+s\lambda)^2+(b+s\mu)^2+(c+s\nu)^2}$$

and the absorption effect of the ray between ET

$$\exp\left(-\int_0^r k\,\rho(H_1)ds\right)$$

is a function of $x_1\,y_1\,z_1$. And further that between EO$'$ is the same to **2** and can be given by a function of R, θ.

Now let, $X_1 Y_1 Z_1$ be a new system of coordinates with its centre at T and parallel to $x_1 y_1 z_1$. Let the coordinates of E referred to the new system be γ, δ, κ, then

$$\gamma = a - x_1, \quad \delta = b - y_1, \quad \kappa = c - z_1$$

and
$$\cos\omega = \frac{\kappa}{r}, \quad \cos\omega' = \frac{\delta}{r}.$$

Since the coordinates of O′ referred to $x_1 y_1 z_1$ system are

$$-a_0 \sin d, \quad 0, \quad a_0 \cos d,$$

then the Z_2 coordinate of O′ referred to $X_2 Y_2 Z_2$ system is

$$N = \frac{1}{r^2}\Big\{\gamma\kappa(-a_0 \sin d - x_1 - \gamma) + \delta\kappa(-y_1 - \delta) - (\gamma^2 + \delta^2)(a_0 \cos d - z_1 - \kappa)\Big\}$$

and that corresponding to $X_2' Y_2' Z_2'$ is

$$N' = \frac{1}{r^2}\Big\{\gamma\delta(-a_0 \sin d - x_1 - \gamma) - (\gamma^2 + \kappa^2)(-y_1 - \delta) + \delta\kappa(a_0 \cos d - z_1 - \kappa)\Big\}.$$

About it the reader must refer to Chapter 1 of (5). Then we get

$$\cos\Omega = \frac{N}{R}, \qquad \cos\Omega' = \frac{N'}{R}.$$

Hence the horizontal intensity due to secondary scattering at O′ contributed by 1 cm^3 atmosphere at E and T

$$dS_2 = \frac{1}{2}\frac{I_0}{D^2}\left(\frac{3}{8\pi}\right)^2 \frac{k^3}{R^2 r^2} \rho_E \, \rho_T \sin\theta$$
$$\times (\sin^2\omega \sin^2\Omega + \sin^2\omega' \sin^2\Omega')$$
$$\times \exp\left(-\int_0^L k\rho(H_2)ds - \int_0^r k\rho(H_1)ds - \int_0^R k\rho(H)ds\right),$$

will be a function of some kinds of coordinates of E by integrating with respect to T by the cylindrical coordinates $r_1 \ \alpha \ x_1$. In this function when we express a, b, c by R, θ, A by coordinate transformation, we get the horizontal intensity from the whole sky by integrating it with respect to E over this domain by the polar coordinates.

The integration of E and T are different at the sun's dip, so discussion shall be proceeded by dividing in some sections as follows.

§ 5 The integration of T when $0 \leqq d \leqq 2d_1$

a) The integration of T when E is in bright zone.

Now, consider a cone with its centre at E and touching the earth (shall be called B cone). The equation of a section of B cone by a plane perpendicular to x_1 axis can be obtained from its equation.

Using the polar coordinates on the plane of section

$$y_1 = r_1 \cos\alpha, \qquad z_1 = r_1 \sin\alpha,$$

the equation of section becomes the form

$$r_1 = \Phi(x_1 \ \alpha).$$

The section of the earth is $y_1^2 + z_1^2 = a_0^2 - x_1^2$, so it can be transformed to the form

$$r_1 = \varphi_2(x_1 \ \alpha) = \sqrt{a_0^2 - x_1^2} = p(x_1).$$

Similarly that of the upper limit of the earth's atmosphere becomes

$$r_1 = \varphi_1(x_1 \ \alpha) = \sqrt{(a_0 + l)^2 - x_1^2} = q(x_1)$$

and that of the earth's shadow

$$r_1 = \varphi_0(x_1 \ \alpha) = a_0.$$

Let the values of α of the intersecting points between

Φ and φ_2 be a_2, a_2'

Φ and φ_1 be a_1, a_1'

Φ and φ_0 be a_0, a_0'.

Let x_c be the greatest value of x_1 of the touching points of B cone and the earth, and x_t that of the points on the circle in which the cone intersects with the sphere representing the upper limit of the earth's atmosphere (shall be called G sphere). Then the integration of T with respect to r_1 a on the plane perpendicular to x_1 axis has different conditions according as x_1 is between $-q(a_0) \sim 0$, $0 \sim x_c$ and $x_c \sim x_t$ respectively as illustrated in Fig. 1 *a*, *b*, *c*, and Fig. 2.

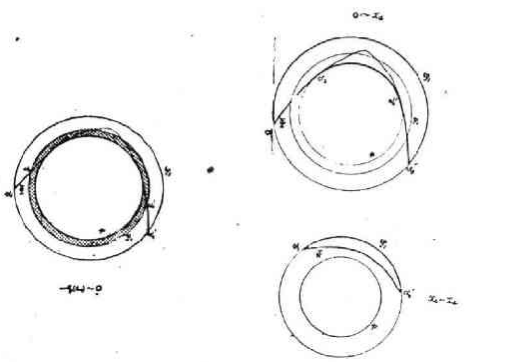

Fig. 1 This makes us easy to understand the integration domains of r_1 and α in three domains of x_1 concerning the integration of T when E is in bright zone.

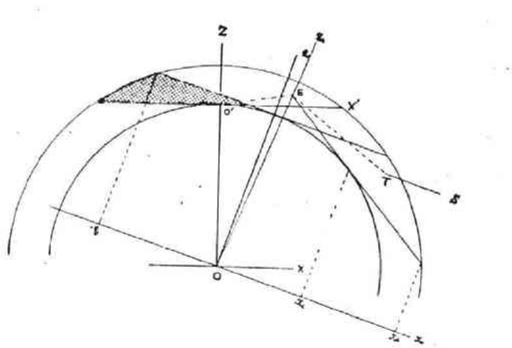

Fig. 2 This makes us easy to understand the lower and upper limits of integration of x_1 concerning the integration of T when E is in bright zone.

Then the intensity of secondary scattering by one E will be

$$\Delta_b = \Big[\int_{-q(a_0)}^{0} dx_1 \Big\{ \int_{a_1}^{a_0} da \int_{\Phi(x_1 a)}^{q(x_1)} r_1 dr_1 + \int_{a_0}^{a_0'} da \int_{a_0}^{q(x_1)} r_1 dr_1 + \int_{a_0'}^{a_1'} da \int_{\Phi(x_1 a)}^{q(x_1)} r_1 dr_1 \Big\}$$

$$+ \int_{0}^{x_c} dx_1 \Big\{ \int_{a_1}^{a_2} da \int_{\Phi(x_1 a)}^{q(x_1)} r_1 dr_1 + \int_{a_2}^{a_2'} da \int_{\varphi_2(x_1 a)}^{q(x_1)} r_1 dr_1 + \int_{a_2'}^{a_1'} da \int_{\Phi(x_1 a)}^{q(x_1)} r_1 dr_1 \Big\} + \int_{x_c}^{x_t} dx_1 \int_{a_1}^{a_1'} da \int_{\Phi(x_1 a)}^{q(x_1)} r_1 dr_1 \Big] dS_2.$$

b) The integration of T when E is in dark zone.

In this case E is not solarradiated. Now, let x_b be the greatest value of x_1 of the points on the intersecting curve between B cone and the cylinder

$$y_1^2 + z_1^2 = a_0^2.$$

Then the partial domains of x_1 corresponding to a) become

$$-q(a_0) \sim x_b, \qquad x_b \sim x_t$$

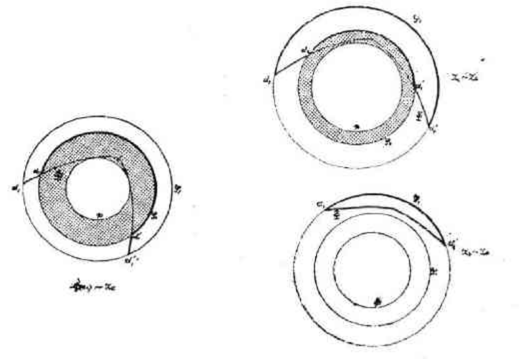

Fig. 3 This makes us easy to understand the integration domains of r_1 and α in three domains of x_1 concerning the integration of T when E is in dark zone.

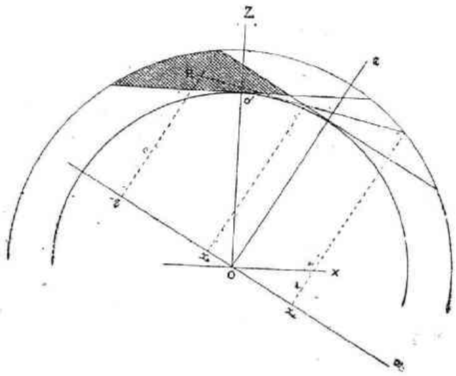

Fig. 4 This makes us easy to understand the lower and upper limits of integration of x_1 concerning the integration of T when E is in dark zone.

as illustrated in Fig. 3 a, b, c and Fig. 4.

Hence in this case will be

$$\Delta_t = \left[\int_{-q(a_0)}^{x_b} dx_1 \left\{ \int_{a_1}^{a_0} da \int_{\Phi}^{q} r_1 d\gamma_1 + \int_{a_0}^{a_0'} da \int_{a_0}^{q} r_1 dr_1 + \int_{a_0'}^{a_1'} da \int_{\Phi}^{q} r_1 d\gamma_1 \right\} + \int_{x_b}^{x_t} dx_1 \int_{a_1}^{a_1'} da \int_{\Phi}^{q} r_1 dr_1 \right] dS_2.$$

§ 6 The integration of E when $0 \leqq d \leqq d_1$.

a) The integration of bright zone

The intersecting point of Z' axis and G sphere (shall be called G' point) is within the bright zone, and so the integration domain of E is identical with § 2

$$E_b = 2 \left[\int_0^{A'} dA \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta \int_{R_0}^{R_1} R^2 dR + \int_{A'}^{\pi} dA \int_{\Theta}^{\frac{\pi}{2}} \cos\theta \, d\theta \int_{R_0}^{R_1} R^2 dR \right].$$

b) The integration of dark zone

In this case it becomes

$$E_d = 2 \left[\int_0^{A'} dA \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta \int_0^{R_0} R^2 dR + \int_{A'}^{\pi} \left\{ \int_0^{\Theta} \cos\theta \, d\theta \int_0^{R_1} R^2 dR + \int_{\Theta}^{\frac{\pi}{2}} \cos\theta \, d\theta \int_0^{R_0} R^2 dR \right\} \right].$$

§ 7 The horizontal intensity due to secondary scattering when $0 \leqq d \leqq d_1$.

Thus it becomes

$$D_2 = E_b \Delta_b + E_t \Delta_t.$$

§ 8 The integration of E when $d_1 \leqq d \leqq 2d_1$.

a) The integration of bright zone

In this case the integration of E is

$$E_b' = 2 \int_0^{A'} dA \int_0^{\Theta} \cos\theta \, d\theta \int_{R_0}^{R_1} R^2 dR.$$

b) The integration of dark zone.

In this case

$$E_t' = 2 \left[\int_0^{A'} \int_0^{\Theta} \int_0^{R_0} + \int_{A'}^{\pi} \int_0^{\frac{\pi}{2}} \int_0^{R_1} \right].$$

§ 9 The horizontal intensity due to secondary scattering when $d_1 \leqq d \leqq 2d_1$.

From above it becomes

$$D_2' = E_b' \Delta_b + E_t' \Delta_t.$$

§ 10 The domain of T when $2d_1 \leqq d \leqq 4d_1$.

In this case all the portion of the total sky will not send secondary scattering.

The circle in which the shadow side of the cylinder $y_1^2 + z_1^2 = a_0^2$ and G sphere intersect shall be called D circle.

That is, the portion enclosed by a cone, guided by D circle and touching the earth, and total sky will not be indifferent to secondary scattering (ref. Fig. 5).

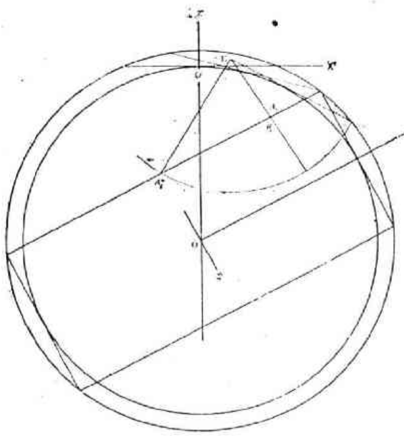

Fig. 5 This figure makes us easy to understand the domain of integration with respect to T when the Sun's dip d is between 2 d_1 and 4 d_1, where d_1 = 6.404.

As for E outside this cone (called G cone), E will send out secondary scattering by receiving primary scattering from T in the portion (hatched portion in Fig. 5) of the solarradiated domain bounded by the cylinder

$$y_1^2 + z_1^2 = a_0^2$$

and the sphere representing the atmospheric upper limit which is outside the cone touching the earth and having E as vertex. And the problem is to determine the region of E of this portion.

G′ cone has the base angle

$$\frac{\pi}{2} - 2d_1 \qquad \text{(here } d_1 > 0\text{)}$$

whose equation is

$$y_1^2+z_1^2=(a_0+(x_1+q(a_0))\mathrm{tg}2d_1)^2.$$

Now let the axes $x_2 y_2 z_2$ be defined by the following relation to XYZ, choosing z_2 axis to OE

	x_2	y_2	z_2
X	$\cos\gamma\cos A$	$-\sin A$	$\sin\gamma\cos A$
Y	$\cos\gamma\sin A$	$\cos A$	$\sin\gamma\sin A$
Z	$-\sin\gamma$	0	$\cos\gamma$,

for simplicity this being exprssed by

	x_2	y_2	z_2
X	l_1	m_1	n_1
Y	l_2	m_2	n_2
Z	l_3	m_3	n_3

Now, let $x_2' y_2' z_2'$ be a new system of coordinates with its centre at E which is parallel to $x_2 y_2 z_2$ system (see the next figure 6 and 7).

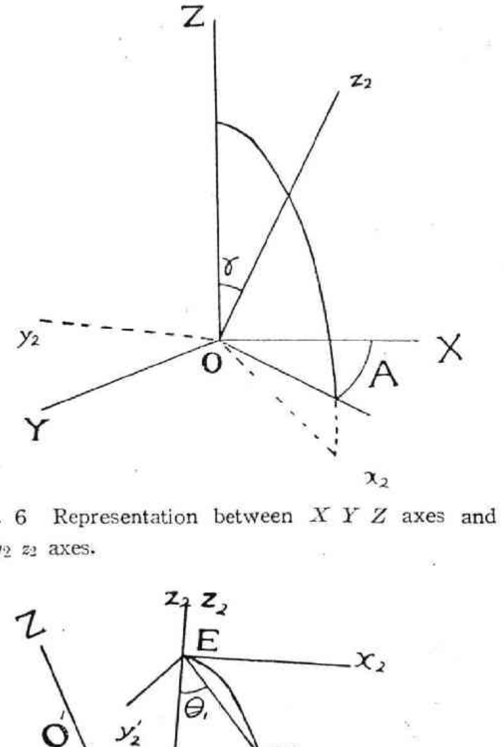

Fig. 6 Representation between $X\ Y\ Z$ axes and $x_2\ y_2\ z_2$ axes.

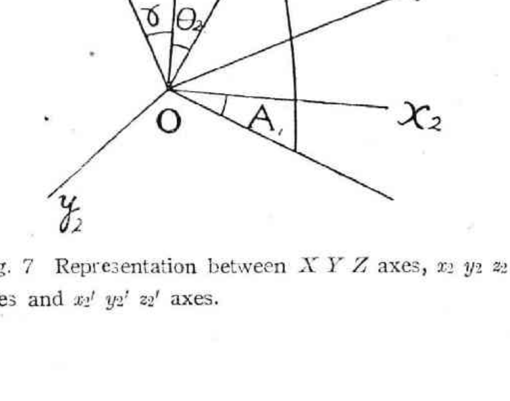

Fig. 7 Representation between $X\ Y\ Z$ axes, $x_2\ y_2\ z_2$ axes and $x_2'\ y_2'\ z_2'$ axes.

Define θ_1 A_1 by

$$x_2'=r\sin\theta_1\cos A_1,\qquad y_2'=r\sin\theta_1\sin A_1,$$
$$z_2'=r\cos\theta_1.$$

In this case the integration of T shall be executed by this polar coordinate.

Let $x_2''\ y_2''\ z_2''$ system be produced by rotating $x_2'\ y_2'\ z_2'$ system by the angle A_1 around z_2' axis.

Find the intersecting point of G circle with $y_2''=0$, by solving both corresponding equations. But there is only a single point because $y_2''=0$ is one sided plane with it's end in OE, and let the value of θ_1 given by this point be θ_1'.

How about then the limits of A_1 ?

Find A_1 by expressing D circle in $x_2'\,y_2'\,z_2'$ system and substituting

$$\theta_1=\theta_0=\sin^{-1}\frac{a_0}{e},\quad r=e\cos\theta_0+a_0\,\mathrm{tg}(+d_1)$$

in it and denote it by A_1', A_1'', here it is so defined that the y_1 value corresponding to A_1'' is greater than that of A_1'. They are the limits required. For the domain from A_1' to A_1'', the value of r, r_1', of the intersected point of the cylinder

$$y_1^2+z_1^2=a_0^2$$

by a line passing through E can be determined by giving θ_1 and A_1.

Then the integration with respect to T for such region of A_1 is

$$\Delta_\delta=\int_{A_1'}^{A_1''}dA_1\int_{\theta_0}^{\theta_1'}\sin\theta_1\,d\theta_1\int_{r_1'}^{r_2}r^2dr\ dS_2.$$

For the rest domain of A_1 there is no integration.

§ 11 The integration of E when $2d_1\leqq d\leqq 3d_1$.

In this case G′ point is outside G cone. Let

$$f(R,\theta,A)=0$$

be the equation of G′ cone expressed in $X'\ Y'\ Z'$ system by the method similar to the above mentioned. we can get R, the distance from O′ to the conical surface, in terms of θ, A. Let it be R_2.

From $f(R,\theta,A)=0$ and $Z'=0$,

that is, by putting $\theta=0$ in $f=0$ and then $R=q(a_0)$, we can get A_0' representing the value A of the intersecting point of horizontal line with the line in which the cone intersects with the horizontal plane. Then for $A>A_0'$, let Θ_0' be the value of θ given by the intersecting point between two sections of the cone and the atmospheric upper limit by a plane containing Z axis whose direction is defined by A, similarly to the above process.

Then the domain of E is

$$E_\delta=2\left\{\int_0^{A_0'}dA\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta+\int_{A_0'}^{\pi}dA\int_{\Theta_0'}^{\frac{\pi}{2}}\cos\theta\,d\theta\right\}\int_{R_2}^{R_1}R^2dR.$$

§ 12 The integration of E when $3d_1\leqq d\leqq 4d_1$.

In this case G′ point is inside G cone. The integration of E will be by using the same notation as in § 11, (but in the condition of using Θ_0' for $0<A<A_0'$).

$$E_\delta'=2\int_0^{A_0'}dA\int_0^{\Theta_0'}\cos\theta\ d\theta\int_{R_2}^{R_1}R^2dR.$$

§ 13 Horizontal intensity due to secondary scattering when $2d_1\leqq d\leqq 3d_1$.

From § 10 and § 11 the intensity will be

$$D_{2\delta}=E_\delta\Delta_\delta.$$

§ 14 Horizontal intensity due to secondary scattering when $3d_1\leqq d\leqq 4d_1$.

From § 10 and § 12 the intensity will be

$$D_{2\delta}'=E_\delta'\Delta_\delta.$$

§ 15 The secondary scattering when $4d_1\leqq d$.

Δ_δ and the domain of E vanish just when d reaches $4d_1$ and the intensity also vanishes.

4 The horizontal intensity of twilight

§ 16 The horizontal intensity when $0 \leqq d \leqq d_1$.

From § 2 and § 7 the intensity will be

$$D = D_1 + D_2.$$

§ 17 The horizontal intensity when $d_1 \leqq d \leqq 2d_1$.

From § 3 and § 9 the intensity will be

$$D' = D_1' + D_2'.$$

§ 18 The horizontal intensity when $2d_1 \leqq d \leqq 3d_1$.

In this case the first scattering will vanish. From § 13 $D_{2\delta}$ is the intensity.

§ 19 The horizontal intensity when $3d_1 \leqq d \leqq 4d_1$.

From § 14 $D_{2\delta}'$ is the intensity.

5 The method of evaluation and its result

The method was executed basing on Chapter 5 of (5), giving full attention to the lower altitude of the sun. Table 1 gives the intensity of first scattering from 1 steradian on each portion of the sky for $d = 6.0$, $d = 6.404$ (in which shadow cylinder passes through G′ point) and $d = 7.0$. Here the figure in the blacket is the power of 10 to be multiplied to the left figure, for example 0.3325 (-3) means 0.3325×10^{-3}. As the above explanation may lead the reader to the misunderstanding as if the intensity were uniform all over that solid angle, so it is good idea to say that the above value multiplied by 10^{-6} is the intensity of the same nature from a solid angle subtended by 1 mm^2 area on sphere with radius 1 meter.

Table 2 is the intensity of secondary scattering from 1 steradian, the figure in the blacket for each A being the same with that of $A = 0$.

Table 3 gives the sum of Table 1 and 2.

In general, secondary scattering is greater than the 1st, and the former becomes far predominant with decreasing p (the trans. coeff.) *i, e,* decreasing wave-length. Several conditions can be easily explained by Table 4 giving the ratio secondary : first scattering.

Table 5 is the radiation falling on a horizontal plane.

However, assuming the pure dry atmosphere there exists a relation between p and the wave-length λ (c. f., Linke: Meteor. Taschenbuch), so the values of λ corresponding $p = 0.9, \cdots 0.6$ are given in the last column of tne table.

The above tables except Table 4 are expressed in unit of the intensity of solar radiation for each of the corresponding wave-lengths received vertically by the upper atmospheric limit of the earth. Consider now the wave-length for $p = 0.6, \quad 0.9$ represents respectively the domain

$0 \sim 0.394\mu$, $0.394\mu \sim 0.419\mu$, $0.419\mu \sim 0.488\mu$, $0.488\mu \sim \infty$.

The contributions of the intensities of above four domains to the total intensity of the sun, calculating from the table by LINKE's book, as follows 0.067, 0.030, 0.104, 0.799.

Summing up these values multiplied by the corresponding figures of Table 5, we can produce Table 6 representing the horizontal intensity in unit of the total intensity of the sun falling normally on the upper limit of the earth's atmosphere.

This is given for example by

$$\frac{1.4\times 10^5}{D^2}\ \text{Lux},\quad \frac{1.940}{D^2}\ \frac{\text{g.cal}}{\text{cm}^2.\text{min}}\ \text{etc.}$$

We can select any one of them as we desire. The horizontal luminosity expressed in Lux of 2nd column from 1st column of Table 6 is in good agreement with the observation by Mr. OOSAWA of 3rd column given in (6).

The energy among the wave-length range

with 200 Å breadth with its centre at each of the above given wave-lengths ($0.532\mu \cdots 0.393\mu$) is respectively 57.0, 58.0, 38.0, 36.3 (10^{-3} g. cal/cm^2.min), and their ratios to the sun's total energy 1.940 g.cal/cm^2. min are respectively 0.0294, 0.0299, 0.0196, 0.0187.

Table 7 is the multiplication of Table 3 and these values. Therefore this is expressed in unit of the sun's total energy, and tells us that the value is greatest at $A = 0$, smallest at $A = 90$ and secondary maximum at $A = 180$ for each θ and λ.

Table 8 is the value of Table 7 in unit of the value for $p = 0.9$ in this table for each θ and A. According to this expression the energy decreases absolutely with decreasing wave-length for every (θ, A), and yet this value (*i.e.* the ratio of energy of p = 0.8, 0.7, 0,6 to that of 0.9) increases exactly with increasing θ for each A and λ, and at $\theta = 0$ the ratio increases with increasing A. But at other θ the ratio has the inclination of increasing with increasing A, but decreasing at $A = 180$ with increasing θ and decreasing λ besides the above one.

Moreover at $\theta = 0$ the ratio decreases with increasing d for each A.

Table 9 is found from Table 3 by the same method of finding Table 6 from Table 5, so the new table is expressed in I_0/D^2 unit. Fig. 8 is the graphical representation of Table 9 multiplied by 10^7 with the curves of isophotens.

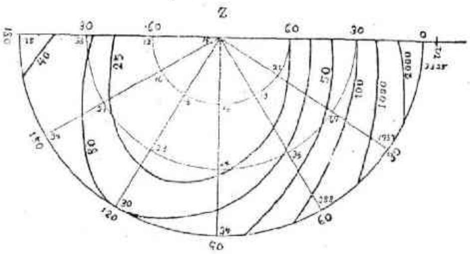

Fig. 8 This figure is the graphical representation of Table 9 and isophotens of the distribution of the twilight intensity when the Sun's dip is 6°. The figures in the semisphere is the intensity from one steradian multiplied by 10^7 in unit of the solar intensity at the upper limit of the atmosphere of the earth (I_0/D^2), S the Sun's position, Z the zenith. The figure on the line passing SZ is altitude on the sky dome, and that on the semi-arc (horizontal line) the azimuth from the Sun. The tough curves are isophotens accompanied by their values indicated by tough letters.

In conclusion the writer expresses his thanks to the Ministry of Education for its financial aid and Professors of the Department of Geophysics and Astronomy of Faculty of Science of Tôhoku University.

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Table 1.

This gives the intensity of first scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, for example 0.3325 (−3) means 0.3325×10^{-3}. θ is the altitude and A the azimuth from the Sun, p the transmission's coefficient. The unit is the intensity of solar radiation received vertically at the upper atmospheric limit of the earth, for each wave-length corresponding to p in table.

$d = 6.0$

θ	$p \diagdown A$	0	30	60	90	120	150	180
0	0.9	0.3325(-3)	0.1896(-3)	0.3652(-4)	0.5978(-7)	No	No	No
	0.8	0.5751(-5)	0.3448(-5)	0.6250(-6)	0.1567(-9)			
	0.7	0.6159(-7)	0.3666(-7)	0.6069(-8)	0.2622(-12)			
	0.6	0.3198(-9)	0.1853(-9)	0.2808(-10)	0.8119(-16)			
30	0.9	0.6270(-5)	0.4221(-5)	0.1501(-5)	0.3386(-6)	0.2987(-7)	No	No
	0.8	0.1917(-5)	0.1249(-5)	0.2878(-6)	0.3865(-7)	0.1128(-8)		
	0.7	0.7161(-6)	0.4351(-6)	0.6996(-7)	0.3627(-8)	0.2612(-10)		
	0.6	0.2246(-6)	0.1205(-6)	0.1085(-7)	0.2000(-9)	0.2872(-12)		
60	0.9	0.7038(-6)	0.5965(-6)	0.3727(-6)	0.2088(-6)	0.1397(-6)	0.8975(-7)	0.7330(-7)
	0.8	0.1361(-6)	0.1127(-6)	0.6087(-7)	0.2860(-7)	0.1443(-7)	0.6704(-8)	0.4713(-8)
	0.7	0.2637(-7)	0.1951(-7)	0.8852(-8)	0.3022(-8)	0.1138(-8)	0.3542(-9)	0.2096(-9)
	0.6	0.3508(-8)	0.2342(-8)	0.8236(-9)	0.1949(-9)	0.5223(-10)	0.9555(-11)	0.5733(-11)
90	0.9	0.8295(-7)						
	0.8	0.8104(-9)						
	0.7	0.3924(-11)						
	0.6	0.1056(-13)						

$d = 6.4$

θ	$p \diagdown A$	0	30	60
0	0.9	0.2541(-3)	0.1535(-3)	0.2436(-4)
	0.8	0.4485(-5)	0.2770(-5)	0.4024(-6)
	0.7	0.4876(-7)	0.3000(-7)	0.9787(-8)
	0.6	0.2527(-9)	0.1531(-9)	0.2334(-10)
30	0.9	0.1346(-5)	0.8144(-6)	0.8779(-7)
	0.8	0.2567(-6)	0.1306(-6)	0.2027(-8)
	0.7	0.4422(-7)	0.1840(-7)	0.3132(-10)
	0.6	0.5247(-8)	0.1641(-8)	0.2491(-12)
60	0.9	0.5993(-7)	0.4385(-7)	0.1912(-7)
	0.8	0.4002(-8)	0.2930(-8)	0.9421(-9)
	0.7	0.1851(-9)	0.1254(-9)	0.2810(-10)
	0.6	0.4344(-11)	0.3233(-11)	0.4290(-12)

$d = 7.0$

θ	$p \diagdown A$	0	30	60
0	0.9	0.1726(-3)	0.8915(-4)	0.1208(-4)
	0.8	0.3139(-5)	0.1696(-5)	0.1793(-6)
	0.7	0.3398(-7)	0.1801(-7)	0.1413(-8)
	0.6	0.1704(-9)	0.8798(-10)	0.4641(-11)
30	0.9	0.9784(-9)		
	0.8	0.5308(-12)		
	0.7	0.7838(-16)		
	0.6	0.2428(-20)		

Table 2.

This is the intensity of secondary scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, (the value for each A being the same to that $A = 0$.) θ is the altitude and A the azimuth from the Sun, p the trans. coeff.

The unit is the intensity of solar radiation received vertically at the upper atmospheric limit of the earth, for each wave-length corresponding to p in table.

$d = 6.0$

θ	$p \diagdown A$	0	30	60	90	120	150	180
0	0.9	822(-7)	295	118	041	036	041	053
	0.8	491(-8)	238	112	062	074	113	226
	0.7	543(-9)	376	272	233	249	308	406
	0.6	170(-9)	134	108	095	097	118	165
30	0.9	520(-8)	344	256	233	251	297	386
	0.8	403(-8)	262	200	169	171	199	266
	0.7	273(-8)	168	122	103	104	119	149
	0.6	192(-8)	133	092	066	060	062	086
60	0.9	205(-8)	176	161	151	151	163	193
	0.8	160(-8)	132	121	119	120	124	142
	0.7	106(-8)	091	083	080	078	080	090
	0.6	756(-9)	646	587	561	565	578	604

$d = 6.4$

θ	$p \diagdown A$	0	30	60	90	120	150	180
0	0.9	543(-7)	231	091	027	013	013	036
	0.8	238(-8)	103	041	018	015	018	029
	0.7	124(-9)	077	050	032	031	038	056
	0.6	130(-10)	092	072	063	070	087	111
30	0.9	368(-8)	258	195	175	190	224	289
	0.8	266(-8)	187	141	126	137	160	197
	0.7	161(-8)	111	081	065	064	078	106
	0.6	102(-8)	061	047	046	047	048	054
60	0.9	150(-8)	131	118	114	112	120	134
	0.8	116(-8)	095	090	089	091	096	107
	0.7	749(-9)	661	603	582	589	621	676
	0.6	508(-9)	435	400	387	390	409	439

		d = 7.0						
θ	p\A	0	30	60	90	120	150	180
0	0.9	270(-7)	100	042	019	014	017	022
	0.8	104(-8)	027	108	006	006	007	009
	0.7	267(-10)	134	068	031	029	034	055
	0.6	477(-12)	294	232	220	239	288	383
30	0.9	253(-8)	181	131	115	120	144	182
	0.8	173(-8)	121	096	081	084	095	116
	0.7	981(-9)	576	480	461	478	526	589
	0.6	566(-9)	362	281	255	258	267	281
60	0.9	972(-9)	825	778	778	830	935	902
	0.8	744(-9)	697	664	644	642	653	677
	0.7	437(-9)	427	420	415	408	406	405
	0.6	288(-9)	268	253	246	237	232	231

θ	p\d	6.0	6.4	7.0
90	0.9	143(-8)	102(-8)	656(-9)
	0.8	118(-8)	798(-9)	507(-9)
	0.7	844(-9)	529(-9)	318(-9)
	0.6	626(-9)	365(-9)	198(-9)

Table 3.

This gives the sum of table 2 and 3, *i.e.* the intensity of total scattering from one steradian on each portion of the sky dome for three dips (d) of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, the value for each A being the same to that of A = 0. θ is the altitude and A the azimuth from the Sun, p the trans. coeff. The unit is the intensity of solar radiation received vertically at the earth's upper atmospheric limit, for each wave-length corresponding to p in table.

d	θ	p\A	0	30	60	90	120	150	180
6.0	0	0.9	0.4148(-3)	0.2191	0.0483	0.0042	0.0036	0.0041	0.0053
		0.8	1.0665(-5)	0.5828	0.1745	0.0615	0.074	0.113	0.2267
		0.7	0.6047(-6)	0.4127	0.2781	0.2326	0.249	0.308	0.4058
		0.6	0.1703(-6)	0.1332	0.1080	0.0948	0.097	0.118	0.1653
	30	0.9	1.148 (-5)	0.7661	0.4061	0.2661	0.2540	0.2970	0.3857
		0.8	0.5944(-5)	0.3869	0.2288	0.1724	0.1709	0.199	0.2664
		0.7	0.3446(-5)	0.2115	0.1290	0.1029	0.1040	0.119	0.1487
		0.6	0.2147(-5)	0.1454	0.0931	0.0658	0.0600	0.062	0.0858
	60	0.9	0.2754(-5)	0.2356	0.1983	0.1721	0.1648	0.1723	0.1999
		0.8	0.1734(-5)	0.1433	0.1269	0.1222	0.1214	0.1247	0.1426
		0.7	0.1085(-5)	0.0930	0.0842	0.0798	0.0781	0.080	0.0904
		0.6	0.7590(-6)	0.6483	0.5878	0.5607	0.5651	0.578	0.6038
6.4	0	0.9	0.3085(-3)	0.1766	0.1154	0.00265	0.0013	0.0013	0.0036
		0.8	0.6863(-5)	0.3800	0.0812	0.0176	0.015	0.018	0.029
		0.7	0.1724(-6)	0.1070	0.0598	0.0320	0.031	0.038	0.056
		0.6	0.1323(-7)	0.0935	0.0722	0.0632	0.070	0.087	0.1105
	30	0.9	0.5021(-5)	0.3394	0.2038	0.1748	0.190	0.224	0.2889
		0.8	0.2921(-5)	0.2001	0.1412	0.1257	0.137	0.160	0.1969
		0.7	0.1654(-5)	0.1128	0.0810	0.0653	0.064	0.078	0.1058
		0.6	0.1024(-5)	0.0612	0.0470	0.0463	0.047	0.048	0.0544
	60	0.9	0.1564(-5)	0.1354	0.1199	0.1137	0.112	0.120	0.1338
		0.8	0.1165(-5)	0.0953	0.0901	0.0892	0.091	0.096	0.1069
		0.7	0.7492(-6)	0.6611	0.6030	0.5821	0.589	0.621	0.6761
		0.6	0.5084(-6)	0.4350	0.4000	0.3873	0.390	0.409	0.4386
7.0	0	0.9	0.1996(-3)	0.1000	0.0163	0.0019	0.0014	0.0017	0.0022
		0.8	0.4174(-5)	0.1970	0.0287	0.0059	0.0055	0.0068	0.0091
		0.7	0.6069(-7)	0.3141	0.0821	0.0312	0.029	0.034	0.0551
		0.6	0.6415(-9)	0.382	0.2366	0.2196	0.239	0.288	0.3832
	30	0.9	0.2529(-5)	0.181	0.131	0.1145	0.120	0.144	0.1822
		0.8	0.1727(-5)	0.121	0.096	0.0808	0.084	0.095	0.1158
		0.7	0.9810(-6)	0.576	0.480	0.4613	0.478	0.526	0.5894
		0.6	0.5657(-6)	0.362	0.281	0.2554	0.258	0.267	0.2810
	60	0.9	0.9719(-6)	0.855	0.799	0.7780	0.796	0.840	0.9016
		0.8	0.7443(-6)	0.697	0.664	0.6442	0.642	0.653	0.6765
		0.7	0.4367(-6)	0.427	0.420	0.4146	0.408	0.406	0.4046
		0.6	0.2876(-6)	0.268	0.253	0.2460	0.237	0.232	0.2312

θ	$p\diagdown d$	6.0	6.4	7.0
90	0.9	0.1518(-5)	0.1017(-5)	0.6558(-6)
	0.8	0 1182(-5)	0.7982(-6)	0.5071(-6)
	0.7	0.8440(-6)	0.5287(-6)	0.3183(-6)
	0.6	0.6260(-6)	0.3645(-6)	0.1977(-6)

Table 4.

This gives the ratio of the value of first scattering to the secondary scattering. θ is the altitude and A the azimuth from the Sun, p the trans. coeff., d the Sun's dip.

d	θ	$p\diagdown A$	0	30	60	90	120	150	180
6.0	0	0.9	0.24	0.16	0.32	68.56			
		0.8	0.85	0.69	1.79	3924.7	∞	∞	∞
		0.7	0 88	10.27	44.81	887109			
		0.6	530.0	717.7	3846.2	0.117×10^{10}			
	30	0.9	0.83	0.81	1.70	6.86	83.94		
		0.8	2.10	2.10	6.96	43.65	1514.2	∞	∞
		0.7	3.80	3.86	17.43	282.6	39816.2		
		0.6	8.50	11.06	85.19	3290.0	0.21×10^{7}		
	60	0.9	2.91	2.95	4.31	7.24	10.79	18.20	26.28
		0.8	11.71	11.71	19.83	41.71	83.19	185.0	301.51
		0.7	40.16	46.66	94.12	262.98	685.4	2258.6	4312.98
		0.6	215.4	276.1	712·4	2874.3	10817.5	60492.0	105320.1
	90	0.9	17.26						
		0.8	1457.3						
		0.7	21508.7						
		0.6	0.592×10^{8}						
6.4	0	0.9	0.21	0.15	0.37				
		0.8	0.53	0.37	1.02				
		0.7	2.53	2.57	5.10				
		0.6	51.36	60.07	309.0				
	30	0.9	2.73	3.17	22.21				
		0.8	10.38	14.32	695.6				
		0.7	36.40	60.30	25862.1				
		0.6	194.2	371.7	0.19×10^{7}				
	60	0.9	25.10	29.88	61.7				
		0.8	290.1	324.4	955.0				
		0.7	4048.0	5272.0	21459.0				
		0.6	117035.0	134563.5	0.93×10^{6}				

Table 5.

This is the radiation due to total scattering falling on a horizontal plane, in unit of the intensity of solar radiation received vertically at the earth's upper atmospheric limit, for each wave-length (λ) corresponding to the trans. coeff. p in the table. d is the dip of the Sun. The figure in the blacket is the power of 10 to be multiplied to the left figure.

$p\diagdown d$	6.0	6.4	7.0	λ (in μ).
0.9	1.0178(-5)	0.5900(-5)	0.3648(-5)	0.5320
0.8	0.6152(-5)	0.4088(-5)	0·2644(-5)	0.4429
0.7	0.3680(-5)	0.2360(-5)	0.1500(-5)	0·3955
0.6	0.2421(-5)	0.1490(-5)	0.0866(-5)	0.3932

Table 6.

This is the intensity falling on a horizontal plane. First column is the calculated value in unit of the total intensity all over the wave-length of the Sun falling normally on the upper limit of the earth's atmosphere.

Second column is the author's calculation in unit of Lux, and the 3rd column is the observed value by OOSAWA in the same unit.

d is the Sun's dip. The figure in the blacket is the power of 10 to be multiplied to the left figure.

d	6.0	6.4	7.0
	0.90477(-5)	0.53112(-5)	0.32939(-5)
	1.27	0.74	0.46
	1.36	0.75	0.43

Table 7.

This is the intensity of total scattering for 200 Å breadth with its centre at each wave-length corresponding to the value of p in table from one steradian in unit of the sun's total intensity for all over the wave length falling normally at the earth's upper atmospheric limit. d is the sun's dip, θ the altitude, A the azimuth from the sun, p the trans. coeff. The figure in the blacket is the power of 10 to be multiplied to the left figure. The former value for each A being the same to that of $A = 0$.

d	θ	$p \diagdown A$	0	90	180
6.0	0	0.9	1.2187(-5)	0.0123	0.0156
		0.8	0.3189(-6)	0.0184	0.0676
		0.7	0.1185(-7)	0.0456	0.0795
		0.6	0.3185(-8)	0.1773	0.3091
	30	0.9	0.3373(-6)	0.0782	0.1133
		0.8	0.1777(-6)	0.0515	0.0797
		0.7	0.6754(-7)	0.2017	0.2914
		0.6	0.4015(-7)	0.1230	0.1604
	60	0.9	0.8091(-7)	0.5056	0.5873
		0.8	0.5185(-7)	0.3654	0.4264
		0.7	0.2127(-7)	0.1564	0.1772
		0.6	0.1419(-7)	0.1049	0.1129
6.4	0	0.9	0.9064(-5)	0.0078	0.0106
		0.8	0.2052(-6)	0.0053	0.0087
		0.7	0.3379(-8)	0.0627	0.1098
		0.6	0.2474(-9)	0.1182	0.2066
	30	0.9	0.1475(-6)	0.0514	0.0849
		0.8	0.8734(-7)	0.3758	0.5887
		0.7	0.3242(-7)	0.1280	0.2074
		0.6	0.1915(-7)	0.0866	0.1017
	60	0.9	0.4595(-7)	0.3341	0.3931
		0.8	0.3483(-7)	0.2667	0.3196
		0.7	0.1468(-7)	0.1141	0.1325
		0.6	0.9507(-8)	0.7243	0.8202
7.0	0	0.9	0.5864(-5)	0.0056	0.0065
		0.8	0.1248(-6)	0.0018	0.0027
		0.7	0.1190(-8)	0.0061	0.0108
		0.6	0.1200(-10)	0.0411	0.0717
	30	0.9	0.7430(-7)	0.3364	0.5353
		0.8	0.5164(-7)	0.2416	0.3462
		0.7	0.1923(-7)	0.0904	0.1155
		0.6	0.1058(-7)	0.0478	0.0525
	60	0.9	0.2855(-7)	0.2286	0.2649
		0.8	0.2225(-7)	0.1926	0.2023
		0.7	0.8559(-8)	0.8126	0.7930
		0.6	0.5378(-8)	0.4600	0.4323

θ	$p \diagdown d$	6.0	6.4	7.0
90	0.9	0.4460(-7)	0.2988	0.1927
	0.8	0.3534(-7)	0.2337	0.1516
	0.7	0.1654(-7)	0.1036	0.0624
	0.6	0.1171(-7)	0.0682	0.0370

Table 8.

This is the intensity of total scattering for 200 Å breadth with its centre at each wave-length corresponding to the value of p in table from one steradian in unit of this value for $p = 0.9$.

d is the sun's dip, θ the altitude, A the azimuth from the sun, p the trans. coeff.

d	θ	$p \diagdown A$	0	90	180
6.0	0	0.9	1	1	1
		0.8	0.02617	0.1496	0.4333
		0.7	0.00097	0.03707	0.0510
		0.6	0.00026	0.01441	0.0198
	30	0.9	1	1	1
		0.8	0.5268	0.6586	0.7034
		0.7	0.2002	0.2579	0.2572
		0.6	0.1190	0.1573	0.1416
	60	0.9	1	1	1
		0.8	0.6409	0.7227	0.7260
		0.7	0.2629	0.3093	0.3017
		0.6	0.1754	0.2075	0.1922
6.4	0	0.9	1	1	1
		0.8	0.02264	0.06795	0.0821
		0.7	0.00037	0.00804	0.0104
		0.6	0.000027	0.00152	0.00195
	30	0.9	1	1	1
		0.8	0.5921	0.7309	0.6934
		0.7	0.2198	0.2490	0.2443
		0.6	0.1298	0.1684	0.1198
	60	0.9	1	1	1
		0.8	0.7580	0.7982	0.8130
		0.7	0.3195	0.3415	0.3371
		0.6	0.2069	0.2168	0.2087
7.0	0	0.9	1	1	1
		0.8	0.02128	0.03214	0.04153
		0.7	0.00020	0.00109	0.00166
		0.6	0.205×10^{-5}	0.00007	0.00011
	30	0.9	1	1	1
		0.8	0.6951	0.7182	0.6467
		0.7	0.2588	0.2687	0.2158
		0.6	0.1424	0.1421	0.0981
	60	0.9	1	1	1
		0.8	0.7793	0.8425	0.7637
		0.7	0.2998	0.3555	0.2994
		0.6	0.1884	0.2012	0.1632

θ	$p \diagdown d$	6.0	6.4	7.0
90	0.9	1	1	1
	0.8	0.7923	0.7989	0.7867
	0.7	0.3708	0.3467	0.3238
	0.6	0.2625	0.2282	0.1920

Table 9.

This is the intensity of the total scattering from one steradian on each portion of the sky dome for the dip d = 6.0 of the sun, in unit of the total intensity of the sun all over the wave-length falling normally on the earth's upper atmospheric limit. θ the altitude, A the azimuth from the sun. The figure in the blacket is the power of 10 to be multiplied to the left figure, the former for every A being the same to that of A = 0.

d = 6.0							
$\theta \backslash A$	0	30	60	90	120	150	180
0	0.3325(-3)	0.1757	0.0388	0.0034	0.0030	0.0034	0.0045
30	0.1004(-4)	0.0668	0.0358	0.0238	0.0228	0.0266	0.0346
60	0.2464(-5)	0.2103	0.1781	0.1564	0.1504	0.1569	0.1813
90	0.1403(-5)						