Evaporation by Natural Convection.

EVAPORATION BY NATURAL CONVECTION

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§ Introduction.

The rate of evaporation in still air is larger than that calculated by molecular diffusion theory. It will be mainly by natural convection due to cooling of water surface by evaporation. If the problem of natural convection is solved in the case of a heated flat plate, one can by analogy between heat transfer and evaporation obtain the solution for the rate of evaporation from a flat evaporating surface. Actually on heat transfer the solution is only obtained at present in the two-dimensional case of a vertical plate and so the corresponding solution for evaporation would be of little practical use in itself. But it is known experimentally that the heat transfer from a horizontal plate is expressed by the same law except with some different numerical coefficient, so that the theoretical formula for the case of evaporation will also have some physical meaning.

§ Calculation of the Rate of Evaporation by Natural Convection from a Vertical Evaporating Surface.

We take x-axis vertically, y-axis horizontally and the evaporating surface on x-axis as shown in Fig. 1. Then the differential equations for the two-dimensional case will be as follows. The equation of motion will be

$$\frac{\partial\Phi}{\partial y}\frac{\partial^2\Phi}{\partial x\partial y} - \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial y^2} = \nu\frac{\partial^3\Phi}{\partial y^3} + g\beta\Theta, \qquad \cdots\cdots\cdots\cdots\cdots (1)$$

and the equation of heat transfer from the evaporating surface will be

$$\frac{\partial\Phi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\Phi}{\partial x}\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}, \quad \cdots (2)$$

and the equation of diffusion will be

$$\frac{\partial\Phi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\Phi}{\partial x}\frac{\partial C}{\partial y} = d\frac{\partial^2 C}{\partial y^2}, \quad \cdots\cdots (3)$$

where Φ = stream function, ν = kinematic viscosity, β = expansion coefficient of air, g = acceleration of gravity, $\Theta = \theta - \theta_\infty$, $T = \frac{\Theta}{\Theta_0} = \frac{\theta-\theta_\infty}{\theta_0-\theta_\infty}$, θ_0 = temperature of water surface, θ, θ_∞ = air temperatures near the surface and at infinity respectively, $C = \frac{C-C_\infty}{C_0-C_\infty}$, C_0, C, C_∞ = respectively vapour concentrations at the surface, near the surface and at infinity, a = coefficient of heat transfer, α = diffusivity. The boundary conditions are:

$$u = \frac{\partial\Phi}{\partial y} = 0,\ v = -\frac{\partial\Phi}{\partial x} = 0,\ T = 1,$$
$$C = 1 \text{ at } y = 0,$$
$$u = 0,\ v = \text{const.},\ T = 0, C = 0 \text{ at } y = \infty.$$

Now we introduce the new variable ξ defined by

$$\xi = \left(\frac{g\beta\Theta}{4\nu^2}\right)^{\frac14}\frac{y}{x^{\frac14}} = A\frac{y}{x^{\frac14}} = \frac{y}{x}\left(\frac{Gr}{4}\right)^{\frac14},$$

where $Gr = \frac{x^3 g\beta\Theta}{\nu^2}$ (GRASHOF number), then

$$\Phi(x,y) = 4\nu A x^{\frac34}\zeta(\xi),$$
$$T(x,y) = t(\xi),$$
$$C(x,y) = \eta(\xi),$$

where ζ, t and η satisfy following differential equations,

$$\zeta''' + 3\zeta\zeta'' - 2\zeta'^2 + t = 0, \quad \cdots\cdots\cdots\cdots (4)$$

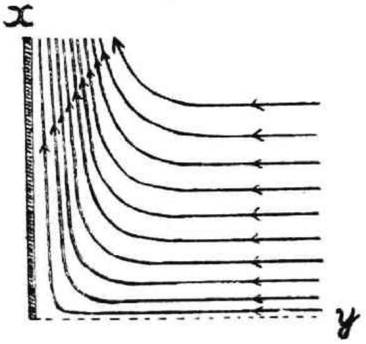

Fig. 1

$$t'' + 3\frac{\nu}{d}\zeta t' = 0, \quad \cdots\cdots (5)$$

$$\eta'' + 3\frac{\nu}{d}\zeta \eta' = 0. \quad \cdots\cdots (6)$$

Boundary conditions are now

$$\zeta = 0,\ \zeta' = 0,\ t = 1,\ \eta = 1 \quad \text{at } \xi = 0,$$

$$\zeta' = 0,\ \zeta'' = 0,\ t = 0,\ \eta = 0 \quad \text{at } \xi = \infty.$$

(4), (5) and (6) can be solved by expanding ζ, t and η in series of ξ. The calculations are to some extent troublesome, but if we assume that the flow due to convection is "slow", by the analogy to the relation which M. ten BOSCH[1] obtained in the case of heat transfer, we can get the relation for evaporation, *i.e.*,

$$Nu'_m = 0.525\,(Gr \times Pr')^{\frac{1}{4}}, \quad \cdots\cdots (7)$$

where $Nu'_m = \frac{x\delta_m}{d}$ (=mean NUSSELT number for evaporation), δ_m = mean coefficient of evaporation, (which is defined by the formula, $E = \delta_m(C_0 - C_\infty)$, where E is the rate of evaporation), x = length of the surface, $Gr = \frac{x^3 g\beta\Theta_0}{\nu^2}$ (= GRASHOF number), $Pr' = \frac{\nu}{d}$ (= PRANDTL number for evaporation). (7) is then written as follows:

$$\frac{x\delta_m}{d} = 0.525\left(\frac{x^3 g\beta\Theta_0}{\nu d}\right)^{\frac{1}{4}}. \quad \cdots\cdots (8)$$

Hence by definition for δ_m

$$\frac{E}{C_0 - C_\infty} = 0.525\, d\left(\frac{g\beta\Theta_0}{x\nu d}\right)^{\frac{1}{4}}. \quad \cdots\cdots (9)$$

(9) gives the rate of evaporation for the vertical evaporating surface. According to M. ten BOSCH the rate of heat transfer from a horizontal disk or plate is given by the formula which has the same form as for the heat transfer from the vertical plate, but in which the radius or half of the length of the plate is taken as x instead of the total length in the case of the vertical plate. Therefore by analogy in the case of evaporation from a horizontal disk or plate (9) will be available in which the radius or half of the length of the plate must be taken as x.

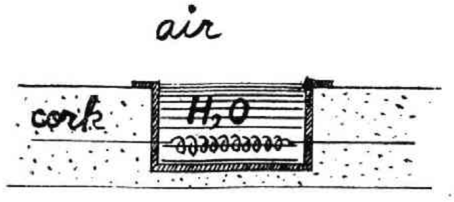

Fig. 2

§ Experiment and Discussion.

Measurement of the rate of evaporation was carried out by employing a glass vessel whose radius is 1.83 cm and depth 2 cm. A nichrome wire was inserted in the vessel as shown in Fig. 2, and by heating it experiment was carried out in the range of $\theta_0 - \theta_\infty$ between 0.1 and 50°C. The rate of evaporation was measured by weighing the water loss by chemical balance whose sensitivity is 1 mg. The vessel was covered by a glass lid to prevent evaporation during weighing. The vessel was also surrounded by cork to protect heat exchange to the surroundings. The temperature of water surface was measured by a thermojunction of copper-constantan, and the vapour pressure and air temperature were measured by ASSMANN's psychrometer before and after the weighing. Experiment was

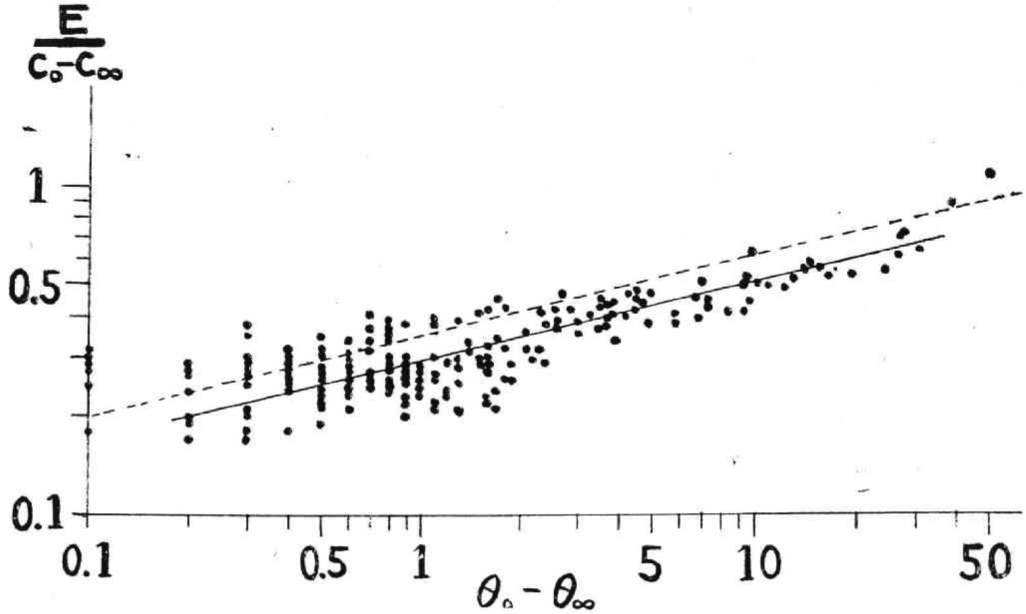

Fig. 3

carried out in a closed chamber at night to avoid the variation of air temperature and vapour pressure as far as possible.

The result of measurement was shown in Fig. 3, in which abscissa expresses $\theta_0-\theta_\infty$ and ordinate $\frac{E}{C_0-C_\infty}$, each in logarithmic scale. The unit of E is $\mathrm{gcm^{-2}\,sec^{-1}}$, and that of C_0 or C_∞ $\mathrm{g\ cm^{-3}}$. The full line in the figure shows

$$\frac{E}{C_0-C_\infty}=0.298\ (\theta_0-\theta_\infty)^{\frac{1}{4}}, \quad \cdots\cdots\cdots (10)$$

which expresses the mean value of the measurement, and the dotted line shows the theoretical value for vertical surface, *i. e.*,

$$\frac{E}{C_0-C_\infty}=0.359\ (\theta_0-\theta_\infty)^{\frac{1}{4}}, \quad \cdots\cdots\cdots (11)$$

which is obtained from (9) by inserting following numerical data corresponding to $\theta_\infty=20°\mathrm{C}$;

$\nu=0.153,\ d=0.257,\ \beta=0.367\times10^{-2}$ and $x=1.83\ \mathrm{cm}$.

It will be shown from the result that the measured rate of evaporation from the horizontal disk by natural convection is about 21% smaller than that predicted by the above theory.

In the case of heat transfer from a horizontal surface by natural convection, HENCKY's experimental result(2) agrees with the theoretical calculation by M. ten BOSCH, whereas WEISE's experimental result(3) shows about 20% smaller rate of heat transfer than BOSCH's calculation.

Our experimental result on evaporation resembles with WEISE's result on heat transfer.

References.

(1) M. ten BOSCH, "*Wärmeübertragung*".
(2) K. HENCKY. *Oldenburg* (1921).
(3) R. WEISE, *Forschung*, 6, 281 (1935).