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ON THE RATE OF EVAPORATION OF SMALL WATER DROPS

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§ 1. Introduction.

Generally the surface of water has been considered to be saturated with water vapour at the temperature of the surface of water. However, in 1938, Frössling (1) made some discussion about the effect of non-saturation of water surface on the rate of evaporation of a water drop. And recently Tsuji (2) has derived a new formula for the rate of evaporation of a water drop assuming that the surface of water is not saturated. His formula gives slower rate of evaporation than those of Frössling (1) and Ogiwara (3), when the radius of the drop becomes small. For instance, when the radius is 5μ the ratio of the rate of evaporation by Tsuji's formula to that of Frössliug is 1/2, and when the radius is 1μ the ratio becomes 1/5. Whereas the experimental investigations on the rate of evaporation of drops, carried out until present (4) (5) (1) were all concerned with relatively large drops to decide whether Tsuji's theory is correct or not. Therefore in the present investigation we measured in the microscopic field the rates of evaporation of small water drops whose radii are smaller than 25μ to examine the problem.

For the rate of evaporation of a water drop in still air, the following formula is well known,

$$\frac{dm}{dt}=-\frac{4\pi DM}{RT}(p_s-p_\infty)a, \quad \cdots\cdots(1)$$

where $-\dfrac{dm}{dt}$ = the rate of evaporation,

m = mass of the drop, a = radius of the drop, D = diffusion coefficient of water vapour through air, M = molecular weight of water molecule, R = absolute gas constant, p_s = saturation vapour pressure at the temperature of the drop, p_∞ = vapour pressure in the air remote from the drop.

On the other hand, Tsuji's formula, derived under the assumption that the surface of water is not saturated, gives the following rate in still air,

$$\frac{dm}{dt}=-\frac{4\pi DM}{RT\left(1+\dfrac{D}{Ka}\right)}(p_s-p_\infty)a, \quad \cdots\cdots(2)$$

where $K=\dfrac{\beta}{1-\dfrac{\beta}{2}}\sqrt{\dfrac{RT}{2\pi M}}$, β = coefficient of condensation of water molecule.

If the time required to evaporate for the drop from radius a_1 to a_2 be expressed by t_{12}, putting $m=\dfrac{4}{3}\pi a^3$, we have from (1),

$$t_{12}=-\int_{a_1}^{a_2}\frac{RT}{DM(p_s-p_\infty)}a\,da$$

$$=\frac{RT}{2DM(p_s-p_\infty)}(a_1^2-a_2^2), \quad \cdots\cdots(3)$$

and from (2),

$$t_{12}=-\int_{a_1}^{a_2}\frac{RT}{DM(p_s-p_\infty)}\left(1+\frac{D}{Ka}\right)a\,da$$

$$=\frac{RT}{2DM(p_s-p_\infty)}(a_1^2-a_2^2)$$

$$+\frac{RT}{KM(p_s-p_\infty)}(a_1-a_2), \quad \cdots\cdots(4)$$

when the radius of the drop is large, t_{12} depends mainly on the term of $(a_1^2-a_2^2)$, and both (3) and (4) give practically same value of t_{12}, but when the radius of the drop becomes small, the term of (a_1-a_2) turns into effective and t_{12} by (4) becomes longer than that by (3).

§ 2. Experiment and Discussion.

At first water drops were attached to fine hairs of cat or rabbit by a sprayer. But even the finest hair was selected as far as possible, its diameter was about 3 μ, which was a little bigger for our purpose. Moreover the spherical drop, initially attached to the hair, often became spindle shaped in the course of evaporation. So we gave up to use hairs and after some groping, fine quartz wires, whose diameters were 1~2 μ, were employed finally. The water drops attached to the cleaned quartz wire has remained for the most part spherically during evaporation. The arrangement of the apparatus was shown in Fig. 1. The experiment was carried out in the closed dark room to avoid variations of temperature and humidity and also to remove the disturbing influence of wind. The water drop was attached to the quartz wire by a sprayer and the wire was hung vertically in the microscopic field. The diameter of the drop was measured by comparing it with the scale inserted in the microscopic field up to 1 μ and the time was measured with a stop-watch up to 0.5 sec.

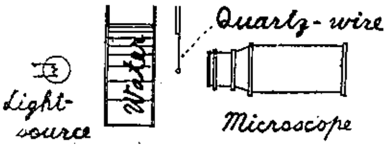

Fig. 1

The temperature and the humidity in the dark room were measured with an Assmann's aspiration psychrometer. The result of experiment was shown in Table 1 and in Fig. 2. In Fig. 2 ordinate indicates a_2^2 and abscissa time. It is seen in Fig. 2 that the experimental curves are not straight lines. Therefore assuming that $t=Aa_2^2+Ba_2$, where the origin of t is now removed to the respective times at which respective drops disappeared by evaporation and the direction of the time axis is reversed, the values of A and B were determined from the experimental values of Table 1, and were shown in Table 2.

Table 1

Radius (μ)	Time (sec.)					
	I	II	III	IV	V	VI
25.0		0.0				
22.5	0.0	6.0				
20.0	4.0	10.5	0.0			
17.5	7.0	15.0	4.0			
15.0	9.5	19.0	8.0	0.0	0.0	3.0
12.5	12.0	22.5	11.0	2.5	3.0	6.0
10.0	13.5	25.5	14.5	5.0	5.0	9.0
7.5	15.5	28.0	17.5	7.0	8.0	12.0
5.0	17.0	30.5	21.0	9.0	9.5	15.0
2.5	18.5	33.0	22.5	10.5	11.5	17.0
0.0	20.0	35.0	23.0	11.5	12.0	17.5
d.b. temp.	5.8°C	5.8°C	6.2°C	5.6°C	5.6°C	6.0°C
w.b. temp	3.8°C	3.0°C	4.6°C	3.7°C	3.6°C	4.3°C

Then on referring (4), and putting $R=8.31\times10^7\,\mathrm{erg/K^\circ}$, $M=18.0$, we have

$$t_{12}=1.74\times10^{-5}\frac{T}{D(p_s-p_\infty)}(a_1^2-a_2^2)$$
$$+4.04\times10^{-4}\frac{\sqrt{T}}{\alpha(p_s-p_\infty)}(a_1-a_2),\cdots\cdots(5)$$

where vapour pressure is measured in mm-Hg and the radius of the drop in μ. More-

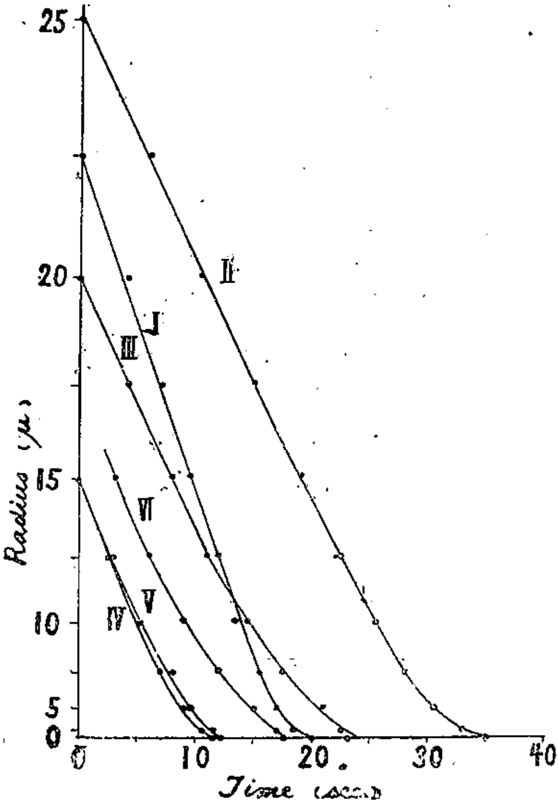

Fig. 2

Table 2.

	I	II	III	IV	V	VI	mean	
A	0.019	0.029	0.035	0.034	0.030	0.033		
B	0.44	0.66	0.47	0.32	0.35	0.44		
p_s-p_∞	1.09	0.71	0.59	0.61	0.69	0,62		calculated values
α	0.014	0.014	0.024	0.034	0.028	0.025	0.023	
p_s-p_∞	1.23	0.90	0.75	0.85	0.89	0.78		measured values
α	0.012	0.011	0.019	0.025	0.022	0.020	0.018	

over we may reasonably assume as T the wet bulb temperature, because the mass, and accordingly, heat capacity, of the drop is very small, and as the diffusion coefficient D, the values advocated by Montgomery$^{(6)}$ (see Taple 3). Then we can calculate p_s-p_∞ and α from our experimental result, and they are listed in Table 2. Further the values of p_s-p_∞, measured directly with Assmann's aspiration psychrometer, together with the values of α, calculated from B and the latter values of p_s-p_∞, are also listed in Table 2. As is seen from Table 2, the values of p_s-p_∞ obtained from experimental curves are to some extent smaller than those obtained from direct measurement with Assmann's aspiration psychrometer. It may probably because the situation where p_∞ was measured with Assmann's aspiration psychrometer was too far from the wire to be appropriate for the present purpose and the vapour pressure at the nearer point to the wire should be taken as p_∞. Corresponding to both values of p_s-p_∞, the values of α, too, differ to some extent as is seen in Table 2.

The values of α hitherto obtained by different researchers indicate considerable discrepancy and we cannot decide the true value of α as yet. Our values of α agree with Alty's$^{(7)}$ values as is seen from the following table. However, our experiment is only an indirect method in determining α, so this coincidence will not have so much mean in deciding the correct value of α.

Table 3

Temp. (°C)	Diffusivity (cm^2 sec^{-1})
−20	0.197
−10	0.211
0	0.226
10	0.241
20	0.257
30	0.273

Our values of α	0.023(3.0−4.6°C)
Alty's values of α	0.036(0°C),0.027 (4°C), 0.016(5.9°C)

In conclusion, our experimental result agreed with the theoretical formula of Tsuji pretty well. So it may be said that the assumption of non-saturation of the surface of water is true. Finally authors express their hearty thanks to the aids given by Dr. K.Terada and Mr. M.Tsuji of the Central Meteorological Observatory. It is also noted that this research was helped by the Scientific Research Expenditure of the Department of Education.

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