EFFECTS OF TURBULENT MIXING ON THE ADIABATIC LAPSE RATE OF THE AIR TEMPERATURE (I. DRY AIR)

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The thermodynamical treatment of the atmosphere assumes adiabatic and quasi-static process which is equivalent to assume the effects of radiation and turbulent mixing neglegible. However these effects, especially the latter, may not be neglected in the case of the convection of the air. Applying the aerodynamical theory of jet from a circular nozzle the author studied the effects of turbulence on the distribution of velocity and temperature in the ascending current with circular section.

When a symmetrical jet is discharged from a nozzle the fluid will spread and mix with the sourrounding fluid as shown in Fig. 1.

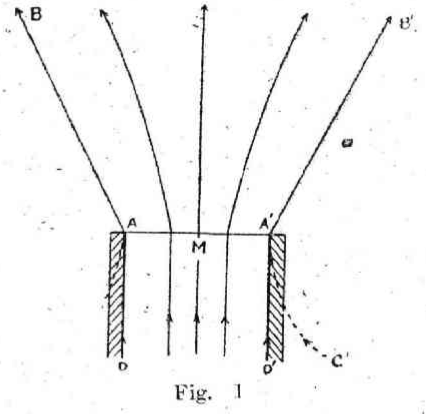

Fig. 1

We consider the ascending air current analogous to the turbulent spreading of a jet and apply the theory obtained by W. Tollmien(1) and L. Howarth(2) on this problem.

Introducing the cylindrical polar coordinates z, r, θ, equations of motion and the heat transfer will be written as follows.

$$\left\{\begin{aligned} & u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}=-\frac{1}{r}\frac{\partial}{\partial r}\left(l^2 r\left(\frac{\partial u}{\partial r}\right)^2\right) \\ & \qquad -g-\frac{1}{\rho}\frac{\partial p}{\partial r} \quad \cdots\cdots(1) \\ & u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r} \quad \cdots\cdots(2) \\ & u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}=-\frac{1}{r}\frac{\partial}{\partial r}\left(l^2 r\left(\frac{\partial T}{\partial r}\right)^2\right) \\ & \qquad +\frac{1}{c_p J}\frac{1}{\rho}\left(u\frac{\partial p}{\partial z}+v\frac{\partial p}{\partial r}\right). \quad \cdots\cdots(3) \end{aligned}\right.$$

The first term on the right-hand side of equation (1) and (3) is the rate of the change of the upward velocity u and the temperature T by turbulent mixing. The mixture lengh l will be considered as proportional to height z (or $l=kz$) as in the case of jet. Equation of continuity is

$$\frac{d\rho}{dt}+\rho\left(\frac{\partial u}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv)\right)=0. \quad \cdots\cdots(4)$$

Neglecting small quantities it reduces to

$$\frac{\partial u}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv)=0. \quad \cdots\cdots(5)$$

With the aid of stream function ψ, u and v are expressed as

$u=\frac{1}{r}\frac{\partial\psi}{\partial r}$, $v=-\frac{1}{r}\frac{\partial\psi}{\partial z}$, and if we put $\psi=z\,F(\eta)$, $\eta=r/z$, then

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}=\frac{1}{z^3\eta}\frac{d}{d\eta}\left(F\left(\frac{F}{\eta}-F'\right)\right).$$

As $-\frac{1}{\rho}\frac{\partial p}{\partial z}-g$ is very small quantity, (1) reduces to

$$\frac{FF'}{\eta}-k^2\eta\frac{d}{d\eta}\left(\frac{d}{d\eta}\frac{F'}{\eta}\right)^2+\frac{1}{2}\frac{d}{d\eta}\left(\frac{F^2}{\eta}\right)=\text{const} \quad \cdots\cdots(6)$$

(1) Z. S. f. angew. Math. u. Mech 6. 1926.

(2) Proc. Camb. Phil. Soc. 34. 1938.

Putting $\eta=k^{\frac{2}{3}}\xi$ and expanding F in power series of ξ we get

$$F=a\xi^2\exp\Big(-\frac{4\sqrt{2}}{21}\xi^{\frac{3}{2}}\Big)-\frac{1}{3\times245}\xi^3-\frac{2}{7}\times\frac{3\sqrt{2}}{44}k^{-\frac{4}{3}}\xi^{\frac{7}{2}}+\frac{2}{9}\times\frac{\sqrt{2}}{1715}\xi^{\frac{7}{4}}+\frac{k^{\frac{4}{3}}}{50\times11\times14}\xi^5+\frac{2}{11}\times\frac{3\sqrt{2}}{320}k^{-\frac{8}{3}}\xi^{\frac{11}{2}}+\frac{37}{6\times240100}\xi^6+\cdots\Big)\cdots\cdots(7)$$

Further if we put

$$T=A-\gamma z+\frac{B}{z}\frac{F'}{\eta}+\frac{1}{z^2}G(\eta),$$

(γ is the dry adiabatic lapse rate)
then from (3) and (7)

$$G=\frac{k^{-\frac{4}{3}}a^2}{c_pJ}\xi^4\Big\{\frac{3\sqrt{2}}{7}\xi^{-\frac{1}{2}}-\frac{131}{22\times72}\xi-\frac{9\sqrt{2}}{88}k^{-\frac{4}{3}}\xi^{\frac{3}{2}}-\Big(\frac{11}{245\times7}+\frac{9}{44\times13}k^{\frac{1}{3}}\Big)\times\sqrt{2}\,\xi^{\frac{5}{2}}-\cdots\cdots\Big\}\exp\Big\{-\frac{8\sqrt{2}}{21}\xi^{\frac{3}{2}}-\frac{3}{3\times245}\xi^3-\frac{3\sqrt{2}}{77}k^{-\frac{4}{3}}\xi^{\frac{7}{2}}-\cdots\Big\}\cdots(8)$$

Boundary conditions are

$$u=u_0,\ v=0,\ \frac{\partial u}{\partial r}=0,\ T=T_0,\ \frac{\partial T}{\partial r}=0 \text{ at } r=0$$

and $z=z_0$; $u=0$, $T=T_1$, at $r=r_0$ and $z=z_0$. T_1 is the temperature of the surrounding air. Thus we can write as

$$\begin{cases} u=\frac{z_0}{2z}u_0\frac{F'}{\xi}, \\ v=k^{\frac{2}{3}}\frac{z_0}{2z}u_0\Big(F'-\frac{F}{\xi}\Big), \\ T=T_1-\gamma(z-z_0)+\frac{z_0}{2z}(T_0-T_1)\frac{F'}{\xi}+\frac{G}{z^2}, \\ a=\frac{z_0}{2}u_0k^{\frac{2}{3}}. \end{cases}\cdots\cdots(9)$$

Experiments on jet show that $k=0.158$ and if we adopt the result also in our case, then $r_0=3.4k^{\frac{2}{3}}z_0=0.214\,z_0$, $l=0.158\,z_0$. In the case of jet $z=z_0$ at the mouth of nozzle. In our case, however, it is legitimate to consider it at the elevation where the ascending current is most intense or at the top of the friction layer, say 500 to 1000m above the ground. Therefore r_0 is the radius of the current at $z=z_0$ and the origin of the coordinate axes is determined by the radius of the ascending current.

The interpretation of (9) is as follows.

(a) Upward velocity of the air decreases as ξ increases and becomes zero at $\xi=3.4$ or at the boundary between the ascending current and the surrounding atmosphere. Therefore the breadth of the ascending current increases linearly with elevation. The streamline of the air is shown in Fig. 2.

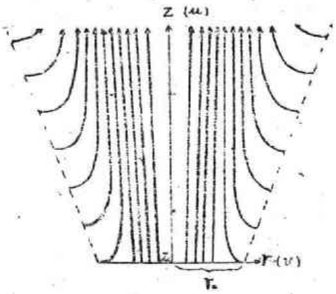

Fig. 2
Streamline of the Air.

(b) As $\frac{G}{z^2}$ is very small quantity, the air temperature and the upward velocity u is similary in any horizontal section of the ascending current and with horizontal velocity v they are shown in Fig. 3.

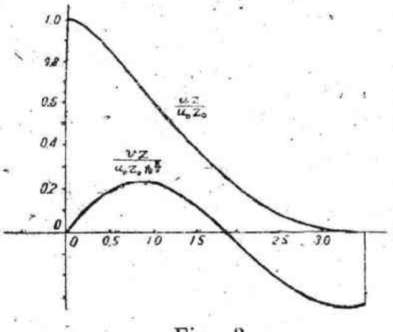

Fig. 3

(c) Upward velocity is largest at the centre and the radial or horizontal velocity is positive (outflow) near the centre of the current ($\xi<1.9$), and the air flows in from the outside of the ascending current.

(d) The temperature and the velocity of the current is inversely proportional to height.

The lapse rate of the temperature is

$$-\frac{\partial T}{\partial z}=\gamma+\frac{z_0}{2z^2}(T_0-T_1)F''-\frac{\partial}{\partial z}\Big(\frac{G}{z^2}\Big)$$

The third term of the right-hand side is

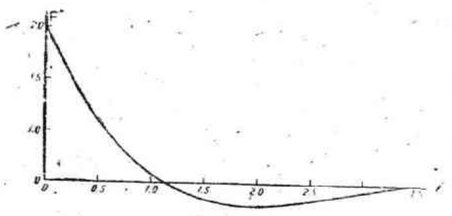

Fig. 4

small compared with other terms. F'' has maximum value at $\xi=0$ or $r=0$ and is positive for $\xi<1.15$ as shown in Fig. 4. Consequently the lapse rate is largest near the centre of the ascending current and is smallest at $\xi=2.1$. Further it is equal to the dry adiabatic lapse rate at $\xi=1.15$ and at the boundary of the current. As an example when $r_0=1\,km$, $T_0-T_1=2°C$, it is 1.05×10^{-4} at $r=0$ and 0.99×10^{-4} at $\xi=2.1$.

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ON THE AIR TEMPERATURE AND THE GROWTH OF CLOUD PARTICLES IN CONVECTIVE CLOUDS (STUDIES ON THE FORMATION OF CLOUD AND RAIN. PART I.)

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The problem of condensation in the ascending air current has been studied thermodynamically by Herz, Neuhoff, Bezold, Fjeldstad etc., but they did not take the influence of condensation nucleus into consideration. In this paper the author studied the distribution of the temperature of the air and the growth of cloud particles in the ascending current which contains condensation nuclei.

(1) When the velocity of ascending current is very small or in the case of quasi-static process.

Consider a parcel of air which contains n nuclei ascends adiabatically. When the nuclei are non-hygroscopic the saturated water vapour tension on them is

$$E' = E e^{\frac{2\alpha M}{R r_0 T}}, \qquad (1)$$

where α is the surface tension of water, M the molecular weight of water, R gas constant; r_0 and T are the radius and the temperature of nucleus respectively, E the water vapour tension on the plane surface of water at T. When the velocity of the ascending current is very small the temperature of nuclei or cloud particles may be considered as equal to that of the surrounding air. As condensation begins when the water vapour tension of the air becomes E', the supersaturation at the base of cloud is given by

$$S_0 = E'/E - 1 = e^{\frac{2\alpha M}{R r_0 T}} - 1. \qquad (2)$$

After the condensation begins the temperature of the air is given by

$$T = T_0 - \gamma h + \frac{L}{c_p} w. \qquad (3)$$

γ is the dry adiabatic lapse rate of the air, h height measured from the base of cloud, c_p specific heat of the air at constant pressure, L latent heat of water vapour, w water content of clouds per unit volume of the air. Further

$$w = \frac{4\pi}{3} r^3, \qquad w + 0.622\left(\frac{E}{p-E} e^{\frac{2\alpha M}{R r T}} - \frac{E_0}{p_0 - E_0} e^{\frac{2\alpha M}{R r_0 T_0}}\right) = 0, \qquad (4)$$