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Two-Loop Renormalization of Gaugino Masses in General Supersymmetric Gauge Models

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We calculate the two-loop renormalization group equations for the running gaugino masses in general supersymmetry (SUSY) gauge models, improving our previous result. We also study its consequences on the unification of the gaugino masses in the SUSY SU(5) model. The two-loop correction to the one-loop relation $m_i(\mu) \propto \alpha_i(\mu)$ is found to be of the order of a few percent.

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It has recently been found [1] that the experimentally measured values of three gauge couplings in the standard model are consistent with the prediction of the supersymmetric (SUSY) SU(5) grand unified theory (GUT) [2]: The three running gauge coupling constants $\alpha_i(\mu) = g_i(\mu)^2/4\pi$ ($i=1,2,3$) are unified in good precision,

$$\alpha_3(m_U)=\alpha_2(m_U)=\alpha_1(m_U)=\alpha_5(m_U), \tag{1}$$

at the GUT unification scale $m_U \sim 10^{16}$ GeV.

In the SUSY SU(5) model, there is another unification condition which is related to the SU(5) gauge symmetry. It is the unification of the soft-SUSY-breaking running masses of three gauge fermions (gauginos) $m_i(\mu)$ ($i=1,2,3$) at the same scale m_U. Namely, the relation

$$m_3(m_U)=m_2(m_U)=m_1(m_U)=m_5(m_U) \tag{2}$$

holds, apart from the threshold correction.

The one-loop renormalization group equations for the gaugino masses take a very simple form [3]:

$$\frac{d}{dt}\left(\frac{m_i}{\alpha_i}\right)=0, \quad t \equiv \ln\mu, \quad i=1,2,3. \tag{3}$$

By combining (2) and (3), we obtain the following well-known results in the leading order [3]:

$$\begin{aligned} m_3(\mu)/\alpha_3(\mu) &= m_2(\mu)/\alpha_2(\mu) = m_1(\mu)/\alpha_1(\mu) \\ &= m_5(m_U)/\alpha_5(m_U). \end{aligned} \tag{4}$$

Note that the relations (3) hold in any SUSY gauge model.

In this paper, we study the two-loop correction to the unification condition (4). This correction, together with the one-loop threshold corrections at m_U [4] and at the weak scale, gives the complete next-to-leading-order correction to the identity (4). There are several reasons for studying the correction of this order. First, since the unification of the gauge coupling constants in the SUSY GUT has already been studied in the next-to-leading order [1,5], consistent treatment of the gaugino mass unification also needs the two-loop renormalization group equations for the running gaugino masses. Second, the next generation of pp and e^+e^- colliders are expected to be able to measure the gaugino masses accurately enough [6] to test the unification condition in this order, just as in the case of the gauge couplings at present e^+e^- colliders. Therefore, the next-to-leading correction to the identity (4) will become important in the future study of the unification condition of the gaugino masses and the test of the SUSY GUT's.

In the previous paper [7], we have shown that the relation (3) is violated in the two-loop order in the model which contains only the vector supermultiplets. In this paper, we extend our previous analysis of the two-loop renormalization of gaugino masses to general SUSY gauge models, by including the contributions of chiral supermultiplets, Yukawa couplings, and soft-SUSY-breaking trilinear scalar couplings (A terms). We also show numerical estimates of the two-loop correction to the gaugino mass unification in the SUSY SU(5) model.

Let us first fix our framework. We consider the SUSY gauge model with a semi-simple gauge group $G=\prod_i G_i$, where G_i's are simple subgroups. The model contains chiral supermultiplets Φ_a in the representations $R_a^{(i)}$ for the subgroup G_i. The superpotential is

$$\mathcal{W}=\tfrac{1}{6}y^{abc}\Phi_a\Phi_b\Phi_c. \tag{5}$$

The soft-SUSY-breaking term in the Lagrangian is

$$\mathcal{L}_{\text{soft}}=-\tfrac{1}{6}A^{abc}y^{abc}\phi_a\phi_b\phi_c-(m_i/2)\lambda_i\lambda_i+\text{H.c.}, \tag{6}$$

where ϕ_a and λ_i denote the scalar component of Φ_a and the gaugino of the group G_i, respectively. The Yukawa couplings y^{abc} and the A terms A^{abc} are defined to be symmetric with respect to the indices a,b,c. We have omitted lower-dimensional terms in (5) and (6) since they are irrelevant for our study.

The two-loop renormalization group equations for the gauge coupling constants g_i [8,9] are then expressed as

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$$\frac{d}{dt}g_i = \frac{g_i^3}{(4\pi)^2}[-3C(G_i)+T_i(\Phi)] + \frac{g_i^5}{(4\pi)^4}2C(G_i)[-3C(G_i)+T_i(\Phi)] + \sum_j \frac{g_i^3 g_j^2}{(4\pi)^4}4T_i(\Phi)C_j(\Phi) - \frac{g_i^3}{(4\pi)^4}y^{abc}y_{abc}\frac{C_i(\Phi_c)}{d(G_i)} . \tag{7}$$

Here we adopt the notations

$$C(G_i)\delta^{AB} = f_{(i)}^{ACD}f_{(i)}^{BCD},\quad T_i(\Phi_a)\delta^{AB} = \mathrm{Tr}R_a^{(i)A}R_a^{(i)B},\quad C_i(\Phi_a)I = R_a^{(i)A}R_a^{(i)A},$$
$$T_i(\Phi) = \sum_a T_i(\Phi_a),\quad T_i(\Phi)C_j(\Phi) = \sum_a T_i(\Phi_a)C_j(\Phi_a),\quad y_{abc} = (y^{abc})^* , \tag{8}$$

where $f_{(i)}^{ABC}$ denotes the structure constant of the group G_i, and $d(G_i)$ is the dimension of G_i.

We obtain the two-loop renormalization group equations for the running gaugino masses m_i in the $\overline{\mathrm{DR}}$ scheme (dimensional reduction [10] with modified minimal subtraction [11]) by evaluating the two-loop diagrams for the gaugino propagators in the Wess-Zumino gauge; see Fig. 1. The results are

$$\frac{d}{dt}m_i = \frac{g_i^2}{(4\pi)^2}[-6C(G_i)+2T_i(\Phi)]m_i + \frac{g_i^4}{(4\pi)^4}8C(G_i)[-3C(G_i)+T_i(\Phi)]m_i + \sum_j \frac{g_i^2 g_j^2}{(4\pi)^4}8T_i(\Phi)C_j(\Phi)(m_i+m_j) - \frac{2g_i^2}{(4\pi)^4}y^{abc}y_{abc}\frac{C_i(\Phi_c)}{d(G_i)}m_i + \frac{2g_i^2}{(4\pi)^4}A^{abc}y^{abc}y_{abc}\frac{C_i(\Phi_c)}{d(G_i)} . \tag{9}$$

The term proportional to $C(G_i)^2$ is the contribution from the diagrams with only the vector supermultiplets, which has been found in Ref. [7].

Here we comment on the renormalization scheme dependence of our results. The two-loop renormalization group equations for the gaugino masses are dependent on the renormalization scheme, whereas those for the gauge coupling constants are independent. In this paper, we adopt the $\overline{\mathrm{DR}}$ scheme [10] since this scheme respects supersymmetry while the usual $\overline{\mathrm{MS}}$ (modified minimal subtraction) scheme [12] does not. Indeed, by using the formulas in Refs. [9,13], we find that the two-loop renormalization group equations for the gauge vector couplings and those for the gaugino couplings to the chiral supermultiplets do not agree in the $\overline{\mathrm{MS}}$ scheme.

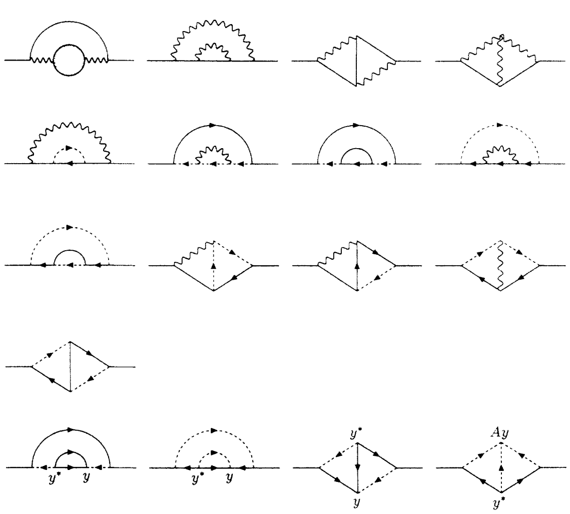

FIG. 1. Two-loop diagrams which contribute to the gaugino mass renormalization. The wavy line, solid line without an arrow, solid line with an arrow, and dashed line represent the gauge vector, gaugino, chiral fermion, and chiral scalar, respectively.

Our results (9) can be expressed as equations for the ratios m_i/α_i, by using Eq. (7). We find

$$\frac{d}{dt}\left(\frac{m_i}{\alpha_i}\right) = \frac{g_i^2}{(4\pi)^3}4C(G_i)[-3C(G_i)+T_i(\Phi)]m_i + \sum_j \frac{g_j^2}{(4\pi)^3}8T_i(\Phi)C_j(\Phi)m_j + \frac{2}{(4\pi)^3}A^{abc}y^{abc}y_{abc}\frac{C_i(\Phi_c)}{d(G_i)} . \tag{10}$$

The right-hand side of (10) contains only the two-loop contributions, in accordance with the one-loop identities (3). We can clearly see that the simple relations (3) are no longer valid at the two-loop level in general SUSY gauge models.

In order to examine consequences of the two-loop corrections, we consider the minimal SUSY standard model with two Higgs doublets and three generations of quarks and leptons. The renormalization group equations (10) for the ratios m_i/α_i then take the form

$$\frac{d}{dt}\left(\frac{m_i}{\alpha_i}\right) = \sum_{j=1}^{3} b_{ij}^{(m)}\frac{g_j^2}{(4\pi)^3}m_j + b_{i,\mathrm{top}}^{(m)}\frac{y_t^2}{(4\pi)^3}A_t , \tag{11}$$

with

$$b_{ij}^{(m)} = \begin{pmatrix} 398/25 & 54/5 & 176/5 \\ 18/5 & 50 & 48 \\ 22/5 & 18 & 28 \end{pmatrix},\quad b_{i,\mathrm{top}}^{(m)} = \begin{pmatrix} 52/5 \\ 12 \\ 8 \end{pmatrix} . \tag{12}$$

Here we have retained only the contributions from the top-quark–Higgs-boson Yukawa coupling y_t with the corresponding A term A_t.

We estimate the two-loop correction to the relation (4) numerically by integrating Eq. (11), while approximating the right-hand side by its one-loop solution. By using the following inputs $\alpha_s(m_Z)=0.12$, $\alpha(m_Z)^{-1}=128$, $m_{\rm SUSY} \sim m_Z$, and the unification conditions (1) and (2), we obtain

$$\begin{pmatrix} m_1/\alpha_1 \\ m_2/\alpha_2 \\ m_3/\alpha_3 \end{pmatrix}(m_Z) = \frac{m_5}{\alpha_5}(m_U)\left[1-\begin{pmatrix}0.043\\0.068\\0.036\end{pmatrix}-\begin{pmatrix}0.87\\1\\0.67\end{pmatrix}\Delta_t\right], \tag{13}$$

where

$$\Delta_t = \frac{0.09 y_t^2(m_U)}{1+11y_t^2(m_U)}\left[1+0.4\frac{A_t(m_U)}{m_5(m_U)}\right]. \tag{14}$$

The main contribution to the above corrections comes from the diagrams with internal gluino lines (see Fig. 1). The contributions from the top-quark–Higgs-boson interactions can be comparable to the gauge interaction contributions only if $|A_t(m_U)/m_5(m_U)|$ is greater than 10.

From Eq. (13), we can clearly see that the deviations from the one-loop identities (4) are of the order of a few percent. There is a possibility to detect these corrections by future precision measurements. We repeat here that the complete next-to-leading-order predictions require threshold corrections at the GUT scale [4] and at the weak scale, which is obtained by relating the running masses and the pole masses in the one-loop order. The latter relation is rather complicated for a realistic case with $SU(2)\times U(1)$ breaking effects, which will be discussed elsewhere.

In summary, we have obtained the two-loop renormalization group equations for the gaugino masses in general SUSY gauge models in the $\overline{\rm DR}$ scheme. We have found that the one-loop proportionality relations (3) and (4) between the gaugino masses and the gauge coupling constants are violated at the two-loop order. The two-loop corrections to the relation (4) have been evaluated numerically in the SUSY SU(5) model and they are found to be detectable in future precision experiments.

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Note added.— After the calculation was completed, we received a preprint [14] in which the renormalization group equations (9) have been given. Our results agree with theirs. They have also discussed the renormalization scheme dependence in detail.

[1] J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **260**, 131 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. B **260**, 447 (1991).

[2] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[3] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1982); **70**, 330(E) (1983); **71**, 413 (1984).

[4] J. Hisano, H. Murayama, and T. Goto, Phys. Rev. D (to be published).

[5] J. Ellis, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. **B373**, 55 (1992); J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. Lett. **69**, 1014 (1992); F. Anselmo, L. Cifarelli, and A. Zichichi, Nuovo Cimento **105A**, 1335 (1992); K. Hagiwara and Y. Yamada, Phys. Rev. Lett. **70**, 709 (1993); P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993); Y. Yamada, Z. Phys. C **60**, 83 (1993).

[6] JLC Group, JLC-I, KEK Report No. 92-16, 1992 (unpublished).

[7] Y. Yamada, Phys. Lett. B **316**, 109 (1993).

[8] D. R. T. Jones, Nucl. Phys. **B87**, 127 (1975).

[9] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B222**, 83 (1983).

[10] W. Siegel, Phys. Lett. **84B**, 193 (1979); D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. **B167**, 479 (1980).

[11] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).

[12] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).

[13] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B236**, 221 (1984).

[14] S. P. Martin and M. T. Vaughn, Northeastern University Report No. NUB-3072-93TH, 1993 (to be published).