Probing internucleus potential with large-angle quasi-elastic scattering

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Probing internucleus potential with large-angle quasi-elastic scattering

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Abstract. Recent measurements for fusion cross section at energies around the Coulomb barrier have systematically indicated a significant deviation of fusion cross sections from a prediction of double-folding model. It has been argued that the deviation can be accounted for if one uses a large value of surface diffuseness parameter for a Woods-Saxon internuclear potential. We investigate this problem using large-angle quasi-elastic scattering, that is a good counterpart of fusion reaction. Towards a reconciliation of the apparent anomaly in the diffuseness parameter for fusion reactions, we also discuss possible ingredients which are missing in present nuclear reaction models. Those include a deviation from the frozen density approximation, the effect of antisymmetrization and the role of coordinate dependent moment of inertia for the relative motion between the colliding nuclei.

Keywords: Quasi-elastic scattering, internucleus potential, antisymmetrization, resonating group method, double folding model, coupled-channels method
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INTRODUCTION

It has been recognized for some time that fusion reactions at energies around the Coulomb barrier require a large value of surface diffuseness parameter if one uses the Woods-Saxon parametrization for an internuclear potential [1–8]. For calculations of elastic and inelastic scattering, which are sensitive only to the surface region of the nuclear potential, the diffuseness parameter of around 0.63 fm has been conventionally employed [9, 10]. This value of surface diffuseness parameter has been well accepted, partly because it is consistent with a double folding potential [11]. In contrast, a recent systematic study has shown that experimental data for heavy-ion fusion reactions require a larger value of the diffuseness parameter, ranging between 0.75 and 1.5 fm, as long as the Woods-Saxon parameterization is employed [8].

In this contribution, we present our recent systematic analyses on this problem using large-angle quasi-elastic scattering [12, 13]. Quasi-elastic scattering (a sum of elastic, inelastic, and transfer channels) is a good counterpart of fusion reaction [14, 15]. Firstly, both are inclusive processes in a sense that the final configurations (channels) of the projectile and target nuclei are all summed up. The former is related to the reflection probability at the Coulomb barrier, while the latter to the penetration probability. Since the reflection and the penetration probabilities are related to each other by the unitarity condition, so are the quasi-elastic and fusion cross sections. Secondly, at energies close to the Coulomb barrier, both are sensitive to the collective inelastic excitations of the colliding nuclei and/or transfer process [16, 17]. Using this fact, the experimental barrier distribution, originating from the channel coupling effects[18], has been extracted for many systems from both fusion and quasi-elastic cross sections [4, 16, 19].

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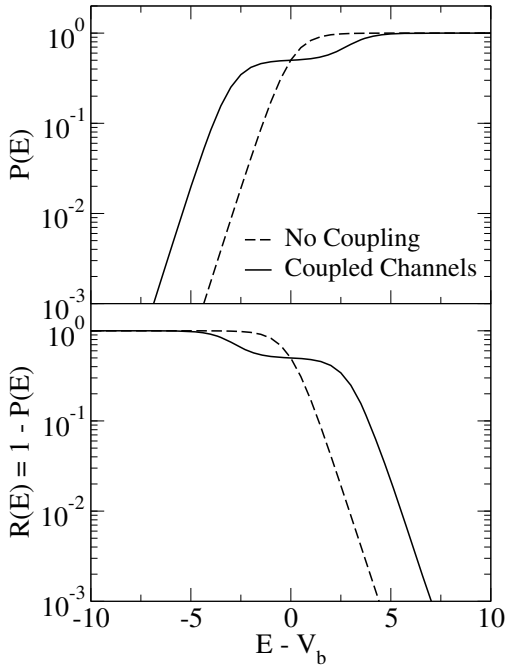

FIGURE 1. The penetrability (the upper panel) and the reflection probability (the lower panel) for a two-channel problem.

LARGE-ANGLE QUASI-ELASTIC SCATTERING AT DEEP SUB-BARRIER ENERGIES

Advantages of using deep sub-barrier data

In investigating internuclear potentials using heavy-ion quasi-elastic scattering, we are particularly interested in the deep sub-barrier region. At these energies, the cross sections of (quasi)elastic scattering are close to the Rutherford cross sections, with small deviations caused by the effect of nuclear interaction. This effect can be taken into account by the semiclassical perturbation theory. The ratio of elastic scattering $\sigma_{\rm el}$ to Rutherford cross sections σ_R at a backward angle θ is given by [14, 20]

$$\frac{d\sigma_{\rm el}(E_{cm},\theta)}{d\sigma_R(E_{cm},\theta)} \sim 1 + \frac{V_N(r_c)}{ka}\,\frac{\sqrt{2a\pi k\eta}}{E_{cm}}, \tag{1}$$

where E_{cm} is the center-of-mass energy, $k = \sqrt{2\mu E_{cm}/\hbar^2}$, μ being the reduced mass, and η is the Sommerfeld parameter. This formula is obtained by assuming that the nuclear potential $V_N(r)$ has an exponential form, $\exp(-r/a)$, around the distance of closest approach, r_c. We see from this formula that the deviation of the elastic cross sections from the Rutherford ones is sensitive to the surface region of the nuclear potential, especially to the surface diffuseness parameter a.

There is another advantage of using the deep sub-barrier data. That is, the effect of channel coupling on quasi-elastic scattering can be disregarded at these energies, since

the reflection probability is almost unity irrespective of the presence of channel couplings, even though inelastic channels themselves may be strongly populated [13]. This is similar to fusion at energies well above the Coulomb barrier, where the penetrability is almost unity [8]. We illustrate this in Fig. 1 for a two-channel problem.

From these considerations, it is evident that the effect of surface diffuseness parameter can be studied in a transparent and unambiguous way using the large-angle quasi-elastic scattering at deep sub-barrier energies.

Spherical systems

Let us now show the results of our analyses. In order to analyze the experimental data at deep sub-barrier energies, we use a one-dimensional optical potential with the Woods-Saxon form. Absorption following transmission through the barrier is simulated by an imaginary potential that is well localized inside the Coulomb barrier. This model calculates the elastic and fusion cross sections, in which the elastic cross sections can be considered as quasi-elastic cross sections to a good approximation at these deep sub-barrier energies.

In order to carry out a systematic study, we estimate the Coulomb barrier height using the Akyüz-Winther potential [9]. We then vary the surface diffuseness parameter while keeping the barrier height. We define the region of "deep sub-barrier energies" as the region where the experimental value of the ratio of the quasi-elastic to the Rutherford cross sections is larger than around 0.94. See Ref. [12] for more details.

Figure 2 compares the experimental data with the calculated cross sections obtained with different values of the surface diffuseness parameter for the ^{32}S + ^{197}Au system (the upper panel) and the ^{34}S + ^{197}Au system (the lower panel). The best fitted values for the surface diffuseness parameter are $a = 0.57 \pm 0.04$ fm and $a = 0.53 \pm 0.03$ fm for the ^{32}S and ^{34}S + ^{197}Au reactions, respectively. The cross sections obtained with these surface diffuseness parameters are denoted by the solid line in the figure. The dotted and the dot-dashed lines are calculated with the diffuseness parameter of a = 0.80 fm and a = 1.00 fm, respectively. It is evident from the figure that these spherical systems favor the standard value of the surface diffuseness parameter, around $a = 0.60$ fm. The calculations with the larger diffuseness parameters underestimate the quasi-elastic cross sections and are not consistent with the energy dependence of the experimental data. We obtain a similar conclusion for the ^{32,34}S + ^{208}Pb and ^{16}O + ^{208}Pb systems[12].

Deformed systems

We next analyse deformed systems, ^{16}O + ^{154}Sm, ^{186}W [12, 13]. For these systems, only a few data points are available at deep sub-barrier energies. We therefore include the experimental data at energies not only well below but also around the lowest barrier in the χ^2 fitting procedure. At these energies, the channel coupling effects start playing an important role in quasi-elastic reactions, and we include the effect of deformation of the target nucleus in our calculations [12, 14, 15].

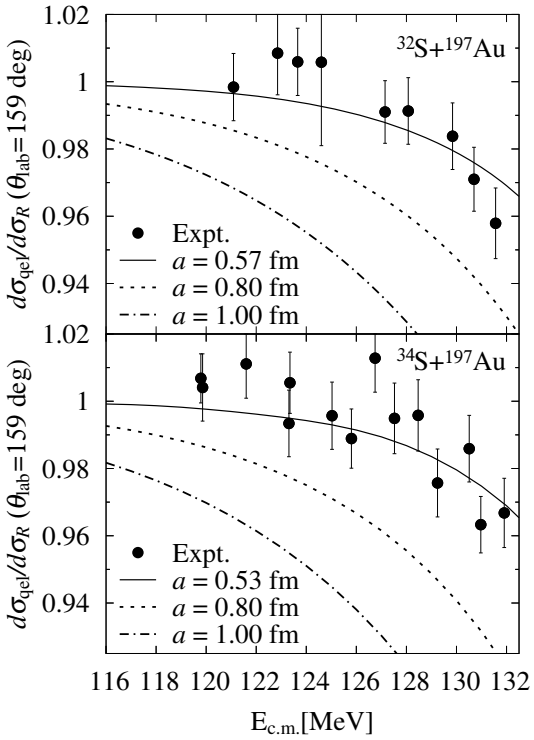

FIGURE 2. The ratio of the quasi-elastic to the Rutherford cross sections at $\theta_{lab} = 159^\circ$ for the ^{32}S + ^{197}Au (the upper panel) reaction and for the ^{34}S + ^{197}Au (the lower panel) reaction.

The best fitted value for the surface diffuseness parameter obtained in this way is $a = 1.14 \pm 0.03$ fm and 0.79 $\pm$0.04 fm for the ^{16}O + ^{154}Sm and ^{16}O + ^{186}W reactions, respectively. For the ^{16}O + ^{154}Sm reaction, the calculated cross sections with the standard value of the surface diffuseness parameter around 0.60 fm are found to be in strongly disagreement with the experimental data.

Although these large values of surface diffuseness parameter are consistent with that extracted from fusion, the origin of the difference between the spherical and the deformed systems is not clear. One should bear in mind, however, that our analyses for the deformed systems are somewhat model dependent as we needed to include the deformation effect in the calculations. Also, there might be a problem of normalization in the experimental data [21]. Since these experiments were performed in aiming at extracting the quasi-elastic barrier distribution, rather than extracting the value of surface diffuseness parameter, the experimental data were arbitrarily normalized to unity at the lowest energy. (Notice that the shape of barrier distribution is rather insensitive to the absolute normalization factor.) If the data had been normalized at a different energy, a different conclusion concerning the value of surface diffuseness parameter might have been achieved.

In order to clarify the difference in the diffuseness parameter between the spherical and the deformed systems, apparently further precision measurements for large-angle quasi-elastic scattering at deep sub-barrier energies will be necessary, especially for deformed systems.

DISCUSSIONS

The value of the surface diffuseness parameter obtained in this study for the spherical systems agrees well with the conventionally used value $a \sim 0.63$ fm. This value is consistent with the double folding potential, suggesting that the double folding model is valid at least in the surface region and for systems which do not involve a deformed target. The discrepancy between the values of the diffuseness parameter determined from fusion data and those from quasi-elastic data must be related with the dynamics inside the Coulomb barrier [8].

In this connection, it is instructive to step back to the original many-body Hamiltonian of the system,

$$H = \sum_i t_i + \sum_{i<j} v_{ij}, \tag{2}$$

where t_i is the kinetic energy for a nucleon i and v_{ij} is a two-body interaction between nucleons. The philosophy of any reaction model is to somehow extract the degree of freedom for the relative motion from the original many-body Hamiltonian (2) and write it in a form of

$$H = H_{\rm rel} + H_{\rm s.p.} + H_{\rm coup}, \tag{3}$$

where $H_{\rm rel}$ is the Hamiltonian for the relative motion which describes the scattering process (the "collective" degree of freedom), $H_{\rm s.p.}$ is the Hamiltonian for other degrees of freedom than the relative motion ("non-collective" degrees of freedom), and $H_{\rm coup}$ is the coupling term between the relative motion and the non-collective degrees of freedom.

Although it is an extremely difficult task to extract the degree of freedom for the relative motion from the original many-body Hamiltonian in a consistent manner, let us assume that it can be achieved in some way. Yet, it is difficult to treat the coupling term, $H_{\rm coup}$ exactly. One usually takes one of the following approaches for this purpose.

- Sudden approach
  This approach is justified if the reaction takes place suddenly and/or the coupling term $H_{\rm coup}$ is negligibly small. The double folding model[9], the optical model, the coupled-channels model[22], and the resonating group method (RGM) [23] are categorized to this approach. In this approach, the constant reduced mass is usually used for the moment of inertia for the relative motion.
- Adiabatic approach
  This is in the opposite limit to the sudden approach, and is justified if the reaction takes place very slowly. The liquid-drop + shell correction model [24] and the adiabatic time-dependent Hartree-Fock (ATDHF) method [26–30] are in this category.

For the region where the colliding nuclei do not significantly overlap with each other, both the sudden and the adiabatic approaches would lead to a similar result. However, for the region inside the Coulomb barrier, one may obtain considerably different results depending on which approach one employs. Since one would not know a priori which approach is more reasonable, one has to investigate both of the two approaches simultaneously. An important thing is that, in either of sudden or adiabatic approach, we are now at the stage where the present nuclear reaction models have to be re-examined by taking into consideration the many-particle nature of nuclear reactions.

Sudden approach: role of antisymmetrization

A typical model in the sudden approach is the double-folding model [9], where the internucleus potential is constructed by convoluting an effective nucleon-nucleon interaction with the ground state density distributions of the projectile and the target nuclei. This potential corresponds to the direct ("Hartree") part of a microscopic ion-ion potential. This model will thus be reasonable for the small overlap region, but is questionable for the large overlap due to the Pauli exclusion principle.

This can be easily understood if one considers a simple two nucleon system, where two nucleons are confined in potential wells whose center is located at $\pm R/2$, respectively. If one ignores the Pauli principle and takes only the direct term, the two particle wave function is given by

$$\Psi(\vec{r}_1,\vec{r}_2) = \phi_R(\vec{r}_1)\phi_L(\vec{r}_2), \tag{4}$$

where ϕ_R and ϕ_L are the ground state wave function for the right and the left hands side of potential wells, respectively. From this wave function, the one-body density reads,

$$\rho(\vec{r}) = \langle\Psi|\delta(\vec{r}-\vec{r}_1)+\delta(\vec{r}-\vec{r}_2)|\Psi\rangle = |\phi_R(\vec{r})|^2+|\phi_L(\vec{r})|^2 \equiv \rho_R(\vec{r})+\rho_L(\vec{r}). \tag{5}$$

This is exactly what one obtains in the so called frozen density approximation. One obtains an unphysically high density matter in the limit of $R \to 0$ in this approximation. In contrast, if one takes into account the Pauli principle, the two particle wave function is given by

$$\Psi(\vec{r}_1,\vec{r}_2) = [\phi_R(\vec{r}_1)\phi_L(\vec{r}_2)-\phi_R(\vec{r}_2)\phi_L(\vec{r}_1)]/\sqrt{2(1-S(R)^2)}, \tag{6}$$

from which one obtains

$$\rho(\vec{r}) = |\phi_+(\vec{r})|^2+|\phi_-(\vec{r})|^2. \tag{7}$$

Here, $S(R) = \langle\phi_R|\phi_L\rangle$ is the overlap integral between the "right" and "left" wave functions, and $\phi_\pm \equiv (\phi_R \pm \phi_L)/\sqrt{2(1\pm S(R))}$. Although the density (7) is reduced to Eq. (5) when the overlap integral $S(R)$ is small, the two densities are considerably different if the overlap is large. In fact, the density given by Eq. (7) leads to the ground state density of the two particle system in a unified single potential well in a natural way in the limit of vanishing R, rather than the unphysical high density matter. An important fact is that the Pauli principle plays an essential role even in the *frozen configuration* approximation.

In the double folding model, the effect of Pauli principle is partly taken into account in a specific manner, *i.e.,* through the so called knock-on exchange potential, where the interacting pair of nucleons are exchanged to each other. There are however many other exchange terms from a microscopic point of view, which are not included in the double folding model. These are the one nucleon exchanges other than the knock-on exchange, the two nucleon exchanges, and so on. These exchange effects would have to be taken into account if one discusses the internucleus potential inside the Coulomb barrier.

All the exchange effects can be incorporated in the microscopic RGM method. A well-known problem of RGM is that it is very difficult to apply it to heavy systems. We have recently developed the no-recoil approximation, where the recoil effect due to the exchange of nucleons between the projectile and target nuclei is neglected [31]. We have

found that the no-recoil approximation works well for the $\alpha + {}^{90}$Zr reaction and heavier systems. We expect that this no-recoil approximation will provide a useful way to apply the RGM method even to heavy systems.

Adiabatic approach: role of coordinate dependent mass

The adiabatic approximation is valid when the reaction takes place so slowly that the non-collective motion adiabatically follows the relative motion at every instant. This corresponds to the case where the excitation energies for the non-collective degrees of freedom are large [32, 33]. In the context of heavy-ion collisions, the adiabatic approximation involves the dynamical change of the density of the colliding nuclei. At energies around the Coulomb barrier, there are good reasons why the adiabatic approach may be reasonable. Firstly, the relative velocity between the colliding nuclei is small around the barrier. Secondly, the dynamical deformation of the densities involves the excitations (on the sudden basis) of the non-collective degrees of freedom. Among them, low-lying collective motions can be explicitly included in the coupled-channels framework, but the remaining excitations are high-lying and the adiabatic treatment should be adequate.

One of the important consequences of the adiabatic treatment is that the moments of inertia become coordinate dependent[32]. That is, the Hamiltonian for the relative motion is now given as

$$H = -\frac{\hbar^2}{2}\frac{d}{dr}\frac{1}{M(r)}\frac{d}{dr} + V(r) + \frac{l(l+1)\hbar^2}{2\Theta(r)}, \tag{8}$$

where $M(r)$ and $\Theta(r)$ are the moment of inertia for the translational and the rotational motions, respectively. (There is an additional complication originating from the way of quantization with the coordinate dependent mass, which we do not discuss in this contribution.)

Using a coordinate transformation, the Hamiltonian (8) can be written in a different form as well. Transforming the coordinate from r to ρ such that $M(r) \to \widetilde{M}(\rho) = \mu$ and $\Theta(r) \to \widetilde{\Theta}(\rho)$[24], one obtains

$$H = -\frac{\hbar^2}{2\mu}\frac{d^2}{d\rho^2} + \widetilde{V}(\rho) + \frac{l(l+1)\hbar^2}{2\widetilde{\Theta}(\rho)}. \tag{9}$$

If one writes this Hamiltonian in a form of

$$H = -\frac{\hbar^2}{2\mu}\frac{d^2}{d\rho^2} + \frac{l(l+1)\hbar^2}{2\mu\rho^2} + \widetilde{V}(\rho) + \frac{l(l+1)\hbar^2}{2\widetilde{\Theta}(\rho)} - \frac{l(l+1)\hbar^2}{2\mu\rho^2}, \tag{10}$$

it implies that the internucleus potential has a strong angular momentum dependence.

So far, the possibility of coordinate dependent mass has not yet been considered seriously in the context of heavy-ion reactions, except for Ref. [34]. It would be an interesting future problem to explore it more systematically in connection to sub-barrier fusion reactions.

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