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Dynamics of Orientational Phase Ordering Coupled to Elastic Degrees of Freedom

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Abstract. We take Ginzburg-Landau approaches to the dynamics of orientational phase ordering coupled to elastic degrees of freedom. In nematic fluid membranes, subdiffusively slow shape relaxation limits the defect coarsening kinetics. In lamellar binary gels, coarsening is frozen by random stress that acts via a coupling between strain and layer orientation.

INTRODUCTION

Dynamics of orientational phase ordering such as those in nematic liquid crystals and vector spin models possess certain universal characters, as was clarified in last decade. In some complex fluids, however, elastic degrees of freedom coupled to the orientational order give rise to non-universal features. Here we give two examples that demonstrate the roles of such couplings in the coarsening dynamics and patterns of topological defects.

NEMATIC MEMBRANES

First we consider model fluid membranes that carry in-plane long-range orientational order. It has recently been observed that the so-called gemini surfactants with chiral counter-ions form ribbon- or helix-like sheets of stacked bilayers [1]. They have been modeled in terms of the coupling between membrane curvature and in-plane nematic order. For simplicity, here we assume a non-chiral membrane with an almost-flat configuration expressed in the Monge gauge as $z = h(x,y)$. The curvature tensor is approximated as $H_{ij} = \partial_i \partial_j h$ ($i, j = x, y$) under the assumption $|\nabla h| \ll 1$. We consider the isotropic-nematic transition described by the order parameter $Q_{ij} = S(n_i n_j - \frac{1}{2}\delta_{ij})$. We write the free energy as the sum of the Landau, Frank, bending, and coupling contributions as

$$F = \int d^2 r \left[f_L(Q_{ij}) + \frac{M}{2}(\partial_i Q_{jk})^2 + \frac{\kappa}{2}H_{ii}^2 + \alpha Q_{ij}H_{ij} \right].$$

where $f_L(Q_{ij}) = \frac{A}{2}Q_{ij}^2 + \frac{C}{4}Q_{ij}^4$. We numerically solved the kinetic equations

$$\frac{\partial Q_{ij}}{\partial t} = -\Gamma_Q \left(\frac{\delta F}{\delta Q_{ij}} \right)_s$$

where s denotes the symmetric traceless part, and

$$\frac{\partial h}{\partial t} = -\Gamma_h \frac{\delta F}{\delta h} = -\Gamma_h \left(\nabla^2 \nabla^2 h + \alpha \partial_i \partial_j Q_{ij} \right).$$

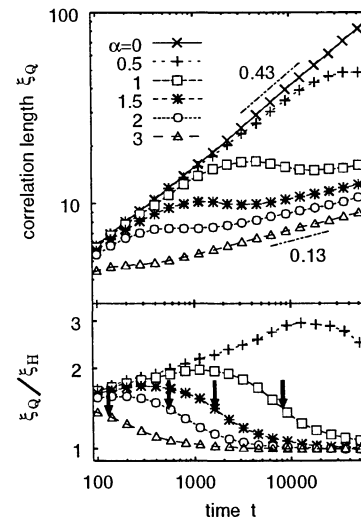In Fig. 1 we show the orientational and curvature corre-

FIGURE 1. Coarsening dynamics of nematic membranes. Top: Orientational correlation length ξ_Q versus time. Bottom: The ratio between orientational and curvature correlation lengths. Arrows indicate the time at which the curvature free energy becomes twice the Frank free energy.

lation lengths ξ_Q and ξ_H versus time. The coarsening is slowed down as the curvature free energy becomes non-negligible compared to the Frank one, and converges to a power law with the exponent 0.12 ± 0.02, which is much smaller than the exponent 0.43 for the XY model. We can interpret the slowing down as follows. In the initial stage, the order parameter and curvature fields coarsen in a different manner, since the former is driven by orientational

diffusion while the membrane height obeys the subdiffusive equation given above. Their coupling becomes important when ξ_Q exceeds $\xi_c = \sqrt{\kappa M}/\alpha$, after which curvature relaxation limits defect coarsening. The former obeys $\xi_H \propto t^{1/4}$ according to a simple dimensional analysis. The deviation of the numerical value from $1/4$ may be partly explained as logarithmic corrections in 2D. Due to the logarithmic dependence of the Frank free energy on defect separation, the apparent coarsening exponent for the XY model is smaller than the scaling value $1/2$. In addition, the effective curvature free energy (after integrating out h assuming local equilibrium) diverges logarithmically. We also incorporated the long-range hydrodynamic interaction due to solvent flow. It slightly accelerates the coarsening in consistency with a dimensional argument.

LAMELLAR BINARY GELS

The dynamics of isotropic-lamellar transition in its late stage can also be regarded as an orientational ordering process. For block copolymer melts, the correlation length ξ of layer normal is known to grow as $\xi \sim t^{1/4}$ [4]. Here we consider microphase separation in binary gels, where crosslinking of the two polymer species in the isotropic phase causes a long-range attraction between them in the lamellar phase. We extend de Gennes's classical model [5] to inhomogeneous systems by incorporating strain-orientation coupling and quenched random stress. Segregation of the two components induces strains that are parallel to the layer normal on average and modulated by the random disorder. This can be formulated in terms of the concentration gradient tensor,

$$Q_{ij} = (\partial_i \psi)(\partial_j \psi) - \frac{1}{d}(\nabla^2 \psi)\delta_{ij}.$$

Under the assumption of incompressibility $\nabla \cdot u = 0$, we write the optimal value of the elastic strain tensor $U_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right)$ as

$$U_{ij}^0(r) = \alpha Q_{ij}(r) + R_{ij}(r).$$

Here, α is the anisotropy parameter and $R_{ij}(r)$ is the (dimensionless) random stress which is spatially uncorrelated and is parametrized by its strength $R = \sqrt{R_{ij}^2}$. We decompose the free energy into two parts as $F = F_\psi + F_{el}$ where F_ψ is the free energy of an orientationally homogeneous system and F_{el} is the elastic contribution due to heterogeneity. Explicitly, they read

$$F_\psi = \int d^d r \left[f_L(\psi) - \frac{K}{2}\psi\frac{1}{\nabla^2}\psi + \frac{L}{2}(\nabla\psi)^2 + \frac{M}{4}(\nabla\psi)^4\right],$$

where $f_L(\psi) = \frac{A}{2}\psi^2 + \frac{C}{4}\psi^4$ and

$$F_{el} = \mu \int d^d r \left(U_{ij} - U_{ij}^{(0)}\right)^2,$$

where μ is the shear modulus. After integrating out the strain field assuming local mechanical equilibrium, we have an effective elastic free energy $\tilde{F}_{el} = \tilde{F}_{el}[\psi(r)]$. We numerically solved the kinetic equation

$$\frac{\partial \psi}{\partial t} = \Gamma \nabla^2 \frac{\delta}{\delta\psi}\left(F_\psi + \tilde{F}_{el}\right)$$

in 2D, and found that the elastic coupling slightly reduces the growth exponents for S_{max} and ξ, from the nonelastic exponents $1/5$ and $1/4$ [4], respectively. We may argue, as in the membrane case, that the deviation is due to a logarithmic anomaly in the effective elastic interaction. When the randomness is turned on, the correlation length converges to a finite value. The dependence of the equilibrium correlation length on the disorder strength is weaker than a power law, which is in contrast with the exponential dependence found in the random field XY model and disordered nematic elastomers [6]. The details of our results will be presented elsewhere.

SUMMARY

In this paper, we have shown that both dynamic and static properties of an elastic medium can affect orientational ordering dynamics through energetic couplings. It might be interesting to seek analougous effects in collective reorientation dynamics of anisotropic elements coupled to elastic media, such as proteins attached to membranes [7] and cells on extra-cellular matrices [8].

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