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Particle production from disoriented chiral condensates described by a coherent state

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The production of pions and sigma mesons from disoriented chiral condensates (DCC's) of finite size is studied using the linear sigma model and a continuous mode coherent state. Two models are used to describe the decay of DCC's. One assumes instantaneous decay and the other assumes radiation from the surface. It is found that, for a spherical DCC of radius of several fm, possible signals related to the momentum spectrum, multiplicity, and the neutral-to-charged pion ratio will be completely masked by the background which dominates multiparticle production in nucleus-nucleus collisions at high energies unless an appropriate cut or binning in the phase space is made on an event-by-event basis. We propose that a signal of DCC's may manifest itself as an abnormal peak in the rapidity distribution if pions with medium and large transverse momenta are cut off. The smearing effect due to sigma meson decay on the characteristic distribution of neutral-to-charged pion ratio is also studied. [S0556-2813(98)04203-4]

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I. INTRODUCTION

Nucleus-nucleus collisions at high energies have been investigated enthusiastically, expecting the possible formation of hot quark-gluon plasma (QGP), the state of matter where quarks and gluons are deconfined and chiral symmetry is restored. However, even if QGP is formed in such a process, it will decay eventually because the system expands and is cooled. As has been discussed by many authors [1–13], there is a possibility that a disoriented chiral condensate (DCC) of finite size may be formed in such a process. It is a finite domain of a vacuum which is misaligned in the chiral space relative to the physical vacuum. The physical vacuum has a finite scalar condensate. In DCC's, however, the pseudoscalar condensate may also have nonzero value.

Rajagopal and Wilczek suggested the possibility of the formation of large DCC's in a nonequilibrium quenching process [4]. They studied the pionic content of the DCC's and obtained a characteristic distribution of the ratio ρ of the neutral pion multiplicity to all pion multiplicities defined as

$$\rho \stackrel{\text{def}}{=} \frac{n(\pi^0)}{n(\pi^+)+n(\pi^0)+n(\pi^-)}. \tag{1}$$

The same distribution was also derived by Anselm and Ryskin [2] within a similar context. This distribution provides a possible explanation of the Centauro events found in cosmic ray experiments [14,15]. Kowalski and Taylor used the ''coherent isospin singlet states'' to study DCC's [5]. They estimated the number of pions produced in DCC decay assuming that the momenta of pions are vanishing. There is, however, no strong reason to believe that the DCC is an isospin singlet (isospin eigenstate in general) even if it is neutral. Pions produced from finite DCC's should have nonzero momenta in general. The signal-to-background ratio has to be estimated before claiming that the large fluctuations of ρ in DCC events may explain the Centauro events and may provide a signal of a chiral phase transition.

The purpose of this paper is to present useful quantitative results of DCC signal analysis which has been only qualitative in most previous works. We study pion production from DCC's of finite size by using the linear sigma model and continuous mode coherent states which are not isospin eigenstates. The momentum spectrum [16–18], the mean pion multiplicity, and the ρ distributions are calculated for a spherical DCC or DCC's with various sizes. The contribution to the energy density or the total energy from the DCC is also evaluated. As there is no established method to describe the decay of DCC's, we consider two models; one is the instantaneous decay model and the other is the radiation model. It is found that the mean multiplicity and the energy density are much smaller than the values expected for ''normal'' events at, say, the RHIC energy region. This implies that possible signals due to a soft momentum spectrum or the peculiar ρ distribution will be completely masked by the overwhelming backgrounds unless an appropriate cut or binning in the phase space is made on an event-by-event basis.

In Sec. II, a general formalism based on a coherent state representation is given. In Sec. III, two decay models are introduced and the physical quantities are calculated. In Sec. IV, charge fluctuations which includes the effect of sigma mesons are calculated. In Sec. V, numerical results are shown for some cases. They are compared with the expected background if relevant. The lifetime of DCC's is estimated by using the radiation model. It is shown in Sec. V that an appropriate cutoff in the transverse momentum may enhance the signal-to-background ratio in the rapidity space. Conclusions and discussions are given in Sec. VI.

II. COHERENT STATE REPRESENTATION

Consider the linear sigma model. The Lagrangian is given by

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$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{\lambda}{4}(\phi^2-v^2)^2+H\sigma, \tag{2}$$

where $\phi=(\sigma,\vec{\pi})$ and the last term on right-hand side is the explicit symmetry-breaking term. Since the distortion of the chiral circle due to the $H\sigma$ term is small, the condensates of the fields at time $t=0$ may be parametrized as

$$\langle \mathrm{DCC}|\sigma(\vec{x})|\mathrm{DCC}\rangle=v\ \cos\theta(\vec{x}), \tag{3a}$$

$$\langle \mathrm{DCC}|\vec{\pi}(\vec{x})|\mathrm{DCC}\rangle=v\ \sin\theta(\vec{x})\vec{n}(\vec{x}), \tag{3b}$$

where $|\mathrm{DCC}\rangle$ represents the vacuum with DCC's, θ is the angle from the sigma direction, and $\vec{n}$ is the unit vector representing the direction in the $\vec{\pi}$ space. Here, the field $\sigma(x)$ does not correspond to a physical particle. Therefore, we make a field translation $\sigma'=\sigma-v$, which corresponds to a physical sigma meson. Equation (3a) is rewritten as

$$\langle \mathrm{DCC}|\sigma'(\vec{x})|\mathrm{DCC}\rangle=v[\cos\theta(\vec{x})-1]. \tag{4}$$

We use a continuous mode coherent state to represent the DCC state:

$$|\mathrm{DCC}\rangle=N_D\exp\left(\sum_{\mu=0}^{3}\int d^3\vec{k}f_\mu(\vec{k})a_\mu^\dagger(\vec{k})\right)|0\rangle, \tag{5}$$

where $|0\rangle$ is the normal vacuum state, N_D is the normalization constant given by

$$N_D^2=\exp\left(-\sum_{\mu=0}^{3}\int d^3\vec{k}|f_\mu(\vec{k})|^2\right), \tag{6}$$

and the annihilation [creation] operator $a_\mu(\vec{k})$ $[a_\mu^\dagger(\vec{k})]$ for σ' and $\vec{\pi}$ fields satisfies

$$a_\mu(\vec{k})|0\rangle=0, \tag{7}$$

$$a_\mu(\vec{k})|\mathrm{DCC}\rangle=f_\mu(\vec{k})|\mathrm{DCC}\rangle, \tag{8}$$

$$[a_\mu(\vec{k}),a_\nu^\dagger(\vec{k'})]=\delta_{\mu,\nu}\delta(\vec{k}-\vec{k'}). \tag{9}$$

By a Fourier transformation, we have another representation of the left-hand sides of Eqs. (4) and (3b):

$$\int d^3\vec{x}e^{i\vec{k}\cdot\vec{x}}\langle \mathrm{DCC}|\sigma'(\vec{x})|\mathrm{DCC}\rangle = \sqrt{\frac{(2\pi)^3}{2\omega_0(\vec{k})}}[f_0(-\vec{k})+f_0^*(\vec{k})], \tag{10a}$$

$$\int d^3\vec{x}e^{i\vec{k}\cdot\vec{x}}\langle \mathrm{DCC}|\pi_i(\vec{x})|\mathrm{DCC}\rangle = \sqrt{\frac{(2\pi)^3}{2\omega_i(\vec{k})}}[f_i(-\vec{k})+f_i^*(\vec{k})], \tag{10b}$$

where $\omega_\mu=\sqrt{\vec{k}^2+m_\mu^2}$, m_0 is the σ' mass and m_i is the pion mass. The functions $f_\mu(\vec{k})$ and the unit vector $\vec{n}(\vec{x})$ are determined by the spatial configuration of the chiral angle $\theta(\vec{x})$. As we are considering a finite DCC, we have the following asymptotic condition:

$$\langle \mathrm{DCC}|\sigma'(\vec{x})|\mathrm{DCC}\rangle,\langle \mathrm{DCC}|\vec{\pi}(\vec{x})|\mathrm{DCC}\rangle\to 0 \tag{11}$$

$$\text{as } |\vec{x}|\to\infty.$$

More conditions are needed to determine the function $f_\mu(\vec{k})$.

As an example, we consider a simple case where the expectation values of the conjugate momenta are vanishing: $\langle\dot{\sigma}'\rangle=\langle\dot{\vec{\pi}}\rangle=0$. Then, we have

$$f_0(\vec{k})=\frac{\sqrt{\omega_0}}{\sqrt{2(2\pi)^3}}\int d^3\vec{x}e^{i\vec{k}\cdot\vec{x}}\langle \mathrm{DCC}|\sigma'(\vec{x})|\mathrm{DCC}\rangle, \tag{12a}$$

$$f_i(\vec{k})=\frac{\sqrt{\omega_i}}{\sqrt{2(2\pi)^3}}\int d^3\vec{x}e^{i\vec{k}\cdot\vec{x}}\langle \mathrm{DCC}|\pi_i(\vec{x})|\mathrm{DCC}\rangle. \tag{12b}$$

Furthermore, we assume that the configuration is spherically symmetric, the pionic part is Gaussian, and $\vec{n}(\vec{x})$ is independent of $\vec{x}$:

$$\langle \mathrm{DCC}|\vec{\pi}(\vec{x})|\mathrm{DCC}\rangle=v\sin\theta(r)\vec{n}=vQ\exp\left(-\frac{r^2}{2R^2}\right)\vec{n}, \tag{13a}$$

$$\langle \mathrm{DCC}|\sigma'(\vec{x})|\mathrm{DCC}\rangle=-v[1-\sqrt{1-Q^2\exp(-r^2/R^2)}]. \tag{13b}$$

The origin of r is taken at the center of the DCC. The parameter Q is the sine of the chiral angle θ at the center which has to be between 0 and $\pi/2$ in order to satisfy Eq. (13a).

The average value of Q is nearly 0.8 because of chiral symmetry. Namely,

$$\overline{\sin\theta(x)}=\frac{\int_0^1 d(\cos\theta)\sin\theta}{\int_0^1 d(\cos\theta)}\approx 0.8. \tag{14}$$

Therefore we take $Q=0.8$ as a typical choice. The radius parameter R is related to the correlation length ξ which appears in the normal ordered correlation functions:

$$\langle:\pi_i(\vec{x})\pi_i(0):\rangle=\langle:\pi_i(0)\pi_i(0):\rangle\exp(-r^2/2R^2) =\langle:\pi_i(0)\pi_i(0):\rangle\exp(-r^2/\xi^2), \tag{15}$$

where the first line is obtained from Eqs. (5), (12b), and (13a) while the second line gives the definition of ξ.

Therefore, we have $\xi=\sqrt{2}R$. The value of ξ has been estimated to be 1–3 fm in a quench scenario [7,8] and from 3 fm to 7 fm in an annealing scenario [9]. We thus take $(Q,R)=(0.8,5.0\text{ fm})$ as a typical choice for numerical calculations and consider also another choice (0.8,0.5 fm) and/or (0.8,1.0 fm) for comparison.

Finally, we have the following results for $|f_\mu(\vec{k})|^2$:

$$|f_0(\vec{k})|^2=\frac{1}{16}v^2Q^4R^6\sqrt{\vec{k}^2+m_{\sigma'}^2}\left\{\sum_{n=1}^{\infty}\frac{(2n-3)!!}{2^n n! n^{3/2}}Q^{2n-2}\times\exp(-R^2k^2/4n)\right\}^2, \quad (16a)$$

$$|f_i(\vec{k})|^2=\frac{1}{2}n_i^2v^2Q^2R^6\sqrt{\vec{k}^2+m_\pi^2}\exp(-\vec{k}^2R^2), \quad (16b)$$

where $(-1)!!=1$ and n_i is the component of $\vec{n}$.

III. ENERGY DENSITY AND DECAY OF DCC's

In this section, two DCC decay models are described and an expression for the energy density is given.

A. Instantaneous decay model and energy density

One needs to specify the mechanism of DCC decay in order to estimate the observable effects. However, we are not aware of any established theory to describe DCC decay. So we consider here a simple model which may be called the instantaneous decay model. It is assumed that a particle characterized by $a_\mu^\dagger(\vec{k})$ in a DCC state becomes a free particle with momentum $\vec{k}$ and energy $\sqrt{\vec{k}^2+m_\mu^2}$ instantaneously. It is then straightforward to calculate physical quantities such as momentum distributions, multiplicity, the total energy of pions, and energy density.

The momentum distribution is given by

$$\frac{d^3N_\mu}{dk^3}=\langle \mathrm{DCC}|a_\mu^\dagger(\vec{k})a_\mu(\vec{k})|\mathrm{DCC}\rangle=|f_\mu(\vec{k})|^2. \quad (17)$$

The mean multiplicity is obtained by integrating $|f_\mu(\vec{k})|^2$ over $\vec{k}$. The total energy in this approximation is

$$E_{\mathrm{free}}(R)=\sum_\mu E_\mu=\sum_\mu\int d^3k\sqrt{\vec{k}^2+m_\mu^2}|f_\mu(\vec{k})|^2. \quad (18)$$

More precise values of the energy (density) can be estimated as follows. The energy density is estimated by taking the expectation value of the Hamiltonian density given by substituting the DCC configuration into the Hamiltonian density:

$$\mathcal{H}=\frac{1}{2}\dot{\sigma}'^2+\frac{1}{2}\dot{\vec{\pi}}^2+\frac{1}{2}(\nabla\sigma')^2+\frac{1}{2}\sum_{j=1}^{3}(\nabla\pi_j)^2+V(\sigma,\vec{\pi}). \quad (19)$$

Using Eqs. (13a) and (13b), we have

$$E_{\mathrm{tot}}(R)=\int d^3\vec{x}\langle \mathrm{DCC}|\mathcal{H}|\mathrm{DCC}\rangle$$
$$=\int d^3\vec{x}\left\{\frac{f_\pi^2}{2}\left[\left(\frac{Q^2r^2}{R^4}\right)\exp\left(-\frac{r^2}{R^2}\right)+\left(\frac{Q^4r^2}{R^4}\right)\frac{\exp(-2r^2/R^2)}{1-Q^2\exp(-r^2/R^2)}\right]+Hf_\pi[1-\sqrt{1-Q^2\exp(-r^2/R^2)}]\right\}, \quad (20)$$

where the energy density is taken to be zero when there is no DCC $(Q=0)$ and v has been replaced with the pion decay constant f_π. On the other hand, the energy density in the instantaneous decay model is obtained by substituting $\vec{\pi}=f_\pi\sin\theta(\vec{x})\vec{n}$ and $\sigma'=f_\pi[1-\cos\theta(\vec{x})]$ into the free part of the Hamiltonian:

$$\mathcal{H}_{\mathrm{free}}=\mathcal{H}_\pi+\mathcal{H}_{\sigma'}, \quad (21a)$$

$$\mathcal{H}_\pi=\frac{1}{2}\sum_{i=0}^{3}\partial_i\vec{\pi}\partial_i\vec{\pi}+\frac{1}{2}m_\pi^2\vec{\pi}^2, \quad (21b)$$

$$\mathcal{H}_{\sigma'}=\frac{1}{2}\sum_{i=0}^{3}\partial_i\sigma'\partial_i\sigma'+\frac{1}{2}m_{\sigma'}^2\sigma'^2, \quad (21c)$$

$$\tilde{E}_{\mathrm{free}}(R)=\int d^3\vec{x}\langle \mathrm{DCC}|\mathcal{H}_{\mathrm{free}}|\mathrm{DCC}\rangle, \quad (21d)$$

where

$$\langle \mathrm{DCC}|\mathcal{H}_\pi|\mathrm{DCC}\rangle=\frac{f_\pi^2Q^2}{2R^4}r^2\exp(-r^2/R^2)+\frac{1}{2}m_\pi^2f_\pi^2Q^2\exp(-r^2/R^2), \quad (21e)$$

$$\langle \mathrm{DCC}|\mathcal{H}_{\sigma'}|\mathrm{DCC}\rangle=\frac{f_\pi^2Q^4}{2R^4}r^2\frac{\exp(-2r^2/R^2)}{1-Q^2\exp(-r^2/R^2)}+\frac{1}{2}m_{\sigma'}^2f_\pi^2[1-\sqrt{1-Q^2\exp(-r^2/R^2)}]^2. \quad (21f)$$

The energy $\tilde{E}_{\mathrm{free}}(R)$ coincides with $E_{\mathrm{free}}(R)$ given by Eq. (18). It will be shown in Sec. V that $E_{\mathrm{free}}(R)$ gives a considerable overestimate compared to the precise value $E_{\mathrm{tot}}(R)$. This is because the contribution from the ''potential'' energy is neglected in the free particle approximation used in the instantaneous decay model. The multiplicity of particles is also overestimated in this model. However, the approximation used in the model may be valid if there is an energy reservoir and the energy that amounts to the ''potential'' energy is provided by the reservoir. Fortunately, there are many background particles as a reservoir in A-A collisions. One possibility of DCC decay in the absence of an energy reservoir is discussed in the next subsection.

B. Radiation model

Energy conservation is apparently violated in the instantaneous decay model discussed above. The simplest model which includes energy conservation is the radiative cooling model. So we consider another simple model where energy conservation holds exactly, treating the DCC system as an isolated system. Although the DCC has a smooth tail, we treat it as if it has a spherical surface of definite radius for simplicity and consider the radiation of particles from the surface.

At first, we give the variables which are used in this model. $E_{\rm tot}(R(t))$ is the energy of a DCC of a time-dependent radius $R(t)$ and is given by Eq. (20) with $R=R(t)$. The surface of a large DCC can be regarded locally as a plane. Particles pass through the surface toward various directions. Then, the flow due to a particle with momentum $\vec{k}$ contains a factor $\cos\theta$ which is the cosine of the angle between $\vec{k}$ and the normal to the surface. The angular average of $\cos\theta$ gives a factor of 1/4. In this subsection, R is a function of t because energy conservation is taken into account. The velocity is $v_\mu(\vec{k})=|\vec{k}|/\sqrt{m_\mu^2+\vec{k}^2}$.

As the volume and the surface area of the DCC are given by $V(t)=4\pi R^3(t)/3$ and $S(t)=4\pi R^2(t)$, respectively, the rate of energy flow through the surface of a DCC of radius $R(t)$ becomes

$$\frac{dE_{\rm tot}(R(t))}{dt}=-\frac{1}{4}\frac{S(t)}{V(t)}\int d^3\vec{k}\times\sum_\mu |f_\mu(\vec{k},R(t))|^2 v_\mu(\vec{k})\sqrt{\vec{k}^2+m_\mu^2}$$
$$=-3\pi\left[\int_0^\infty dk k^3\left(\sum_{\mu=0}^{3}|f_\mu(\vec{k},R(t))|^2\right)R^{-1}(t)\right]. \quad (22)$$

The net momentum distributions of emitted particles are then given by

$$\frac{dN_{\pi^j}}{dk}=3\pi\frac{k^3}{\sqrt{k^2+m_\pi^2}}\int dt|f_j(\vec{k},R(t))|^2R^{-1}(t), \quad (23)$$

$$\frac{dN_{\sigma'}}{dk}=3\pi\frac{k^3}{\sqrt{k^2+m_{\sigma'}^2}}\int dt|f_0(\vec{k},R(t))|^2R^{-1}(t). \quad (24)$$

IV. CHARGE FLUCTUATIONS

One of the characteristic properties of DCC's is the charge fluctuations represented by the peculiar $1/\sqrt{\rho}$ distribution [3–5]. [See Eq. (1) for the definition of ρ.] This distribution was derived either classically [3,4] or quantum field theoretically [5]. The derivation in quantum field theory was done by using an isospin coherent state with $I=I_z=0$. The isospin coherent state is constructed by isospin projection:

$$|I\ \ I_z\rangle\propto\int d\Omega\ \ Y_{I,I_z}|\mathrm{DCC}\rangle, \quad (25)$$

where Y_{I,I_z} is a spherical harmonic function. The $1/\sqrt{\rho}$ distribution was derived by using the $|I=0,\ I_z=0\rangle$ state. However, it is not clear if the choice of an isospin coherent state with $I=I_z=0$ is essential. In fact, this distribution was derived even in the classical picture where isospin projection is not used. The classical picture corresponds to the coherent state, but not to the isospin coherent state. In realistic processes of nucleus-nucleus collisions, there is no reason to fix the state to isosinglet space [19–22]. It is natural to consider that isospin distributes in general. The isospin of a DCC in our case distributes such that

$$\langle\mathrm{DCC}|I^2|\mathrm{DCC}\rangle=2\tilde{n},\quad\langle\mathrm{DCC}|\vec{I}|\mathrm{DCC}\rangle=0, \quad (26)$$

where $\tilde{n}$ is the number of pions. The $I=0$ component does not dominate in a DCC which contains many pions. Thus we consider a coherent state without isospin projection.

The coherent state is expanded in terms of the number state $|n,p,q,r,s\rangle$ as follows:

$$|\mathrm{DCC}\rangle=N_D\sum_n\sum_{\substack{p,q,r,s\\p+q+r+s=n}}\times\sqrt{\frac{(T_0)^p(T_+)^q(T_-)^r(T_3)^s}{p!q!r!s!}}|n,p,q,r,s\rangle, \quad (27)$$

where

$$|n,p,q,r,s\rangle=\frac{(b_0^\dagger)^p(b_+^\dagger)^q(b_-^\dagger)^r(b_3^\dagger)^s}{\sqrt{p!q!r!s!}}|0\rangle,$$
$$n=p+q+r+s. \quad (28)$$

Furthermore, p is the number of sigma mesons, and q,r,s are the numbers of π^+,π^-,π^0, respectively. The quantities T_μ and b_μ are defined as follows ($\mu=0,+,-,3$):

$$n_i^2K=\int d^3\vec{k}|f_i|^2\quad\text{for}\quad i=1,2,3, \quad (29)$$

$$T_0=\int d^3\vec{k}|f_0(\vec{k})|^2, \quad (30a)$$

$$T_\pm=\frac{K}{2}\sin^2\phi, \quad (30b)$$

$$T_3=K\cos^2\phi, \quad (30c)$$

$$b_\mu^\dagger=\frac{\int d^3\vec{k}f_\mu(\vec{k})a_\mu^\dagger(\vec{k})}{\sqrt{T_\mu}}, \quad (31)$$

$$f_\pm(\vec{k})=[\mp f_1(\vec{k})+if_2(\vec{k})]/\sqrt{2}, \quad (32)$$

$$a_\pm^\dagger(\vec{k})=[\mp a_1^\dagger(\vec{k})-ia_2^\dagger(\vec{k})]/\sqrt{2}, \quad (33)$$

TABLE I. Numerical values of physical quantities associated with either large DCC's or closely packed small DCC's with typical angle parameter Q and radius R: $N_{\rm tot}$ is the total number of pions, $\langle p\rangle$ is the averaged momentum of produced pions, $E_{\rm tot}$ and $E_{\rm free}$ are the energies of the DCC (system) given by Eqs. (20) and (18), respectively, $\epsilon_{\rm max}$ is the maximum value of the energy density, and $(dN_{\rm ch}^{{\rm direct}\ \pi}/dy)_{y=0}$ is the central height of the rapidity distribution of direct charged pions. Typical backgrounds for $N_{\rm tot}$ and $(dN_{\rm ch}^{{\rm direct}\ \pi}/dy)_{y=0}$ estimated from the parton cascade model [23] for Au-Au collisions at a total c.m. energy of 200A GeV are also shown for comparison.

	Large DCC's	Small DCC's	Parton cascade model (Au-Au)
Q	0.8	0.8	–
R(fm)	5.0	0.5	–
$N_{\rm tot}$	58	150	11480
$\langle p\rangle$ (GeV/c)	1.3×10^{-1}	5.0×10^{-1}	–
$E_{\rm tot}$ (GeV)	5.7	71.7	(200A)
$E_{\rm free}$ (GeV)	12	79	–
$\epsilon_{\rm max}$ (MeV/fm^3)	9	32.5	–
$(dN_{\rm ch}^{{\rm direct}\ \pi}/dy)_{y=0}$	51	42	1200 ($y\sim$ 0)

and $b_\mu, b_\nu^\dagger$ satisfy the commutation relation $[b_\mu, b_\nu^\dagger] = \delta_{\mu,\nu}$.

The probability distribution is

$$P(n,p,q,r,s)=|\langle n,p,q,r,s|{\rm DCC}\rangle|^2 = N_D^2\frac{(T_0)^pK^{q+r+s}}{p!q!r!s!}\frac{1}{2^{q+r}}(\sin^2\phi)^{q+r}(\cos^2\phi)^s, \tag{34}$$

where ϕ is the polar angle of $\vec{n}$ in $\vec{\pi}$ space.

We consider first the case where there is no sigma meson at all. The unit vector $\vec{n}$ in $\vec{\pi}$ space is chosen at random for each DCC to evaluate the distribution for many DCC samples. This means an angular average in $\vec{\pi}$ space. The result is given as

$$\overline{P}(n,0,q,r,s)=\frac{1}{4\pi}\int d\Omega P(n,0,q,r,s) = C(n)\frac{(q+r)!}{q!r!}\frac{(2s)!}{2^s(s!)^2}, \tag{35}$$

$$C(n)=N_D^2\frac{K^n}{(2n+1)!!}. \tag{36}$$

We evaluate the n,s distribution by summing up for q and r:

$$\mathcal{P}(n,s)=\sum_{\substack{q,r\\ q+r+s=n}}\overline{P}(n,0,q,r,s)=2^nC(n)\frac{(2s)!}{4^s(s!)^2} \propto\frac{1}{\sqrt{s}}\quad \text{for}\ \ 1\ll s\ \ (\ll n). \tag{37}$$

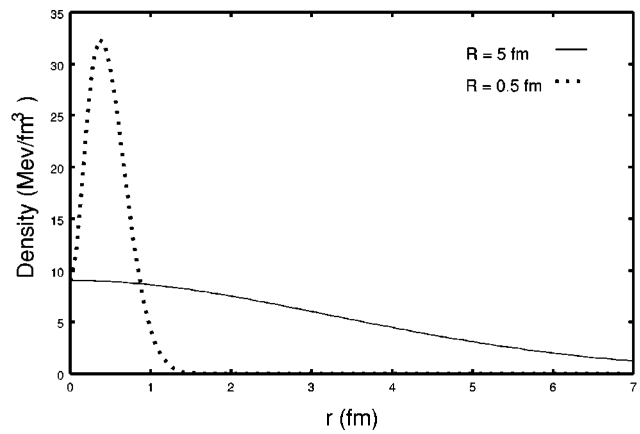

FIG. 1. Energy density distribution of DCC's.

The average value of the neutral pion number given by Eq. (37) is one-third of the total pion number.

Now we check the effect of sigma mesons. We start from Eq. (34). The decay mode of a sigma meson is $\sigma'\rightarrow\pi^+ +\pi^-$ or $2\pi^0$. We fix the total number $N=n_\pi+2n_\sigma$ of the final state pions including those from sigma decay and the number of neutral pions, m. The distribution after averaging in $\vec{\pi}$ space is

$$\mathcal{P}(N,m)=N_D^2\sum_{\substack{j={\rm mod}(N,2)\\ \text{and } j\geqslant{\rm mod}(m,2)}}^{N}{}' \times\frac{(2K)^j}{(2j+1)!!}\left(\frac{2T_0}{3}\right)^{(N-j)/2} \times\sum_{t=\max(\langle(m-j)/2\rangle,0)}^{\min((N-j)/2,[m/2])}\frac{8^t}{t![(N-j)/2-t]!} \times\frac{[2(m-2t)]!}{4^m[(m-2t)!]^2}, \tag{38}$$

where $[x]$ is the maximum integer which is not larger than x. For example, $[4.56]=4$. Here $\langle x\rangle$ is equal to $-[-x]$. The primed sum is

$$\sum_{n=n_0}{}' a_n = a_{n_0}+a_{n_0+2}+a_{n_0+4}+\cdots.$$

Examples of the primed sum in Eq. (38) are

$$\mathcal{P}(2,0)\rightarrow\sum_{j=0}^{2}{}', \quad \mathcal{P}(2,1)\rightarrow\sum_{j=2}^{2}{}',$$

$$\mathcal{P}(4,1)\rightarrow\sum_{j=2}^{4}{}'.$$

V. NUMERICAL RESULTS

We made numerical calculations of various quantities related to the DCC and its decay using the formula presented in the preceding sections. We consider the following two

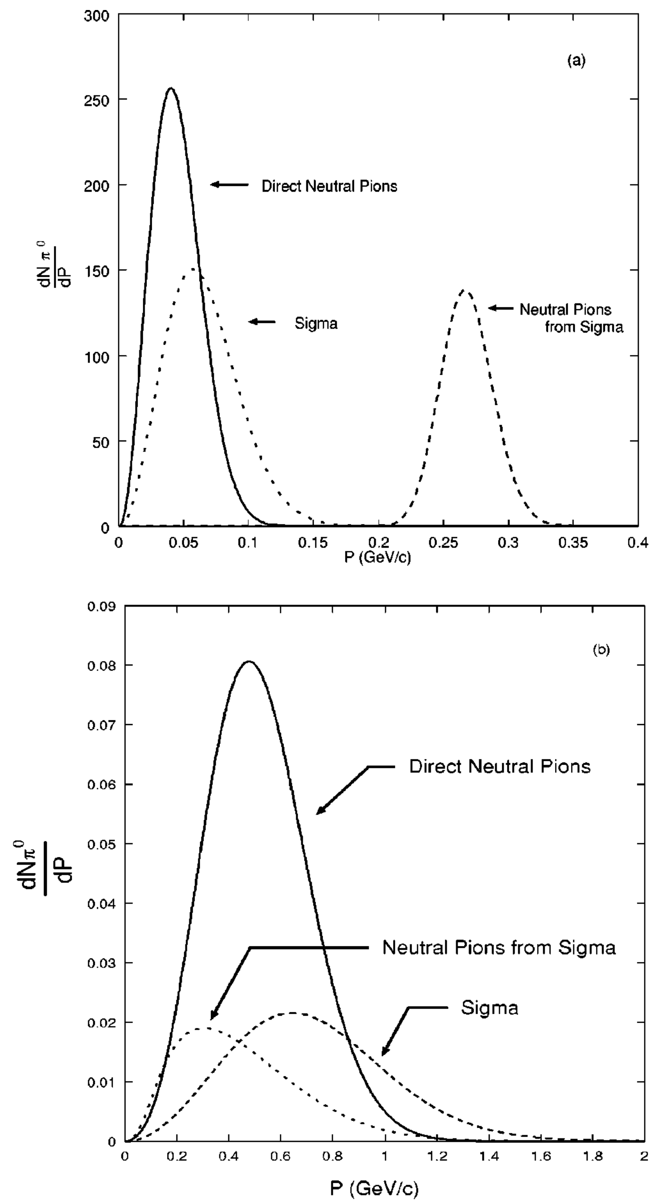

FIG. 2. (a) Momentum distributions of neutral pions and sigma mesons coming from a large DCC (R = 5 fm). (b) Momentum distributions of neutral pions and sigma mesons coming from a single small DCC (R = 0.5 fm).

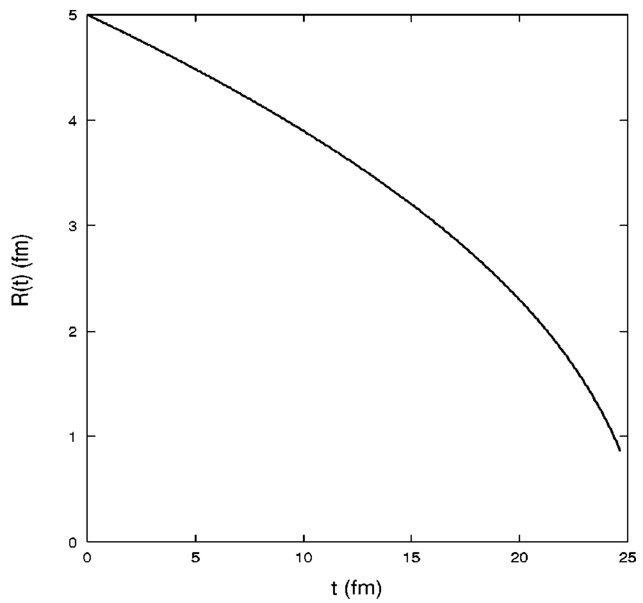

FIG. 3. Time dependence of the radius of a DCC in the radiation model. The curve terminates at the point where the energy of the DCC becomes smaller than the pion mass (140 MeV).

cases[1] as typical examples: a single large DCC of $R=5$ fm and many small DCC's of $R=0.5$ fm closely packed in the same volume. The three parameters λ, v, and H of the linear sigma model are determined by requiring that m_π = 140 MeV, $m_{\sigma'}$ = 600 MeV, and f_π = 93 MeV. The result is $\lambda=19.7$, $v=$ 87.5 MeV, and $H=(122\ \text{MeV})^3$. Some numerical characteristics of pions stemming from DCC's in the instantaneous decay model are shown in Table I. The mean total pion multiplicity from closely packed small DCC's is some 2.5 times larger than that from a large DCC. This is because the particle (pion and sigma meson) multiplicity is approximately proportional to R^2 for small R (surface effect) while proportional to R^3 for large R (volume effect) and also to Q^2 for pions and Q^4 for σ' mesons as can be seen in Eqs. (16a) and (16b). In any case, the mean total multiplicity of pions from DCC's is much smaller than that of pions (hadrons) which will be produced by a normal mechanism in nucleus-nucleus collisions at high energies. As an example, we have shown in Table I the result of a parton cascade model for central Au-Au collisions at RHIC energy [23]. It is obvious that the multiplicity alone cannot be a good signal of DCC formation.

The total particle energy E_{free} in two cases is shown in Table I. It should be noted here that E_{free} for small DCC's is

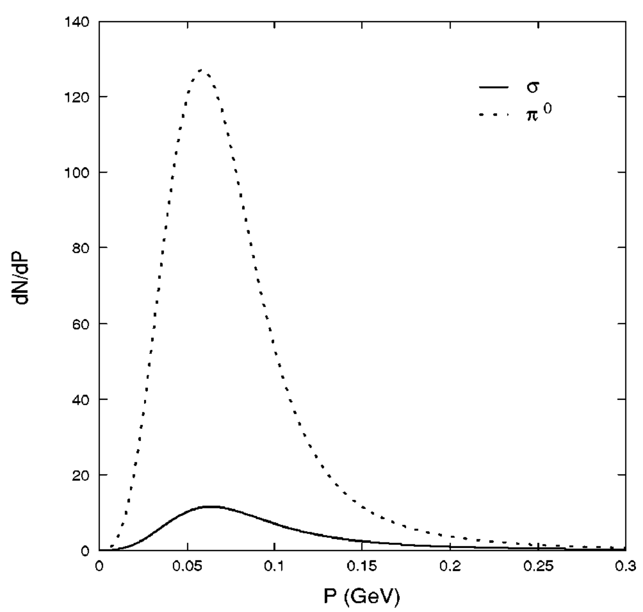

FIG. 4. Momentum distributions of σ' and π^0 from a DCC of R = 5 fm in the radiation model.

[1]We consider here ideal cases for clarity. Several DCC's of different sizes may be produced in different locations of phase space in general.

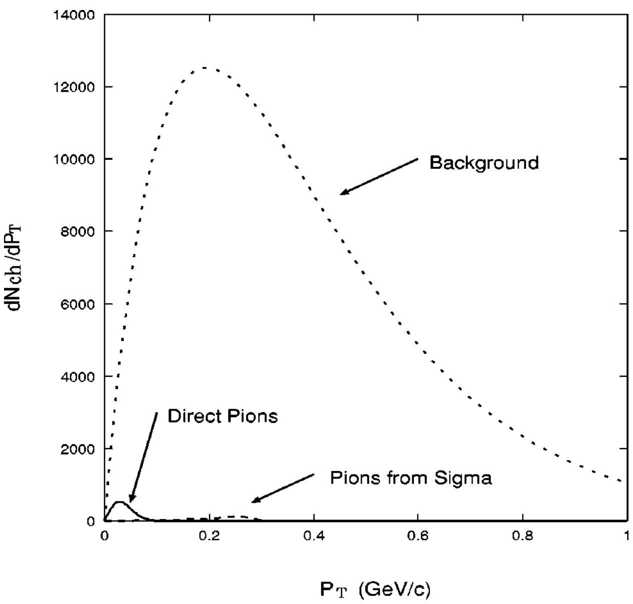

FIG. 5. Transverse momentum distributions of charged pions. The background is estimated by using the result of Geiger [23] for the multiplicity with the assumption that $dN_{\rm ch}/dP_T \propto P_T \exp[-P_T/(2\langle P_T\rangle)]$ with $\langle P_T\rangle = 400$ MeV.

equal to 10^3 times $E_{\rm free}$ $(R=0.5)$ calculated from Eq. (18). The energy density of a single DCC as a function of radius is calculated using Eqs. (21a), (21d), (21e), and (21f) and the result is shown in Fig. 1. The maximum energy density is given in Table I. In any case, both the energies and energy densities associated with DCC's are so small in comparison with the total collision energy and the expected energy density in central collisions of heavy nuclei at RHIC and higher energy regions. This means that there is no difficulty in forming DCC's from the point of view of energy conservation.

The momentum distributions [cf. Eq. (17)] of direct π^0's and that of σ''s for two cases are shown in Figs. 2(a) and 2(b). The average momenta of pions in the two cases are shown in Table I. It is remarkable that the averaged momentum in the case of large DCC's is much smaller than the normal value (say, 0.4–0.5 GeV/c) of the mean transverse momentum observed in general high energy nuclear collisions. Furthermore, the momentum of a pion coming from a σ' meson is much larger than that of a direct π when R is large. As seen in Table I, the mean momentum of pions from small DCC's is much larger than that from large DCC's.

As already discussed in Sec. III, the energy $\widetilde{E}_{\rm free}(R)$ calculated from Eqs. (21a), (21d), (21e), and (21f) overestimates the total energy of the DCC system. A more precise value can be estimated by using the radiation model. First of all, one can evaluate the lifetime of DCC's by solving Eq. (22) with Eq. (20) for $R(t)$. The numerical result for $R(t)$ is shown in Fig. 3, from which one sees that the lifetime is, for example, 25 fm for $R=5$ fm and some 10 fm for $R=3$ fm. Equations (23) and (24) allow us to evaluate momentum distributions of π^0's and σ's and the result for $R=5$ fm is shown in Fig. 4. The direct π^0 distribution is considerably reduced in comparison with that in the instantaneous model, while the σ' distribution is much more reduced. The mean total pion multiplicity in this case is about 29.

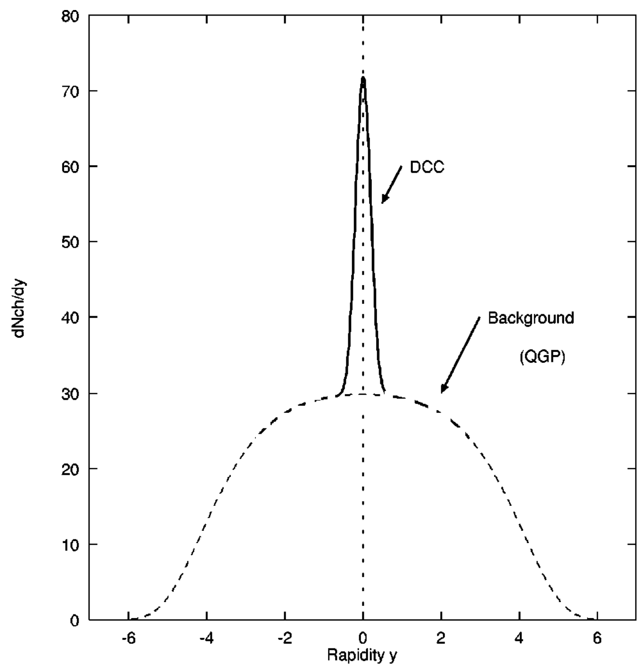

FIG. 6. Rapidity distribution of charged pions from $R = 5$ fm DCC's with a transverse momentum cutoff ($P_{T\,\rm cut}=0.055$ GeV). The background is estimated by assuming that the rapidity and P_T distribution are factorized. For the distribution of rapidity it is assumed that $dN_{\rm ch}/dy \propto \exp(-{\rm const}\times \cosh y)$.

The energy-weighted integration of the distribution gives the total energy $E_{\rm tot}$, of which the numerical result is also shown in Table I. It is remarkable that the difference $E_{\rm free} - E_{\rm tot}$ is almost independent of R. This means that the "potential energy" of DCC's per unit volume is almost independent of R.

The soft momentum spectrum of pions [16,17] produced by decay of large DCC's provides the possibility to use it as a signal of DCC formation. We show the transverse momentum distribution of charged pions (direct and indirect) [18] stemming from a DCC of $R=5$ fm in the instantaneous decay model together with the huge background expected for central Au-Au collisions at $\sqrt{s}$ = 200 GeV in Fig. 5. It should be noted here that one can enhance considerably the signal-to-noise ratio by cutting off the high transverse momentum component. The expected rapidity distribution of charged pions (or charged hadrons) after a p_T cut with cutoff momentum = 0.055 GeV/c is shown in Fig. 6. A sharp peak emerges at $y=0$ on top of a broad background. In general, a sharp peak will appear at such a rapidity which is equal to the rapidity of the DCC. Such a peak (or peaks) as shown in Fig. 6 may be observed as a clear signal of DCC formation if the rapidity bin size is chosen most appropriately and the effects of statistical fluctuations are correctly evaluated [24].

One of the most characteristic properties of DCC's is charge fluctuations, i.e., fluctuations of ρ defined by Eq. (1). We have already shown in Sec. IV that a coherent pion state without isospin projection and without a sigma component yields a $1/\sqrt{\rho}$ distribution when the total multiplicity is not so small [cf. Eq. (37)]. The distributions for $N=10$ and 80 are shown in Fig. 7 in comparison with the asymptotic $1/2\sqrt{\rho}$ distribution. The distribution for $N=80$ is already very close to the asymptotic distribution and even the distribution for

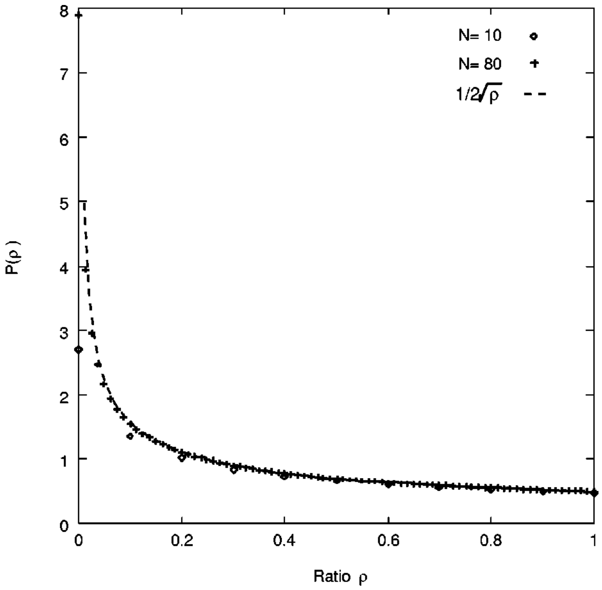

FIG. 7. The probability distribution of neutral-to-total ratio ρ when there is no sigma meson.

$N=10$ is not much different from the asymptotic one.

However, there is a smearing effect due to the contribution from σ mesons. We have calculated the distribution using Eq. (38). The results for a single DCC of $R=5$ fm and 1 fm with $N=50$ are shown in Fig. 8. The smearing effect is so strong that the distributions deviate significantly from the $1/2\sqrt{\rho}$ distribution. However, an appreciable enhancement still remains at the small ρ region in comparison with the binomial distribution which corresponds to the case where all the pions come from 25 σ mesons. One can see also that the distribution is insensitive to R.

Finally, we have examined the dependence of the ρ distribution on the total pion number N. The distributions for $N=20$ and 50 with $R=5$ fm are shown in Fig. 9. It is found that the enhancement at small ρ is larger for smaller N. This suggests that the relative contribution from σ mesons depends on N. In order to confirm this point, we have calculated the fraction of indirect pions, i.e., the mean multiplicity of indirect pions divided by the total pion multiplicity N as a

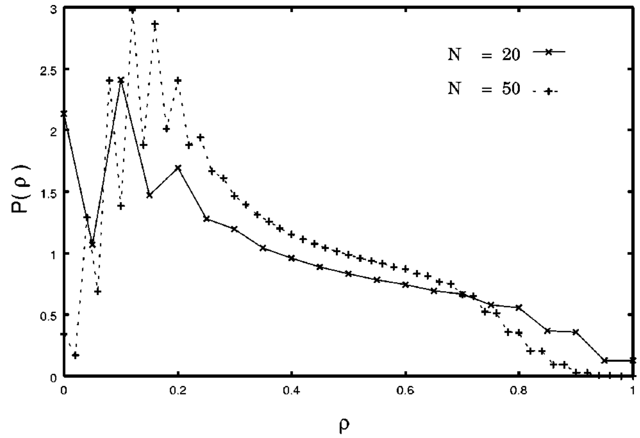

FIG. 9. The N dependence of the ρ distribution.

function of N for various R. The result is shown in Fig. 10. They are monotonically increasing as functions of N while monotonically decreasing as functions of R for a fixed N.

VI. CONCLUSION

We investigated the properties of the DCC which is described by a coherent state. The particle distributions are evaluated using two models of DCC decay: the instantaneous decay model and radiation model. The results are compared with possible backgrounds in order to find possible signals of DCC's. The radiation model gives us a rough estimate of the lifetime of DCC's. It is found that the multiplicity of particles from DCC's is so small that it cannot be a signal of DCC's. In order to attain a sufficiently large signal-to-noise ratio, we note the fact that the momentum distribution coming from a large DCC is much softer than that of pions produced by an ordinary mechanism. This gives us the possibility to find a clear signal of DCC's in a rapidity distribution by cutting off the large P_T component. An event-by-event analysis with an adequate trigger and a varying rapidity bin will be most useful. A correlation analysis of charge fluctuations and rapidity fluctuations will also be useful.

On the other hand, the $1/(2\sqrt{\rho})$ distribution will be considerably smeared if there are sigma mesons. However, the

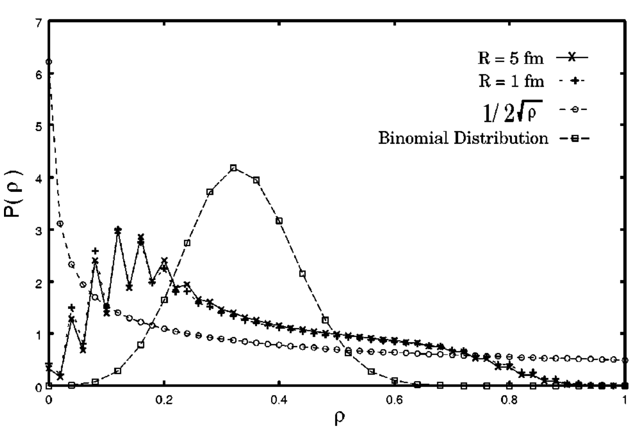

FIG. 8. ρ distribution when there are sigma mesons.

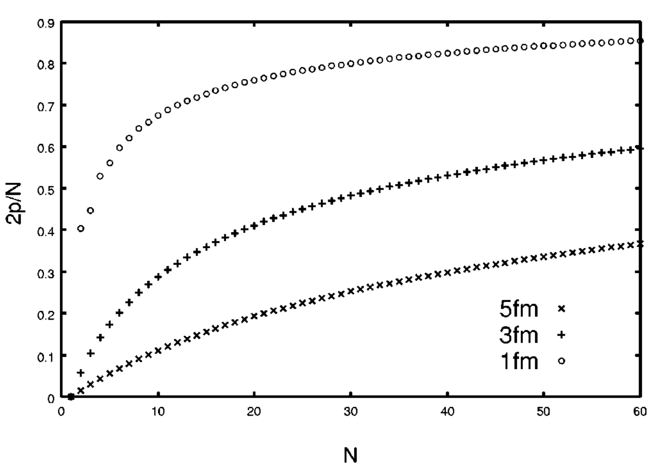

FIG. 10. N dependence of the average fraction of indirect pions.

distribution at small ρ may still show an observable enhancement in comparison with the binomial distribution provided that effects from other backgrounds are not so large.

We have considered that DCC's may be produced in nucleus-nucleus collisions at high energies. It may also be possible that they are produced in hadron-hadron collisions [25]. However, the formation of large DCC's is unlikely in this case. It is then obvious that it is difficult to explain the Centauro events in terms of DCC's, whether they are induced by nucleus-nucleus collisions or hadron-hadron collisions.

There are many problems about the formation and decay of DCC's. In the present paper, we have simply assumed the formation of a DCC and have studied its decay by using simple models. In a more complete theory, formation and decay of DCC's should be studied in a unified manner as a sequence of a dynamical process or a nonequilibrium statistical process [8,12,13,16]. These problems are under investigation.

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