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Mitsuhiro Arikawa,[1] Yasuhiro Saiga,[2] and Yoshio Kuramoto[3]

[1]*Max-Planck-Institut für Physik komplexer Systeme, Nöthnizer Strasse 38, D-01187 Dresden, Germany*
[2]*Institute for Solid State Physics, University of Tokyo, Kashiwa-no-ha 5-1-5, Kashiwa 277-8581 Japan*
[3]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

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The electron addition spectrum $A^+(k,\omega)$ is obtained analytically for the one-dimensional (1D) supersymmetric t-J model with $1/r^2$ interaction. The result is obtained first for a small-sized system and its validity is checked against the numerical calculation. Then the general expression is found which is valid for arbitrary size of the system. The thermodynamic limit of $A^+(k,\omega)$ has a simple analytic form with contributions from one spinon, one holon, and one antiholon—all of which obey fractional statistics. The upper edge of $A^+(k,\omega)$ in the (k,ω) plane includes a delta-function peak which reduces to that of the single-electron band in the low-density limit.

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The concept of spinons and holons, both of which obey the fractional statistics [1], has turned out to be useful in approaching 1D electron systems. In terms of these quasiparticles one can inquire into not only the low-energy and low-wavelength limit, but the global feature of the dynamics. Hence special interest has been cherished in the global dynamics from both theoretical and experimental points of view. For example, angle resolved photoemission [2] has revealed some evidence of the spin-charge separation by resolution of holon and spinon contributions. On the theoretical side, numerical studies have been performed for the 1D t-J model for a small number of lattice sites [3] and some structures have been ascribed to spinons and holons. For deeper understanding of the overall dynamics, demand is growing for analytic theory which can go to the thermodynamic limit. Partly analytic theory is available for the single-particle spectral functions of the t-J model in the $J \to 0$ limit [4]. A notable feature is that a satellite band is observed whose intensity is comparable to that of the main band. It is natural to ask how the finite J influences the dynamics.

In the supersymmetric t-J model with $1/r^2$ interaction [5], spinons and holons appear in the simplest manner. In fact, exact thermodynamics for the model [6] can be interpreted in terms of free spinons and holons. Ha and Haldane [7] analyzed numerical results for dynamics in finite-sized systems, and found that only a few elementary excitations contribute to spectral functions. They proposed a momentum-frequency region where each spectral function takes nonzero values in the thermodynamic limit, but they did not obtain the spectral functions themselves. Recently, exact results have been derived for a particular component [8], and for a particular momentum range of the spectral weight [9].

In this paper we report on the analytical result of the electron addition spectrum for the t-J model at zero temperature. The electron addition spectral function is relevant to the angle resolved inverse photoemission spectroscopy. Our result constitutes the first analytical knowledge for dynamical quantities of lattice electrons with no restriction on the system size, the density, and the momentum-frequency range. Although we cannot provide the formal proof for the exactness, the analytic results show complete agreement with numerical results for finite systems with various sizes. Hence our result in the thermodynamic limit is also expected to be exact.

We consider the supersymmetric t-J model given by

$$\mathcal{H}_{tJ} = \mathcal{P}\sum_{i<j}\Bigg[-t_{ij}\sum_{\sigma=\uparrow,\downarrow}(c^{\dagger}_{i\sigma}c_{j\sigma} + \text{h.c.}) + J_{ij}\left(\boldsymbol{S}_i\cdot\boldsymbol{S}_j - \frac{1}{4}n_i n_j\right)\Bigg]\mathcal{P}, \quad (1)$$

where $c_{i\sigma}$ is the annihilation operator of an electron at site i with spin σ, n_i is the number operator, and $\boldsymbol{S}_i$ is the spin operator. The projection operator $\mathcal{P}$ excludes double occupation at each site. The transfer and exchange energies are given by $t_{ij} = J_{ij}/2 = tD_{ij}^{-2}$ where $D_{ij} = (N/\pi)\sin[\pi(i-j)/N]$ with even N being the number of lattice sites, and the lattice constant is unity. The electron addition spectral function with the ground state $|0\rangle$ is defined by

$$A^+(k,\omega) = \sum_{\nu}|\langle\nu;N_{\mathrm{e}}+1|c^{\dagger}_{k\sigma}|0;N_{\mathrm{e}}\rangle|^2 \times \delta[\omega - E_\nu(N_{\mathrm{e}}+1) + E_0(N_{\mathrm{e}}) + \zeta], \quad (2)$$

where N_{e} is the total electron number, ζ the chemical potential, $c^{\dagger}_{k\sigma} = N^{-1/2}\sum_l c^{\dagger}_{l\sigma}e^{ikl}$, and $|\nu\rangle$ denotes an eigenstate of the Hamiltonian with energy E_ν.

We first give our main result and then its derivation. The addition spectrum includes the dispersion relations $\epsilon_{\mathrm{s}}(q)$ of spinons, $\epsilon_{\mathrm{h}}(q)$ of holons, and $\epsilon_{\mathrm{a}}(q)$ of antiholons. They are given in units of t by [6,7] $\epsilon_{\mathrm{s}}(q) = q(v_{\mathrm{s}} - q)$, $\epsilon_{\mathrm{h}}(q) = q(v_{\mathrm{c}} + q)$, and $\epsilon_{\mathrm{a}}(q) = q(2v_{\mathrm{c}} - q)/2$, where $v_{\mathrm{s}} = \pi$ and $v_{\mathrm{c}} = \pi(1-\bar{n})$ with $\bar{n} = N_{\mathrm{e}}/N = 2k_{\mathrm{F}}/\pi$.

Analytical expression of $A^+(k,\omega)$ with $0 \le k < 2\pi$ consists of the following components:

$$A^+(k,\omega) = A_{\mathrm{R}}(k,\omega) + A_{\mathrm{L}}(k,\omega) + A_{\mathrm{U}}(k,\omega), \quad (3)$$

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where

$$A_{\rm R}(k,\omega) = \frac{1}{4\pi}\int_0^{k_{\rm F}} dq_{\rm h}\int_0^{k_{\rm F}-q_{\rm h}} dq_{\rm s}\int_0^{2\pi-4k_{\rm F}} dq_{\rm a}\,\delta(k-k_{\rm F}-q_{\rm s}-q_{\rm h}-q_{\rm a})\delta[\omega-\epsilon_{\rm s}(q_{\rm s})-\epsilon_{\rm h}(q_{\rm h})-\epsilon_{\rm a}(q_{\rm a})] \times \frac{\epsilon_{\rm s}(q_{\rm s})^{g_{\rm s}-1}\epsilon_{\rm h}(q_{\rm h})^{g_{\rm h}-1}\epsilon_{\rm a}(q_{\rm a})^{g_{\rm a}-1}}{(q_{\rm h}+q_{\rm a}/2)^2}, \quad (4)$$

and $A_{\rm L}(k,\omega) = A_{\rm R}(2\pi-k,\omega)$. In Eq. (4) we have $g_{\rm s} = 1/2$, $g_{\rm h} = 1/2$, and $g_{\rm a} = 2$, which correspond to statistical parameters of spinons, holons, and antiholons, respectively [6,7]. Thus the matrix element in Eq. (4) can be interpreted in terms of the fractional statistics as in the case of correlation functions in related continuum systems [10]. The third component is given by

$$A_{\rm U}(k,\omega) = \sqrt{\frac{\epsilon_{\rm a}(k-2k_{\rm F})}{\epsilon_0(k)}}\,\delta[\omega-\omega_{\rm aU}(k)], \quad (5)$$

which contributes only in the region $2k_{\rm F} \le k \le 2\pi - 2k_{\rm F}$. Here $\epsilon_0(k) \equiv k(\pi - k/2)$ describes the spectrum of noninteracting electrons, and $\omega_{\rm aU}(k) \equiv \epsilon_{\rm s}(k_{\rm F}) + \epsilon_{\rm a}(k-2k_{\rm F})$. The $A_{\rm U}(k,\omega)$ can be regarded as contribution from antiholons with fixed energies of spinons and holons. In the dilute limit ($k_{\rm F} \to 0$), $\omega_{\rm aU}(k)$ as well as $\epsilon_{\rm a}(k-2k_{\rm F})$ tend to $\epsilon_0(k)$. Hence the coefficient in Eq. (5) becomes unity, and $A_{\rm U}(k,\omega)$ tends to the spectral intensity of noninteracting electrons. The other contributions $A_{\rm R,L}(k,\omega)$ can be neglected in this limit.

Figure 1 shows the spectral edges by solid lines together with the spectral intensities for a finite system as explained later. The threshold behavior in $A_{\rm R}(k,\omega)$ is derived as follows: As the frequency approaches an upper edge given by $\omega_{\rm s}(k) \equiv \epsilon_{\rm s}(k-k_{\rm F})$ with $k_{\rm F} \le k \le 2k_{\rm F}$, $A_{\rm R}(k,\omega)$ diverges as $[\omega_{\rm s}(k)-\omega]^{-1/2}$. Along the connecting upper edge for $2k_{\rm F} \le k \le 2\pi - 2k_{\rm F}$, $A_{\rm R}(k,\omega)$ has a stepwise discontinuity such as $\theta[\omega_{\rm aU}(k)-\omega]$. Here $\theta(x)$ is a Heaviside step function. On the other hand, near a lower edge given by $\omega_{\rm h}(k) \equiv \epsilon_{\rm h}(k+3k_{\rm F}-2\pi)$ with $2\pi - 3k_{\rm F} \le k \le 2\pi - 2k_{\rm F}$, $A_{\rm R}(k,\omega)$ behaves as $[\omega-\omega_{\rm h}(k)]^{3/2}$. The feature is in contrast to the result of Ref. [4] where the spectrum is enhanced near $\omega_{\rm h}(k)$. Finally near another lower edge given by $\omega_{\rm aL}(k) \equiv \epsilon_{\rm a}(k-k_{\rm F})$ with $k_{\rm F} \le k \le 2\pi - 3k_{\rm F}$, there is a stepwise discontinuity such as $\theta[\omega-\omega_{\rm aL}(k)]$. We note that the asymptotic behavior of $A^+(k,\omega)$ is fully consistent with the conformal field theory [5].

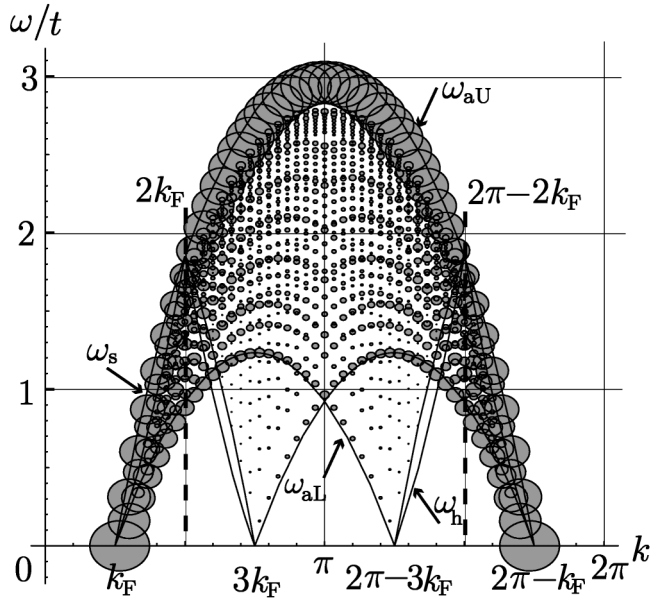

FIG. 1. The electron addition spectrum $A^+(k,\omega)$ in the case of $N = 60$, $N_{\rm h} = 29$, and $N_{\rm s} = 15$ with Fermi momentum $k_{\rm F} = \pi/4$. The intensity is proportional to the area of each oval. The solid lines are determined by dispersion relations of the elementary excitations in the thermodynamic limit.

Let us describe derivation of $A^+(k,\omega)$. We represent a state vector $|\psi\rangle$ in the t-J model by

$$|\psi\rangle = \sum_{x^{\rm h},x^{\rm s}} \psi(x^{\rm h},x^{\rm s}) \prod_{i\in x^{\rm s}} S_i^- \prod_{j\in x^{\rm h}} h_j^\dagger |F\rangle, \quad (6)$$

where $|F\rangle$ is the fully up-polarized state, $S_i^- = c_{i,\downarrow}^\dagger c_{i,\uparrow}$, and $h_i^\dagger = c_{i,\uparrow}$. The set of coordinates $(x^{\rm h},x^{\rm s}) \equiv x$ represents $x = (x_0^{\rm h},\ldots,x_{N_{\rm h}}^{\rm h},x_1^{\rm s},\ldots,x_{N_{\rm s}}^{\rm s}) = (x_0,x_1,\ldots,x_n)$ with $n = N_{\rm h} + N_{\rm s}$. Here $x_i^{\rm h}$ is for the ith hole and $x_j^{\rm s}$ is for the jth down-spin electron. In order to derive eigenfunctions of the system, it is convenient to use the Sutherland model [11] with the SU(1, 1) supersymmetry as an auxiliary [12,13]. The merit of using the Sutherland model is that much more is known about properties of the eigenfunctions than those for the t-J model. The eigenfunctions with the coupling parameter $\beta = 1$ correspond to such part of the set $\{\psi(x^{\rm h},x^{\rm s})\}$ that belongs to the Yangian highest weight states (YHWS) with the SU(2, 1) supersymmetry [14]. In making the correspondence one restricts the coordinates x to integer variables. It is convenient to introduce the complex coordinate by $z_j = \exp(2\pi i x_j/L)$ where L $(= N)$ is the length of the ring-shaped system. The YHWS can be expressed by polynomials of z_j where the degree of each variable lies in the range $[-N/2, N/2]$. The YHWS form a subset of the Fock space and the other states can be generated by successive actions of Yangian generators on the YHWS.

The ground-state wave function for the t-J model with $N_{\rm h} + 1$ holes and $N_{\rm s} = N_{\rm e}/2$ down-spin electrons is given by [5]

$$\Psi_{\rm GS} = \prod_{i\in[0,n]} z_i^{-\beta n/2} \prod_{i\in I_{\rm s}} z_i^{-(N_{\rm s}-1)/2} \times \prod_{i<j\in[0,n]} (z_i - z_j)^\beta \prod_{i<j\in I_{\rm s}} (z_i - z_j), \quad (7)$$

where we have introduced the interval $I_s = [N_h + 1, N_h + N_s]$ for the suffixes i and j. We have assumed that both N_s and N_h are odd so that the ground state is nondegenerate. In the following we always take $\beta = 1$. We fix a hole position at $x_0^h = 0$ in Ψ_{GS} which corresponds to adding an up-spin electron at this site. In terms of the ground-state wave function $\tilde{\Psi}_{GS}$ for the system with N_h holes and N_s down-spin electrons, we can represent the resultant state by

$$\Psi_{GS}|_{x_0=0} = \sum_{m=0}^{n} (-1)^m e_m(z) \tilde{\Psi}_{GS}\,, \tag{8}$$

where $e_m(z) = \sum_{i_1<i_2<\cdots<i_m \in I} z_{i_1} \cdots z_{i_m}$ is the elementary symmetric function of order m. Note that the interval $[0, n]$ in Eq. (7) is replaced by another interval $I = [1, n]$ in Eq. (8).

The spectrum of the Sutherland model $\mathcal{H}_{Su}$ is conveniently analyzed with the use of a similarity transformation generated by $\mathcal{O} = \prod_{i<j\in I}(z_i - z_j)^\beta \prod_{i\in I} z_i^{-(\beta n + N_s - 1)/2}$. We obtain $\hat{\mathcal{H}} \equiv \mathcal{O}^{-1}\mathcal{H}_{Su}\mathcal{O}$ as follows:

$$\hat{\mathcal{H}} = \frac{1}{2}\left(\frac{2\pi}{L}\right)^2 \sum_{i=1}^{n}\left(\hat{d}_i + \frac{\beta n}{2} - \frac{N_s - 1}{2}\right)^2, \tag{9}$$

where $\hat{d}_i$ is called the Cherednik-Dunkl operator [15]. It is known that the set $\{\hat{d}_i\}$ can be diagonalized simultaneously with real eigenvalue $\bar{\lambda}_i$ for each $\hat{d}_i$. The resultant eigenfunctions are polynomials $E_\lambda(z;\beta)$ of z_i and are called the nonsymmetric Jack polynomials [15]. The polynomial $E_\lambda(z;\beta)$ has a *triangular* structure with respect to a certain ordering on the set of $\{\lambda\}$ [13,15]; if one expands $E_\lambda(z;\beta)$ in terms of monomials $\Pi_i z_i^{\nu_i}$, the expansion coefficient is zero unless $\lambda \succeq \nu$, where $\succeq$ describes the ordering relation in the set.

Since we are dealing with identical particles, eigenfunctions should satisfy the following conditions of the SU(1, 1) supersymmetry: (i) symmetric with respect to exchange of z_i^h's; (ii) antisymmetric with respect to exchange of z_i^s's. By taking linear combination of $E_\lambda(z;\beta)$, we can construct another polynomial $K_\lambda(z;\beta)$ with the SU(1, 1) supersymmetry [16–18]. We specify the set of momenta as $(\lambda^h, \lambda^s) \equiv (\lambda_1^h, \ldots, \lambda_{N_h}^h, \lambda_1^s, \ldots, \lambda_{N_s}^s)$ with $\lambda_1^h \geq \cdots \geq \lambda_{N_h}^h, \lambda_1^s > \cdots > \lambda_{N_s}^s$. In this way we can parametrize $K_\lambda(z;\beta)$ by using $\lambda \equiv (\lambda^h, \lambda^s)$. At the ground state with $2N_s + 1$ electrons we have $\lambda = \tilde{\lambda}_{GS} = (\lambda_{GS}^h, \lambda_{GS}^s)$ with $\lambda_{GS}^h = [(N_s - 1)/2, (N_s - 1)/2, \ldots, (N_s - 1)/2]$, and $\lambda_{GS}^s = (N_s - 1, N_s - 2, \ldots, 0)$.

We define the inner product $\langle f, g\rangle_0$ for complex functions $f(z)$ and $g(z)$ as the constant term in the Laurent expansion of $f(z)^* g(z)$. As is clear from the definition of the transformation $\mathcal{O}$, $K_\lambda(z;\beta)\mathcal{O}(z)$ is orthogonal with respect to the above inner product. In order to derive the norm of $K_\lambda(z;\beta)$, we generalize the procedure taken in Refs. [18–20]. The result of lengthy calculation is given by

$$\langle K_\lambda\mathcal{O}, K_\lambda\mathcal{O}\rangle_0 = N_h!\, N_s!\, \rho_\lambda^{-1} \langle E_\lambda\mathcal{O}, E_\lambda\mathcal{O}\rangle_0\,, \tag{10}$$

where $\langle E_\lambda\mathcal{O}, E_\lambda\mathcal{O}\rangle_0$ denotes the norm of the nonsymmetric Jack polynomials. We refer to the literature [15] for the explicit form of the latter norm since it requires many lengthy combinatorial quantities. The quantity $\rho_\lambda = \rho_\lambda^h \rho_\lambda^s$ is given by

$$\begin{aligned} \rho_\lambda^h &= \prod_{i<j\in I_h} \frac{\bar{\lambda}_i - \bar{\lambda}_j + \beta}{\bar{\lambda}_i - \bar{\lambda}_j}\,, \\ \rho_\lambda^s &= \prod_{i<j\in I_s} \frac{\bar{\lambda}_i - \bar{\lambda}_j - \beta}{\bar{\lambda}_i - \bar{\lambda}_j}\,, \end{aligned} \tag{11}$$

where we use the intervals I_s and $I_h = [1, N_h]$ for the suffixes i and j.

In order to calculate $A^+(k,\omega)$, we need to know the expansion coefficient c_λ which appears in

$$e_m(z)\tilde{\Psi}_{GS} = \sum_\lambda c_\lambda K_\lambda(z;\beta)\mathcal{O}(z)\,. \tag{12}$$

Using the coefficient c_λ, $A^+(k,\omega)$ can be expressed as

$$A^+(k,\omega) = N_h \sum_\lambda{}' \delta(\omega - \Delta E_\lambda)\,|c_\lambda|^2 \frac{\langle K_\lambda\mathcal{O}, K_\lambda\mathcal{O}\rangle_0}{\langle \Psi_{GS}, \Psi_{GS}\rangle_0}\,, \tag{13}$$

where $\Delta E_\lambda = E_\lambda - E_0 - \zeta$, the chemical potential ζ for the finite N is given by $\zeta/\pi^2 = -(3\bar{n}^2 - 6\bar{n} + 4)/12 - (\bar{n} - 2)/(2N) + 1/(3N^2)$, and the prime means that the summation over λ is restricted by the momentum conservation $|\lambda| = |\tilde{\lambda}_{GS}| + m$ with $|\lambda| = \sum_j \lambda_j$ and $k = 2\pi[m + (N_s + 1)/2]/N$. The norm $\langle \Psi_{GS}, \Psi_{GS}\rangle_0$ can be obtained by a procedure similar to that described above.

From the *triangular* structure inherited to K_λ, the contributions to the excited states are limited to the cases where $\lambda^s = \lambda_{GS}^s + (1^{\lambda_s}, 0^{N_s - \lambda_s})$. Here p^α means the sequence $p, \ldots, p$ with the number of p's being α. Therefore the spin part can be parametrized by the single variable λ_s. We first calculate K_λ by a brute force for small systems (up to $N_h = 5$ and $N_s = 15$) in the small momentum region $m \leq (N_s - 1)/2$. In this region, we have $\lambda^h = \lambda_{GS}^h + \nu$. The coefficient c_λ is zero if the partition ν contains $s = (2, 2)$ where $s = (i, j)$ denotes a square in the Young diagram [15]. We find that the results for K_λ are expressed in the following form:

$$c_\lambda = \prod_{s\in\nu} \frac{-a'(s) + 1 + \beta'[l'(s) - 1]}{a(s) + 1 + \beta' l(s)}\,, \tag{14}$$

where $\beta' = \beta/(\beta + 1) = 1/2$, $a(s) = \nu_i - j$ (arm length), $a'(s) = j - 1$ (arm colength), $l(s) = \nu_j' - i$ (leg length) with ν_j' the length of the column, and $l'(s) = i - 1$ (leg colength). Thus, we can parametrize ν as $\nu = (\lambda_h, 1^{\lambda_a - 1}, 0^{N_h - \lambda_a})$ in this small momentum region. Namely $A^+(k,\omega)$ is determined by the three parameters λ_s, λ_h, and λ_a, which are related directly with momenta of the excitations. We have checked numerically that the expression given by Eq. (14) can in fact be extended beyond the small momentum region. Note that c_λ is independent of λ_s which, however, enters $A^+(k,\omega)$ through the norm $\langle K_\lambda\mathcal{O}, K_\lambda\mathcal{O}\rangle_0$.

We thus obtain the finite-size version of Eq. (4):

$$A_{\mathrm{R}}(k,\omega) = \sum_{\lambda}{}' I_{\mathrm{R}}(\lambda)\delta(\omega - \Delta E_{\lambda}) \tag{15}$$

and similar results with the suffix R replaced by L and U. In Eq. (15) we have

$$I_{\mathrm{R}}(\lambda) = \frac{1}{2[\Gamma(1/2)]^2}\frac{\Gamma(\lambda_{\mathrm{h}} - 1/2)\Gamma(\lambda_{\mathrm{h}} + N_{\mathrm{h}}/2)}{\Gamma(\lambda_{\mathrm{h}})\Gamma[\lambda_{\mathrm{h}} + (N_{\mathrm{h}} + 1)/2]} \times \frac{\lambda_{\mathrm{a}}(N_{\mathrm{h}} - \lambda_{\mathrm{a}} + 1)}{(2\lambda_{\mathrm{h}} + \lambda_{\mathrm{a}} - 1)(2\lambda_{\mathrm{h}} + \lambda_{\mathrm{a}} - 2)} \times \frac{\Gamma(\lambda_{\mathrm{s}} + 1/2)}{\Gamma(\lambda_{\mathrm{s}} + 1)}\frac{\Gamma(N/2 - \lambda_{\mathrm{s}})}{\Gamma[(N + 1)/2 - \lambda_{\mathrm{s}}]}, \tag{16}$$

$$I_{\mathrm{U}}(\lambda) = \frac{\Gamma[(\lambda_{\mathrm{a}} + 2)/2]\Gamma[(N_{\mathrm{h}} - \lambda_{\mathrm{a}} + 2)/2]}{\Gamma[(\lambda_{\mathrm{a}} + 1)/2]\Gamma[(N_{\mathrm{h}} - \lambda_{\mathrm{a}} + 1)/2]}\frac{\Gamma[(N_{\mathrm{s}} + \lambda_{\mathrm{a}})/2]\Gamma[(N_{\mathrm{s}} + N_{\mathrm{h}} - \lambda_{\mathrm{a}} + 2)/2]}{\Gamma[(N_{\mathrm{s}} + \lambda_{\mathrm{a}} + 1)/2]\Gamma[(N_{\mathrm{s}} + N_{\mathrm{h}} - \lambda_{\mathrm{a}} + 3)/2]}, \tag{18}$$

where $Nk/(2\pi) = \lambda_{\mathrm{a}} + N_{\mathrm{s}}$. The excitation energy ΔE_{λ} is given by Eq. (17) in the case $(\lambda_{\mathrm{s}}, \lambda_{\mathrm{h}}, \lambda_{\mathrm{a}}) = [(N_{\mathrm{s}} - 1)/2, 1, \lambda_{\mathrm{a}}]$. Thus in terms of I_{R}, I_{L}, and I_{U} we obtain the finite-size version of $A^{+}(k,\omega)$.

In Fig. 1, we present the result for $N = 60$, $N_{\mathrm{h}} = 29$, and $N_{\mathrm{s}} = 15$. We have checked the validity of Eqs. (16) and (18) by comparison with numerical results up to $N = 16$ [21]. In the special case $\nu = (0^{N_{\mathrm{h}}})$, momenta of the holon and the antiholon are both zero, and we obtain a form different from Eq. (16). However, this case can be neglected in the thermodynamic limit. Following the same procedure as that in the spinless Sutherland model [22,23], we obtain the expressions (4) and (5) in the thermodynamic limit.

We hope that the quasiparticle structure discussed in this paper is found in a future experiment of inverse photoemission.

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with $Nk/(2\pi) = \lambda_{\mathrm{h}} + \lambda_{\mathrm{a}} + \lambda_{\mathrm{s}} + (N_{\mathrm{s}} - 1)/2$, and

$$\Delta E_{\lambda} = \frac{2\pi^2}{N^2}[\lambda_{\mathrm{s}}(N - 1 - 2\lambda_{\mathrm{s}}) + \lambda_{\mathrm{h}}(2\lambda_{\mathrm{h}} + N_{\mathrm{h}} - 2) + (\lambda_{\mathrm{a}} - 1)(N_{\mathrm{h}} - \lambda_{\mathrm{a}})]. \tag{17}$$

The *triangularity* leads to $0 \le \lambda_{\mathrm{s}} + \lambda_{\mathrm{h}} \le (N_{\mathrm{s}} - 1)/2$. The parameter λ_{a} varies from zero to N_{h}. For the case where $\lambda_{\mathrm{s}} > (N_{\mathrm{s}} - 1)/2$, we use the reflection symmetry about $k = \pi$. We obtain $I_{\mathrm{L}}(\lambda) = I_{\mathrm{R}}(\bar{\lambda})$ where the components in $\bar{\lambda} = (\bar{\lambda}_{\mathrm{s}}, \bar{\lambda}_{\mathrm{h}}, \bar{\lambda}_{\mathrm{a}})$ are given by $\bar{\lambda}_{\mathrm{s}} = N_{\mathrm{s}} - \lambda_{\mathrm{s}}$, and by the relation $\lambda^{\mathrm{h}} + (0^{N_{\mathrm{h}} - \bar{\lambda}_{\mathrm{a}}}, 1^{\bar{\lambda}_{\mathrm{a}} - 1}, \bar{\lambda}_{\mathrm{h}}) = \lambda^{\mathrm{h}}_{\mathrm{GS}} + (1^{N_{\mathrm{h}}})$. We obtain the allowed range $0 \le \bar{\lambda}_{\mathrm{s}} + \bar{\lambda}_{\mathrm{h}} \le (N_{\mathrm{s}} + 1)/2$.

Finally we have the case where $\lambda_{\mathrm{s}} = (N_{\mathrm{s}} - 1)/2$ should be considered separately. In this case, the *triangular* structure requires that c_{λ} should be unity, and that $\lambda^{\mathrm{h}} = \lambda^{\mathrm{h}}_{\mathrm{GS}} + (1^{\lambda_{\mathrm{a}}}, 0^{N_{\mathrm{h}} - \lambda_{\mathrm{a}}})$ with $1 \le \lambda_{\mathrm{a}} \le N_{\mathrm{h}}$. We obtain for this special case

[1] F. D. M. Haldane, Phys. Rev. Lett. **66**, 1529 (1991).
[2] C. Kim, Z.-X. Shen, N. Motoyama, H. Eisaki, S. Uchida, T. Tohyama, and S. Maekawa, Phys. Rev. B **56**, 15 589 (1997); H. Fujisawa, T. Yokoya, T. Takahashi, S. Miyasaka, M. Kibune, and H. Takagi, Solid State Commun. **106**, 543 (1998).
[3] T. Tohyama, P. Horsch, and S. Maekawa, Phys. Rev. Lett. **74**, 980 (1995).
[4] K. Penc, K. Hallberg, F. Mila, and H. Shiba, Phys. Rev. Lett. **77**, 1390 (1996); Phys. Rev. B **55**, 15 475 (1997).
[5] Y. Kuramoto and H. Yokoyama, Phys. Rev. Lett. **67**, 1338 (1991).
[6] Y. Kuramoto and Y. Kato, J. Phys. Soc. Jpn. **64**, 4518 (1995); Y. Kato and Y. Kuramoto, J. Phys. Soc. Jpn. **65**, 1622 (1996).
[7] Z. N. C. Ha and F. D. M. Haldane, Phys. Rev. Lett. **73**, 2887 (1994).
[8] Y. Kato, Phys. Rev. Lett. **81**, 5402 (1998).
[9] M. Arikawa, T. Yamamoto, Y. Saiga, and Y. Kuramoto, J. Phys. Soc. Jpn. **68**, 3631 (1999).
[10] Y. Kato, T. Yamamoto, and M. Arikawa, J. Phys. Soc. Jpn. **66**, 1954 (1997).
[11] B. Sutherland, J. Math. Phys. **12**, 246 (1971); **12**, 251 (1971); Phys. Rev. A **4**, 2019 (1971); **5**, 1372 (1972).
[12] D. F. Wang, J. T. Liu, and P. Coleman, Phys. Rev. B **46**, 6639 (1992).
[13] Y. Kato and Y. Kuramoto, Phys. Rev. Lett. **74**, 1222 (1995).
[14] See, e.g., J. C. Talstra, Ph.D. thesis, Princeton University, 1995, and references therein.
[15] I. G. Macdonald, *Symmetric Functions and Orthogonal Polynomials* (American Mathematical Society, Providence, R.I., 1997).
[16] T. H. Baker and P. J. Forrester, Nucl. Phys. **B492**, 682 (1997).
[17] C. F. Dunkl, Commun. Math. Phys. **197**, 451 (1998).
[18] Y. Kato and T. Yamamoto, J. Math. A **31**, 9171 (1998).
[19] K. Takemura and D. Uglov, J. Phys. A **30**, 3685 (1997).
[20] Y. Kato (private communication).
[21] Y. Saiga and Y. Kuramoto, J. Phys. Soc. Jpn. **68**, 3631 (1999).
[22] Z. N. C. Ha, Phys. Rev. Lett. **73**, 1574 (1995); **74**, 620(E) (1995); Nucl. Phys. **B435**, 604 (1995).
[23] F. Lesage, V. Pasquier, and D. Serban, Nucl. Phys. **B435**, 585 (1995).