Fall 10-4-2017

The production of phantom partials due to nonlinearities in the structural components of the piano

Thomas R. Moore
Department of Physics, Rollins College, TMOORE@rollins.edu

Eric Rokni
Rollins College

Lauren Neldner
Rollins College

Camille Adkison
Rollins College

Published In

The Journal of the Acoustical Society of America 142, EL344 (2017); doi: http://dx.doi.org/10.1121/1.5006351

The production of phantom partials due to nonlinearities in the structural components of the piano

Eric Rokni, Lauren M. Neldner, Camille Adkison, and Thomas R. Moore
Department of Physics, Rollins College, 1000 Holt Avenue, Winter Park, Florida 32789, USA
erokni@rollins.edu, lneldner@rollins.edu, cadkison@rollins.edu, tmoore@rollins.edu

Abstract: Phantom partials are anomalous overtones in the spectrum of the piano sound that occur at sum and difference frequencies of the natural overtones of the string. Although they are commonly assumed to be produced by forced longitudinal waves in the string, analysis of the sound of a piano produced by mechanically vibrating the soundboard while all the strings are damped indicates that phantom partials can occur in the absence of string motion. The magnitude of the effect leads to the conclusion that nonlinearity in the non-string components may be responsible for some of the power in the phantom partials.

[DMC]
Date Received: July 25, 2017 **Date Accepted:** September 23, 2017

1. Introduction

The presence of anomalous overtones in the sound of the piano has been known since the mid-20th century. Originally referred to as *clang tones* by Knoblaugh,[1] Conklin later coined the currently used term of *phantom partials* to identify these frequency components.[2] These partials occur at the sum and difference frequencies of overtones generated by the transverse motion of the string, and Conklin attributed them to forced longitudinal waves in the string that result from the transverse displacement.[3]

There has been significant theoretical work on the coupling between transverse and longitudinal waves in stretched strings since the initial identification of phantom partials in 1944, and the process is well understood. However, to our knowledge the contribution of other components of the piano to the generation of phantom partials has not been investigated. Since it has been demonstrated that these overtones are detectable by a listener,[4] a thorough understanding of the process is necessary before a complete model of the piano can be formulated.

While it is clear that transverse motion of the string produces longitudinal waves at the frequencies that correspond to those of the detectable phantom partials, and it is likely that these vibrations are transferred to the soundboard through the bridge, it is not clear that the majority of the power in these frequency components originates in the string. Indeed, the work reported here indicates that significant sound power can be produced in phantom partials through a nonlinear mechanism associated with the structural components of the piano.

2. Background

The theory describing the generation of frequency components in the sound of the piano that do not coincide with any transverse or longitudinal resonance frequencies of the struck string has been developed in detail. A complete description of the process can be found in several references.[5–11] The analysis indicates that the anomalous frequency components can be driven by both quadratic and cubic nonlinearities in the string, resulting in the presence of sum and difference frequencies of the transverse string resonance frequencies, as well as sum and differences of the associated harmonics.

A recent experimental investigation of the process indicated that the amplitudes of these nonlinearly generated frequency components are proportional to the product of the amplitudes of the two constituent frequencies, implying that a quadratic nonlinearity is responsible for the creation of phantom partials.[12] In the experiments described in Ref. 12, a string was driven directly by two electromagnetic shakers and measurements were made of the radiated sound. However, while these experiments provided empirical evidence that appeared to validate the accepted theory, both the theoretical development and the experimental verification ignored the possibility that parts of the piano other than the string may contribute to the production of phantom partials.

The lack of consideration of nonlinear processes originating in the structural components of the piano seems to be justified because no other part of the piano appears to have an amplitude of oscillation large enough to induce a significant geometrical nonlinearity. The string can vibrate with an amplitude exceeding its diameter, but the most likely structural component to contribute to nonlinear coupling, the soundboard, typically has an amplitude of oscillation that is two to three orders of magnitude smaller than its thickness. The amplitudes of other structural components are even smaller.

Although there is good reason to believe that phantom partials are almost exclusively produced in the string, there are also reasons to believe that non-string components may contribute to their production. For example, theoretical work by Mamou-Mani has shown that the stress produced by the force of the strings on the crowned soundboard can lead to a geometric nonlinearity.[13] This nonlinearity is likely to be small, however, the overlap of the deflection shapes of the soundboard associated with different frequencies can be significant, especially if the frequencies in question are below 200 Hz. Therefore, it is possible that the large overlap may compensate for the small effect and produce significant power in the nonlinearly generated partials. Additionally, it is well known that cracks, interfaces, and stresses in wood can result in nonlinear effects at ultrasonic frequencies,[14,15] although to our knowledge there has been no investigation of these effects at audible frequencies.

In what follows, we describe experiments similar to those reported in Ref. 12, but with the string motion damped and the shakers driving oscillations of the soundboard. The results of these experiments indicate that there is a significant nonlinear process occurring in the piano structure that contributes to the production of phantom partials.

3. Experiment

To investigate the contribution of non-string components of the piano to the production of phantom partials, two electromagnetic shakers (Wilcoxon model F4) were attached to the soundboard of a 6-ft Steinway grand piano. The shakers were placed approximately 40 cm apart and contacted the soundboard on the side opposite the bridge and strings. One shaker was placed near the position of the bass bridge while the other was placed near the edge of the soundboard.

The driving frequencies of the two shakers were chosen to be the second and third overtones of the A_0 string, which were approximately the third and fourth harmonics of the fundamental frequency 27.5 Hz. The driving frequencies were determined by plucking the string and analyzing a power spectrum produced from the recorded sound. The frequencies were slightly inharmonic, which is expected due to the effects of the string stiffness on the resonance frequencies. Spectral analysis revealed that the two overtones of the string occurred at frequencies of $f_3 = 82.5$ Hz and $f_4 = 110.3$ Hz, with uncertainties of ± 0.1 Hz.

To determine the response of the piano to the excitation from the drivers, an accelerometer (PCB model 352B10) was glued to the soundboard approximately midway between the shakers. Electronic speckle pattern interferograms of the soundboard were observed while it was being vibrated at frequencies f_3, f_4, and $(f_3 + f_4)$ independently to determine the appropriate position for the accelerometer. Excitation at each of these frequencies produced a deflection shape with a single antinode near the center of the soundboard, and the position of the antinode did not change significantly between frequencies. The accelerometer was placed in the region of this antinode and a microphone (ACO model 7052E) was positioned approximately 30 cm from the soundboard and approximately level with the accelerometer. Additionally, the shaker that was driven at frequency f_3 was equipped with an impedance head that included an accelerometer and force sensor (Wilcoxon model Z820WA).

To investigate the production of phantom partials, all the strings were damped with the felt dampers associated with the piano mechanism. Additionally, strips of cloth were woven through the strings and some strings were damped with pieces of rubber. The A_0 string was damped with three felt blocks 12.5 cm in length and 5.0 cm wide that were tightly wedged between the string and the iron frame. During the experiment one shaker was driven with a constant amplitude while the amplitude of the other was increased linearly over a period of 200 s. The signals from the microphone, accelerometers, and force sensor were sampled at a rate of 10 kHz and stored for later analysis.

The signal from the accelerometer attached to the soundboard was integrated twice to determine the maximum amplitude of the soundboard vibration, which was approximately 30 μm. The maximum sound power level measured 50 cm from the soundboard was approximately 77 dBA. This sound level was estimated by a trained musician to correspond to approximately mezzo-forte in musical terms.

The signals derived from the transducers were analyzed by calculating the power spectrum in 1-s intervals, each segment individually windowed with a Hanning window. This resulted in 200 individual measurements while the amplitude of vibration of the shaker oscillating at f_3 was held constant and the amplitude of the shaker oscillating at frequency f_4 was gradually increased. Figure 1 is a plot of the sound power in the frequency components f_3, f_4, and the sum frequency (f_3+f_4) versus the product of the powers in the driving frequencies. Note the significant power present in the phantom partial even in the absence of string motion. Analysis of the signal from the accelerometer attached to the soundboard produced similar results.

The product of the powers in the two driving frequencies is used for the abscissa in Fig. 1 on the assumption that the responsible nonlinearity is quadratic. The slope of the linear portion of the plot of the power in the phantom partial determined by linear regression is 1.000 ± 0.002, where the uncertainty represents the standard error of the slope.

To provide an estimate of the contribution of the string to the production of the phantom partial, the felt blocks damping the A_0 string were removed and the experiment was repeated with the string left free to vibrate. The power in the phantom partial vs the product of the powers in the driving frequencies is plotted for both cases in Fig. 2. There are differences between the power in the phantom partial produced in each case, but they are small. Interestingly, allowing the string to vibrate increases the power in the phantom partial at low amplitudes of vibration, but reduces the power as the amplitude of the driver increases. However, some care is required when interpreting these results. In these experiments, the soundboard motion is driving the string motion, which is not the case when the piano is played in the usual manner. Therefore, the displacement of the string relative to the displacement of the soundboard will be smaller in the experimental arrangement than it will be when the piano produces a similar average sound power during performance. Due to these differences, the results presented in Fig. 2 do not lead to the conclusion that the string plays a minimal role in the production of phantom partials under normal playing conditions. Determining the relative contributions of the string and structural components to the production of phantom partials during normal play will require further investigation.

To ensure that the nonlinearity is occurring in the piano and not in the shakers, the power in the sum frequency (f_3+f_4) was plotted against the product of the powers in the driving frequencies as measured by each of the two accelerometers independently. This allows a comparison of the power in the phantom partial relative to the power in the driving frequencies at the position of the antinode on the soundboard, and independently at the position of the shaker. The shakers were positioned approximately 20 cm on either side of the antinode, therefore, the impedance of the soundboard was significantly higher at the driving points than at the position of the accelerometer attached to the soundboard. If the sum frequency was being generated in the

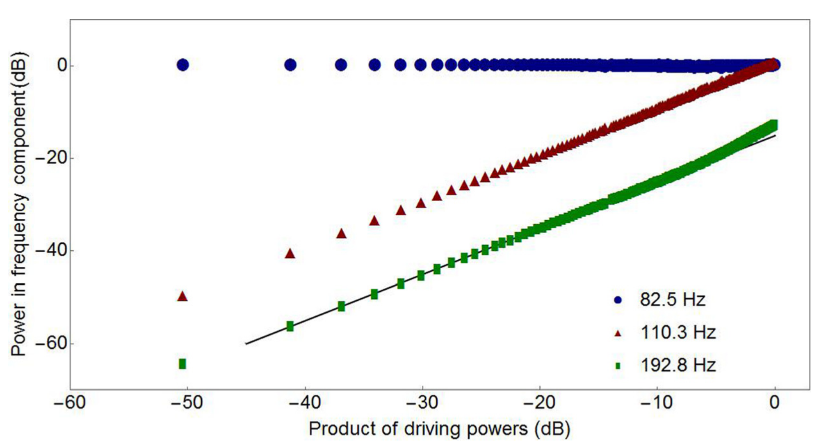

Fig. 1. (Color online) Power in the driving frequencies f_3 (circles), f_4 (triangles), and the phantom partial $f=f_3+f_4$ (squares) vs the product of the power in the driving frequencies. The strings are all damped. The line represents a fit to the points in the linear regime.

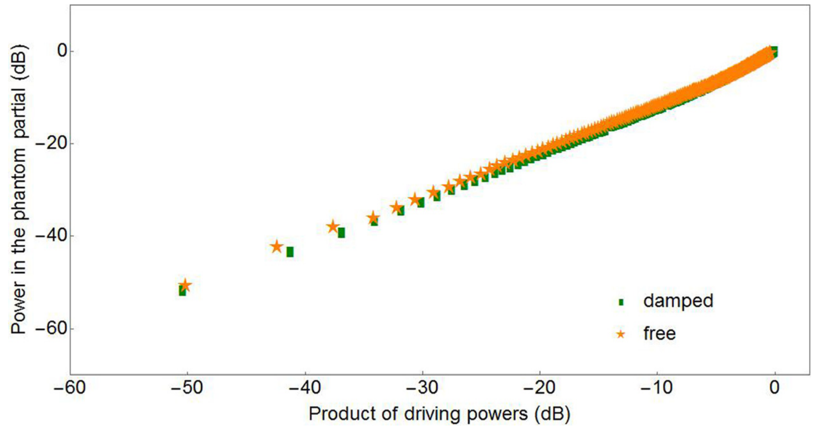

Fig. 2. (Color online) Power in the phantom partial vs the product of the power in the driving frequencies when the A_0 string is damped (squares) and when it is left free to vibrate (stars).

shaker, one would expect the power in the sum frequency relative to the power in the driving frequencies measured by the accelerometer at the driving point to be similar to that same ratio recorded by the accelerometer placed in the antinodal region of the soundboard. However, if the power in the sum frequency is generated elsewhere, the power in the sum frequency relative to the power in the driving frequencies recorded at the shaker will be lower than that measured in the antinodal region of the soundboard due to the higher impedance at the position of the shaker.

As is shown in Fig. 3, the ratio of the power in the sum frequency to the product of the powers in the driving frequencies in the soundboard exceeds that same ratio recorded by the accelerometer in the impedance head by approximately 26 dB. This difference between the plots indicates that the power in the sum frequency recorded in the shaker is dominated by oscillations driven by a process that is not spatially coincident with the shaker. If the responsible nonlinearity was in the shaker, the power in the sum frequency relative to the power in the driving frequencies would be similar to that measured in the antinodal region of the soundboard.

To further ensure that nonlinearities in the shakers are not producing any significant power at the sum frequency, the shaker with the impedance head was separated from the soundboard, then the two driving signals were added together and input into the shaker. In this manner, the coupling of the signals in the shaker was maximized. The power in the sum frequency plotted against the product of powers in the driving frequencies measured in this configuration is also shown in Fig. 3. In this case the maximum power in the sum frequency relative to the power of the product of the driving frequencies is approximately 56 dB below that recorded by the accelerometer on the soundboard when the shaker is driving the soundboard. Although removing the shaker from contact with the soundboard changes the load impedance, the lack of

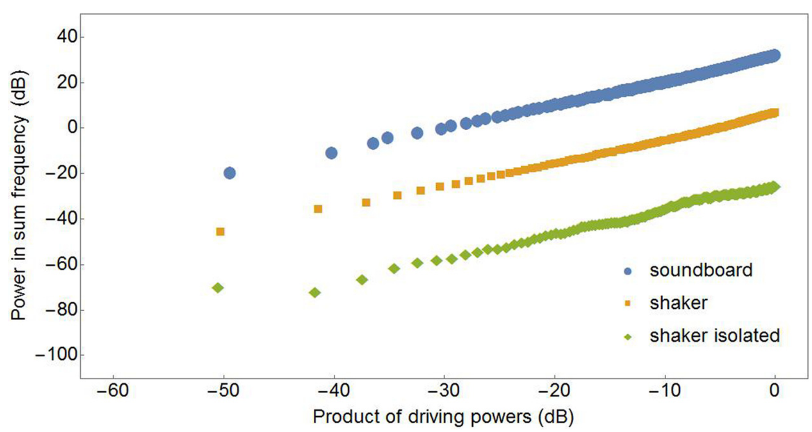

Fig. 3. (Color online) Power in the sum frequency vs the product of the powers in the two driving frequencies measured using the accelerometer on the soundboard (circles) and the accelerometer in the impedance head (squares) when attached to the soundboard. The diamonds represent the power measured in the impedance head when separated from the piano and driven at both frequencies simultaneously.

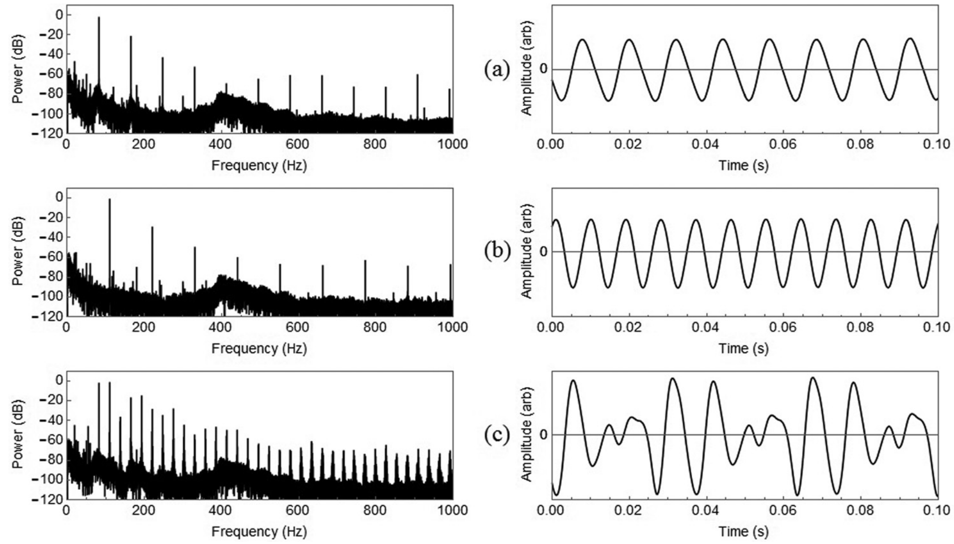

Fig. 4. Power spectra and waveforms of the sound produced by the piano when driven by (a) the shaker oscillating at frequency f_3 only, (b) the shaker oscillating at frequency at f_4 only, and (c) both shakers oscillating simultaneously. In each case the shakers were oscillating at the maximum available amplitude. All strings were damped.

significant power in the sum frequency is consistent with the assumption that the responsible nonlinearity is in the structure of the piano and not in the shaker.

To ensure that the results described here are not unique to the individual piano or driving mechanism, the experiments described above were also performed on an upright piano with the shakers attached to two of the bridge pins near the A_0 string. The results of the experiments were similar to those reported here. Additionally, experiments have been performed using piezoelectric drivers in place of the electromagnetic shakers on both upright and grand pianos. Piezoelectric drivers have a much higher mechanical impedance than electromagnetic shakers and are therefore less susceptible to nonlinear effects stimulated by external mechanical motion (e. g., motion of the soundboard), although the maximum displacement is typically much smaller. Experiments on both pianos, with piezoelectric drivers attached to the soundboard or bridge pins, produced results similar to those shown in Fig. 1. Because the efficiency of the generation of the sum frequency is dependent on both the absolute and relative powers in the driving frequencies, the effect of the reduced maximum displacement of the piezoelectric driver was evident in a reduced power in the sum frequency relative to the driving powers. However, the linear dependence evident in Fig. 1 was maintained.

4. Conclusion

The results presented here indicate that the production of phantom partials in the piano is not limited to geometric nonlinearities in the string, as has been generally assumed in the past. The nonlinear mechanism has yet to be identified, and the responsible component may be any or all of the structural components of the piano. However, the magnitude of the effect may be comparable to that attributable to the string and is possibly greater.

It is likely that the efficiency of the nonlinear process that produces phantom partials in the piano structure is dependent on the values of the two driving frequencies. The complex deflection shapes of the soundboard that occur above 200 Hz imply that any process dependent on the motion of the piano structure will be frequency dependent. We have not performed experiments using driving frequencies above 150 Hz in the experiments described here, but experiments involving higher driving frequencies will be required to completely characterize and understand the production of phantom partials by the structural components of the piano.

Although the phantom partials constitute only a small portion of the total sound power produced by a piano, it is important to include this structural nonlinearity in future models of the instrument. Not only because the phantom partials are perceivable to the listener, but also because the nonlinear effects are not restricted to producing phantom partials. Power spectra of the sound produced when the shakers are driven at maximum amplitude and all the strings are damped are shown in Fig. 4, along with plots of the associated waveforms. The plot in Fig. 4(a) represents the case where only the shaker driven at frequency f_3 was oscillating, Fig. 4(b) represents the case where only the shaker driven at frequency f_4 was oscillating, and Fig. 4(c) represents the case where both shakers were oscillating simultaneously. The presence of harmonic overtones is evident in Figs. 4(a) and 4(b), while those harmonics as well as higher-order phantom partials are evident in Fig. 4(c). How important these overtones are to the final sound has yet to be determined, but the nonlinear process responsible for their production will presumably need to be included to formulate an accurate model of the piano.

Acknowledgments

This work was supported by Grant No. PHY-160749 from the U.S. National Science Foundation. The authors thank C. Gibson and N. Etchenique for assistance in performing some of the experiments, and N. Giordano for helpful discussions.

References and links

[1] A. F. Knoblaugh, "The clang tone of the pianoforte," J. Acoust. Soc. Am. **16**, 102 (1944).
[2] H. A. Conklin, "Piano strings and 'phantom' partials," J. Acoust. Soc. Am. **102**, 659 (1997).
[3] H. A. Conklin, "Generation of partials due to nonlinear mixing in a stringed instrument," J. Acoust. Soc. Am. **105**, 536–545 (1999).
[4] B. Bank and L.-M. Lehtonen, "Perception of longitudinal components in piano string vibrations," J. Acoust. Soc. Am. **128**, EL117–EL123 (2010).
[5] P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (McGraw-Hill, New York, 1968), pp. 856–863.
[6] A. Watzky, "Non-linear three-dimensional large-amplitude damped free vibration of a stiff elastic stretched string," J. Sound Vib. **153**, 125–142 (1992).
[7] A. Watzky, "On the transverse-axial mode coupling in the non-linear vibration of strings," in *Proceedings of the International Symposium on Musical Acoustics 1995* (Le Normont, Dourdan, France, 1995), pp. 189–195.
[8] B. Bank and L. Sujbert, "Generation of longitudinal vibrations in piano strings: From physics to sound synthesis," J. Acoust. Soc. Am. **117**, 2268–2278 (2005).
[9] S. Bilbao, "Conservative numerical methods for nonlinear strings," J. Acoust. Soc. Am. **118**, 3316–3327 (2005).
[10] J. Chabassier and P. Joly, "Energy preserving schemes for nonlinear Hamiltonian systems of wave equations: Application to the vibrating string," Comput. Meth. Appl. Mech. Eng. **199**, 2779–2795 (2010).
[11] A. Chaigne and J. Kergomard, *Acoustics of Musical Instruments* (Springer, New York, 2016), pp. 411–419.
[12] N. Etchenique, S. R. Collin, and T. R. Moore, "Coupling of transverse and longitudinal waves in piano strings," J. Acoust. Soc. Am. **137**, 1766–1771 (2015).
[13] A. Mamou-Mani, J. Frelat, and C. Besnainou, "Prestressed soundboards: Analytical approach using simple systems including geometric nonlinearity," Acta Acust. Acust. **95**, 915–928 (2009).
[14] D. Donskoy, A. Sutin, and A. Ekimov, "Nonlinear acoustic interaction on contact interfaces and its use for nondestructive testing," NDT&E Int. **34**, 231–238 (2001).
[15] I. Yu. Solodov, N. Krohn, and G. Busse, "CAN: An example of nonclassical acoustic nonlinearity in solids," Ultrasonics **40**, 621–625 (2002).