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Laurie A. Stephey

Department of Physics, Rollins College

Thomas R. Moore

Department of Physics, Rollins College, TMOORE@rollins.edu

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Experimental investigation of an American five-string banjo

Laurie A. Stephey and Thomas R. Moore^{a)}

Department of Physics, Rollins College, Winter Park, Florida 32789

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An experimental investigation of an American five-string banjo is reported. The acoustic spectra of the strings are analyzed and it is found that in many ways banjo strings behave in a manner similar to those on a piano. The terminating impedance presented to the strings at the bridge is reported and it is found to be generally much lower than the characteristic impedance of the strings. It is also shown that, for a properly tuned banjo, the mode shapes of the membrane are symmetric about the center even though there is a large asymmetric force due to the off-center placement of the bridge. Finally, high-speed electronic speckle-pattern interferometry was used to study the motion of the membranous head after a string is plucked, and it is found that the motion is dominated by a rocking motion of the bridge. © 2008 Acoustical Society of America. [DOI: 10.1121/1.2982371]

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I. INTRODUCTION

The American five-string banjo is an instrument that consists of five metal strings coupled to a circular membrane through a wooden bridge. Its origins are in Africa, but the modern American banjo is the result of changes made to the instrument after its arrival in the United States over 200 years ago. While there are myriad publications on the motion of plucked strings and struck membranes, to our knowledge, there are a very limited number of reports of investigations into the physics of a coupled string and membrane as is the case with the banjo. Recently, the investigation of a simple model of the banjo was reported by Dickey,¹ this was followed by a report of some experimental observations by Rae and Rossing.² The recent publication of at least two conference abstracts also indicates a growing interest in the subject.^{3,4}

When one attempts to investigate the physics of the banjo, there are two difficulties that are immediately encountered: the difficulty in modeling the instrument due to the complexity of the interactions of the various parts and the lack of experimental data with which to compare any theoretical results. Here we attempt to remedy the latter of these problems in the hope that it will spur efforts to address the former.

In what follows, we present the results of experiments designed to lend some insights into the physics of the banjo. We present measurements of the driving point impedance at the bridge, analysis of the time-averaged and time-resolved acoustic spectra, images of the normal modes of the membrane, and results of studies of the motion of the bridge and membrane after a string has been plucked. These measurements form a foundation for future theoretical investigations that will hopefully result in a complete description of the dynamics of this complex musical instrument.

II. EXPERIMENTS

A photograph of the banjo studied in this investigation can be found in Fig. 1. The banjo head is comprised of a membrane stretched over a wooden ring (commonly called a rim) and held in place by a circular metal flange. Connected to the bottom left edge of the rim is a metal handrest and fitted into the rim on the underside of the membrane is a metal tone ring. Some banjos also have a removable wooden back. The rim is attached to a wooden neck and at the top of the neck are tuning pegs to adjust the tension of the strings; the fifth string, however, is shorter than the other four and is adjusted with a tuning peg located at a position near the middle of the neck. Each string passes over a small piece of wood or bone on the neck called a nut, which can be treated as the terminating point of the string. At the base of the head, the strings are attached to the rim by a piece of metal referred to as a tailpiece. A small wooden bridge is placed on the membrane in a position such that a string terminated at the 12th fret has a fundamental frequency one octave higher than that of the same string when terminated at the nut. The strings are stretched over the bridge, which places a downward force on it and ensures continuous contact with the membrane.

All of the experiments reported here were performed on a Gold Tone MC-150R banjo manufactured circa 2005. It has a frosted mylar membrane of thickness 0.21 ± 0.01 mm and density of approximately 1200 kg/m^3 . It also has a standard three-footed bridge, shown in Fig. 2, with a length of 80.4 ± 0.1 mm and mass 2.5 ± 0.1 g. It has five metal strings and an arch-top tone ring. The neck is 501 ± 1 mm long, and between 31.9 ± 0.1 mm and 49.6 ± 0.1 mm wide. The radius of the head is 135 ± 1 mm and the rim is 52.3 ± 0.1 mm tall and 20.0 ± 0.1 mm thick. There are 24 screws around the rim that were used to tighten the flange and keep the membrane under tension.

The large number of screws securing the flange around the membrane gives the player the ability to adjust the tension of the head in several places. Most players agree that a symmetric head tension is desirable, and uniform tension can be achieved using several methods, including the use of a

^{a)}Electronic mail: tmoore@rollins.edu



FIG. 1. (Color online) A photograph of the American five-string banjo used in this investigation.

torque wrench or a drum-head tension gauge.⁵ Many players, however, tune the head by tapping various points on the membrane and listening to the pitch. They then tighten or loosen the screws to obtain the same pitch at all points on the head. All three methods result in a banjo with approximately symmetric head tension.

In all of the experiments reported here, the wooden back and the metal handrest were removed from the instrument and, unless otherwise noted, the strings were damped with strips of cloth. The head was tuned to approximately 275 Hz (approximately a D4-flat) and a drum head tuner was used to ensure that the membrane remained at uniform tension throughout the experiments. The strings were tuned in the standard G-tuning manner described in Table I.

When the banjo is played in concert, it is often played with the wooden back attached (sometimes called the “resonator”), and there is little doubt that the addition of the back causes a coupling of the air modes and the modes of the membrane. This coupling can shift the resonance frequencies of the membrane, and this effect has been addressed in detail for the case of the tympani;⁶ one can reasonably assume that

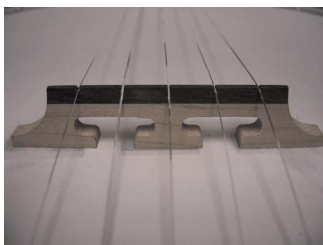


FIG. 2. (Color online) A photograph of the three-footed bridge used in this investigation.

TABLE I. The standard G-tuning of a five string banjo.

String	Note	Fundamental frequency (± 0.1 Hz)
1	D	293.7
2	B	246.9
3	G	196.0
4	D	146.8
5	G	392.0

the effect of the cavity will be similar for the banjo. However, while the removal of the back will change the resonance frequencies of the membrane, we have found that the presence of the resonator does not change the deflection shapes of the modes and the configuration investigated in this work allows the experimentalist to address a common situation that can be reproduced with almost any banjo. Additionally, the removal of the back allowed a precise measurement of the driving point impedance as described below.

A. Driving point impedance at the bridge

One of the most important physical parameters for any stringed musical instrument is the impedance presented to the string. As is the case with most stringed instruments, the banjo string is terminated at one end by the nut, which has significantly greater impedance than the characteristic impedance of the string and in most cases can be considered infinite. The end of the string opposite the nut can usually be considered to terminate at the wooden bridge, although it extends approximately 5 cm beyond the bridge to the tailpiece. When a string is plucked, the energy in the string is transferred to the resonating membrane through the bridge. This transfer of energy is governed by the impedance presented to the string at the bridge.

The impedance at the bridge is both a function of the shape and material of the bridge and the characteristics of the membranous head to which it is attached. It is dependent on the driving frequency, and to understand why the banjo produces the sound it does it is critical to know how the impedance presented to the strings varies as a function of frequency. To determine this, we have measured the driving point impedance at the bridge of a fully assembled banjo.

The driving point impedance is the mechanical impedance at the point of excitation and is defined as the driving force divided by the velocity at the point of contact. To measure this quantity, it is essential to obtain simultaneous measurements of both the driving force on the bridge and the velocity of the bridge without significantly altering the instrument. The experimental arrangement used for these measurements is depicted in Fig. 3.

Screws that traditionally are intended to attach the banjo to the wooden back were instead used to attach the banjo to several aluminum posts that were in turn securely mounted on an optical table. The mounting arrangement was such that the neck of the banjo was perpendicular to the table, leaving the area behind the banjo free of any obstruction. The bridge was driven in a sinusoidal manner using an electromagnetic shaker. A load cell designed to measure forces less than 0.01 N was placed between the electromagnetic driver and

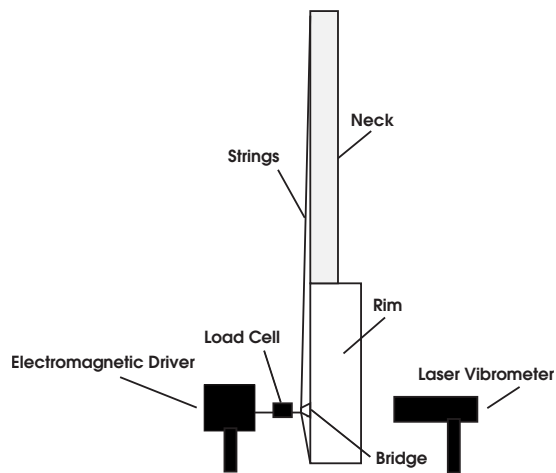


FIG. 3. The experimental arrangement used to measure the impedance presented to the strings at the bridge.

the bridge to measure the force exerted on the bridge. The velocity of the resulting motion was measured using a laser vibrometer, which was placed behind the banjo with the beam focused at the point on the membrane directly under the bridge. Measuring the velocity in this manner, rather than the more common method of attaching an accelerometer near the driving point, avoided the possibility of introducing uncertainties into the measurement caused by adding the mass of the accelerometer to the mass of the bridge. Measurements of velocity and force were simultaneously recorded by a computer in real time.

Measurements of impedance were made from 50 to 2500 Hz with 1 Hz resolution. By increasing the driving amplitude of the electromagnetic shaker, three to six measurements of increasing force and velocity were made at each frequency. Because force is linearly related to the velocity through the impedance, the impedance at each frequency was calculated by performing a linear regression on the several measurements of force versus velocity. The uncertainty in the measurements was found by calculating the uncertainty in the slope of this regression and was typically less than 1 kg/s. A plot of the driving point impedance versus frequency is shown in Fig. 4. The data shown in Fig. 4 were taken with the strings left free to vibrate. The effects due to the strings is small but noticeable and the local minima at the fundamental frequencies of the strings are noted in the figure.

B. Acoustic power spectra

The impedance mismatch between the string and the bridge determines how efficiently energy is transferred from the vibrating string to the vibrating membrane. If the kinetic energy was only transferred from the string to the head, it would be a simple matter to model the interaction and determine the dynamics of the string motion. However, because energy can be transferred both to and from the string, accurately accounting for this energy transfer will be one of the more difficult aspects of modeling the banjo. Arguably, the most important result of the two-way energy flow is the length of time that the string continues to vibrate after being

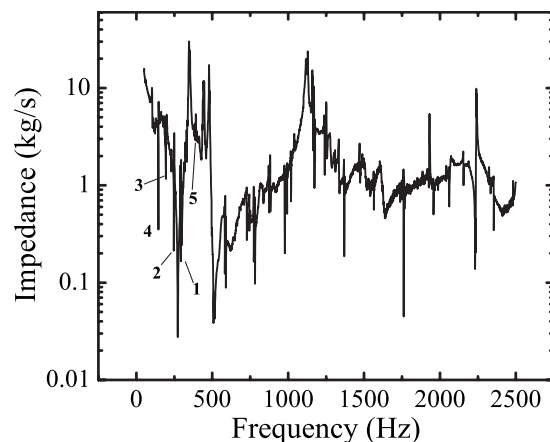


FIG. 4. A plot of the driving point impedance presented to the strings at the bridge vs driving frequency. The numbers indicate the minima at the fundamental frequency of each of the five strings.

plucked. Accurately modeling this aspect of the banjo sound will be imperative for both understanding and electronically reproducing the banjo sound, so it is useful to study the manner in which the sound of a plucked string decays.

We have measured the acoustic power in the first several harmonics of each string while damping the remaining four strings. During these experiments the banjo was mounted in the same way as detailed above. A microphone was placed approximately 40 cm from the front of the banjo head and each string was plucked with a piece of synthetic hair, which was pulled until the hair broke. Each string was plucked in the same manner and the same place, approximately 7 cm above the bridge.

A logical decomposition of the string motion is to assume a linear superposition of motions that are parallel to and perpendicular to the plane of the membrane. The acoustic signal that results from plucking the string in each of these two directions is different, and it is worthwhile to study the sound produced by the banjo in each case. Although the sound produced by plucking each string in each of the two directions is unique to the string, there are a few common characteristics that appear to be important. We only present spectra from the first two strings here because they clearly demonstrate all of these phenomena.

Figure 5 contains the time-averaged power spectra of the first two strings that result from plucking each string vertically and horizontally. Also included are the graphs of the time evolution of the first three harmonics in each case. The decay of many of the resulting harmonics is well described by an exponential function and can be modeled in the usual form $P = P_0 e^{-\alpha t}$, where P_0 is the initial power, α is the decay constant, and t is the time. However, there are also instances of cyclic behavior as well as cases where there are two distinct decay constants for the same string. We will address these issues more fully in Sec. III below.

C. Mode shapes of the membrane

Much can be learned by observing the shapes and frequencies of the normal modes of vibration of a membrane. In the case of the banjo head, the shape of the modes of vibra-

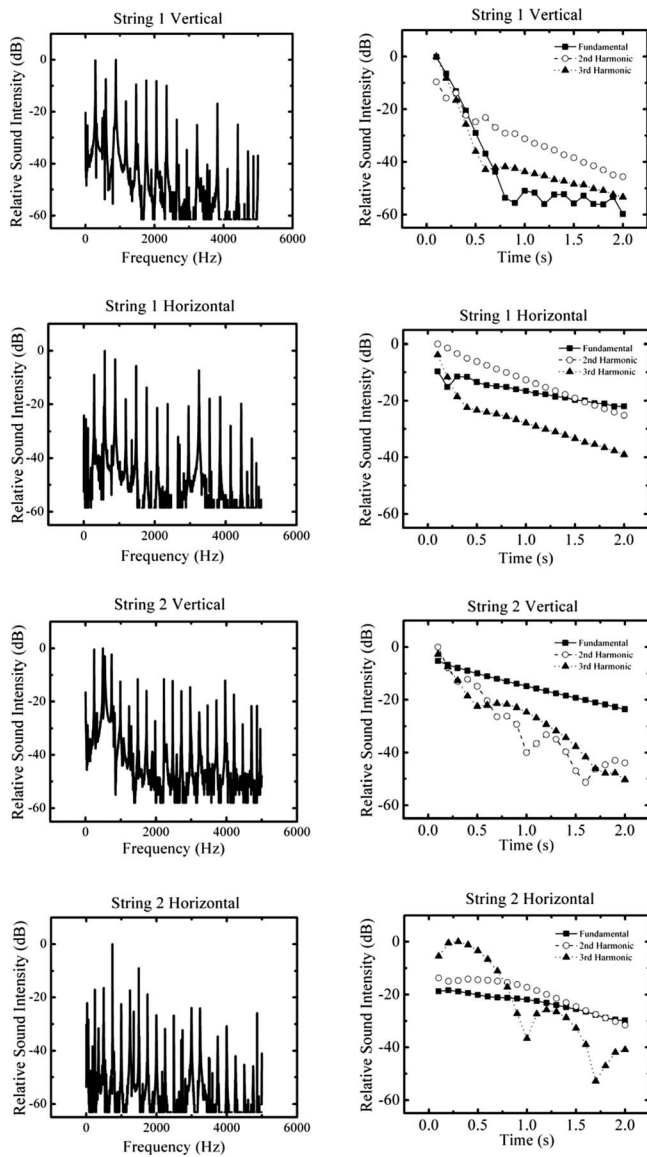


FIG. 5. The time-averaged power spectra resulting from vertically and horizontally plucking each of the first two strings and the plots of the decay of the first three harmonics for each string as a function of time.

tion can lend some insights into the propensity for acoustic short circuiting of the sound, as well as the impact of changing the placement of the bridge on the head. Furthermore, knowledge of the shapes and frequencies of the normal modes can help explain some of the structure observed in the plot of the driving point impedance at the bridge shown in Fig. 4.

The mode shapes of the banjo head were determined using time-averaged electronic speckle-pattern interferometry. The design of this particular interferometer is described elsewhere.⁷ The motion of the head was driven acoustically by a speaker connected to a function generator, ensuring that an antinode was not forced at any particular position as can be the case when the vibrations are driven by making contact with the head. The function generator produced a sinusoidal wave with a precision of 0.1 Hz and the speaker was placed approximately 1 m from the front of the banjo. Measurements were made with the strings both damped and un-

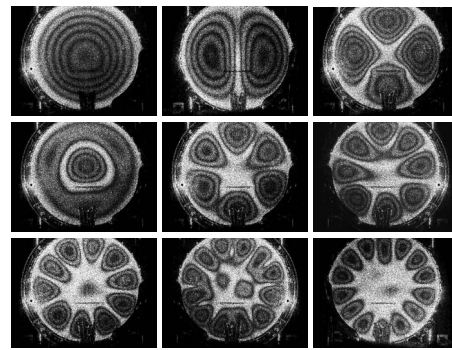


FIG. 6. Time-averaged electronic speckle-pattern interferograms of the membrane motion at nine of the resonant frequencies. Note that the final three interferograms depict deflection shapes with strong components of a normal mode of a circular membrane, but anomalous antinodes are also present near the center. The resonance frequencies for each mode are listed in Table II.

damped; however, it was found that the resonances of the strings did not noticeably affect the resonances of the head. Interferograms that indicate the shapes of nine of the normal modes are shown in Fig. 6 and the frequencies at which these normal modes occur have been cataloged in Table II. In general, the width of the resonances were less than 10 Hz; however, the (0,0) mode could be easily excited over a range of almost 100 Hz.

It is difficult to ensure that the patterns shown in interferograms such as those in Fig. 6 are indeed the shapes of the normal modes of the membrane. The best that can often be said is that the interferograms represent deflection shapes; however, the mode shapes of a circular membrane are well established in the literature and it is reasonable to assume that the shapes shown in Fig. 6 are indeed those of the normal modes. Additionally, there exists an orthogonal sister mode to two of the modes shown in Fig. 6 due to a slight asymmetry in the head tension.

D. Motion of the bridge and membrane

Having cataloged the most easily excited modes of the head, it is useful to ask if these modes are preferentially excited by a plucked string. Most other membranaphones are struck, and after the transient motion has subsided, modes

TABLE II. Catalog of the nine most easily excited normal modes of the banjo membrane. For modes with a pronounced doublet, the frequency of the nondegenerate mode is also shown. The asterisk indicates that the interferogram demonstrates a strong component of the mode, but the shape was not exactly that expected of a circularly symmetric membrane.

Mode	Frequency 1 ± 10 Hz
(0,0)	277
(1,0)	550, 580
(2,0)	831, 842
(0,1)	978
(3,0)	1084
(4,0)	1320
(5,0)*	1549
(6,0)*	1771
(7,0)*	1991

that do not have nodes at the point of excitation are left to radiate. In contrast to this usual membranous behavior, the banjo has a membrane that is driven continuously at the three points of bridge contact, sometimes over the course of several seconds. It is therefore important that those wishing to model the complete instrument understand how the bridge actually excites the head motion. To investigate this motion, high-speed electronic speckle-pattern interferometry was employed to study the time-resolved motion of the head.

Electronic speckle-pattern interferometry is a common method for studying the time-averaged motion of vibrating objects, and the usefulness of this technique is demonstrated above. However, in situations where the transient response to excitation is important, the time-averaged deflection shapes do not provide enough information. To study the response of the banjo to a plucked string, an electronic speckle-pattern interferometer was outfitted with a high-speed camera. The result was a system that could resolve displacements of less than 150 nm with a time resolution of 100 μ s.

Each string was plucked both vertically and horizontally approximately 9 cm above the top of the rim. The speckle patterns resulting from the motion of the head during and after plucking the string were recorded by the high-speed camera, and the resulting images were processed to yield a series of interferograms. These interferograms represent contour maps of the instantaneous displacement of the membrane and show how the deflection shapes of the banjo head change within the first second after the pluck. An example of the time-resolved behavior of the membrane is shown in Fig. 7 and examples of some instantaneous deflection shapes are shown in Figs. 8–10. The interferograms look the same whether the displacement of the membrane is into or out of the plane; therefore, only one-half of a cycle of the fundamental frequency is shown in Fig. 7.

III. DISCUSSION OF RESULTS

When taken together, the data presented above provide a basic understanding of the response of the banjo to transient and steady-state excitation. In general, the measurements conform to what one may expect of a string-membrane interface, but there are many interesting effects that deserve to be noted.

It is common to estimate the impedance of the string by assuming it is close to the characteristic impedance of an infinite string, which is given by the square root of the product of the tension and linear density. For a banjo string, this value is on the order of several hundred kg/s; thus the impedance presented to the string at the bridge generally appears to be significantly lower than the impedance of the string itself. Because the characteristic impedance of the string is generally larger than the measured impedance at the bridge, this impedance mismatch should lend itself to efficient energy transfer at most frequencies, but it should also lead to the presence of strong odd harmonics. Since we do not see this evident in the spectrum, it probably indicates that the characteristic impedance of the string is not a good estimate of the actual impedance in this case.

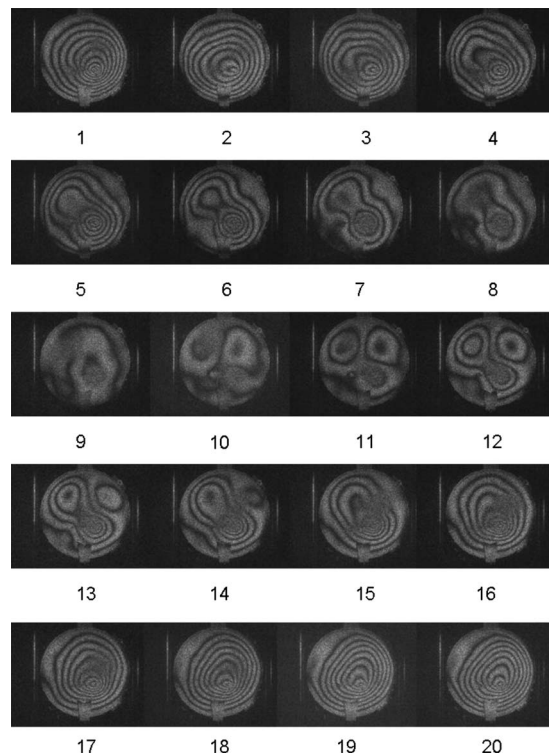


FIG. 7. An example of a series of interferograms of the time evolution of the vibrating head. This motion is the result of the second string plucked vertically and depicts a complete cycle of the second harmonic. Each interferogram depicts the membrane 100 μ s after the previous frame. The initial frame occurs 0.4152 s after the pluck. Note that the bridge pivots about its left foot.

The measurements of the impedance presented to the string at the bridge, which are discussed in Sec. II A and shown in Fig. 4, indicate that there are very few frequencies at which the impedance is above 25 kg/s. However, there are two frequency ranges in which the impedance is relatively large: between 300 and 500 Hz and again between 1000 and 1200 Hz. It is interesting to note that the first peak in impedance roughly corresponds to the range in which most of the fundamental resonances of the strings lie.

It is often more useful to study the admittance, the inverse of impedance, rather than the impedance itself. A graph of the mechanical admittance at the bridge is shown in Fig. 11. As one might expect, the frequencies of several of the

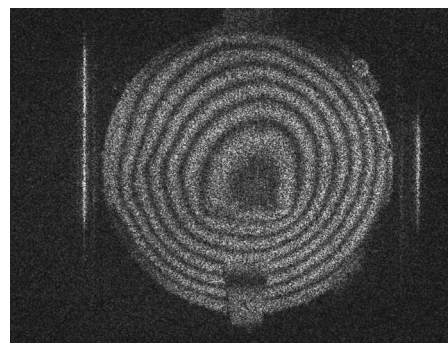


FIG. 8. A single image extracted from a series of time-resolved interferograms. This image shows that vertically plucking the first string results in a strong component of the (0,0) mode. Note that, in this case, the bridge is not the location of an antinode.

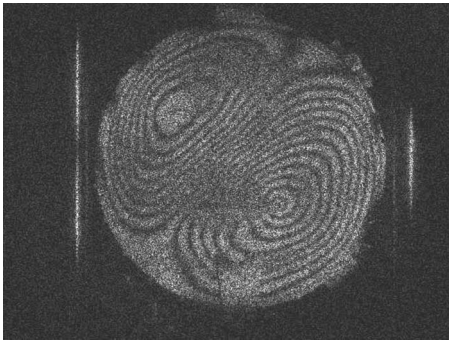


FIG. 9. A single image extracted from a series of time-resolved interferograms. This image shows that vertically plucking the fourth string results in a strong component of the (1,0) mode. Note that the right foot of the bridge is on an antinode but the left foot is not.

admittance maxima are close to those of normal modes of the membrane, and these coincidences have been noted in Fig. 11. We note that these measurements differ considerably from those presented in Ref. 2, and while we have no explanation for this difference, it is probably not due to the structure of the bridge itself since the work reported in Ref. 2 indicates that very little of the structure in the admittance curve can be attributable to the bridge.

The measurements of the acoustic power after a string is plucked are also noteworthy. The power spectra that result from plucking a string horizontally and vertically show that the power generally decreases with increasing harmonic number. This decrease in the power appears not to be a simple function of frequency and is unique to each string. Of particular interest when studying the acoustic power spectra is the placement of the strongest harmonic. The balance of power within the harmonics is a function of the details of the instrument as well as the position on the string at which the pluck occurs, so one must be careful when extending the results to a general conclusion about the sound of any particular banjo; however, the location of the pluck in these experiments was approximately 7 cm above the bridge and was deliberately chosen so as not to fall on a node of any of the first five harmonics of the string. It is also approximately the position that a player normally plucks the string. As one would expect, the direction of the pluck (i.e., horizontally or

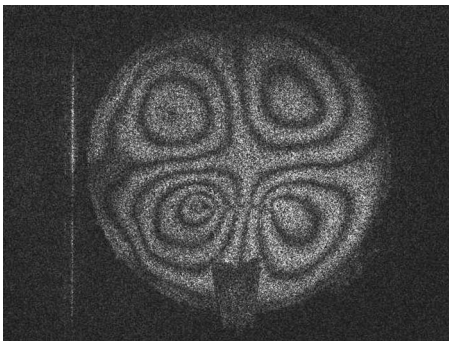


FIG. 10. A single image extracted from a series of time-resolved interferograms. This image shows that vertically plucking the fifth string excited a strong component of the (2,0) mode. Note that each end of the bridge rests on an antinode of the mode shape.

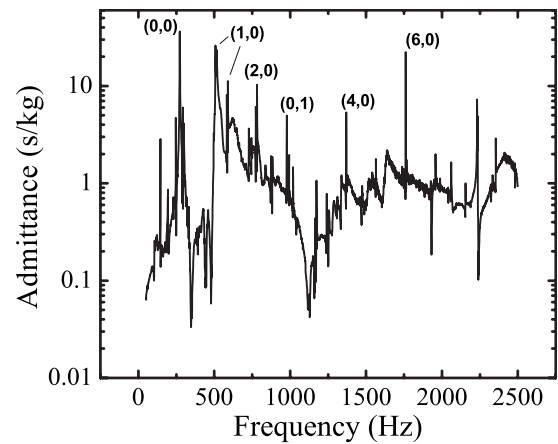


FIG. 11. A plot of the admittance presented to the strings at the banjo bridge vs driving frequency. Admittance maxima which coincide with a normal mode of the membrane are labeled with the mode designation.

vertically) alters the ratio of the power in each harmonic since the motion of the bridge is largely determined by the direction of string motion.

The results shown in Fig. 5 also indicate that the direction of the pluck strongly affects the time it takes for each harmonic to decay. As a rule, horizontally plucked strings decay more slowly than vertically plucked strings. It is also clear that the harmonics decay in one of three ways: exponentially, exponentially with two distinct rates of decay, or with a series of distinct periodic maxima and minima. The direction of the pluck appears to have little impact on which of the functional forms describes the decay of the sound.

The nonexponential decay with evident periodicity is most likely indicative of a slight difference in phase between the sound generated by the vertical and horizontal motions, of string vibration. Since this phase difference varies depending on distance from the instrument, the appearance of the nonexponential decay can change depending on the location of microphone. This also indicates that the horizontal and vertical motions are not necessarily coupled, although evidence for some coupling is discussed below. This lack of strong coupling has also been observed in the high-speed analysis of the deflection shapes of the membrane, where it was determined that plucking the string vertically excited significantly different deflection shapes than plucking the string horizontally.

The harmonics that demonstrate a decay at two distinct rates, such as those evident in the analysis of the first string, most likely correspond to frequencies at which there is a significant difference between the decay rate for the two distinct polarizations of string motion. Thus the change in the slope of the decay occurs as a result of the superpositions of the two motions, each with a significantly different decay rate. Although the plucking mechanism ensured that the initial motion of the string was primarily in the horizontal or vertical direction, high-speed photography indicated that there was usually a transition to elliptical motion, demonstrating that the two directions of motion are not completely decoupled. Indeed, we found it impossible to ensure that the string motion occurs only in one direction using the method of plucking described above.

These observations indicate that the decay characteristics of banjo strings are remarkably similar to those of piano strings. In a study of the decay of piano strings, Weinreich observed both nonexponential periodic decay and twofold exponential decay.⁸ The fact that both instruments are comprised of vibrating strings with two independent degrees of motion and coupled oscillators (the head and strings in the banjo and multiple strings in the piano) probably accounts for their similar behavior.

The symmetric shapes of the normal modes of the membrane have been noted previously by Rae and Rossing and the mode shapes shown in Fig. 6 are similar to those reported in Ref. 2. The significant difference between the interferograms in Ref. 2 and those presented here lies primarily in the fact that in producing the interferograms shown in Fig. 6 there was no discrete driving point. That is, in the work reported here, the positions of the antinodes were entirely determined by the physics of the instrument because the vibrations of the membrane were driven acoustically rather than by direct contact with the membrane.

The method used to drive the vibrations is important because there can be significant differences in the orientation of the mode shapes, or even the presence of certain modes, depending on where the membrane is excited. Thus, when the interferograms in Fig. 6 are compared with those in Figs. 8–10, which were produced when the motion is driven by a plucked string, the differences are meaningful. Driving the motion of the head acoustically rather than mechanically also ensures that harmonic distortion of the driving signal is minimized and there is no concern that the excitation is occurring on the nodal line of a mode.

The symmetric deflection shapes evident in Fig. 6 are indicative of the fact that when tuning the banjo head the artist attempts to account for the asymmetric force on the head created by the bridge. Therefore, while the presence of symmetric mode shapes may appear surprising, it is the expected result of a well tuned banjo head. This is fortunate for those attempting to model the complex interaction between the string and membrane; the presence of symmetric mode shapes of the head that are consistent with those expected of a circular membrane indicates that the normal modes of a circular membrane are a logical basis set to use, should one wish to model the head motion using a modal decomposition.

Since the motion of the membrane plays an integral part in producing the final sound, the time-resolved interferograms are some of the most interesting results reported here. Analysis of the interferograms described in Sec. II D shows that the two outer feet of the bridge are extremely important in driving the membrane, whereas the middle foot appears to play a minimal role. This may indicate that accurately modeling the complex shape of the bridge is not necessary to produce a reasonable model of the instrument. Indeed, it is possible that a solid rectangular bridge may be adequate as a first-order approximation for the purpose of modeling the banjo.

The time-resolved interferometry also revealed that it may not be difficult to obtain an intuitive understanding of the bridge motion after a string has been plucked. When a

string that terminates on the left side of the bridge was plucked, the rocking motion of the bridge appeared to pivot about the right foot. Conversely, when a string that terminates on the right side of the bridge was plucked, the opposite was true. This pivoting motion can be observed in Fig. 7, where the bridge pivots about the left foot over one complete cycle of the second harmonic. A clearly defined rocking motion of the bridge was present in all but one case: when the third string, located at the center of the bridge, was plucked vertically the bridge did not rock significantly.

As one might also expect, plucking any string excited the membrane into a series of complex deflection shapes, and in most cases the bridge was the location of largest displacement. Most often, the two outer feet of the bridge were antinodes, although in some cases additional antinodes of comparable or greater amplitude were excited elsewhere in the membrane. Generally, the deflection shapes were a complex superposition of the normal modes of a circular membrane; however, in some interesting cases, there was a strong component of a single mode present. Analysis of the head motion as a result of horizontal and vertical plucking, in conjunction with the acoustic data, indicates that when there were strong components of a normal mode of the membrane they were excited by the close proximity of the string harmonics and head resonances, as one might expect.

For example, when the first string was plucked vertically it excited a strong component of the lowest resonance of the head, seen in Fig. 8, due to the close proximity of the fundamental frequency of the string and the resonance frequency of the head. The rapid decay of the fundamental due to this coupling is evident in Fig. 5. Plucking the fourth string vertically resulted in a strong (1,0) component in the motion of the head, as seen in Fig. 9, this is probably due to the close proximity of the resonance of that mode to the third harmonic of the fourth string. Likewise, plucking the fifth string vertically resulted in a strong (2,0) component in the motion of the membrane, seen in Fig. 10, due to the proximity of the frequency of the (2,0) mode to the second harmonic of the fifth string. The interferograms did not indicate any strong isolated components of modes above (2,0).

It is important to note that the interferograms shown in Figs. 8–10 are individual time-resolved frames, not time-averaged interferograms such as those shown in Fig. 6. A superposition of other mode shapes was clearly visible in each time-resolved sequence, but the modes shown in these figures represent a strong component that was visible in isolation at several times during the oscillation. These mode shapes are similar to those shown in Fig. 6, but note the different orientation that results from the fact that the motion was driven by the string rather than an acoustic signal. The orientation of the antinodes in Fig. 9 is particularly interesting because only one of the antinodes corresponds to the position of the a foot of the bridge, but when the membrane is driven freely the two exterior feet of the bridge both fall on an antinode of the motion. Thus, the rocking motion of the bridge pivots about the left foot even though the natural motion of the membrane would have the motion rock symmetrically about the center of the bridge, as is the case shown in Fig. 6. The fact that the bridge motion forces an orientation

of the mode shape that is not normally present when the head is freely vibrating indicates that if the head is not under uniform tension the sound produced by plucking some strings may be altered more significantly than others.

IV. CONCLUSIONS

The banjo is a complex instrument that is just now beginning to be investigated by the community of musical acousticians. Because of the complex interaction between the string, bridge, and membrane, any successful model will require extensive validation. A complete model of this instrument will not only assist in understanding the complex physics responsible for its unique sound, but will also open the door for electronic synthesis of the sound based on first principles. To encourage this process, we have presented the results of several studies of the banjo that lend some insights into the physics and provide a firm basis on which to validate future models. We hope that these results will lead to further theoretical work on this unique instrument.

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