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# The Effects of Bell Vibrations on the Sound of the Modern Trumpet

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## Summary

The acoustic spectrum of a modern trumpet with the bell section heavily damped has been compared to the spectrum of the same instrument with the bell section left free to vibrate. Measurements of the amplitude of vibration indicate that the damping significantly reduces the movement of the metal, and a corresponding change in the acoustic spectrum between the two cases is found. It is shown that the relative power in the fundamental may change by more than 3 dB when the vibrations in the bell section are damped. Two possible causes for the effects are considered: a change in input impedance, and the transfer of mechanical vibrations through the instrument to the lips. Results of modelling and experiments are presented that indicate the latter is the more plausible explanation; however, the etiology of the effect is still unknown.

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## 1. Introduction

The effect of the vibrations of the bell on the sound of brass instruments, or lack thereof, has been the topic of debate for many years. Musicians almost universally believe that the material and thickness of the bell section have a great effect on the sound of the instrument due to the enhancement or damping of the resonant vibrations of the metal. Yet there is conflicting evidence in the scientific literature.

Pyle has reported that something seemingly as insignificant as lacquering the bell of a French horn can make a noticeable difference in the perceived sound [1, 2]. However, Smith has reported that something as important as changing the material of the bell has no effect on the sound of trombones [3]. Lawson and Lawson [4] have reported that the effect of annealing the bell of a French horn on the radiated sound is exceptionally strong, and Wogram [5] has reported differences in timbre that are related to the composition of the metal; yet Hoekje, et al. have theoretically investigated the contribution of bell vibrations to the sound field of a trombone and concluded that vibrations make only a very small difference (30 to 40 dB less than the contribution due to the air column) [6, 7, 8].

These conflicting reports leave both the musician and the musical acoustician in the position of being unable to conclusively comment on the importance of bell vibrations to the sound of the instrument. It is generally accepted that wall vibrations have a negligible effect on the sound of clarinets and organ pipes [9, 10, 11], yet no such conclusion can be drawn about brass instruments. Indeed, the be-

lief that the material of which brass instruments are made, how they are braced, and other aspects related to wall vibrations and not directly associated with the shape of the air column have led to some interesting and thoroughly unsubstantiated claims by musicians and experts in musical instrument repair [12, 13].

In order to determine the effects of bell vibrations on the acoustic spectrum of brass instruments we have studied how damping the bell vibrations of a modern trumpet affects the sound. In what follows we begin by presenting the results of experiments that indicate the bell vibrations do indeed have a large effect on the acoustic spectrum. We then discuss two possible causes for the observed effects.

## 2. Experiment

In order to determine the influence of the bell vibrations on the acoustic spectrum of brass wind instruments we chose the modern trumpet as a typical instrument upon which to experiment. The trumpet used in our experiments was a *Silver Flair* model made by the King Instrument Company circa 1970.

The instrument was securely mounted to an optical table and played with a set of artificial lips. The lips used were different from those reported by Gilbert and others in that these lips were made of solid rubber and formed from molds of actual human lips [14, 15, 16]. The lips were mounted on a mouth made of a machined aluminum box as shown in Figure 1. Unlike the artificial lips of Gilbert, the lips used in these experiments have no method for changing the embouchure beyond physically repositioning the lips before applying the pressure of the mouthpiece to the lips. The internal mouth pressure was approximately 20 kPa, which is similar to that reported for loud playing by human subjects [17].

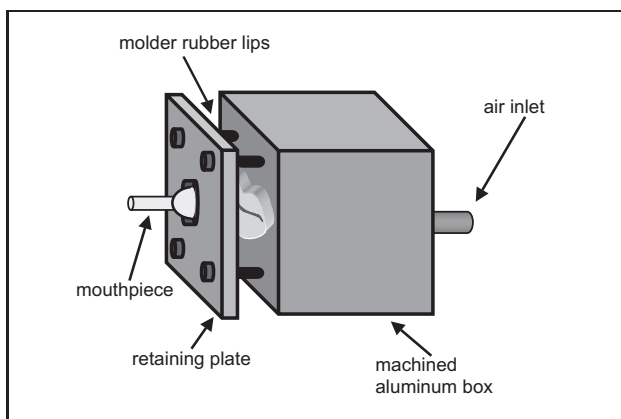


Figure 1. Schematic of the artificial mouth. The lips were made of solid rubber and cast from molds of human lips. The mouthpiece was held in place by an aluminum retaining plate.

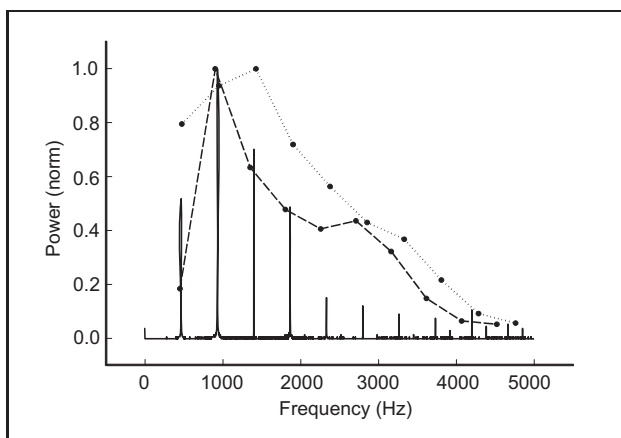


Figure 2. Typical power spectrum of the sound produced by the trumpet as played by the artificial lips. The frequency of the fundamental is 466 Hz. Also shown are the envelopes of the spectra of the same note played by an amateur player (dashed line) and by a professional player (dotted line).

Throughout the experimentation it was necessary to measure the power spectrum of the trumpet. During these experiments the sampling rate of the instrumentation was 10 kHz, resulting in a Nyquist frequency that exceeded the tenth harmonic of the fundamental frequency used in the experiments; however, typically over 99 percent of the power was contained in the first eight harmonics. The power spectrum was calculated in real time and recorded for later analysis.

Figure 2 shows a typical spectrum of the sound from the trumpet as played by the artificial lips. The note is a concert Bb4 (466 Hz). The sound intensity level was approximately 103 dB(A) measured one meter from the bell. This spectrum compares well with the spectrum of the same instrument played by a human player; however, in recording the spectra of several players of various skill levels we have found that the spectra vary so significantly between players that a definitive statement about the similarities is difficult. For example, the envelope of the spectrum of a beginning trumpet player playing a concert Bb4 on the in-

strument at a similar volume is also shown in Figure 2, as is the envelope of the spectrum of the trumpet as played by a professional player. Note that the relative powers in the harmonics is very different between the two human players. The details of the spectra of other players of varied skill levels were also quite different from one another.

The one thing that can be said unequivocally is that the spectrum of the note played by the artificial lips has relatively less power in some of the upper harmonics than is present when the trumpet is played by human lips at high volume. While our study was not exhaustive, in every case we have studied the human players produced more relative power in the harmonics above the fourth than the artificial lips did when played at a similar volume. An extensive investigation of the movement of these lips and the detailed differences between their movement and those of a representative sample of human lips has yet to be undertaken, but based on the spectra shown in Figure 2 one may assume that the modulation of the air flow through the artificial lips is somewhat similar to the air flow through human lips.

To determine the effects of the bell vibrations on the sound that the instrument produces the acoustic power spectrum was measured approximately one meter from the bell. Measurements were made with the bell free to vibrate, and then again with the bell heavily damped by sandbags that were placed under, around, and over the bell section. After conclusion of the experiment the power in each harmonic was determined by integrating over the portion of the power spectrum encompassing the peak. The power in each harmonic was normalized to the total power, and the normalized power in each harmonic measured with the bell of the trumpet damped was divided by the power in that harmonic measured with the bell freely vibrating.

During any individual experiment the orientation of the lips and the internal pressure of the mouth cavity were not changed. By using an artificial mouth and lips we were able to damp the vibrations of the bell while the instrument was playing, thus eliminating the need for multiple human subjects and the inherent uncertainty associated with them. To ensure that the position of the trumpet relative to the lips did not change during the course of the experiments the entire trumpet was clamped to a solid base which was mounted to the optical table.

Although the bell section is usually considered to extend from the rim of the bell to the first bend, a distance of approximately 50 cm, for the purposes of these experiments we have defined the bell section as extending from the rim to the bell brace, a distance of approximately 14 cm. This shorter distance was determined to be the length over which most of the bell vibrations occur by observing the bell vibrations using electronic speckle pattern interferometry.

The amplitude of the vibrations of the trumpet bell approximately 2.5 cm from the rim was measured during play and found to be  $200 \pm 20 \mu\text{m}$ . When the bell was damped with the sandbags the bell vibrations were reduced to  $20 \pm 10 \mu\text{m}$ .

It is well known that the trumpet radiates in all directions during play; it is therefore plausible that the addition of the sandbags themselves may affect the acoustic spectrum measured in front of the bell, even when the experiments are performed in an anechoic chamber. In order to eliminate this concern, and ensure that any difference between the two spectra were actually attributable only to the vibrations of the bell, the microphone used to detect the sound was placed inside a small anechoic chamber.

The anechoic chamber measured approximately  $1 \times 1.3 \times 1.5 \text{ m}^3$  and was completely tiled inside with sound absorbing foam. The chamber was placed on the optical table holding the instrument so that the trumpet was mounted outside of the chamber, with the bell of the instrument flush with the outside wall. A hole approximately two mm in diameter larger than the bell allowed the radiation from the bell to enter into the chamber. This arrangement ensured that any changes detected by the microphone were due to a change in the instrument and not due to a change in the environment. Experiments with the trumpet inside and outside of the chamber produced identical results, indicating that this concern was unjustified; however, because of the ease of having the instrument outside of the chamber when the bell needed to be damped all of the experiments were eventually performed with the trumpet outside of the chamber and only the microphone inside. A photograph of the experimental arrangement is shown in Figure 3.

Every effort was made to ensure that any effects on the acoustic spectrum attributable to bell vibrations were maximized. The trumpet was played at a fortissimo level, with the sound intensity level being between 100 dB(A) and 104 dB(A) measured one meter from the bell. The frequency of play was chosen by comparing the frequencies of the modes of the air column to those of the normal modes of vibration of the bell and choosing a near coincidence.

To determine the resonance frequencies of the air column the input impedance was measured at the mouthpiece as a function of frequency. The measurement of the input acoustic impedance of pipes has been well established for many years (see for example [18] and references therein), and the input impedance of the trumpet was measured using an adaptation of the method described by Benade [19]. The acoustic driver was a 2.5 cm diameter piezoelectric transducer that was attached to the mouthpiece rim. A 1.4 mm diameter instrumentation microphone was used to detect the pressure in the mouthpiece cup at the position of the rim. The tip of the microphone was inserted into the cup at the rim through a small hole drilled for that purpose. The response of the piezoelectric transducer was measured prior to performing the experiments and these data were later used to calibrate the results.

The microphone recorded the pressure response at the rim of the mouthpiece as the piezoelectric transducer was driven at frequencies from 10 to 2000 Hz. The power spectrum was calculated in real time from the signal received from the microphone, and only the power in the driving

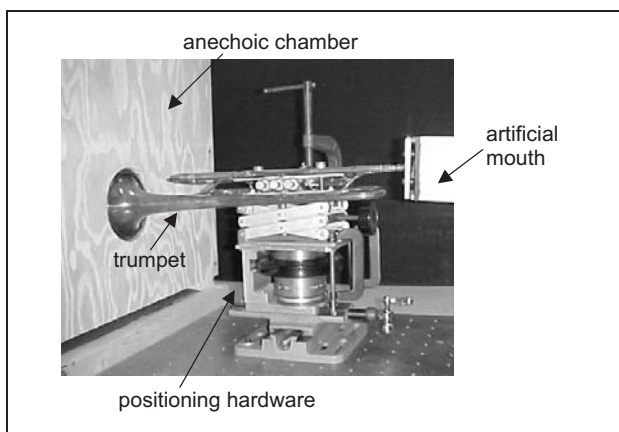


Figure 3. Photograph of the arrangement for recording the power spectra from the trumpet. The trumpet was located outside of a small anechoic chamber with the bell facing a hole in the wall of the chamber. Only a microphone was inside the chamber. The clamps are to ensure that the positioning hardware does not move when the sandbags are applied to the trumpet bell.

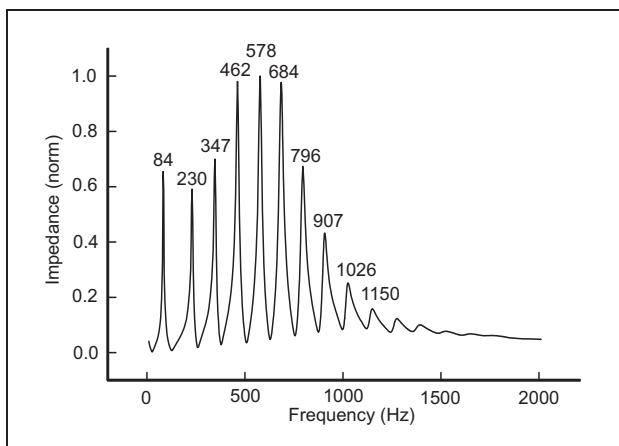


Figure 4. Impedance spectrum of the trumpet used for the experiments. The frequencies of the impedance maxima are annotated over each peak.

frequency was recorded to ensure that the data were unaffected by the presence of higher harmonics. The precision was determined by the sampling time, which was three seconds. The impedance spectrum of the trumpet with all valves open is shown in Figure 4, with the frequencies of the peaks annotated.

The frequencies of the normal modes of vibration of the trumpet bell were found by driving the vibrations of the instrument with a small speaker and observing the bell with an electronic speckle pattern interferometer as discussed in reference [20]. Interferograms showing the deflection shapes of the trumpet bell at these frequencies are shown in Figure 5 along with the resonant frequencies. The interferograms shown in Figure 5 are looking directly into the orifice from the front of the bell. The light areas represent areas of movement while the black areas occur at nodes. Note that the first detectable resonance of the bell and the Bb4 resonance of the air column are nearly coincident. The two can be brought into coincidence by adjustment of the

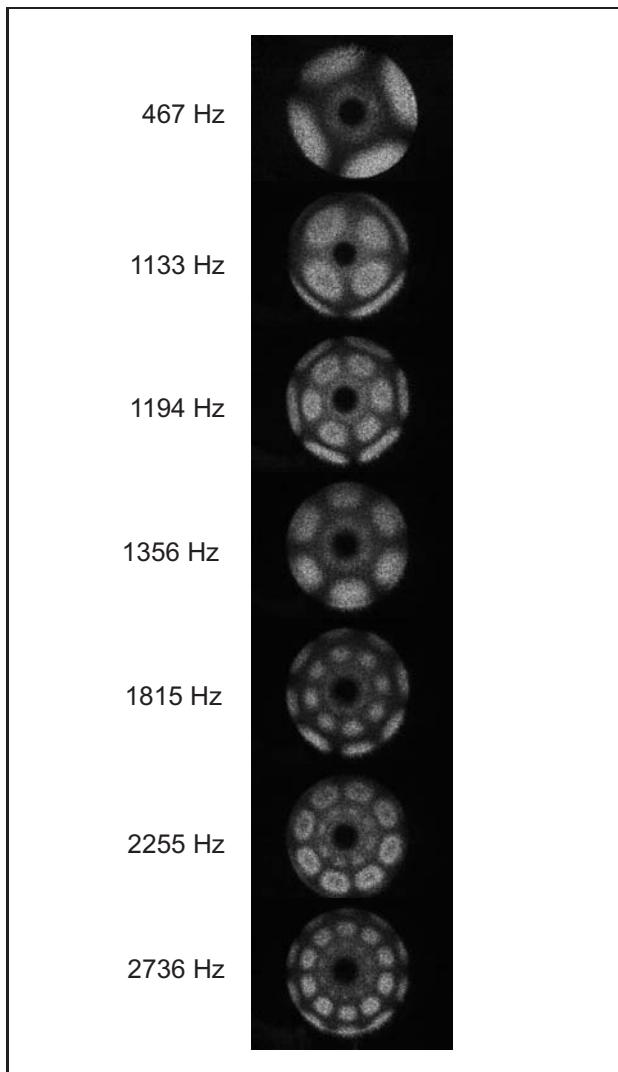


Figure 5. Electronic speckle pattern interferograms showing the deflection patterns associated with the modes of vibration of the trumpet bell used in the experiments. The white areas indicate antinodes and the black areas indicate nodes. The view is from the front of the trumpet, looking directly into the bell orifice.

tuning slide on the trumpet so that both of the resonances occur near 467 Hz. This frequency was chosen as the appropriate frequency to conduct the experiments.

The absolute adjustment of the tuning slide to bring the two resonances into coincidence clearly depends on temperature, but once set we found no need to vary the position of the tuning slide over the course of the experiments. Although the temperature of the room housing the experiment was not rigorously controlled, the room did not contain any heating or cooling vents and the temperature was shown to be stable within approximately  $0.2^\circ\text{C}$  over the short time period required to perform any individual experiment (less than 5 minutes). Any measurable shift in the frequency due to a change in temperature would have been manifest as an overall shift in the spectrum that became progressively larger as the experiment progressed, which we never observed. We believe

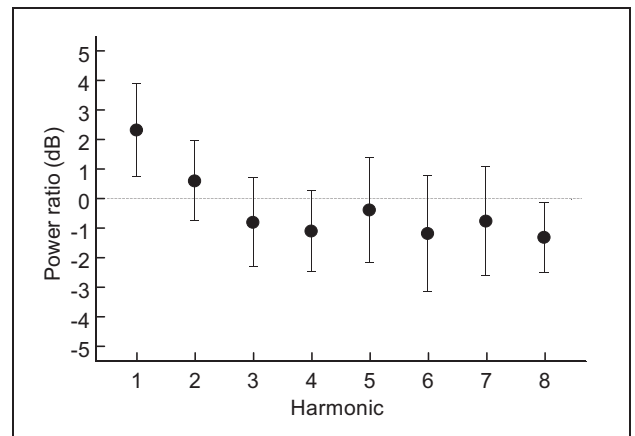


Figure 6. The ratio of the normalized power in each of the first eight harmonics with the bell vibrations damped to the power present when the bell is free to vibrate. These data are compiled from the individual experimental trials shown in Figure 7.

that this indicates that whatever slight variations in temperature there were had no measurable effect on the results.

Over many trials it was discovered that the spectrum, and how it changed when the bell section was damped, was extremely sensitive to the placement of the lips. More importantly, lip orientations that did not exhibit stability in the power spectrum over long periods of play had much more variability in the results than those orientations that were stable, even if the measurable instability was on a time scale much longer than the time needed to take data. For example, instability in the power spectrum on a time scale of fifteen minutes resulted in inconsistent data, even though the experiments were performed within a period of less than five minutes.

This variability was manifest in large changes in the power ratio of each harmonic, as well as large variations in the total output power. Due to the presence of this instability, the lips were tested prior to experimentation by observing the spectrum for a period of at least one hour before recording data. Only after the power spectrum showed stability over this period of time were the data accepted as valid.

Typical results of these experiments are shown in Figure 6, where the ratio of the power in each harmonic with the bell damped to that with the bell vibrating is plotted for each of the first eight harmonics. We note that these data are significantly less uncertain than those that were briefly reported in reference [21]. This improvement can be traced to minor changes in the construction of the artificial mouth that resulted in increased stability of the lips.

The data in Figure 6 represent thirty different experimental trials, recorded with three different lip orientations. The frequency of the fundamental was kept within four hertz of the desired 467 Hz bell resonance. Within each experiment the lips continued to play without adjustment as the bell was damped and then undamped sequentially ten times. The vertical bars in Figure 6 represent the standard deviation of the ratios of the powers in each harmonic.



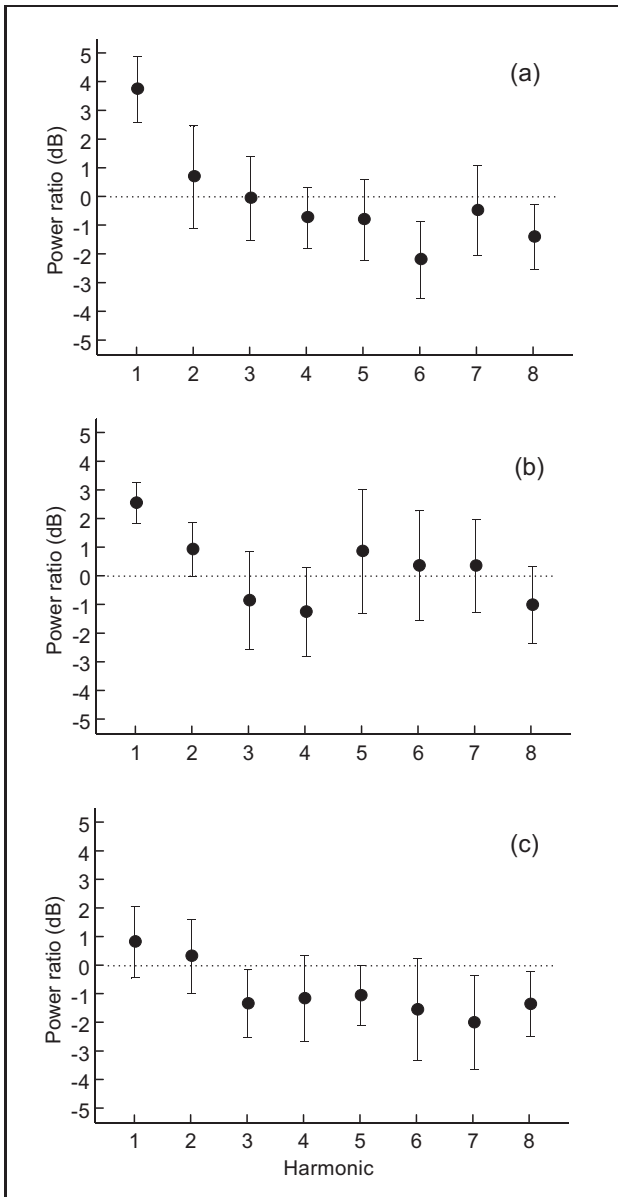


Figure 7. The ratio of the normalized power in each of the first eight harmonics with the bell vibrations damped to the power present when the bell is free to vibrate. Each graph represents data collected with a different orientation of the lips. The aggregate data of all three trials are plotted in Figure 6.

The uncertainty represented in Figure 6 is largely a function of the differing orientation of the lips between the three trials. Attempts were made to ensure that the lips were similarly oriented for each trial; however, as noted above, there is no mechanism to adjust the lips after they come into contact with the mouthpiece. Therefore, there were small but significant differences in the orientation of the lips used for each trial. Plots of the three individual trials are shown in Figure 7, where it is clear that the uncertainty in the power gain in the fundamental can be significantly reduced by limiting the experiments to a single orientation of the lips.

The results shown in Figure 7 are derived from data taken with stable lip configurations; however, as noted

above these results are not typical for lip configurations that are not stable over long periods of time. Indeed, there is no “typical” ratio of the spectra if the spectra are not stable over long time periods. When the lips are extremely stable over the course of the experiment the data are well represented by Figure 6; furthermore, as can be seen in Figure 7(b), if measurements are made using only a single orientation of the lips the uncertainty in the data can be considerably reduced. The uncertainties in the ratios of the power in the fundamental for each of the individual experiments shown in Figure 7 are 25%, 18%, and 28%, respectively, while the uncertainty of the aggregate data shown in Figure 6 exceeds 35%. Uncertainties in the ratio at the fundamental frequency as low as seven percent over ten consecutive measurements have been observed; however, those data were taken before the requisite one-hour delay between the onset of lip motion and the acquisition of data and are therefore not included in the data presented in Figure 6.

Damping the bell vibrations also resulted in a slight shift in frequency. When the lips were in a stable configuration damping the bell resulted in an increase in the frequency level of  $1.0 \pm 0.3$  cents. However, if the lips were not stable over long periods of time, shifts in the frequency on the order of five cents were occasionally observed; in these cases the shift could be to either higher or lower frequencies depending upon the orientation of the lips.

Although no attempt was made to rigorously perform perceptive tests with human subjects, the difference in timbre of the instrument when the bell is damped is obvious to even the untrained ear. To demonstrate this the digitized sound of the trumpet in both the damped and undamped configurations were presented randomly to several people, none of whom were trained musicians. The subjects were asked to listen to the sound through headphones and to identify only if a sound was different from the one played before it. Although these subjects could not articulate the differences in the sound, in every case where the different files were presented to the subject he or she identified them as being different. Likewise, in every case where the two files were identical the subject could not distinguish between the two sounds. While this is clearly not a rigorous experiment, it does highlight that the change in spectrum that results from damping the bell vibrations can result in a noticeable difference in the sound perceived by the listener.

### 3. Analysis

The results shown in Figure 6 demonstrate that the vibrations of the bell section of a trumpet can affect the acoustic spectrum. More significantly, these results demonstrate that damping the bell vibrations results in an increase in the power in the fundamental frequency component. Although the frequency of one of the modes of vibration of the bell is very close to the Bb4 resonance of the air column, it has been reported that the sound radiated by the bell due to the coupling of vibrations to the air column and mechanical vibrations of the lips is approximately 40 dB

lower than that produced by the air column [22]. However, it has been noted that there may be enhanced radiation of the sound in cases where there are asymmetries in the bell [7, 8].

Figure 5 clearly shows that all of the normal modes of vibration of the trumpet bell used in these experiments are symmetric about the center of the bell. This indicates that there is little air to bell coupling due to the short circuiting between adjacent anti-nodes of the bell, suggesting that the effects of the sound attributable to direct radiation by the bell vibrations cannot account for the large changes in the spectra observed in the experiments described above.

As noted previously, the spectrum can vary significantly between experiments, especially if the performance of the lips is not stable over time. This indicates that the change in relative power in any given harmonic that is directly attributable to damping the bell is dependent upon the actual motion of the lips. We take this to indicate that the presence or lack of bell vibrations changes the manner in which the lips move, further supporting the conclusion that it is not radiation from the bell that changes the spectrum.

In considering exactly how the bell vibrations may affect the motion of the lips there appear to be two plausible explanations: feedback through the air column, and feedback through the metal walls of the instrument. Feedback to the lips via the air column may be possible if the vibrations of the bell somehow change the input impedance of the trumpet. Alternatively, the bell vibrations may be mechanically coupled to the lips through the metal, resulting in a modification of the motion. We address both of these possibilities below.

### 3.1. Input impedance effects

It is often thought that the response of brass wind instruments is almost completely determined by the input impedance at the mouthpiece, while the actual sound produced during play is dependent upon both the input impedance and the physiology of the player [23]. In the experiments described above the lips are passive and the orientation with respect to the trumpet is constant throughout the experiment; therefore, in this case it is logical to assert that any variation in the sound production is due to a change in the input impedance, which results in a change in the motion of the lips and a commensurate change in the sound.

The input impedance of brass musical instruments has been studied for some time, and it is known that it is primarily a function of the shape of the walls of the instrument. Gautier and Tahani have theoretically studied the effects of wall vibrations on the impedance of cylindrical pipes, but to our knowledge this is the only report of such an investigation [11]. In a simple model of a clarinet they have shown that small shifts in the resonance frequency may result from wall vibrations, but the power coupled to the vibrations is small and the change in the spectrum is minimal.

In order to investigate possible effects of bell vibration on the input impedance of a trumpet a simple simula-

tion was developed that is capable of calculating the input impedance of a system of connected cylindrical pipes. This method has been used for many years to calculate the input impedance of musical instruments, and the limitations of the method have been studied by Caussé, et al. [24, 25, 26]. This simulation circumvents the issues raised by Gautier and Tahani by allowing fundamental parameters to be changed without accounting for the actual origin of the changes; for the purposes of investigating the etiology of changes in the impedance structure this model is quite useful.

In applying this simulation to the problem, we have not investigated how the vibrations actually change the physical parameters that lead to a variation in the impedance structure. Instead, we have used the model to investigate the possible effects that can result in the appropriate changes in the impedance. A rigorous study must eventually be undertaken to attribute any proposed changes in a physical parameter to the vibrating bell, but is not necessary for a heuristic understanding of the process.

The model used for the simulation follows the development of Benade [27, 28]. We assume that the input impedance of a cylindrical pipe is given by

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos kL + iZ_0 \sin kL}{iZ_L \sin kL + Z_0 \cos kL} \right], \quad (1)$$

where  $Z_0$  is the characteristic impedance of the short section of pipe,  $Z_L$  is the terminating impedance of the pipe (i.e., the input impedance of the previous pipe section),  $k$  is the propagation constant, and  $L$  is the length of the pipe section.

Viscous and thermal boundary layer effects are included in the simulation by including them in the calculation of the wave speed and absorption constant, both of which are used in calculating the propagation constant. The propagation constant is given by

$$k = \frac{\omega}{v} + i\alpha, \quad (2)$$

where  $v$  is the speed of sound in the pipe,  $\omega$  is the angular frequency, and  $\alpha$  is the absorption coefficient. Both  $v$  and  $\alpha$  can be approximated [27, 29]:

$$v = c \left[ 1 - \frac{1}{\sqrt{2}r_v} - \frac{\gamma - 1}{\sqrt{2}r_t} \right] \quad (3)$$

and

$$\alpha = \frac{\omega}{c} \left[ \frac{1}{\sqrt{2}r_v} + \frac{\gamma - 1}{\sqrt{2}r_t} \right], \quad (4)$$

where  $c$  is the velocity of sound in free air,  $\gamma$  is the ratio of the specific heat at constant pressure to that at constant volume, and  $r_v$  and  $r_t$  are the ratio of the pipe radius to the viscous and thermal boundary layers respectively. For room temperatures these parameters are well approximated by

$$r_v = 632.8a\sqrt{f} (1 - 0.0029\Delta T) \quad (5)$$

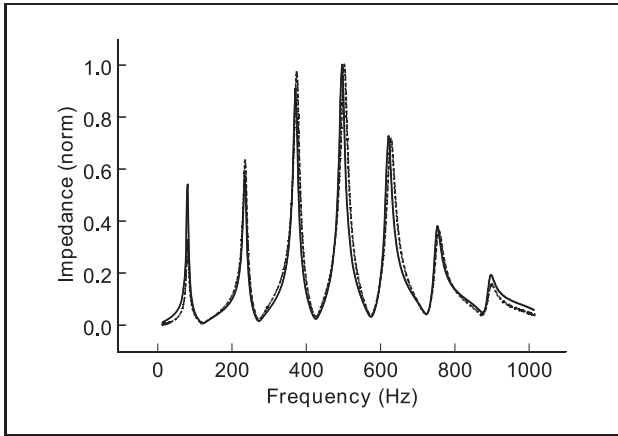


Figure 8. Comparison of measured and calculated impedance structure for a mouthpiece attached to a leadpipe and one meter of straight tubing. The theory is indicated by a solid line and the actual measurement is indicated by a dashed line.

and

$$r_t = 532.8a\sqrt{f}(1 - 0.0031\Delta T), \quad (6)$$

where  $a$  is the radius of the pipe section,  $f$  is the frequency of the sound wave, and  $\Delta T$  is the difference between the temperature of the pipe section and 300 K.

Using the above equations, the input impedance of any series of cylindrical pipe sections may be calculated as long as the radius and length of each section is known. In order to model the trumpet, which has a continuously varying diameter for much of its length, the instrument was assumed to be comprised of many small cylindrical sections of varying radius. The length of each pipe section was chosen such that a reduction in the length of the sections by a factor of ten did not change the resulting calculated impedance of the entire trumpet by more than one part in  $10^3$ . In real terms, the final simulations were completed using steps equivalent to  $25 \mu\text{m}$ .

In modelling the trumpet it was necessary to produce a set of equations that described the variation in the radius of the instrument as a function of position. Typically, the mouthpiece, leadpipe, and bell of any trumpet are hand-made and it is extremely difficult to accurately describe the variation in diameter on the scale of this simulation. The mouthpiece used in our experiments, however, was machined on a computer controlled lathe by GR Technologies and the equations were supplied by the manufacturer. GR Technologies also supplied the equations for a leadpipe design. The bell was modelled as a slightly modified Bessel horn [28].

The accuracy of the simulation was validated by calculating the input impedance for a mouthpiece connected to a leadpipe and one meter of straight tubing and comparing it to the measured impedance of such a structure. The equations used to describe the test structure in the simulation were the same as those used by the machinist to manufacture the mouthpiece and leadpipe. The impedance of the actual mouthpiece/leadpipe configuration is shown with

the calculated values in Figure 8. Comparison between the simulation and the experiment indicated that the simulation quite accurately predicts the impedance spectrum of the test system. Maxima in the impedance typically occurred within one hertz of the predicted value and the error in magnitude was less than 10 percent for frequencies above 100 Hz.

Having verified the accuracy of the simulation a modelling effort was undertaken to determine what parameter may affect the impedance structure in a manner consistent with the results shown in Figure 6. As one may expect, changes in the radius of the bore of the instrument near the bell of a magnitude on the order of the amplitude of the vibrations present during play ( $\sim 200 \mu\text{m}$ ) does not significantly affect the impedance spectrum. However, the simulation indicated that a change in the viscous boundary layer in the final 14 cm of the trumpet can significantly affect the input impedance.

The thickness of the viscous boundary layer is given by [27]

$$\delta = \sqrt{\frac{\eta}{\rho\omega}}, \quad (7)$$

where  $\eta$  is the shear viscosity, and  $\rho$  is the density of air. Using typical values for these parameters for air at room temperature, the thickness of the viscous boundary layer is approximately  $50 \mu\text{m}$ . The thickness of the thermal boundary layer can be found by multiplying the viscous boundary layer by the square root of the Prandtl number, which for air is approximately 0.84. As can be seen from equations (3) and (4), the magnitude of the viscous and thermal boundary layers affect both the velocity of sound and the absorption coefficient, and the input impedance is dependent upon both of these values.

In light of this dependence it is plausible that the motion of the air near the vibrating bell changes the thickness of the boundary layer, which in turn significantly changes the input impedance of the trumpet. It is reasonable, however, to ask if a change in the boundary layer in the last 14 cm can significantly affect the input impedance of an instrument that is comprised of approximately 1.4 m of tubing.

Figure 9 is a graph of the normalized impedance of a trumpet calculated using the simulation described above. Also shown in Figure 9 is the normalized impedance of the same trumpet with the viscous and thermal boundary layer increased by a factor of ten in the final 14 cm of the bell section. Note that there is a significant difference in the two impedance curves, with the input impedance being lower in the presence of the extended boundary layer.

It is possible that the lowering of the impedance due to the increased boundary layer decreases the feedback to the lips at the lower frequencies, thus producing a decrease in the power contained in the fundamental frequency when the boundary layer is increased. There is no equivalent reduction of the impedance in the higher frequencies because the thickness of the boundary layer is proportional to the inverse of the square-root of the frequency as can be seen in equation (7), and therefore the increase in the



boundary layer has significantly less effect at frequencies above 1 kHz.

Although an increase in the boundary layer near the end of the bell of a factor of ten seems rather large, it may be reasonable if one naively considers the boundary layer to be estimated by the magnitude of vibration of the metal. Since the the amplitude of vibration is approximately  $200\ \mu\text{m}$ , the range of motion of the bell has a total displacement that is approximately eight times the estimated boundary layer thickness for a rigid-walled tube. A comparison of the calculated normalized input impedance for the Bb4 resonance of a trumpet is shown in Figure 10, with the change in input impedance shown for increases in the boundary layer of the final 14 cm by a factor of five, ten, and fifty. Note that the change in the impedance varies nonlinearly with the boundary layer thickness, and that much of the decrease in impedance occurs within the first factor of five.

Having shown that it is possible for a change in the boundary layer in the bell section to significantly affect the input impedance of the trumpet, it is important to ask if indeed the bell vibrations can cause a change in the boundary layer. If one could measure the input impedance during play it would be possible to determine whether the bell vibrations do indeed change it in some manner. Unfortunately, to our knowledge there is no method for measuring the input impedance of a brass wind instrument while the instrument is being played. Therefore, an effort was made to measure the input impedance while changing the boundary layer in the bell section by other means.

The input impedance of the trumpet was measured as described above while attempts were made to change the boundary layer in the bell section, but none produced a measurable change in the input impedance. These efforts included attaching piezoelectric transducers to the outside of the bell and driving them at the fundamental frequency of vibration of the bell, and spraying flocking inside the bell to create a surface roughness on the scale of a few hundred micrometers.

It is possible that turbulence at the boundary may also be created by the flow of air through the bell that occurs during play. Since it is possible that this may account for the change in impedance, the impedance of the trumpet was measured with air flowing through it. An air flow was introduced at the mouthpiece through a tube attached between the piezoelectric driver and the mouthpiece. Even with air flowing through the instrument, no difference in impedance structure could be detected that was attributable to a change in the inner surface of the bell on the order of several hundred micrometers. This was true even when the inner surface of the bell was made rough by spraying flocking on it. It is important to note, however, that the flow rate at which the noise due to turbulence in the mouthpiece is reduced to a level below that of the input from the acoustic driver is approximately five liters per minute; therefore, flow rates in excess of this value were not investigated. The flow rate through the lips in the experiments used to obtain the data in Figure 6 was approx-

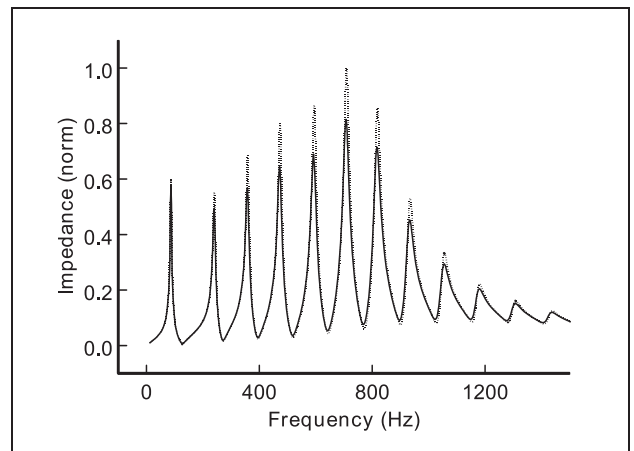


Figure 9. The calculated normalized impedance for a trumpet with a normal viscous boundary layer (dotted line), and for the same instrument with the boundary layer increased by a factor of ten in the final 14 cm of the bell section.

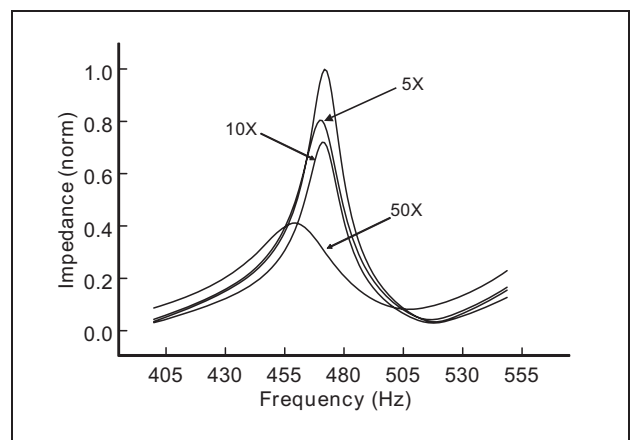


Figure 10. The calculated normalized impedance for the Bb4 resonance of a trumpet, and for the same instrument with the boundary layer increased by a factor of five, ten, and fifty in the final 14 cm of the bell section.

imately 20–30 liters per minute, which is similar to the flow rate that was measured when humans played the instrument at a similar volume.

The experiments described above indicate that it is extremely difficult to influence the impedance structure by changing the boundary layer in the bell section. Since the experiments were not made with air flowing through the trumpet at a rate commensurate with the level of play used to procure the data shown in Figure 6, there still exists the possibility that this effect accounts for the change in spectrum and therefore this explanation cannot be completely discounted. Additionally, we have not taken into account the nonlinear interaction between the lips and the air column. However, the magnitude of the physical change in the surface of the bell required to make a significant change in the impedance spectrum indicates that it is unlikely that the effects shown in Figure 6 can be accounted for by a change in the input impedance of the trumpet.

### 3.2. Mechanical feedback

An alternative explanation to that described above posits that the data shown in Figure 6 can be explained by mechanical feedback to the lips. That is, it is plausible that the vibrations of the bell are transmitted to the mouthpiece through the metal tubing and the vibration of the mouthpiece interacts with the vibrating lips to change the power spectrum.

One interesting consequence of this theory is that the result of damping the bell vibrations may depend strongly on the orientation of the lips. This is in stark contrast to the theory above that posits a change in impedance structure, where it is necessary that the power in the harmonics change in a similar manner regardless of the lip orientation. That is, a reduction in the impedance at the fundamental frequency that accompanies an increase in the magnitude of the boundary layer must always result in a decrease in the power contained in that frequency when the bell section is freely vibrating. Yet a theory that posits mechanical feedback need not have this restriction, and it is entirely conceivable that contradictory results would be obtained depending upon the orientation and motion of the lips, as was observed in the experiments described above.

There are two difficulties in developing a model for the effects of mechanical feedback to the lips. First, the motion of the lips is very complicated and not completely understood, although researchers have made great strides recently in modelling the behavior of a brass player's lips [30, 31, 32, 33, 34, 35, 36, 37, 38]. The second problem is that it is difficult to determine the effects of the mechanical feedback. That is, depending upon the magnitude and phase of the vibrations the harmonic motion of the lips may be enhanced, reduced, or unchanged for any particular lip orientation.

Although it appears that currently there is no complete model for the lip movement, it is possible to gain an intuitive understanding of the possibilities by assuming that the lip motion is adequately described as being sinusoidal. Although it is well known that this is not strictly true, evidence indicates that it is often an acceptable approximation [39]. However, since these experiments all involved extremely loud playing we assume that the sinusoidal motion saturates at some maximum amplitude as proposed by Fletcher and Tarnopolski [17].

Assuming that the lip opening can be modelled by a saturated sinusoid, the flow through the lips can be modelled in accordance with references [17] and [40], and the harmonic structure can be calculated from the power spectrum of the flow. Unfortunately, since this results in a prediction of the power spectrum of the flow inside the trumpet, and not what will be measured outside of the bell, it cannot be directly compared to the results of the experiments described above. However, changes in the power of the fundamental component and second harmonic of the spectrum inside the trumpet are indicative of the effects that will be detected in those components outside of the trumpet. Furthermore, the higher harmonics all occur near or above the cutoff frequency of the bell, which is approx-

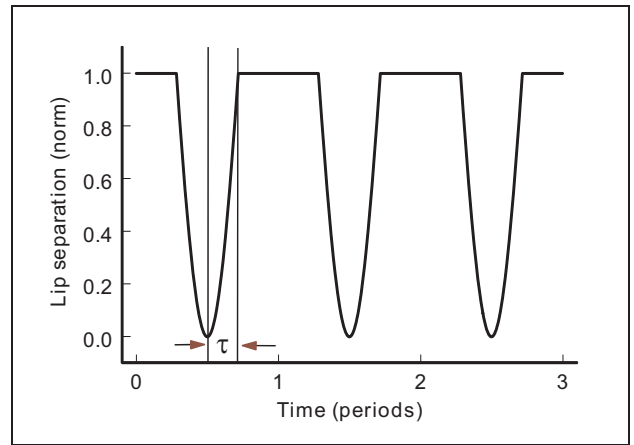


Figure 11. The assumed form of the lip opening used for calculating the flow through the lips. The function is a saturated sinusoid; the time to saturation is indicated by  $\tau$ .

imately 1400 Hz, and therefore the relative power in the harmonics will be similar inside and outside of the instrument.

Assuming that the movement of the lips can be modelled, the flow through the lips can be approximated by [17]

$$U = \frac{B^2 R x^3}{2} \left[ \sqrt{1 + \frac{4p}{B^2 R^2 x^3}} - 1 \right], \quad (8)$$

where  $B$  is a constant,  $p$  is the blowing pressure,  $x$  is the lip separation, which we assume to be a saturated sinusoid such as is shown in Figure 11, and  $R$  is the (purely resistive) input impedance. (When applying equation (8) to the situation presented here we shall assume that  $R$  does not vary significantly for the frequencies of interest.) In deriving equation (8) one must assume some model of how the opening varies with the lip separation. In keeping with reference [17] we assume that the area of the opening is well approximated by the lip separation raised to the  $\frac{3}{2}$  power.

It is not obvious how one is to apply mechanical feedback to equation (8) because it is possible that the phase of the vibrations of the metal may have any phase relative to the lip motion, although the fundamental frequency of vibration is the same. We have addressed this problem by assuming that the vibrations of the bell are primarily driven by the movement of the lips. Therefore, the vibrations of the trumpet that provide positive feedback to the lips and enhance the lip movement will be driven more efficiently than those with a phase that provide negative feedback.

Clearly there is some transfer of energy from the air column to the bell, and it is most efficient when the resonance of the air column is close to the resonance of the bell. However, vibrations of the bell that can be attributed to the movement of the lips is probably significantly larger than those attributable to the air column since the coupling between the lips and mouthpiece is extremely efficient, and the amplitude of vibration of the lips is quite large compared to the amplitude of vibration of the walls

of the instrument. This assumption is supported by observations made during the experiments cited above which cataloged the modes of vibration of the bell by interferometry. In these experiments the amplitude of vibration of the bell was seen to increase by approximately an order of magnitude when the speaker was actually touching the mouthpiece when compared to results achieved when the two were slightly separated. Additionally, in experiments using a trombone mouthpiece connected to a 70 cm long straight pipe Whitehouse has shown that the contributions from the lip motion to the wall vibrations of the pipe significantly exceed the contributions from the air column [41].

Since it is assumed that the lip opening saturates, positive feedback to the lips will decrease the time to saturation rather than increasing the magnitude of the lip opening. Figure 12 contains a plot of the fraction of the total power in each harmonic component derived from the power spectrum of equation (8) assuming the lip motion shown in Figure 11. The power is plotted as a function of time to saturation of the lip opening, which is scaled to be in terms of one half of the period of the lip motion.

In calculating the power in each harmonic we have assumed that the input impedance  $R$  is independent of frequency, which is clearly not the case for the instrument used in our experiments. However, it is evident from the data shown in Figure 4 that the impedances at the frequencies of the fundamental and first three overtones of the Bb4 resonance are comparable. Therefore, a constant impedance is a reasonable approximation for at least these harmonics.

Since the slope of the power in each harmonic shown in Figure 12 changes with the time to saturation, it is necessary to determine the approximate time to saturation before any statement can be made about the behavior of the sound field as the time to saturation is increased or decreased. The ratio of the power in the third harmonic to the power in the fourth harmonic as measured outside of the instrument can be used to make this estimate since both are near or above the cutoff frequency of the bell. (Note that because the frequencies of the fundamental and second harmonic are both well below the cutoff frequency it is not possible to use these in this context.) The ratio of the power in the third and fourth harmonics in the experiments described above indicates that the time to saturation falls between 0.1 and 0.3 half-periods. This range is indicated on the graph in Figure 12.

Although it is unwise to put too much confidence into interpreting these predictions, in light of the data presented in Figure 6 it is obvious that the variation in the power in the fundamental and lower harmonics do indeed perform as expected in the case of positive feedback to the lips. As the time to saturation is reduced the power in the fundamental and second harmonic monotonically decrease, the power in the third harmonic changes little, and the power in the fourth harmonic monotonically increases.

The behavior of the higher harmonics also indicates good agreement with the data; however, a detailed com-

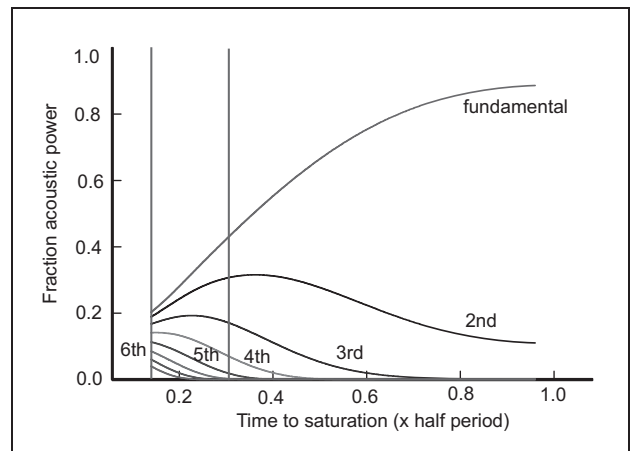


Figure 12. The normalized power in each harmonic component as a function of time to saturation of the lip opening, assuming the lip opening has the form shown in Figure 11. The curves are derived by calculating the power spectrum of equation (8). The vertical lines indicate the plausible range of the time to saturation in the experiments.

parison with the data is not useful for harmonics above the fourth for several reasons. As noted above, the model does not account for the variation in input impedance with frequency, which is significantly reduced at the frequencies corresponding to the higher harmonics. The behavior of the system at these higher frequencies is also increasingly sensitive to the model of the lip opening. Finally, there is a lack of agreement between the relative power in the upper harmonics of the sound produced by the artificial lips and that produced by human lips.

In order to test the theory that a change in the magnitude of the bell vibrations results in a change in the magnitude of the vibrations of the trumpet at the lips, the amplitude of vibration of the trumpet at the point where the mouthpiece is inserted into the instrument (i.e., the mouthpiece receiver) was measured during play. The measurements were made by connecting the anode from a dc power supply to the trumpet and connecting the cathode to an oscilloscope. A micrometer screw was connected to the other terminal of the oscilloscope so that when the metal screw made contact with the trumpet the potential was observed on the oscilloscope. The amplitude of vibration was determined by measuring the displacement of the micrometer necessary to make contact with the trumpet during play and subtracting from it the displacement necessary to make contact while at rest.

Measurements were made both with the bell vibrations damped and with the bell left free to vibrate. With the bell free to vibrate the amplitude of the vibrations perpendicular to the line created by the joining of the lips and perpendicular to the direction of air flow was found to be  $20 \pm 10 \mu\text{m}$ . With the bell damped these vibrations were reduced to  $6 \pm 3 \mu\text{m}$ . The trumpet was not free to vibrate in the direction of the airflow due to the secure mounting of the valve section to the positioning hardware, therefore no at-

tempt was made to measure the movement in the direction of the airflow.

While these experiments have shown that damping the bell vibrations can reduce the vibrations of the mouthpiece receiver by as much as a factor of three, indicating support for a theory that posits the effects of mechanical feedback to the lips, the magnitude of the vibrations can lead one to question whether they are large enough to actually induce an effect.

#### 4. Conclusions

We have shown that the vibrations of the bell of a trumpet do indeed affect the acoustic spectrum. The use of an artificial mouth eliminated the variations that can be attributed to human subjects and allowed objective measurements of the acoustic spectrum as the vibrations of the bell were damped. The data clearly indicate a large difference in the spectra when the bell vibrations are damped compared to when they are not, and the results are in agreement with the anecdotal evidence presented by trumpet players; that is, damping the bell vibrations appears to produce a “darker sound”, which we take to mean an increase in the relative power in the fundamental and a commensurate decrease in the power in one or more of the higher harmonics.

It is not immediately clear why the bell vibrations have such an effect on the spectrum. Studies have shown that the radiation from the bell is not sufficient to explain more than a very small portion of the observed difference in spectra; however, we have proposed two possible explanations and demonstrated that either one may account for the variation in sound. The first possibility is that an increase in the magnitude of the viscous boundary layer may occur when the bell is free to vibrate. This increase in the boundary layer will decrease the input impedance at the fundamental frequency, leading to a reduced output at that frequency. This decrease in impedance is not seen in the higher harmonics due to the inverse square-root dependence of the boundary layer on the frequency. Experiments attempting to demonstrate this effect, or some variant of it, have shown that this explanation is unlikely, but they have not completely eliminated the possibility.

The more likely explanation is that there is mechanical feedback from the bell to the lips through the metal. A simple theory based on this explanation has been presented which predicts results that are in qualitative agreement with the data. This theory also can account for the wide variation in the results, especially when the lips are in an orientation that is unstable over time. This theory also has the added advantage of possibly explaining why some artists claim that adding mass to various parts of a trumpet changes the sound [12]. Additionally, this explanation is supported by the recent results of Bertsch that indicate that the response of a trumpet as described by the player is often not in agreement with what one would expect from a measurement of the impedance spectrum [42].

Measurements of the vibrations of the trumpet near the mouthpiece have shown that there is indeed a correla-

tion between the bell vibrations and the vibrations of the mouthpiece receiver. Damping the bell vibrations can reduce the magnitude of vibration of the mouthpiece receiver by a factor of three; however, the magnitude of these vibrations is less than  $25 \mu\text{m}$  and this leads one to question the plausibility of such a large effect.

Although there is some evidence to support a theory of mechanical feedback to the lips, a comprehensive theoretical understanding will have to wait until a complete model of lip motion is developed. Additionally, it is possible that the presence of slight shifts in the higher harmonics may indicate a small contribution to the impedance as proposed by Gautier and Tahani [11], and this may be indicative of the fact that the results presented here are actually a combination of the two effects.

There are, of course, other plausible explanations. Even though work by others indicates that wall vibrations do not significantly affect the spectrum of organ pipes and woodwind instruments, there may be some nonlinear coupling effects between the vibrating bell and the air column. However, experiments described here which measured the input impedance of the trumpet while the bell was vibrated indicate that the effect, if it exists, requires the high volume created by actual playing of the instrument or a volume flow commensurate with actual performance.

In concluding we note that regardless of the etiology of the effects reported here, one must be troubled by the contradiction of previously reported results from well established experimentalists. It seems unlikely that experimentalists could arrive at such mutually exclusive results as, for example, Pyle [2] and Smith [3] if indeed the physics of playing the instruments were the same. The work reported here clearly agrees with the conclusions of those reporting acoustic effects attributable to bell vibrations; however, it is possible that the contradictory nature of the literature is inherent in the choice of instruments.

A review of the literature reveals that authors claiming a lack of importance of bell vibrations have performed their experiments on trombones, while those claiming an importance of bell vibrations have performed their experiments on instruments with mouthpieces that are significantly smaller in size (i.e., French horns and trumpets). Should the effects of bell vibrations be attributable to mechanical feedback to the lips, one would expect that instruments with smaller lip openings would exhibit more sensitivity to these effects than those requiring larger openings. Therefore, trumpets and French horns may exhibit significant effects attributable to bell vibrations while trombones and tubas may not.

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