Representing Terrain with Mathematical Operators

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The Problem

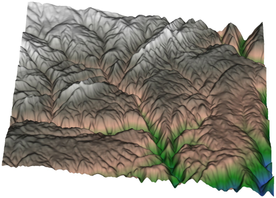
Is there a terrain representation that

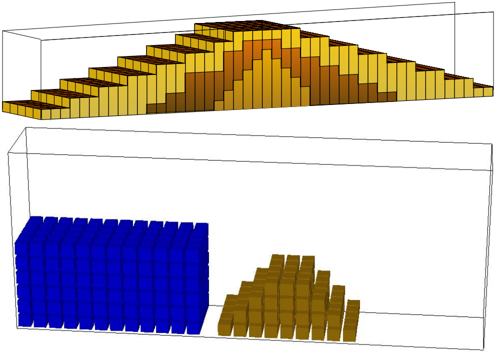
- mimics physical manipulation of terrain by storing data procedurally?
- facilitates prohibition of local minima?
- allows encoding of complex features (caves, cliffs)?

Broader Impact:
More hydrographically valid datasets which can be compressed and manipulated for storage on mobile devices running GIS applications.

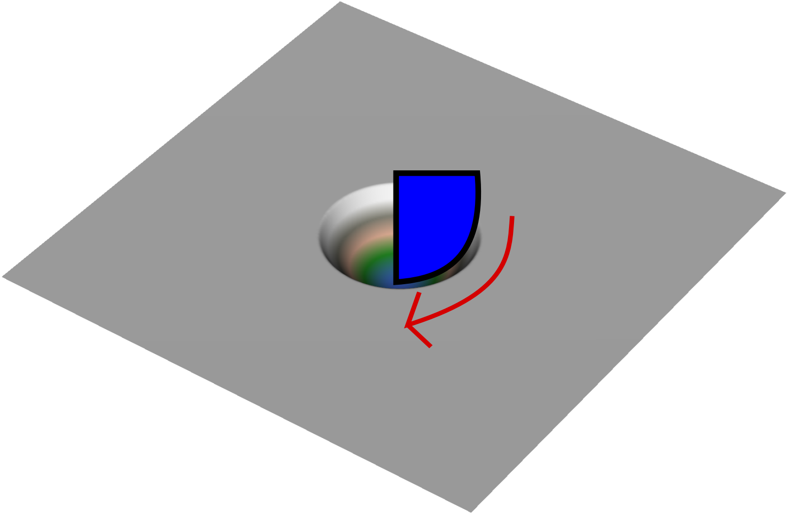
Popular Terrain Surface Representations

- Height Fields (DEMs)
- Triangulated Irregular Networks (TINs)
- Fourier Series
- Splines
- Volumetric Representations:





The Drill Operator



Questions to Answer

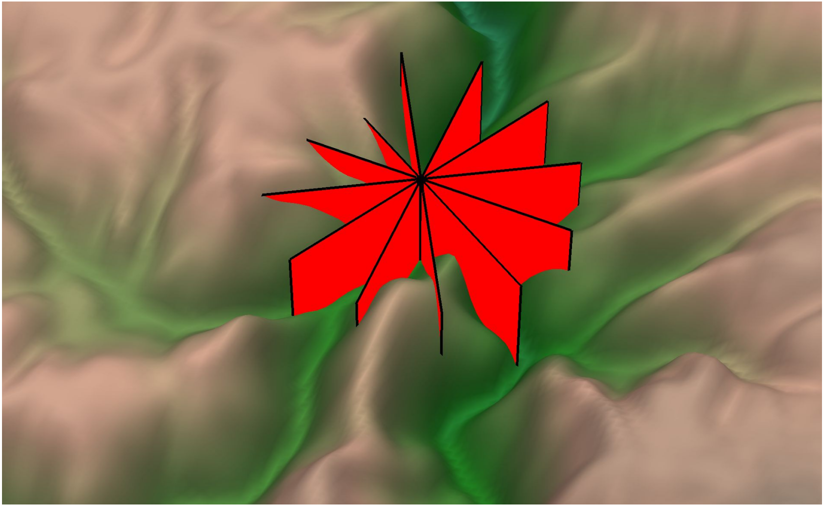
1. How can we fit a drill to a pixel **p**?
2. How can we represent a terrain surface by a series of drill operations?
3. How can we regenerate a surface from a series of drill operations?

Fitting a Drill to Pixel **p**

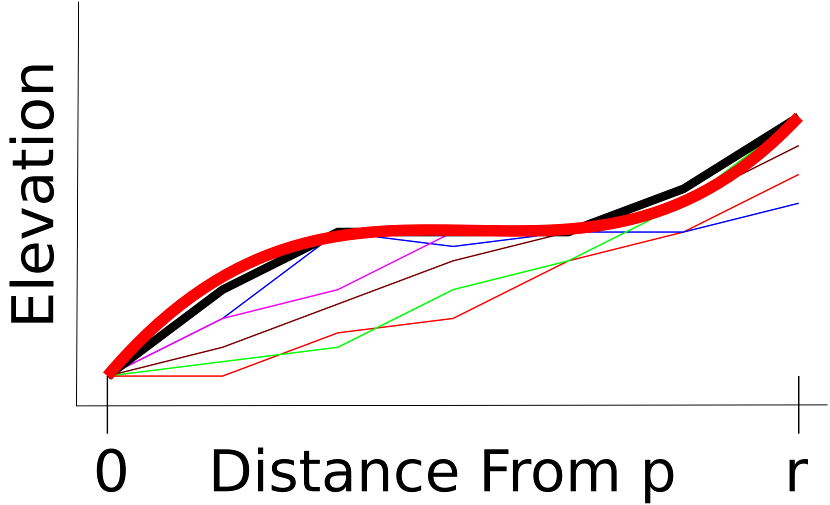
For several possible radii r:

1. Find S, terrain profile of size r
2. Fit function F to S

Calculating S

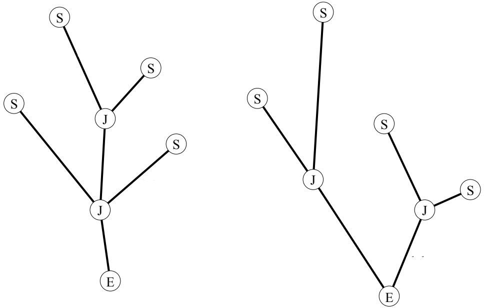


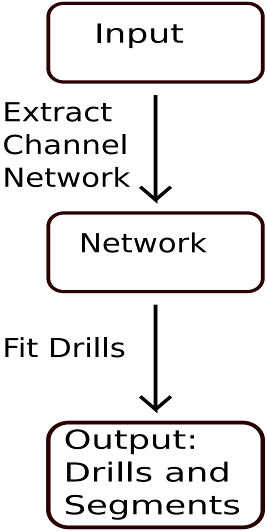
Finding Best-Fit Polynomial for S



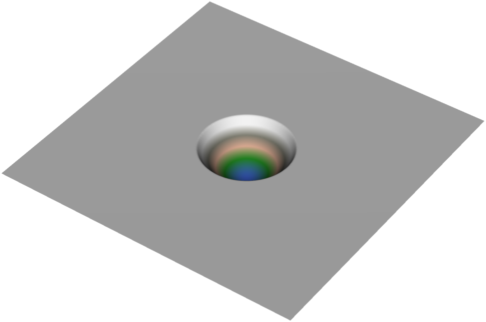
Representing Terrain T with Drills

1. Extract the terrain's channel network
2. Find best drill shape at each start, junction, and end point
3. Store channel segments and drills





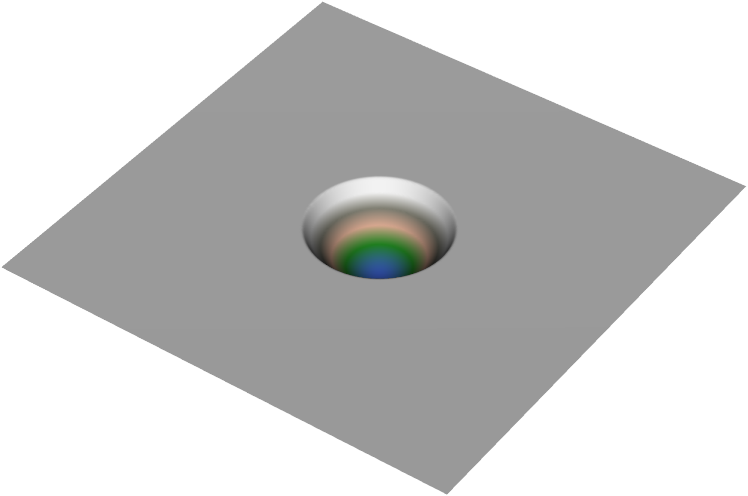
Regenerating the Terrain



$$T_0(X, Y) = \infty$$
$$T_i = \min(T_{i-1}, D_i)$$

where T_i is the i^{th} step of the terrain regeneration,
and D_i is the i^{th} drill matrix, an $X \times Y$ matrix of elevation values resulting from evaluating F_i at each pixel's distance from **p**.

Video!



Terrain Distances

Root Mean Squares Error:

$$RMSE(T_0, T_1) = \sqrt{\frac{\sum_{x<X}\sum_{y<Y}(T_0(x,y) - T_1(x,y))^2}{X * Y}}$$

Ridge-River Error:

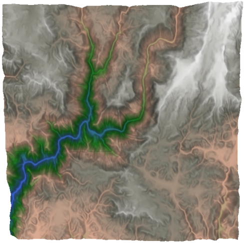
$$EnergyDown = \sum_{x<X}\sum_{y<Y} \max(0, T_0(x,y) - T_0(r(x,y)))$$

$$EnergyUp = \sum_{x<X}\sum_{y<Y} \max(0, T_0(r(x,y)) - T_0(x,y))$$

$$RRE(T_0, T_1) = \frac{EnergyUp}{EnergyDown}$$

where $r(x,y)$ is the pixel receiving flow from (x,y).

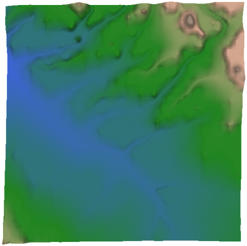
Height Fields Used in this Work



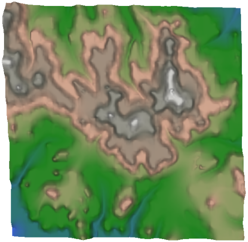
MTN1

MTN2

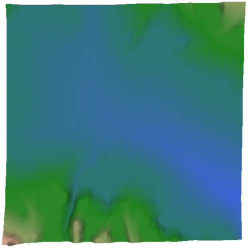
MTN3



HILL1



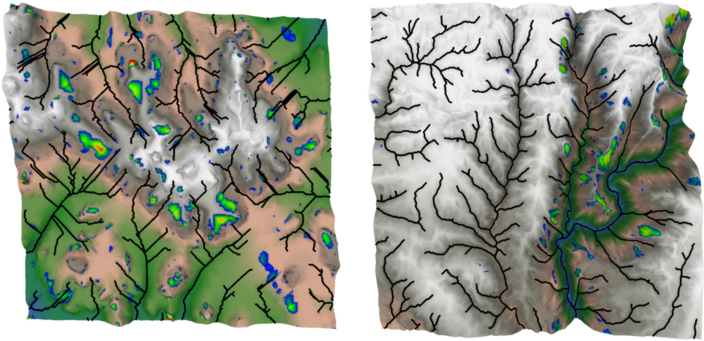
HILL2



HILL3

Accuracy Tests

	RMSE	SSE	RRE
HILL1	6.62 m	9.38°	0.012
HILL2	10.69 m	16.38°	0.008
HILL3	6.80 m	7.20°	0.015
MTN1	7.05 m	14.65°	0.003
MTN2	6.80 m	14.31°	0.003
MTN3	10.41 m	15.51°	0.005



Terrain Compression Using the Drill Operator

	RMSE	SSE	RRE
HILL1	7.01 m	10.605°	0.019
HILL2	11.54 m	18.031°	0.012
HILL3	8.00 m	8.513°	0.023
MTN1	9.57 m	18.553°	0.011
MTN2	9.62 m	18.026°	0.007
MTN3	11.48 m	18.403°	0.011

	Binary	ASCII	Drill	7Zip	PNG	JPEG
HILL1	320.0	781.3	31.4	70.4	89.0	13.7
HILL2	320.0	781.3	30.4	101.6	138.6	20.8
HILL3	320.0	781.3	17.5	48.7	57.7	9.3
MTN1	320.0	625.4	17.2	115.7	182.4	39.9
MTN2	320.0	651.1	12.1	130.5	145.7	36.0
MTN3	320.0	625.3	22.0	116.0	158.6	39.9

Conclusion

The drill operator is a viable representation of terrain data that

- guarantees hydrographically valid terrain with no pits.
- can represent complex terrain features (anything modeled with a mathematical curve).
- stores elevations procedurally.
- can be used to compress features of the terrain where other schemes oversmooth the surface.

Future Work

- Additional drill shapes, and use 3D terrain data to model caves and cliffs
- Automatic detection of ideal drill parameters
- Temporal and spatial optimization
- (Lofty) Develop in conjunction data collection methods that take advantage of drill flexibility

Acknowledgements

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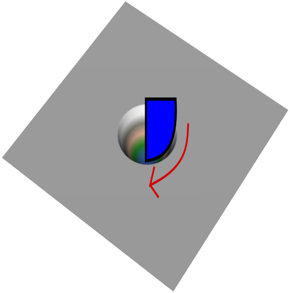
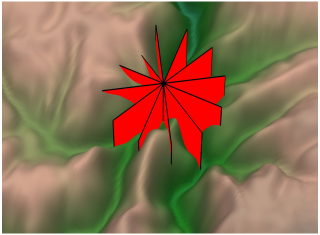
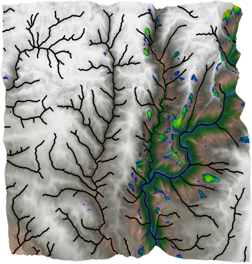
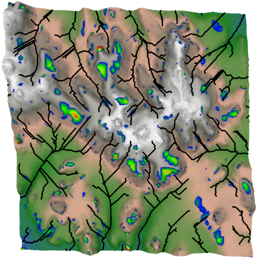
- We would like to thank the reviewers for their helpful comments.

Thank you! Questions?









Complex Terrain Formations

Surface representations cannot model complex terrain formations:

- Cliff faces
- Caves
- Overhangs

Volumetric representations have a large a memory footprint and are dependent on grid resolution.

Post-Process T_{new}

ODETLAP (OverDETermined LAPlacian) applied to T_{new}:

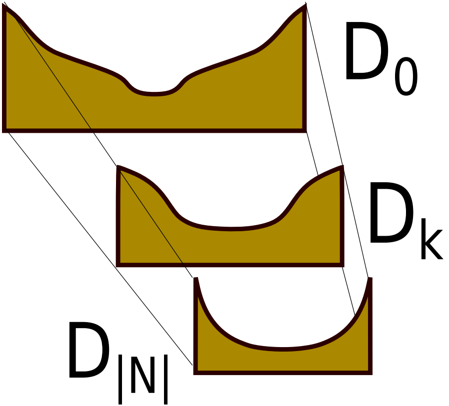
$$T(x,y) = \frac{T(x+1,y) + T(x-1,y) + T(x,y+1) + T(x,y-1)}{4}$$
$$T(x,y) = T_s(x,y)$$

Calculating S

$S = \text{zeros}\left(r + 1\right)$
for $\mathbf{p}_j \in R$ **do**
  if $S\left[\text{floor}\left(d\left(\mathbf{p}, \mathbf{p}_j\right)\right)\right] < T\left(\mathbf{p}_j\right)$ **then**
    $S\left[\text{floor}\left(d\left(\mathbf{p}, \mathbf{p}_j\right)\right)\right] = T\left(\mathbf{p}_j\right)$
  end if
  if $S\left[\text{floor}\left(d\left(\mathbf{p}, \mathbf{p}_j\right) + 0.999\right)\right] < T\left(\mathbf{p}_j\right)$ **then**
    $S\left[\text{floor}\left(d\left(\mathbf{p}, \mathbf{p}_j\right) + 0.999\right)\right] = T\left(\mathbf{p}_j\right)$
  end if
end for

Determining D_i

for $j \in |N|$ **do**
  for $k \in |N^j|$ **do**
    $t \leftarrow \frac{k-1}{|N^j|-2}$
    $D^j_k \leftarrow (t-1) * D^j_0 + t * D^j_{|N^j|}$
  end for
end for

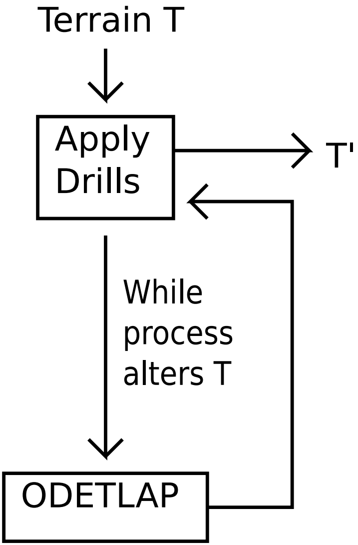


For the k^{th} pixel of the j^{th} segment of our network, N^j, interpolate the drill matrix between the source drill (D^j_0) and sink drill ($D^j_{|N^j|}$).

Post-Process T_{new}

PostProcessTerrain(terrain T, drillSet D)

$oldT \leftarrow zeros()$
while $oldT \neq T$ **do**
  $oldT \leftarrow T$
  for $d \in D$ **do**
    $T \leftarrow applyDrill(d, T)$
  end for
  $ODETLAP(T)$
end while



Terrain Compression Using the Drill Operator

1. Compress each channel network segment
   1. Freeman Chain Codes
   2. Line Generalization
2. Encode each drill
3. Compress the data (binary encoding)
4. (Optional) Compress with archiving algorithm, such as 7Zip.