# Redesign of Three-Echelon Multi-Commodity Distribution Network 

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#### Abstract

This research studies the distribution network redesign of an actual electronics company. The problems are formulated based on multi-echelon capacitated Location Routing Problem (LRP) with two commodities: home products and service items. The objective function consists of three components: facility cost, closing cost of facility and transportation cost. We propose solution method based on clustering technique. The problem is decomposed into the Facility Location Allocation Problem (FLAP) and the Multi-Depot Vebicle Routing Problem (MDVRP). MDVRP is solved by clustering method and feed the results to the modified FLAP to allocate the demand nodes to facilities and configure all distribution networks, for the $2^{\text {nd }}$ and $3^{\text {rd }}$ echelon. The distribution is divided into five region zones. Previously, each region was operated independently but this research compares the solutions from solving each region independently and solving all five zones simultaneously. The results indicate that the proposed solution method can achieve computation time and total cost that are comparable to ones obtained from solving the problem to optimality. Exact approach can only solve small and medium problems, whereas the proposed solution method provides the acceptable solution of real-life largest problem in limit of computation time. Finally, we perform sensitivity analysis on the results.


Keywords: Location routing problem, multi-echelon, distribution network redesign, clustering technique.

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## 1. Introduction

In today's world, the distributions of goods from manufacturing sites to customer 's sites are more complicated and the number of distributed products also increases. This forces companies that operate their own distribution networks to face higher logistics cost as well as longer delay and over-capacity problems. It leads to uncontrollable situations. In order to maintain distribution efficiency and support demand expansion, companies have to redesign the distribution networks. Hence, this research topic is an interesting topic for companies to remain competitiveness in terms of cost advantage.

As mention above, the locations and the number of facilities in distribution network are the significant factors which affect distribution efficiency as well as a delivery routing [1]. Facility Location Problem (FLP) is the problem to identify the optimal number of facilities and to locate them in proper candidate sites. However, most of mathematical models in previous researches of FLP ignored the round-trip delivery in design phase. Consequently, the configuration of distribution network is not suitable for real world cases, which use the tour transportation for replenishing products to demand sites. Also, many researchers proved in their works that making decision separately will lead to suboptimal configuration for distribution network design [2, 3]. In order to obtain higher efficiency, in terms of overall distribution cost, facility location decision and designing route should be integrated. Mathematical model, considering both of dominated aspects, is called Location Routing Problem (LRP). LRP is a combination of two different managerial levels of decision, which are a facility location problem (long term decision) and a vehicle routing problem (tactical term decision), and inherently recognized as a Non-deterministic Polynomial-time (NP) hardness problem. Many researchers and practitioners have proved that solution obtained from LRP can reduced distribution cost and time [2]. Application of LRP can be applied on many cases, for example, a designing an emergency service, an Automated Teller Machine (ATM) location and replenishment network, and also planet exploration and a general commodity distribution network [3].

Furthermore, there are two delivery items in this research; products and service parts. Both of them generally apply different distribution networks. But distribution networks have been designed for flows of products. Therefore, applying those networks to flows of service parts consumes higher cost and longer time [4]. In case of a separating network, it consumes excess expenses and resources. Since sharing distribution resource concept has been arisen, the networks flow of products and service parts for maintenance purposes are also simultaneously planned [5]. It is obvious that a facility location and a vehicle routing decision should be considered as a multi-commodity and multi-echelon problem.

Moreover, there are likely the existing facilities, which is being operated in real-world case study. The decision makers must consider the effect or the consequence of open or closure existing facilities, especially cost of moving facility to a new location. The saving cost earned from selling existing asset or property must be considered. As detailed above, the problem is formulated as three-echelon transportation (links between two adjacent layers of supply chain facility or demand node), multi-commodity LRP with considering of closing cost of facility.

To solve the complex LRP, many researchers applied heuristics or meta-heuristics. Therefore, in this research, we propose the solution method based on sequential heuristic approach. Hence, a main contribution of this research is to propose a mathematical model which can provide a quality configuration of distribution network for a real-world case study. Another one is to develop a solution algorithm due to complexity of the three-echelon multi-commodity LRP, especially for a large-scale case study problem that depends on number of echelons, layers of supply chain, candidates of facilities location and etc. Finally, we prove the outcome of distribution policy that allow replenishment product across different zones.

In this paper, we present into 6 sections. Section 2 is a literature review about a location routing problem and a solution approach by focusing on a clustering technique. Section 3 describes about a problem statement and how to formulate the model. Then, Section 4 explains about the proposed solution approach. In Section 5 , the results are discussed and compared with solving by CPLEX software. Also, a sensitivity analysis is reported. Finally, the conclusion of this research is indicated in Section 6.

## 2. Literature Review

A fundamental issue of formulating model is problem size that affect to the difficulty of solving, especially for LRP, which is recognized as NP-hard problem [2]. With more echelons, the problem is harder to solve. There are some researchers studied a three-echelon problem. For example, Ambrosino and Scutella [6]
extended a mathematical model of Perl and Daskin [1], which merely has single echelon and excluding inventory from consideration into three echelons (four layers) including; plants, central depots, transit points and clients. They applied CPLEX to solve the problems. This research confirmed that the commercial solver is suitable for small-sized LRP by providing the optimal solution in reasonable computation time. But for medium and large-scale problems, commercial solver could not find any feasible solution in limited time.

To obtain solution from this class of problem, some researchers decomposed LRP into subproblems, which easier to solve separately. For instance, Perl and Daskin [1] separated the original warehouse location routing problem to three subproblems, which are, the $1^{\text {st }}$ multi-depot vehicle dispatch problem, the $2^{\text {nd }}$ warehouse location-allocation problem, and the $3^{\text {rd }}$ multi-depot routing allocation problem. To obtain efficient solution, they developed iterative solution method for particular subproblems.

Aksen and Altinkemer [7] studied a location routing problem by applying Lagrangian Relaxation (LR) based solution approach to solve the "click and mortar" case study. The model structure involved three parts including of: (1) pure Facility Location Allocation Problem (FLAP) (2) pure Multi-Depot Vebicle Routing Problem (MDVRP) and (3) FLAP and MDVRP bundle constraints. The LR based heuristic decomposed problems into two subproblems by relaxing FLAP and MDVRP bundle constraint. Furthermore, subtour-elimination, capacity and deadline time constraints in subproblem MDVRP were relaxed again. Finally, they identified LR multiplier by subgradient optimization.

Clustering algorithm is one of the useful heuristic methods to establish the groups of transportation routes. The main idea of this algorithm is to divide demand vertex into clustered groups, and then designing delivery route for each group. This reduces the number of decision variables, related to customer vertex as well as the number of the vehicle routing constraints. [2, 8].

Lin and Kwok [9] proposed clustering-based metaheuristic to solve multi-objective LRP. The proposed approach combines a three-phase method. The first phase is a location phase. The minimum number of required facility is calculated by the ratio of total demand to facility capacity. Then they ranked facilities by the lowest distance to customer sites. After that they applied Greedy method in order to select the set of facilities. In the second phase, they constructed the routes by various version of saving algorithm and the nearest neighbor rule. This research improved transportation routes by insertion and swap algorithms. In final phase, the routes were assigned to the vehicle of each facility as a bin packing problem.

Mehrjerdi and Nadizadeh [10] proposed the hybrid heuristic (greedy method and Ant Colony Optimization (ACO)) to solve the fuzzy demands of capacitated location-routing problem. They applied greedy algorithm to cluster customers and constructed routes by solving Traveling Salesman Problem (TSP) with ACO.

Nadizadeh and Hosseini Nasab [11] applied the greedy based algorithm to cluster customers depended on customer demand and vehicle capacity. In allocation step, they ranked the depots by their capacities and fixed opening cost equation. Then, the customer clusters were ranked by Euclidean distance of gravity center to a top ranked depot. Then, proposed algorithm allocated the group of customers until it reached depot capacity. Furthermore, they applied the ant colony method to solve TSP for specifying a routing in each cluster.

Later, Kchaou Boujelben et al. [12] applied the Mixed Integer Programming (MIP) for LRP. This research introduced the minimum volume constraints in their model. The one of main solution methods is clustering algorithm that clustered customers to particular group depending on the distance among customers and the capacity of vehicle. The modified original problem involved only the representative particular route to reduce complexity of allocating each customer to opened distribution centers. Due to large-scale MIP, they proposed the partial linear relaxation and removed some constraints with three algorithms to reintroduce them back. The result indicated that the solution approach can provide a good quality solution for a large-scale problem in reasonable computation time.

In clustering step, most researchers defined a member of each cluster by closeness among the customers and the vehicle capacity. Basically, the closeness distance in research literatures is formed on Euclidean distance with several proximity measures. Barreto et al. [13] referred to six proximity measures with four clustering techniques proposed in this work. Incorporating between the proximity measures (one of them is grouping average proximity measure) and the various clustering techniques showed that there were no outstanding pairs in terms of performance to construct the optimal route.

Besides, in problem of designing distribution network, some researches also brought existing facilities into consideration. In this case, costs of facility relocation are also considered due to the assumption that opening or closing existing facility could affect total cost.

Melachrinoudis and Min [14] studied the real-world case study of single-echelon warehouse network problem. They formulated the model by applying mixed-integer linear programming to relocate and identify
proper set of operating warehouses. This study considered the cost of relocation, the cost saving of closure existing warehouse, and the fixed cost of maintenance facility over one year planning horizontal. They solved problem by commercial solver (LINGO 7.0). The interesting points mentioned by the authors are the redesign problem should be expanded into more echelon and the multi-commodity problem.

Melo et al. [15] conducted a multi-period logistics network redesign work. The authors formulated the problem that allowed facility relocation in several periods. Therefore, they identified fixed cost of closure facility in each period and proposed two phases of solution approach. In order to reduce computation time, the first phase of linear rounding strategy aimed to round fractional location decision variable. In second phase, the heuristic was used in case of infeasible solution or unsatisfactory solution from the first phase.

From literature reviews, most of previous studies considered only single commodity. There are some researchers studied multi-commodity. For example, Sadjady and Davoudpour [16] and Nezhad et al. [17] studied multi-commodity FLP. Kchaou Boujelben et al. [12] and Nekooghadirli et al. [18] studied multicommodity LRP. Kchaou Boujelben et al. [12] studied multi-commodity problem of car distribution, while Nekooghadirli et al. [18] did not specify a product type in their study (general commodities). But, all of them studied only one or two echelons distribution network.

Comparing to our research, the problem characteristics are the distribution network of multi-commodity with three echelons transportation. The concerned delivery items involve two types of product family; the home products and the service items. Therefore, we apply real-world information to formulate problem with existing facilities in two layers of supply chain. Finally, we develop sequential solution approach for this complex LRP. Table 1 shows the comparing of problem characteristics and solution techniques from related previous studies with this research.

Table 1. Summary of related previous studies.

| Author | Problem | Objective | Planning Horizon | Commodity | Solution Technique |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $[1]$ | 1-E LRP | Cost | Single | Single | Decomposition based and saving heuristic |
| $[6]$ | LRP + ILRP | Cost | Single and Multiple | Single | Commercial Solver (CPLEX) |
| $[7]$ | 2-E LRP | Cost | Single | Single | Lagrangian relaxation heuristic <br> Clustering technique, Tabu search and <br> Simulated Annealing |
| $[9]$ | 1-E LRP | Cost and workload <br> imbalance <br> Cost and additional <br> distances <br> Cost and additional | Single | Single | Sultiple |
| $[10]$ | 1-E LRP | Single | Single | Single | Clustering algorithm and Ant Colony method |
| $[11]$ | 1-E LRP | 2-E LRP | Cost | Single | Multiple |

## 3. Problem Statement and Formulation

### 3.1. Problem Statement

Our case study is an electronics company in Thailand that produces and distributes two product families, which are electrical products and service parts. The company requires to redesign the current distribution network to support demand expansion in the future. Due to complexity of current distribution network that transfer items through depot, warehouses retailers/service centers to the customers, the redesign for the company becomes difficult to identify proper solution. The company classifies products into 2 families; home electrical products and shop products (products used in retailers, such as, food shop-window refrigerators, vending machines, etc.). There are approximately 85 SKUs of products. The service parts are spare parts for maintenance purposes, which there are approximately 300 SKUs as shown in Fig. 1.

The company currently distributes each home product through its network to satisfy demands at retailers as illustrated in Fig. 2. For both of shop products and service parts, the maintenance technicians must bring
these items to particular customer for on-site service purposes in layer 3. To redesign of distribution network for this case study, we divide products and parts into two commodities based on destination, consists of product items and service items as shown in Fig 1. That can reduce complexity of mathematical formulation.


Fig. 1. Product families and parts.
Because of differences on types and sizes of products and parts, we convert all demand quantities into equivalent unit by standard volume. These equivalent unit is also use as facility capacity parameter for warehouses and service centers.

Based on coding principle of Laporte [19] mentioned by Ambrosino and Scutella [6], we denote our problem as $4 / R / T / T$. Number 4 refers to a number of layers of supply chain as shown in Fig. 2 (Layer 0, 1, 2 and 3). There are three layers of facilities and one layer of customers in the distribution network, including of a single depot, the set of warehouses, the set of retailers (plus retailers with a service center) and customers, respectively. Furthermore, there are three-echelon of transport route that links between two adjacent layers as denoted R, T, T (R stands for Replenishment trip and $T$ stands for a Tour trip). For the $1^{\text {st }}$ echelon, company performs a replenishment route $(R)$ that truck transfers cargos directly from the depot to the single warehouse and directly return to the depot. For the $2^{\text {nd }}$ echelon, the truck circulates cargos by milk run distribution (Tour trip: $T$ ) to several retailers in the same trip. For the $3^{\text {rd }}$ echelon, the service of each round is a tour trip ( $T$ ), starting from service center to visit customer sites and return to original service center as shown in Fig. 2.


Fig. 2. Three-echelon distribution network.
To distribute all items to the entire area of Thailand, the company establishes five isolated distribution zones and distributing across zone are not allowed. We summarize the number of facilities and customer nodes of particular zone in Table 2.

Note that each zone consists existing facilities, which have been operated on current distribution network, for both of warehouse and service centers. For example, there are two existing warehouses, eight retailers, and three existing service centers in the central and west zone (Problem Z1). Moreover, a number of candidate warehouses are identified by zone managers, as shown in column three. All retailers are able to manage as service center candidate site. We identify the locations of customers by points instead of planar areas. Each point represents customers located in the same district area due to company history sales plan.

Table 2. Number of node in particular zone.

| Zone (Code) | Number of Warehouses |  | Number of Retailer/service centers |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Existing | New Candidate | Retailer | Existing Retailer with <br> Service center | Customer |
| 1. Central and West Zone (Z1) | 2 | 4 | 8 | 3 | 28 |
| 2. East Zone (Z2) | 2 | 3 | 4 | 2 | 15 |
| 3. South Zone (Z3) | 2 | 4 | 10 | 2 | 27 |
| 4. North-eastern Zone (Z4) | 3 | 3 | 13 | 4 | 36 |
| 5. North Zone (Z5) | 2 | 4 | 10 | 2 | 27 |
| All Zones (ZA) | 11 | 18 | 45 | 13 | 133 |

Next, we consider a trade-off between the facility costs and the transportation costs to redesign this distribution network. Their cost components are shown in Table 3 and 4. We identify the facility cost based on the company account and the previous study of Melachrinoudis and Min [14]. The main idea of formulating objective function is to convert all costs into accumulation cost per year. Hence, we calculate the facility annual depreciation cost for both of the existing sites and the new candidate sites, due to the company accounting policy. The main different, between the ownership site and the rental site, is that we use the annual rental cost instead of depreciation cost for maintaining the existing rental site and renting a new site as shown in Table 3.

If the closing cost, obtained from selling property, is greater than the total cost from laid-off employees and moving to a new location, the closing cost of closure ownership site will be minus (saving cost). In other words, the company will gain benefit from this situation. However, the closing cost of existing rental site is always greater than zero, from the combination of the rental contract terminating cost, the laid-off employees cost and the moving cost. Finally, the variable cost is also taken into account. This study calculates variable cost by summation of the variable cost, as mentioned in Table 3, divided by the facility capacity.

Table 3. Component of facility cost for company ownership location and rental location.

|  | Type | Ownership Location | Rental |
| :---: | :---: | :---: | :---: |
| Fixed Cost | Opening Cost | - Depreciation Cost <br> - Maintenance cost <br> - Operator labor cost <br> - Information system license cost | - Rental cost <br> - Maintenance cost <br> - Operator labor cost <br> - Information system license cost |
|  | Closing Cost | - Cost saving from sold property <br> - Laid-off employees cost <br> - Moving cost | - Rental Contract Terminating Cost <br> - Laid-off employees cost <br> - Moving cost |
| Variable Cost |  | - Wage of temporary operators <br> - Fuel cost of equipment <br> - Electricity cost of equipment <br> - Water charge |  |

According to Table 4, transportation fixed cost is derived from truck operating, which calculated by annual depreciation, salary and equipment cost. Variable cost is dependent on the mileage maintenance cost and fuel cost. This research converts variable cost to annual cost per distance by multiplying with average frequency of travel to each node per year.

## Table 4. Component of transportation cost.

| Type | Detail |
| :--- | :--- |
|  | - Depreciation cost of vehicle |
| Transportation Fixed Cost | - Salary of driver and operator |
|  | - Equipment cost |
| Transportation Variable Cost | - Fuel cost |
|  | - Maintenance cost |

Total transportation fixed cost is annual cost per one vehicle. It is not be able to apply in the model because one vehicle can be operated more than one route. Therefore, it is necessary to estimate the number of routes which be able to assign to one vehicle. From company actual data, in the $1^{\text {st }}$ echelon, company can only operate 12 routes per replenishment period by using 2 container trucks, therefore each vehicle can handle 6 routes. In the $2^{\text {nd }}$ echelon (small truck) and the $3^{\text {rd }}$ echelon (pick-up truck), the average routes per vehicle is 3 routes. Hence fixed cost per year in the $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ must be divided by 6,3 and 3 , respectively.

Based on these characteristics of the problem that mentioned above, the case study is very interested in the point of business to find the way to reduce cost of distribution and the point of academic to develop the mathematical model and new solution method for solving this class of the problem to optimality.

The redesign distribution network requires two types of decision. First, binary decision to identify operating or closure facility locations and selecting the proper transportation routes. Second, the quantity of transferred items through facility is another one of the solutions that required from the model (continuous decision variables). Hence, this research formulates the location routing problem by developing mixed integer linear to identify simultaneously solutions of three main problems;

- Which warehouses and service centers should be operated/closed?
- Which warehouse fill up demand for particular retailers/service centers and which service center supports end customers?
- How to distribute each product/part through distribution network?

Other characteristics of this research problem and assumptions can be described below;

- Depot plays role as source of supply node which can provide all of products and parts.
- Only location of warehouse and service center are allowed to decide to be operated or closed.
- Candidate locations of a warehouse are discrete and finite number including existing locations and new candidate locations identified by company.
- Candidate locations of a service center are discrete and finite number, which can be identified from locations of retailer site.
- Demand of a retailer and each on-site service customer are deterministic and locate on each vertex.
- This research concerns a single planning horizon along with deterministic environment.
- Single-sourcing strategy is considered, which allows a retailer and an end customer to be served from single closest layer facility.
- Each route must start and end at the same facility location.
- Standard volume is given to convert demand quantity and facility capacity.
- Limit number of drop point per transportation route is used instead of vehicle capacity and distance.
- Model is formulated for a period of one year. Therefore, demand, capacity, costs are annual unit.


### 3.2. Notation

The indexes, parameters and variables used in mathematical model are shown below;
Indexes
I:
$I_{1}: \quad$ set of existing warehouse locations, indexed by $i$.
\% set of retailers to open service centers (candidate and existing location), indexed by $j$.
$J_{i}$ : set of existing retailers with service centers, indexed by $j$.
E: $\quad$ set of service customers, indexed by $e$.
$K, K 1, K 2$ : set of routes, $K 1, K 2$ for the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelon distribution respectively, indexed by $k$.

## Parameters

$\alpha_{i}:$
$\beta$;
operating cost of warehouse on potential location $i$.
operating cost of service center on retailer $j$.
$\gamma i \quad$ cost of closure existing warehouse on location $i$.
$\delta j$ : cost of closure existing service center on retailer $j$.
$\eta i, \lambda i: \quad$ variable cost of warehouse $i$ for product/service item, respectively.
$\mu_{j} \quad \quad \quad$ variable cost of service center $j$.
$\varphi_{i} \quad$ fixed and distance cost (including head haul and back haul) from depot to warehouse $i$.
$\sigma_{k}: \quad$ fixed cost of route $k$.
$\tau_{i j}, \tau_{j e}: \quad$ distance cost in arc $i-j$ and $j$-e respectively.
$d_{j}, q_{e}$ : deterministic demand of product/service item on retailer $j$, customer $e$, respectively.
$N_{1}, N_{z}$. limit of number of drop point in the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelons, respectively.
$\theta_{i}$. capacity of warehouse which is operated on location $i$.
$\omega_{j} \quad \quad \quad$ capacity of service center which is operated on retailer $j$.
M: constant large value that big enough for using in valid inequality constraints that bigger than flow from warehouse $i$ to retailer $j$ (in this research, $M$ is equal to $1,000,000$ ).

Binary decision variables
$x_{i j k}, y_{j e k}=1$ if arc operated by route $k, 0$ otherwise for the $2^{\text {nd }}$, the $3^{\text {rd }}$ echelon, respectively.
$h_{k} \quad=1$ if route $k$ is used, 0 otherwise.
$w_{i} \quad=1$ if warehouse $i$ is operated, 0 otherwise.
$s_{j} \quad=1$ if service center is operated on retailer $j, 0$ otherwise.
$z_{i j} \quad=1$ if customer $j$ is allocated to warehouse $i, 0$ otherwise.
$z_{j e} \quad=1$ if customer $e$ is allocated to service center on retailer $j, 0$ otherwise.
Continuous decision variables

| $f_{i}, g_{i}$ | flow of products/service item transfers from central depot to warehouse $i$. |
| :--- | :--- |
| $r_{i j}$ | flow of service item transfers from warehouse $i$ to service center on retailer $j$. |

### 3.3. Mathematical Model

We develop mixed integer linear programming of LRP as node-arc formulation that defined as a directed graph $G=(V, A)$. Set of nodes $(V)$ involve node of warehouse $(I)$, node of retailers and service centers ( $J$ ) and node of service customers $(E)$. A is the set of arcs. Hence, our model is modified and extended from Perl and Daskin [1] and Nguyen, et al. [20] as shown below;

$$
\begin{align*}
\operatorname{Min} Z_{1}= & \sum_{i \in I} \alpha_{i} w_{i}+\sum_{j \in J} \beta_{j} s_{j}+\sum_{i \in I_{1}} \gamma_{i}\left(1-w_{i}\right)+\sum_{j \in J_{1}} \delta_{j}\left(1-s_{j}\right)+\sum_{i \in I} \eta_{i} f_{i}+\sum_{i \in I} \lambda_{i} g_{i}  \tag{1}\\
& +\sum_{i \in I} \sum_{j \in J} \mu_{j} r_{i j}+\sum_{i \in I} \varphi_{i} w_{i}+\sum_{k \in K} \sigma_{k} h_{k}+\sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K_{1}} \tau_{i j} x_{i j k} \\
& +\sum_{j \in J \cup E} \sum_{e \in J \cup E} \sum_{k \in K_{2}} \tau_{j e} y_{i j k}
\end{align*}
$$

Subject to

$$
\begin{array}{ll}
\sum_{i \in I \cup J} \sum_{k \in K_{1}} x_{i j k}=1 & \forall j \in J \\
\sum_{j \in I \cup J}^{j} x_{i j k}-\sum_{j \in I \cup J} x_{j i k}=0 & \forall i \in I \cup J, \forall k \in K_{1} \\
\sum_{i \in S, j \in S} x_{i j k} \leq|S|-1 & S \subseteq J,|S| \geq 2, \forall k \in K_{1} \\
\sum_{j \in J} \sum_{i \in I \cup J} x_{j i k} \leq N_{1} & \forall k \in K_{1} \\
\sum_{i \in I} \sum_{j \in J} x_{i j k} \leq h_{k} & \forall k \in K_{1} \\
-z_{i j}+\sum_{u \in I \cup J}\left(x_{i u k}+x_{u j k}\right) \leq 1 & \forall i \in I, \forall j \in J, \forall k \in K_{1} \\
f_{i}-\sum_{j \in J} d_{j} z_{i j}=0 &
\end{array}
$$

$$
\begin{aligned}
& g_{i}-\sum_{j \in J} r_{i j}=0 \\
& r_{i j} \leq M z_{i j} \\
& \sum_{j \in J} r_{i j}+\sum_{j \in J} d_{j} z_{i j} \leq \theta_{i} w_{i} \\
& \sum_{j \in J \cup E} \sum_{k \in K_{2}} y_{j e k}=1 \\
& \sum_{e \in J \cup E} y_{j e k}-\sum_{e \in J \cup E} y_{e j k}=0 \\
& \begin{array}{l}
\sum_{j \in S, e \in S} y_{j e k} \leq|S|-1 \\
\sum_{e \in E} \sum_{j \in J \cup E} y_{e j k} \leq N_{2}
\end{array} \\
& S \subseteq E,|S| \geq 2, \forall k \in K_{2} \\
& \forall k \in K_{2} \\
& \forall k \in K_{2} \\
& \forall j \in J, \forall e \in E, \forall k \in K_{2} \\
& \forall j \in J \\
& \forall j \in J \\
& \forall i \in I \cup J, \forall j \in I \cup J, \forall k \in K_{1} \\
& \forall j \in J \cup E, \forall e \in J \cup E, \forall k \in K_{2} \\
& \forall k \in K \\
& \forall i \in I, \forall j \in J, \forall e \in E \\
& \forall i \in I \\
& \forall j \in J \\
& \forall i \in I \\
& \forall i \in I, \forall j \in J \\
& \forall i \in I, \forall j \in J \\
& \forall i \in I \\
& \forall e \in E \\
& \forall j \in J \cup E, \forall k \in K_{2} \\
& \sum_{j \in J} \sum_{e \in E} y_{j e k} \leq h_{k} \\
& -z_{j e}+\sum_{u \in J \cup E}\left(y_{j u k}+y_{u e k}\right) \leq 1 \\
& \forall j \in J, \forall e \in E, \forall k \in K_{2} \\
& \sum_{i \in I} r_{i j}-\sum_{e \in E} q_{e} z_{j e}=0 \\
& \forall j \in J \\
& \sum_{e \in E} q_{e} z_{j e} \leq \omega_{j} s_{j} \\
& \forall j \in J \\
& x_{i j k} \in\{0,1\} \\
& y_{j e k} \in\{0,1\} \\
& h_{k} \in\{0,1\} \\
& z_{i j}, z_{j e} \in\{0,1\} \\
& w_{i} \in\{0,1\} \\
& s_{j} \in\{0,1\} \\
& f_{i}, g_{i} \geq 0 \\
& r_{i j} \geq 0
\end{aligned}
$$

The objective function (Eq. (1)) minimizes the overall cost $\left(Z_{i}\right)$ consisting of fixed opening costs of warehouses and service centers $\left(\sum_{i \in I} \alpha_{i} w_{i}+\sum_{j \in J} \beta_{j} s_{j}\right)$, fixed closing costs of existing warehouses and existing service centers $\left(\sum_{i \in I_{1}} \gamma_{i}\left(1-w_{i}\right)+\sum_{j \in J_{1}} \delta_{j}\left(1-s_{j}\right)\right.$ ), variable costs of warehouses $\left(\sum_{i \in I} \eta_{i} f_{i}+\right.$ $\sum_{i \in I} \lambda_{i} g_{i}$ ), variable costs of service centers ( $\sum_{i \in I} \sum_{j \in J} \mu_{j} r_{i j}$ ), delivery cost from central depot to particular warehouse in $1^{\text {st }}$ echelon $\left(\sum_{i \in I} \varphi_{i} w_{i}\right)$, fixed cost of operating transportation route $\left(\sum_{k \in K} \sigma_{k} h_{k}\right)$ and delivery cost for the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelon ( $\sum_{i \in I U J} \sum_{j \in I \cup J} \sum_{k \in K_{1}} \tau_{i j} x_{i j k}+\sum_{j \in J U E} \sum_{e \in J U E} \sum_{k \in K_{2}} \tau_{j e} y_{j e k}$ ), respectively.

Equations (2) - (4) are the set of constraints for constructing route on the $2^{\text {nd }}$ echelon distribution. Equation (2) ensures that each retailer is replenished from a single route. Equation (3) requires that the route entered to particular warehouse/retailer must leave from that warehouse/retailer, in other words, balance inout for the route in particular node. Equation (4) guarantees that each route for the $2^{\text {nd }}$ echelon transportation must visit a warehouse (subtour-elimination constraints for the $2^{\text {nd }}$ echelon route). Equation (5) ensures that the number of visiting points, in each route, cannot be exceeded the allowable number of retailers. Equation (6) specifies that a single route can be operated exactly one time for the $2^{\text {nd }}$ echelon route. Equation (7) is added to assign a retailer to a warehouse which has a route from warehouse to that retailer. Equations (8) and (9) refer to conservation of flows at particular warehouse. Equation (8) ensures that the quantity of products shipped from each warehouse to be equal to the demand at specific retailers, which assigned to that warehouse, Eq. (9) refers to the quantity of service parts. Equation (10) ensures that only flow of service
item can be transferred from assigned warehouse. Flow through particular warehouse must be less than or equal to maximum capacity expressed by Eq. (11).

Equation (12) ensures that each customer is served from a single route. Equation (13) requires that the route entered to particular service center/customer must leave from that service center/customer. Equation (14) guarantees that each route for the $3^{\text {rd }}$ echelon transportation must visit a service center (subtourelimination constraints for the $3^{\text {rd }}$ echelon route). Equation (15) ensures that the number of visiting points in each route cannot be exceed the allowable number of customers. Equation (16) specifies that a single route can be operated exactly one time for the $3^{\text {rd }}$ echelon route. Equation (17) is added to assign a customer to a service center which has route connection. Equation (18) ensures that the flows of the service parts through each service center must satisfy all demands of its served customers (conservation of flow at service center). Flow through particular service center operated at the retailer location must be less than or equal to the capacity of service center expressed by Eq. (19). Finally, Eq. (20) - (27) are the decision variables.

## 4. Solution Approaches

### 4.1. Exact Method

The main consequence of the node-arc formulation is the size of problem that grows exponentially as illustrated in Table 5. Our largest zone is the Problem Z4, which consists of 6 candidate warehouse sites, 17 retailer sites and 36 customer sites as shown in Table 2. The formulated mathematical model Eq. (1) - (27) consists of 66,324 decision variables and 196,863 constraints. The smallest problem is the Z2, which consists only 5,914 decision variables and 5,925 constraints. The Problem " $Z A$ " is the special problem that allows the distribution across different zones and redesign them in one problem. The Problem ZA contains 29 warehouse sites, 58 retailer sites and 133 customer nodes. This problem generates 1,804,313 decision variables and approximately $18,859,609$ constraints.

Table 5. Number of decision variables and constraints.

| Problems | Number of binary <br> decision variables (1) | Number of continuous <br> decision variables (2) | Total number of <br> decision variables (1+2) | Number of <br> Constraints |
| :--- | :--- | :--- | :--- | :--- |
| Zone 1 (Z1) | 31,965 | 114 | 32,079 | 75,285 |
| Zone 2 (Z2) | 5,134 | 65 | 5,199 | 5,914 |
| Zone 3 (Z3) | 27,919 | 120 | 28,039 | 58,216 |
| Zone $4(Z 4)$ | 66,174 | 150 | 66,324 | 196,863 |
| Zone 5 (Z5) | 27,919 | 120 | 28,039 | 61,639 |
| All Zones (ZA) | $1,802,573$ | 1,740 | $1,804,313$ | $18,859,609$ |

Note that: - The number of variables is generated from formulating mathematical model by using Eq. (1) - (27) with information of our case study as shown in Table 2.

- binary decision variables are including of: variable $x_{i j k}, y_{j e k}, h_{k}, w_{i j}, s_{j} Z_{i j}$ and $z_{j e}$.
- continuous decision variables are including of: variable $f_{i}, g_{i}$ and $r_{i j}$.

Due to past studies [ $6,21,22$ ], an exact method can solve only small and medium size problems of LRP. We obtained only the feasible solutions but cannot solve to optimality when we apply exact method to the Problems Z1-Z5 in our test. Specially for the largest Problem ZA, we cannot obtain any feasible solution in approximately four hours runtime limit. (Later, we show the results of exact method in Section 5.) Therefore, we develop new solution approach that the dominant part is a clustering technique to deal with larger problem as shown in Section 4.2.

### 4.2. Heuristic Approach

This study develops the heuristic approach that emphasizes on clustering-based algorithm. The proposed method consists four main phases as shown in Fig. 3. Phase 1, we decompose the original problem into two subproblems, as mentioned before, that the structure of Location Routing Problem (LRP) consists of Facility Location Allocation Problem (FLAP) and Multi-Depot Vehicle Routing Problem (MDVRP).


Fig. 3. Main proposed solution method for three-echelon multi-commodity LRP.
Phase 2, we perform cluster first - route second concept to establish transportation routes for the $3^{\text {rd }}$ echelon. After that, in Phase 3, the customer clusters and the demand of each clusters are brought into the modified FLAP as representative nodes to allocate a service center. In this phase, the customer nodes are grouped into clusters, hence we can identify the route and calculate distance cost from facilities to demand node in particular cluster easier by formulating Traveling Salesman Problem (TSP). TSP is problem to identify the shortest route that visits all nodes and return to origin node. The original TSP, which is applied in this research formulated by applying integer linear programming, is shown in Laporte [23] study. Moreover, TSP can be easily solved by exact methods to optimality due to the low number of members in particular cluster. Hence, to determine TSP route can reduce calculation time comparing to solving the master problem with MDVRP.

To solve three-echelon multi-commodity LRP, we construct the $3^{\text {rd }}$ echelon transportation routes first and identify the operating service centers that are assigned to serve the customer nodes by performing Phase 2 and Phase 3. It is important to note that the demands of clustered customers in layer 3 are added to particular retailer with a service center before constructing the $2^{\text {nd }}$ echelon transportation route. Hence, we update the demand of customers to its served service center. Then, we repeat the process of Phase 2 and Phase 3 to solve the $1^{\text {st }}$ and $2^{\text {nd }}$ echelon in order to construct retailer-cluster and assign a warehouse to particular cluster. After the entire steps are performed, we obtain routes and cost (both distance cost and fixed cost) from TSP within the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelon and combine all parts together to create a completed distribution network in Phase 4. Finally, we describe the detail of proposed algorithms of Phase 2 and 3 in Section 4.2.1 and 4.2.2 respectively.

### 4.2.1. Phase 2: Clustering-based approach

Due to tackle with the large Problem ZA that mentioned in Section 4.1, we develop solution method that dominate in terms of calculation time. Comparing to iterative method, hierarchical method, the algorithm is performed by allowing feed some information to previous phase to improve the results and run process again from beginning. Although both of them can provide better solution than clustering technique, the computation time can be longer, significantly. To prevent drawback the result from previous step as iterative/hierarchical method do, we propose sequential clustering-based approach to solve MDVRP in this study to reduce computation time. We develop this phase based on the past research of Lin and Kwok [9] and Kchaou Boujelben et al. [12]. There are two main algorithms that consist of initial-grouping algorithm and clustering algorithm as described in following parts.

- Phase 2.1: Initial-grouping algorithm

The main idea of initial group algorithm is to reduce the size of the search space by using a facility location as a reference point to construct an initial group as shown in Fig. 4. First, we open the initial set of
operating facilities to be dispersed over the entire distributed zone and being near a demand node as close as possible. The key parameter of this algorithm is the coverage distance, which defines the catchment area within which customers are allocated to individual warehouse. Hence, we determine suitable value by varying the distance from 100 to 300 km (with a step size of 50 km ) for the Problem Z1-Z5 and 100-600 for the Problem ZA. The coverage distance that returns the best solution will be selected. We present the initial group phase algorithm in following part.

Parameter: facility and demand nodes, facility capacity, demand quantity, coverage distance and distance between nodes

Step 1: Compute number of initial operating facilities, which is derived from the ratio of total demand to average facility capacity.

Step 2: Identify the member demand nodes of each facility, which distance from demand node to each facility is less than or equal to the coverage distance. Next, sort facilities in descending order of the average distance from each facility to its member demand nodes.

Step 3: Select a facility that provides minimum average distance and bring all member demand nodes to the next step.

Step 4: Update the average distance between the member demand nodes and facilities but excluding the demand nodes, which already grouped. Then repeat Steps 2 and 3 if a number of selected facility are equal to a number of initial operating facility.

Step 5: Swap each demand node to a nearest selected facility. This step is to confirm that every demand node is assigned to the nearest facility.

Step 6: If there are unassigned demand nodes which are located out of the coverage distance, assign them to the nearest selected facility.


Fig. 4. Phase 2.1: Initial-grouping algorithm.

- Phase 2.2: Clustering algorithm

The purpose of clustering algorithm is to establish the cluster of transportation route for both of retailers and customers from particular group that obtained from initial grouping phase. Due to our largescale problem, which cannot be solved by exact method, we develop this step by using technique that dominate in terms of computation time and can provide efficient solution. NNA is proper technique in terms of calculation time [24]. Nearest Neighbor Algorithms (NNA) is not only used to solve TSP but also be able to be applied to cluster the member of nodes [25]. Hence, we perform NNA in this step to group the demand nodes into initial cluster. The step starts from identifying the first node, then process will identify the nearest node and group it into the same cluster. The process runs until all demand nodes are clustered. However, NNA usually provided non-optimality solutions [24,25]. Hence, we also adapt exchange algorithm to enhance the cluster of transportation route in terms of shorter distance before feeding the results to next phase as shown in Fig. 5.


Fig. 5. Phase 2.2: Clustering algorithm.
We introduce the allowable demand quantity per cluster to prevent combining nodes with large size of demands that leads to inability to allocate the facilities in the phase of solving modified FLAP. The suitable value of allowable demand quantity is derived from the best solution of $30 \%, 40 \%$ and $50 \%$ of average facility capacity. The following section explains the steps of clustering demand nodes.

Parameter: initial groups from the $1^{\text {st }}$ phase, distance, the allowable number of drop point and the allowable demand per cluster

Step 1: Randomly choose a group to create a cluster.
Step 2: Pick up the farthest demand node from the selected facility location of chosen group. To assemble cluster from nodes at boundary first can prevent bias from grouping far nodes together [13].

Step 3: Perform NNA by selecting the nearest demand node next to the latest member node, then add this demand node into same cluster if:

- member of the cluster is less than allowable number of the drop point and
- total demand is not greater than the allowable demand quantity.

Then, the latest location of member is used as a reference in order to find the next nearest demand node.
Step 4: Repeat Steps 2 and 3 until there is no demand node left.
Step 5: Improve the quality of the transportation route by exchanging demand nodes between the different clusters that are adjacent. We define closeness of each route by group average proximity measure [13]. This process performs $(1,1)$ exchange move; swaps a demand node from one cluster to another cluster and (1,0) exchange move; removes a demand node from one cluster and insert to another cluster [26]. But this is applied only in the case of:

- member of the cluster is less than allowable number of the drop point,
- total demand is not greater than the allowable demand quantity,
- the total distance, solved by TSP starts from selected facility, is improved.

Step 6: Repeat Steps 1, 2 and 3 until all initial groups are solved.

### 4.2.2. Phase 3: Modified facility location allocation problem

Cluster representative nodes from previous phase will be reassigned to new facility by solving the modified FLAP. After decomposing problem into FLAP, we modify demand nodes to the cluster representative nodes and add a new constraint to impose any cluster node to be served by only one facility. Moreover, the cost of transportation, from facility to each cluster representative node, is calculated by constructing the TSP route. Therefore, the models to allocate the facility of each echelon are presented in the following parts:

## Index

$C$ set of clustered customer or retailer, indexed by $c$.
Parameters
$d_{c}$ : demand of product of clustered $c$ in the $2^{\text {nd }}$ echelon.
$I_{c}: \quad$ demand of service part of clustered $c$ in the $2^{\text {nd }}$ echelon.
$q_{c} \quad$ demand of service part of clustered $c$ in the $3^{\text {rd }}$ echelon.
$\tau_{i c}, \tau_{j c}$ fixed transportation cost and distance cost from warehouse $i /$ service center $j$ to cluster $c$ obtained by TSP route, respectively.
Binary decision variables
$z_{i c}, \quad=1$ if cluster $c$ is allocated to warehouse $i, 0$ otherwise.
$z_{j c}=1$ if cluster $c$ is allocated to service center on retailer $j, 0$ otherwise.

- Facility Location Allocation Problem (FLAP) for the $2^{\text {nd }}$ echelon

$$
\begin{equation*}
\operatorname{Min} Z_{2}=\sum_{i \in I} \alpha_{i} w_{i}+\sum_{i \in I_{1}} \gamma_{i}\left(1-w_{i}\right)+\sum_{i \in I} \varphi_{i} w_{i}+\sum_{i \in I} \sum_{c \in C}\left(\left(d_{c} \eta_{i}\right)+\left(l_{c} \lambda_{i}\right)+\tau_{i c}\right) z_{i c} \tag{28}
\end{equation*}
$$

Subject to

$$
\begin{array}{ll}
\sum_{i \in I} z_{i c}=1 & \forall c \in C \\
\sum_{\substack{ \\
z_{i c}}}\left(d_{c}+l_{c}\right) z_{i c} \leq \theta_{i} w_{i} & \forall i \in 1\}
\end{array}
$$

$$
\begin{equation*}
w_{i} \in\{0,1\} \tag{32}
\end{equation*}
$$

$$
\forall i \in I
$$

The objective function (Eq. (28)) minimizes the overall cost $\left(Z_{2}\right)$ consisting of the fixed operating costs of warehouses $\left(\sum_{i \in I} \alpha_{i} w_{i}\right)$, fixed closing costs of existing warehouses $\left(\sum_{i \in I_{1}} \gamma_{i}\left(1-w_{i}\right)\right)$, delivery cost from central depot to particular warehouse $\left(\sum_{i \in I} \varphi_{i} w_{i}\right)$, in the $1^{\text {st }}$ echelon, variable costs of operating the open warehouses and transportation cost from warehouse to cluster $\left(\sum_{i \in I} \sum_{c \in C}\left(\left(d_{c} \eta_{i}\right)+\left(l_{c} \lambda_{i}\right)+\tau_{i c}\right) z_{i c}\right)$, respectively.

Equation (29) impose that each retailer cluster is replenished from a single warehouse. Eq. (30) ensure that flow through the particular warehouse must be less than or equal to maximum capacity. Eq. (31) - (32) are the decision variables.

- Facility Location Allocation Problem (FLAP) for the $3^{\text {rd }}$ echelon

$$
\begin{equation*}
\operatorname{Min} Z_{3}=\sum_{j \in J} \beta_{j} s_{j}+\sum_{j \in J_{1}} \delta_{j}\left(1-s_{j}\right)+\sum_{j \in J} \sum_{c \in C}\left(\left(q_{c} \mu_{j}\right)+\tau_{j c}\right) z_{j c} \tag{33}
\end{equation*}
$$

Subject to

$$
\begin{array}{ll}
\sum_{j \in J} z_{j c}=1 & \forall c \in C \\
\left.\sum_{\substack{c \in C \\
z_{j c} \in\{0,1\} \\
q_{c} z_{j c} \leq \omega_{j} s_{j}}} \begin{array}{ll} 
& \forall j \in J \\
s_{j} \in\{0,1\} & \forall j \in J, \forall c \in C \\
\end{array}\right\} j \in J
\end{array}
$$

The objective function (Eq. (33)) minimizes the overall cost $\left(Z_{3}\right)$ consisting of fixed operating costs of service centers $\left(\sum_{j \in J} \beta_{j} s_{j}\right)$, fixed closing costs of existing service centers $\left(\sum_{j \in J_{1}} \delta_{j}\left(1-s_{j}\right)\right)$, variable costs of operating service centers and transportation cost to cluster $\left(\sum_{j \in J} \sum_{c \in C}\left(\left(q_{c} \mu_{j}+\tau_{j c}\right) z_{j c}\right)\right.$, respectively.

Equation (34) ensure that each customer cluster is replenished from a single service center. Equation (35) ensure that flow through the particular service center must be less than or equal to the capacity of service center. Eq. (36) - (37) are decision variables.

When the problems of Eq. (28) - (32) and Eq. (33) - (37) are solved, the overall cost $\left(Z_{1}\right)$ of distribution network in master problem can be calculated by summation values of $Z_{2}$ and $Z_{3}$.

## 5. Computational Study and Results

### 5.1. Case Study and Scenario

This work studies the redesign of distribution network of real-life five-zone case study, as mentioned in Section 3 and 4 , involving zone 1 (Z1), zone 2 (Z2), zone 3 (Z3), zone 4 (Z4) and zone 5 (Z5) and one special problem that involves all zones together (ZA). The detail and code of scenario are presented in Table 6. First, we test all problems (Problems Z1-ZA) with normal demand pattern that collected from our case study. For these set of problems, we code as P1, i.e., the Z1P1 refers to problem of zone 1 and deal with realistically based demand pattern.

Then, we perform sensitivity analysis for all problems by varying demand quantity, facility cost and transportation cost. These will prove whether or not the solutions of proposed LRP are robust. Due to the company planning, they set the target to expand their sale approximately $20 \%$ per year from the previous planning horizon. Hence, the sensitivities of demand quantities are $20 \%, 44 \%, 72 \%$ and $107 \%$ of based pattern for both of products and parts. For these set of problems, we code as P2-P5, respectively (P1 refers to base problem of Eq. (1) - (27)).

Next, we observe $25 \%, 50 \%$ and $75 \%$ sensitivities of facility costs as well as coefficient of: $\alpha_{i j} \beta_{j} ; \gamma_{i}, \delta_{j}$ $\eta_{i} \lambda_{i}$ and $\mu_{j}$ in each scenario, the same as sensitivity of transportation costs, which vary coefficient of: $\varphi_{i}, \sigma_{k,}$
$\tau_{i j}$ and $\tau_{j e}$. We define code as $a 25, a 50$ and $a 75$, respectively, for sensitivities of facility cost and $t 25, t 50$ and $t 75$ for sensitivities of transportation cost. Totally, we test 66 scenarios as shown in Table 7.

Table 6. Scenario coding and Sensitivity analysis detail on demand, facility cost and transportation cost.

| Sensitivity | Scenario (scenario code $=\%$ sensitivity) |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| Demand | $\mathrm{P} 1=0 \%$ | $\mathrm{P} 2=20 \%$ | $\mathrm{P} 3=44 \%$ | $\mathrm{P} 4=72 \%$ | $\mathrm{P} 5=107 \%$ |
| Facility cost | $\mathrm{a} 25=25 \%$ | $\mathrm{a} 50=50 \%$ | $\mathrm{a} 75=75 \%$ |  |  |
| Transportation Cost | $\mathrm{t} 25=25 \%$ | $\mathrm{t} 50=50 \%$ | $\mathrm{t} 75=75 \%$ |  |  |

Table 7. All test scenarios.

| Zones | Zone1 |  | Zone 2 |  | Zone 3 |  | Zone 4 |  | Zone 5 |  | All Zones |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| List of Scenarios | Z1P1 | Z1a25 | Z2P1 | Z2a25 | Z3P1 | Z3a25 | Z4P1 | Z4a25 | Z5P1 | Z5a25 | ZAP1 | ZAa25 |
|  | Z1P2 | Z1a50 | Z2P2 | Z2a50 | Z3P2 | Z3a50 | Z4P2 | Z4a50 | Z5P2 | Z5a50 | ZAP2 | ZAa50 |
|  | Z1P3 | Z1a75 | Z2P3 | Z2a75 | Z3P3 | Z3a75 | Z4P3 | Z4a75 | Z5P3 | Z5a75 | ZAP3 | ZAa75 |
|  | Z1P4 | Z1t25 | Z2P4 | Z2t25 | Z3P4 | Z3t25 | Z4P4 | Z4t25 | Z5P4 | Z5t25 | ZAP4 | ZAt25 |
|  | Z1P5 | Z1450 | Z2P5 | Z2t50 | Z3P5 | Z3t50 | Z4P5 | Z4t50 | Z5P5 | Z5t50 | ZAP5 | ZAt50 |
|  |  | Z1t75 |  | Z2t75 |  | Z3t75 |  | Z4t75 |  | Z5t75 |  | ZAt75 |

### 5.2. Experimental Results

To solve all scenarios, all solution approaches run on PC with Intel Core i7 3.9 GHz processor, with 16 GB of RAM and 400 GB of hard disk. IBM ILOG CPLEX 64-bit version 12.4 with C\# Concert technology is the commercial solver that we apply exact solution method as referent solutions for evaluating the qualities of heuristic approach (computation time and $\%$ gap of objective value). We code all algorithms on Microsoft Visual Studio 2015.

To solve these scenarios by CPLEX, this research determines CPLEX runtime limitation at 15,000 seconds ( 250 minutes) and relative $\%$ gap tolerance at $0.01 \%$. We run all node selected strategies and choose the best for particular scenario.

For proposed heuristic approach, we set the key parameter as coverage distance equal to 200 km with allowable demand at $30 \%$ for all Z1, Z2 and Z5 problems. While the best setup for the Z3 and the Z4 problem is the coverage distance equal to 250 km with allowable demand at $40 \%$. For the ZA problem, the coverage distance 300 km with allowable demand at $40 \%$ is the most suitable value. To solve the modified FLAP and TSP, CPLEX runtime is set a limitation at 500 seconds and relative $\%$ gap tolerance at $0.01 \%$.

After a proper tuning, we compare the results of CPLEX runtime that found best known solution with computation time of heuristic approach as shown in Fig. 6.

According to Fig. 6, X -axis refers to each scenario arranging in ascending order of their sizes of problem. Due to the high difference of computation time between CPLEX and proposed solution method, Y-axis is the computation time in logarithm base 10. For CPLEX, we record runtime when the best known solution is found.


Fig. 6. The comparison between computation time of CPLEX and proposed heuristic approach.
Solving the mathematical model of Eq. (1) - (27) using commercial solver, e.g., CPLEX, is suitable for small and medium size problem, e.g., Problem Z2. Especially the Problems Z2, CPLEX found the best known solutions at runtime 108 to 1,947 seconds as shown in Fig. 6 (the shortest runtime is 108.4 seconds for the Scenario Z2P1). A closer inspection of the results reveals that CPLEX spends a significant amount of time trying to close the gap. This is due in part to the weak LP relaxation bound of formulation model of Eq. (1) - (27). Hence, the gaps for Problems Z2 are around $18.1 \%-36.9 \%$ as shown in Fig. 7 .

For larger instances, CPLEX can only obtain feasible solutions when the solution time reaches time limit ( 250 minutes). The runtime of larger problems, until the best known solutions are found, is extremely greater (especially, the Problems Z4). For special largest Problem ZA, we cannot solve this problem by commercial solver. The operating system reported that there was out of storage memory ( 400 GB of hard disk) with no return of any feasible solution.

In contrast, proposed heuristic approach provides the feasible solutions for all scenarios. Computation times of heuristic approach vary from 10 to 185 seconds, which are extremely lower than CPLEX runtimes. Exclusively, the effectiveness of proposed approach indicates that it can solve the largest Problem ZA in computation time varies from 160 to 185 seconds. Later, we will show the quality of solution for Problem ZA that solved by proposed solution method in Section 5.4.

Next, we define the quality of CPLEX by evaluating $\% \mathrm{gap}$ of best known integer solution comparing to the best lower bound solution of Linear Programming (LP) relaxation. Moreover, we evaluate the quality of heuristic approach by calculating the \%gap between CPLEX result and heuristic result using Eq. (38)

$$
\begin{equation*}
\% \text { gap }=\frac{\left(z_{\text {Heuristic }}-z_{\text {CPLEX }}\right)}{z_{\text {CPLEX }}} \tag{38}
\end{equation*}
$$

The $Z_{\text {Hemristicit }}$ refers to objective value from proposed heuristic approach and the $Z_{\text {CPLEX }}$ refers to objective value from CPLEX solving.

The quality of CPLEX and heuristic approach are shown in Fig. 7. The result indicates that the Z2 solutions provide the best gap around $18 \%-39 \%$. Furthermore, when the size of problem is increasing, CPLEX provides the worst gap due to large number of binary variables and constraints, especially subtour elimination constraints.

However, most of the results from heuristic approach provide better quality than CPLEX that vary from $-4.84 \%$ (Z4P1) to $4.86 \%$ (Z3P4). For a small size problem (all Scenarios Z2), heuristic solutions provide the similar results and the total cost as CPLEX solutions. For a medium size problem like the Scenarios Z1, Z3, Z4 and Z5, heuristic also provides better solutions. According to the quality of proposed approach, clustering phase can reduce the number of binary decision variables in formulation modified FLAP (Eq. (28) - (37)). For example, there are 17 and 26 of demand nodes in the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelon in Scenario Z4, respectively. After establishing the transportation routes, the number of clusters of the $2^{\text {nd }}$ and the $3^{\text {rd }}$ echelon is reduced to $6-7$ routes and 13-15 routes, respectively. Moreover, performing the cluster first -route second concept, this let us can determine each route by TSP. Therefore, we can solve both of modified FLAP and TSP to the
optimality in every scenario with acceptable computation time. In summary, when the problems are larger in terms of binary decision variable and the number of constraint, heuristic can provide better solutions than CPLEX as shown in Fig. 7.


Fig. 7. CPLEX solution quality and \%gap CPLEX solution compares to heuristic solution.
*We did not show the proposed quality of Scenarios ZA because we cannot obtain any solution from CPLEX solving.
In contrast, CPLEX provides the better solution than heuristic method for the Scenarios Z3P3 and Z3P4. This is because the heuristic approach generates one more transportation route than CPLEX, which is the consequence of the parameter of allowable demand quantity. Although we try the higher value of allowable demand quantity to reduce the number of route, but we found that the increasing of allowable demand quantity leads to the higher number of open facilities and total cost.

For the Scenarios Z1P1, Z1a25, Z1a50 and Z1a75, the proposed heuristic approach returns lower quality of solution due in part to the number of operating service centers from heuristic is greater than the solutions from CPLEX. To observe these scenarios in CPLEX solutions, we found that the capacities of service centers are very tight, which compares to assigned demand quantity. Hence, the clustering transportation route first - location allocation second subsequence process of the proposed heuristic approach let the model open an excess service center (the combination of clustered routes cannot be served by the similar number of service centers that equal to CPLEX solution).

### 5.3. Solutions and Sensitivity Report

### 5.3.1. Solutions

In this section, we report the solutions of all Scenario P1 (base problem) in terms of the number of opening, retaining and closing facilities (both of warehouse and service center) as shown in Fig. 8. We obtain the results from solving Scenarios Z1P1-Z5P1 by the proposed heuristic approach as mentioned in Section 5.2. Note that the number of operating facilities in each zone must be the summation of number of open new facilities and retaining existing facilities. For example, the Scenario Z3P1, the solution suggests to open a new location site and to retain an existing warehouse. Totally, the number of operating warehouses is equal to two sites as shown in Fig. 8. Moreover, there are only a closure existing warehouse and two service centers for this problem.

To investigate solutions in Fig. 8, we can see that only the Scenarios Z2P1 and Z3P1 suggest to close existing warehouses. For the Scenario Z2P1, one of closing warehouse is company ownership site and this redesign of distribution network can consequently gain benefit from saving cost. However, a closure warehouse in the Scenario Z 3 P 1 is a rental site, therefore, there is no cost saved from closure in this zone. Moreover, to open new sites of warehouse can provide lower facility cost and lower average distance to all retailers (comparing to current distribution network). Last, the number of operating warehouses is equal to
number of existing warehouses in particular zone. Hence, we can note that there is no excess warehouse opened from heuristic solving for the Problems Z1-Z5.


Fig. 8. Number of open and closure if facilities in particular scenario of P1.
Due to Fig. 8, there are closing service centers in the Scenarios Z2P1 and Z3P1. Although all of them are rental locations that lead to additional cost of contract terminating and moving to new candidate site, the open new sites of service center can contribute significantly lower transportation cost. Moreover, there is a closure ownership service center exclusive in the Scenario Z4P1 that can earn benefit from closure facility in term of saving cost.

For the Scenarios Z3P1, Z4P1 and Z5P1, the new location sites of service center are selected from lower distance cost even though all of them provide not much different in cost of facility comparing to existing sites. For Scenario Z4P1, it is important to refer that we only require three sites to support the customers' demand. The solution suggests to operate three service centers, whereas there are four existing service centers in current distribution network (as-is model). This means that the redesign of distribution network can offer lower fixed opening cost of service center.

Note that there is no closure facility in the Scenario Z1P1. However, the solution advises to open one more excess service center to support customers' demand. This lead to additional cost of operating facility for this zone as mentioned in previous section. Due to the solutions of all Scenarios Z1-Z5 in Fig. 8, the number of operating sites is equal to the number of existing service centers in total.

Hence, we can conclude that the number of closure service centers is greater than the number of closure warehouses, based on the lower closing cost and larger number of alternative service centers, which located nearer to customer sites.

According to result of ZAP1, solving all zones simultaneously provides lower number of operating warehouses than solving separately. It suggests to operate nine warehouses in the ZAP1, while the total number of operating warehouses in solving each zone separately is ten sites. Because it allows distribution across zones, therefore excess capacity of warehouses can share properly and opening cost of warehouse is high. The solution of selected service centers is not different in number but it is different in location. Most of selected ones are new locations, which provide lower facility cost or transportation cost. Later, we present the discussion on cost of this special problem in Section 5.4.

### 5.3.1. Sensitivity analysis

In this section, we perform sensitivity analysis on demand, facility cost and transportation cost. Figs. 9-11 illustrate the results of each cost component separated by geographic zones. The interesting details are described as following part.

## Zone 1 (Z1)

The solutions have been changed when we perform sensitivity analysis on demand in the Scenarios Z1P3Z1P5 $(44 \%, 72 \%$ and $107 \%)$ as shown in Fig. 9. The models suggest to open three warehouses (two of them are the same locations to base problem). The four service centers, similar to the Scenario Z1P1, are selected and a new service center is opened more on the rental site of retailer in the Scenario Z1P3. In the Scenarios Z1P4 and Z1P5, two rental service centers are selected more compare to the Z1P1 solution. In summary, the reason to open new facilities is to support the demand expansion and the selected warehouses/service centers in base scenario are still selected on the Scenarios Z1P2-Z1P5.

About sensitivity of facility cost, it suggests not to open the company's ownership warehouse and select another site instead when saving cost rises higher to $75 \%$ of base scenario. This saving cost covers higher transportation cost, when compares to the base solution. Total cost of the Scenario Z1a75 saves $0.13 \%$ if the model applies the solution from the base problem to the Z1a75 run as shown in Fig. 10.

After the sensitivity analysis on transportation cost as shown in Fig. 11., the results indicate that nothing has changed in all runs ( $25 \%, 50 \%$ and $75 \%$ sensitivities).

## Zone 2 (Z2)

This zone requires two warehouses for supporting flow of products and service parts when we perform demand sensitivity in Scenarios Z2P2-Z2P5. Hence, each scenario suggests to retain a warehouse on similar location of the Scenario Z2P1 solution and open one more rental existing site (while still close one company ownership existing warehouse for a saving cost). It is reasonable results because both of them can provide the lowest facility cost and the lowest average distance to all retailers. The solutions of selected service centers have been changed when the demand quantity has risen over $72 \%$ of base demand in Scenarios Z2P4 and Z2P5. Due to demand expansion, Both of the Scenarios Z2P4 and Z2P5 are opened three service centers. Two of them are located in the similar location to the Problem Z2P1, another one is rental location.

No solution is changed in $25 \%, 50 \%$ and $75 \%$ on facility cost/transportation cost sensitivity as shown in Figs. 10-11.

## Zone 3 (Z3)

There are more open rental service centers when demand is expanded to $20 \%$ in the Scenario Z3P2 as shown in Fig. 9. In the Scenario Z3P4, the solution suggests to open one more warehouse and one more service center for the same reason. One of company ownership service center is still closed for the benefit of saving cost similar to the base problem.

The solutions of $25 \%$ and $50 \%$ of facility cost sensitivity have provided the similar distribution networks as base problem (Z3P1), as shown in Fig. 10. Except, the model relocates the service center to an existing rental one when the facility cost is higher than $75 \%$, despite of slightly higher transportation cost but total cost is still reduced $0.24 \%$ if use the same solution from the Z3P1.

In $75 \%$ sensitivity on transportation cost, the solution provides lower transportation cost and closing cost at 494,345 baht. This lower cost covers the higher facility cost of service center (increase cost by 406,871 baht) as shown in Fig. 11.

## Zone 4 (Z4)

According to Fig. 9, increasing the number of warehouses and service centers in the Scenarios Z4P2Z4P5 is to support the demand expansion. Moreover, we can conclude that the relocation of service center is easier than relocation of warehouse, due to the solutions from the Scenarios Z4P2-Z4P5 are divergent.

The solution has been changed when facility cost is increased more than $25 \%$ (Z4a25) from base problem. All of three existing service centers are now rented, despite of the higher total transportation cost and the total fixed opening cost. It is reasonable solution because we can receive more saving cost of closing the existing service center and the lower facility variable cost (overall, save cost $0.13 \%$ if use the same result from base problem) as shown in Fig. 10.

When the transportation cost increases over $50 \%$ ( Z 4 t 50 ) from base problem, the solution is changed. The provided solutions have reduced $0.89 \%$ and $0.12 \%$ (in $50 \%$ and $75 \%$ sensitivity analysis respectively) comparing to the base solution. But there is no change in warehouse solutions as shown in Fig. 11.


Fig. 9. Cost component of demand sensitivity separated by zone problem.


Fig. 10. Cost component of facility cost sensitivity separated by zone problem.


Fig. 11. Cost component of transportation cost sensitivity separated by zone problem.

## Zone 5 (Z5)

The number of open warehouses has been changed when demand sensitivity increases to $44 \%$ and $107 \%$ of base quantity (Z5P3, Z5P5). All existing warehouses are selected in all demand sensitivity. Hence, there is no closing cost of warehouse, as shown in Fig. 9. However, the set of operating service centers is sensible across the demand sensitivity in term of number and location to support increasing of demand.

We perform sensitivities at $25 \%, 50 \%$ and $75 \%$ on facility cost/transportation cost but nothing has changed in facility locations. However, a number of transportation routes have been changed in Scenarios Z5P4 and Z5P5 due to allowed demand quantity parameter in clustering phase. This prevent to open more excess facility.

## All Zones (ZA)

The solutions of ZAP1-ZAP5 show that increasing the number of operating facilities is to support demand expansion. Both of chosen warehouse and service center are different locations across scenarios. Moreover, we still benefit from closure existing warehouses, whereas slightly suffer from closure existing service centers across demand sensitivity. Because most closure service centers are the rental sites.

The existing warehouse is closed when the saving cost of closure the existing site is raise to $50 \%$ and $75 \%$ from the base problem, despite of the higher transportation cost. Overall, the provided solution reduces total cost $0.10 \%$ and $0.17 \%$ if use the similar result from base problem.

Solution suggests to reopen existing warehouses and service centers when the transportation cost is increased more than $50 \%$ from original setting, despite of higher facility cost. Moreover, we can receive more saving cost of closing existing warehouse and lower facility variable cost. In summary, the model can reduce cost $0.35 \%$ and $0.24 \%$ in $50 \%$ and $75 \%$.

### 5.4. Discussion

This section analyzes the results from solving each zone individually and all zones simultaneously. The results are shown in Table 8. Columns 2-3 present total costs from solving each zone individually for both of CPLEX
and proposed heuristic approach, respectively. Column 4 show total costs of solving all zones simultaneously by proposed heuristic approach. Finally, Columns 5-7, we present \%different total cost comparing with particular result that mentioned before.

The results of allowing the distribution across different zones (ZA) provide the best solutions, except Scenario ZAP5. Its cost is lower than solutions from solving each region independently for both of CPLEX and heuristic approach, averaged $3.33 \%$ and $1.96 \%$ respectively. Moreover, the summation of total cost from solving individual zone by heuristic also provides lower costs than CPLEX with an average lower cost of $1.39 \%$.

Table 8. Comparison on the solutions from solving each region independently and allowing the distribution across different zones.

| Scenario | Sum of total cost for all zones |  |  | \% different total cost |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Solving each zone individually by CPLEX <br> (1) | Solving each zone individually by heuristic (2) | Across different zones problem (ZA) by heuristic <br> (3) | $\frac{(2)-(1)}{(1)} \%$ | $\frac{(3)-(1)}{(1)} \%$ | $\frac{(3)-(2)}{(2)} \%$ |
| P1 | 56,612,155 | 55,640,866 | 54,391,788 | -1.72\% | -3.92\% | $-2.24 \%$ |
| P2 | 61,244,280 | 60,242,243 | 57,647,024 | -1.64\% | -5.87\% | -4.31\% |
| P3 | 68,358,552 | 67,261,172 | 66,592,059 | -1.61\% | -2.58\% | -0.99\% |
| P4 | 75,399,872 | 74,417,148 | 74,363,330 | -1.30\% | -1.43\% | -0.07\% |
| P5 | 82,881,784 | 82,481,008 | 82,506,539 | -0.48\% | -0.50\% | 0.03\% |
| a25 | 63,267,542 | 62,534,828 | 61,079,684 | -1.16\% | -3.46\% | -2.33\% |
| a50 | 70,406,804 | 69,392,015 | 67,700,273 | -1.44\% | -3.84\% | -2.44\% |
| a75 | 77,308,234 | 76,204,695 | 74,325,207 | -1.43\% | -3.86\% | -2.47\% |
| t25 | 63,389,865 | 62,674,654 | 61,301,839 | -1.13\% | -3.29\% | -2.19\% |
| t50 | 71,595,667 | 69,689,974 | 67,974,168 | -2.66\% | -5.06\% | -2.46\% |
| t75 | 77,100,369 | 76,552,064 | 74,939,002 | -0.71\% | -2.80\% | -2.11\% |
| Average |  |  |  | -1.39\% | -3.33\% | -1.96\% |

The solutions from the Problem ZA have fewer number of operating facilities. As the sharing of facilities across zones is allowed, the utilization of each facility is increased. Also, locations of operating facilities are moved to more proper locations. The lower cost of solving across different zones problem has two cases;

- With smaller number of operating facilities: The cost saved from fixed facility cost can compensate for the increasing of transportation distance cost. Therefore, the total cost is lower than the sum of individual zone.
- With the equal number of operating facilities: Although the number of operating facilities is the same as solving each zone individually in some scenarios, but the operating facilities are more properly assigned, especially for the boundary node. Therefore, the Problems ZA provide significantly lower cost of transportation than solving each zone individually.

However, allowing distribution across zone is against the company original policy. In order to get benefit from these results, the company needs to re-zone the distribution to comply with solutions, especially the demand nodes in the boundary.

Next, we compare the result of current distribution network (as-is) to base problems that solving by proposed solution method as shown in Table 9. Once again, allowing distribution across different zones provides the lowest total cost. This confirms by overall cost is lower than the current distribution network at $6.13 \%$. To focus on zone 1 in Table 9, this zone, which is located in the middle area and has boundary linked to other zones, has largely changed on distribution network for Problem ZA. A warehouse in zone 1 is assigned to distribute products to serve a retailer in zone 3 . Another warehouse and service center in zone 1 also distributes goods to retailers and customers in the boundary of zone 4 . Hence, the number of operating warehouses is lower than the current one because the current utilizations of warehouses in this zone are not density. Hence, the total demand of zone 1 is increased, whereas the total demand of zone 4 is reduced as shown in final column of Table 9 . The advantage of this situation is the responsibility of regional manager of these zones is more balanced.

Table 9. Comparison on the solutions to current distribution network.

| Zone | Cost (Baht) |  |  |  | Number of facilities |  |  | Number of routes |  | Demand (Units) |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | $(3) * * *$ | $(1)$ | $(2)$ | $(3)$ | $(1) * *$ | $(2)$ | $(3)$ | $(1) \&(2)$ | $(3)$ |
| 1 | $11,627,478$ | $11,528,717 *$ | $12,202,777$ | 5 | 6 | 4 | - | 16 | 18 | 68,408 | 98,028 |
| 2 | $7,584,569$ | $6,083,007$ | $6,273,042$ | 4 | 3 | 3 | - | 8 | 9 | 42,540 | 42,869 |
| 3 | $11,255,476$ | $10,688,392$ | $10,915,463$ | 4 | 4 | 5 | - | 16 | 16 | 59,473 | 53,551 |
| 4 | $16,401,261$ | $16,057,958$ | $14,419,073$ | 6 | 6 | 5 | - | 22 | 19 | 112,010 | 96,429 |
| 5 | $11,159,067$ | $11,021,509$ | $10,657,432$ | 5 | 4 | 5 | - | 15 | 14 | 71,472 | 63,026 |
| Summation | $58,027,851$ | $55,379,583$ | $54,467,788$ | 24 | 23 | 22 | - | 77 | 76 | 353,903 | 353,903 |

Note that (1) current distribution network, (2) result from individual zone and (3) result of across different zones solving.

* Solution from CLEX that provide better result than heuristic.
** We cannot collect the actual number of routes for current distribution network.
*** We separate zone of ZAP1 solution based on warehouse locations and their networks.
The result also indicates that the remaining capacity of facilities in zone 4 is assign to serve some retailers and customers in zone 5 . Most distributions in zone 2 and 3 are similar to the problem of solving each zone individually, because of their regions are located in isolated zones. Nonetheless, the purpose of increasing number of open facilities (service center) in zone 3 and 5 is to reduce distance cost. However, the solving simultaneously across different zones is still open less facility sites than the current one and solving each zone individually. In conclusion, most of the changing occur in the boundary area. Therefore, it is reasonable and easy to modify distribution networks in order to get lower cost.


Fig. 12. Different cost between the current distribution network and solutions from proposed LRP.
According to Fig. 12, we compare cost from solutions that derived from two types of solving (solving each zone individually and solving all zones simultaneously) with current distribution network. If the particular cost from proposed LRP is lower than the current one, the value will be minus. Fig. 12 reveals that most of different cost come from warehouse cost in Layer 1 of supply chain (both of fixed opening/closing cost and variable cost), especially, solution from solving all zones simultaneously. Furthermore, these results definitely confirm that the proposed solution method based on clustering technique can provide proper solution in reasonable computation time. Because all transportation cost of the $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ echelon is lower than the current one.

In contrast, the current locations of service center are not good enough to support the new customers that located on boundary of each distribution zone in recent year. The relocations of service center can help the planner to redesign efficient routes in daily-operation. The consequence of relocation service center leads to higher cost of facility than the current one. However, higher fixed and variable cost of service center can cover by lower cost of transportation in the $3^{\text {rd }}$ echelon as shown in Fig. 12. Finally, different cost in the $3^{\text {rd }}$ echelon transportation is lower than other echelons due to higher number of routes and visiting nodes. In deep investigation in the $1^{\text {st }}$ and the $2^{\text {nd }}$ echelon, the results reveal that number of operating warehouses from proposed model is lower than the current one. It seems that the distribution distance should be higher due
to coverage area of each facility. However, moving warehouses to proper locations can help to shorten the distance between facility and its served retailers/service centers (exclusively, zone 3 and zone 5), according to Fig. 12, the different costs in the $1^{\text {st }}$ and $2^{\text {nd }}$ echelon are minus.

## 6. Conclusion

This research studies the redesign of distribution network by developing mixed integer linear program of three-echelon multi-commodity of LRP. Due to real-life case study, we introduce the objective function that concerned closing cost of the closure of existing facilities. This work also proposes solution approach to cope with large-scale problem. This method decomposes the problem into MDVRP and modified FLAP and solve them sequentially. The clustering algorithm is effective in establishing the groups of transportation route. This helps to solve the modified FLAP optimality for all scenarios.

In summary, the research finding reveals that when the models formulated in node-arc formulation, the problem size growth exponentially due to subtour elimination constraints. The exact method can solve only small-sized and medium-sized problems, which conform to previous researches. However, the proposed solution method can tackle these class of LRP by achievement of computation time and total cost with comparable quality to the exact method. Especially, it can solve the largest problem that allows the distribution across different zones, which cannot be solved by the commercial solver (such as CPLEX).

The solutions of Problems Z1-Z5 are different from solving Problems Z1-Z5 independently. The retailers/service centers and customers, located on the boundary, are served by new allocated zone. This leads to lower cost compared to solving each zone separately. The consequences are decreasing of the number of demand nodes in some zone and improving the balance of allocating customer to each zone. Finally, redesigned solution can provide lower overall cost of distribution than the current distribution network can.

To verify the solutions in dynamic environment, some parameters, which are applied in the objective function and constraints, are performed sensitivity analysis on demand, facility cost and transportation cost. This study performs $20 \%, 44 \%, 72 \%$ and $107 \%$ on demand sensitivity, $25 \%, 50 \%$ and $75 \%$ on facility cost and transportation cost sensitivity. The results indicate that the proposed models still provide solid solutions across sensitivity analysis. There are some slightly changes in solutions of particular scenario. However, most selected location sites of warehouse in base problem are still selected across demand sensitivity analysis. The divergent solution occurs significantly only for service center locations due to lower moving cost and lots of candidate location sites.

The main contribution of this research in business aspect is the systematic design of distribution network that provide lower cost for real-life problem. Furthermore, the entire distribution network of all zones is redesigned in two distinctive ways in this study. First, the separately solving particular zone is performed. The benefit of this solving way is that models can provide the small and medium size problems, which are easier to solve. Another way is allowing the model to search proper locations of facilities across different zones and solve it simultaneously in one problem. The benefit of this solving way is that model can provide theoretically better solution than another one in terms of cost. This research can prove and provide the complete and quality solution from the second way solving, which can be implemented to the real-life case study. The conclusion helps business to realize that distribution zoning policy can obstruct the efficiency of distribution, which may lead to higher cost.

The one of the important issues to be developed is the model formulation. The ultimate obstacle of our node-arc formulation is a large number of subtour-elimination constraints. Therefore, future research should develop new formulation for this real-life case study and exact method to provide better solution. The heuristic method performs better if facility capacity is not tight. Proposed solution method should be developed in order to deal with problems with large-sized demand node. Finally, future research should improve the solution method into iterative/hierarchical solving, in order to re-route if number of opening facilities are greater than the minimum required to get optimality.

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