

# ENGINEERING JOURNAL

*Article*

## Overhaul Resource Planning for Rolling Stock using MIP Models

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**Abstract.** In term of maintenance, parts must be maintained to satisfy operating conditions. Although, maintenance is costly and unprofitable, it is indispensable. Thus, reducing maintenance costs without reducing maintenance is one of the critical issues. Since maintenance costs mainly come from resources, they should be properly managed to minimize the cost. Hence, the goal of this paper is to find the optimal number of resources for required maintenance activities. Two mixed-integer programming models are developed. The first model is used for a long-term plan to find a proper number of resources while the second one generates a maintenance schedule for a shorter time frame to verify feasibility of the plan.

**Keywords:** Rolling-stock, overhaul maintenance, scheduling, MIP.

**ENGINEERING JOURNAL** Volume 21 Issue 5

Received 4 December 2016

Accepted 13 March 2017

Published 29 September 2017

Online at <http://www.engj.org/>

DOI:10.4186/ej.2017.21.5.145

## 1. Introduction

One of the main public transportations is a rail system due to its ability to transport a large amount of entity, people and goods. For metropolitan zone, Mass Rapid Transport is widely implemented due to its speed, reliability and convenience. For instance, Bangkok has 4 major HRT (Heavy-Rail-Transit) with other 8 extended lines in the future [1-2] and Vancouver has 3 lines (Expo line, Millennium Line and Canada line). A rail system investment is, however, massive in aspects of land acquisition for rail line construction, operation of service, set-up of maintenance depot and cost of trains. Hence, to be in controlled of the cost, the service operator is required to obtain the highest efficiency from the system and also respond to the user demand [3].

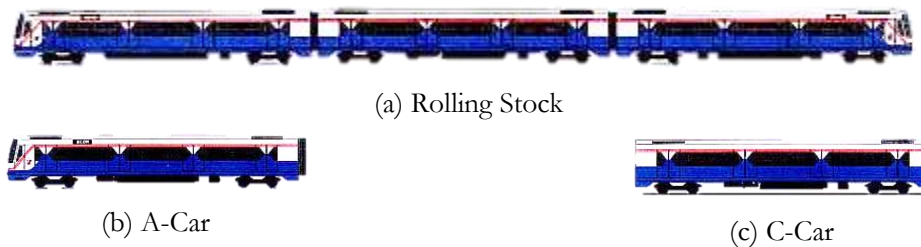


Fig. 1. Type of car in rolling stock [4].

A train or rolling stock, which is a structure of rail transport, consists of  $n$  vehicles or  $n$  cars that move through rail track in order to transport goods or passengers (Fig. 1). Maintenance activity depends on the type of car in each train [5]. Cars are divided into three types [5].

- A-Car is a cabinet with electric drive systems and a control system or cockpit.
- B-Car is a cabinet with electric drive systems but no a control system or cockpit.
- C-Car is a cabinet with no traction control or cockpit but has supply electricity for air conditioning and lighting systems inside the cabin.

A maintenance process is the procedure to repair or replace parts of a rail system to warrant that risk of accident is limited. Although the process is very costly, time consuming and non-profit, it is inevitably undertaken to certify safety of the service [6]. Reducing maintenance cost without reduction in quality is, therefore, the key for service operator to gain more profit. Fundamentally, there are 3 categories of maintenance including corrective maintenance, preventive maintenance and overhaul [7]. Corrective maintenance is employed in the case of accident or unplanned event. Preventive and Overhaul maintenance are time based maintenance or condition based maintenance. Fixed time or time based maintenance refers to maintenance that is carried out at fixed intervals. Condition based maintenance is a maintenance strategy that monitors the actual condition and should be maintained when indicators have signs of decreasing performance or impending failure. Preventive maintenance is scheduled as routine precaution. Overhaul is the scheduled major maintenance such as changing transmission fluid annually and a/c maintenance every  $n$  years. Normally, overhaul maintenance has interval time equal to or more than 1 year [1, 5].

Major costs occur in the maintenance process include labor, part and machine costs. These costs can be reduced by limiting the amount of resources and managing them efficiently. Machine cost is one of the major fixed costs that should be considered; especially machines that are used in overhaul activities. Since many trains from the same operator start their services at the same time, and hence, the overhaul schedules are always contiguous. This results in peak of maintenance demand for some periods, low occupancy in others and low machine utilization accordingly. Sharing overhaul machines among operators can be a way to average out the fixed cost of the machines.

This paper focuses on finding the optimal number of machines for overhaul activities. The number of machines depends on the scheduled maintenance tasks, the number of trains, and a method to schedule tasks. Since performing maintenance tasks too early or too late will increase costs [8]. All maintenance tasks for every train must be well scheduled to optimize the machines while keeping all maintenance tasks within acceptable time intervals. During maintenance, trains could not be normally operated. Therefore, another issue that should be considered while scheduling maintenance tasks is to minimize the number of

trains from the same line to perform maintenance at the same time. This objective provides the plan that reduces interruption of the normal service.

## 2. Literature Review

As a mass-transport-system depends heavily on its warrant of time-reliable and safety of transportation, an importance of an appropriate maintenance is recognized as prominent criteria to deliver those promises [3, 6]. A decent maintenance is to preserve trains to their good condition. This process comprises the arrangement of human-resource, material-resource, machine-resource, part-resource, database and working capital with regard to provide operation flexibility, train quality and operation safety which are key performance indicators of an operational profit [7]. Various maintenance tasks require diverse of methodologies and principal of services, thus it results in discrepancy of service time for various tasks. Different maintenance tasks operate on dissimilar service schedule as seen examples in Table 1

Table 1. Examples of train maintenance tasks [7].

Modules	Sub-modules	Tasks	Intervals	Car Types	Service Time (hrs/train)
<b>Bogie</b>	Gear Box	Gear lubricant change	Every 1 Year	A-Car	32
<b>Bogie</b>	Wheel set	Wheel change and Check the cracks of axle	Every 5 Years	A-Car & C-Car	42
<b>Bogie</b>	Bogie Frame	Maintenance bogie frame	Every 12 Years	A-Car & C-Car	84
<b>Air condition</b>	Air condition	Maintenance air condition	Every 6 Years	A-Car & C-Car	18
<b>Battery</b>	Battery	Maintenance battery	Every 6 Months	C-Car	4
<b>Air compressor</b>	Air compressor	Maintenance air compressor	Every 1 Year	A-Car & C-Car	3

This paper focuses on reducing maintenance cost by determining an obligatory number of maintenance machines. The optimal number of maintenance machines depends deeply on the maintenance requirements and the schedule of those.

Scheduling is defined as an approach to allocate limited resource for the undergo operations within finite timeframe with the purpose of achieving organization target [9, 10]. For the realms of this paper, the spotlight resources are maintenance machines. There are many studies about scheduling the preventive maintenance. Since the preventive maintenance is one of the importance jobs, it becomes one of the jobs for scheduling into the machines. Gopalakrishnana et al. [11] scheduled preventive maintenance using a tabu search. It aimed to maximize the total priority of maintenance tasks which were assigned to machines under the condition that the assigned tasks must not exceed the machine availability. Go et al. [12] scheduled preventive maintenance of container ships. Scheduling preventive maintenance in transportation business, such as railway and container ship is different from other business since the available time slots for maintenance tasks are limited because they absolutely related to travel schedule of the vehicles. Worrall et al. [13] classified preventive maintenance into emergency and non-emergency tasks. Their maintenance scheduling is arranged according to 1) priority of the task, calculated based upon the remaining time, allocated weight and importance; 2) the number of on-hand tasks; and 3) the number of anticipated tasks. Giacco et al. [14] scheduled maintenance of rolling stocks under service constraints. All maintenance tasks must be done within the lower and upper bounds of maintenance schedule and during the time that trains do not provide services.

Since maintenance tasks use several machines simultaneously, scheduling those tasks must consider availability of the machines. Many scheduling papers [15-20] have similar characteristics as this paper. However, their applications are different. Cheng et al. [15], Wang et al. [16], and Qin et al. [17] scheduled films in order to minimize waiting time of actors. Actors who involve in any pieces of film must be

available when those pieces are shot. However each paper used different techniques to get the optimal solution. Smith et al. [18], Sakulsom et al. [19-20] scheduled music rehearsal in order to minimize waiting time of musicians. Music pieces can be rehearsed only when the musician who play those pieces are available. This paper schedules maintenance tasks, tasks can be maintained only when the required machines are available. However, this paper differs from other scheduling papers where machines can be added when needed. Therefore, number of machines are not the same for all periods during the planning horizon.

To solve the problem, a mathematical model is one of the tools used to find solutions that are the best under defined conditions. In this paper, a mathematical model is employed to determine the minimum number of machines to serve the set of maintenance tasks regarding to conditions and constrains including time of service and allocation of service resources. However, this paper is a long-term plan for resource requirement. The planning horizon includes many periods; therefore, there are many decision variables involved which is difficult to be solved. Many papers proposed methods to simplify the problem. Budai et al. [6] grouped tasks to reduce the number of decision variables. While other papers [21-22] reduced the problem size by dividing it into sub problems and sequentially solve them.

### 3. Problem Description

Overhaul maintenance for rolling stocks is defined as a preventive maintenance that maintenance interval is more than one year. Normally, overhaul maintenance requires expensive machines in operation. This paper interests in scheduling overhaul maintenance to minimize the number of machines needed to satisfy all overhaul activities. Rolling stock maintenance primarily focuses on security, and hence, the maintenance is required to be on time before impending failure. Since many rolling stocks start their services during the same time, the overhaul maintenance will be needed approximately the same time also. However, practically, two main reasons for not being able to schedule overhaul maintenance of many rolling stocks at the same time are: 1) the number of rolling stocks must be enough to provide services; 2) It is too expensive to invest in overhaul machines with low utilization. Therefore, instead of setting maintenance tasks exactly on time, tasks must be satisfied within a time interval. For instance, a task that is scheduled in month 12 is allowed to do during months 12-15.

Scheduling maintenance and projecting a number of required machines rely on 2 major criteria which are the number of rolling stocks and their compulsory maintenance tasks. Table 2 shows examples of maintenance activities for each car. Table 3 shows examples of train information.

In Table 2, there are 13 tasks which require 10 machines. Since, a train normally consists of n cars and maintenance activity depends on the types of cars in each train, each train will have difference total service time. For example, if a train consists of 3 cars: 2 A-Car and 1 C-Car, for Task 02, it takes 24 hours to service by using machines 1, 2, 4, 5, 7 and 9.

Table 2. Details of maintenance tasks.

Tasks	Intervals	Car Types	Service Time (hour/car)	Machines
<b>Task 01</b>	1 Year	A-Car	8	2, 5, 7, 9
<b>Task 02</b>	3 Years	A-Car & C-Car	8	1, 2, 4, 5, 7, 9
<b>Task 03</b>	5 Years	A-Car	10	3, 4, 5, 6, 9
<b>Task 04</b>	5 Years	A-Car	14	2, 5, 7
<b>Task 05</b>	1 Year	C-Car	16	2, 5, 7, 9, 10
<b>Task 06</b>	5 Years	A-Car & C-Car	8	3, 5, 6, 8, 9
<b>Task 07</b>	6 Years	A-Car	40	2, 3, 4, 5, 6, 7, 9
<b>Task 08</b>	3 Years	C-Car	32	1, 2, 4, 5, 7, 9
<b>Task 09</b>	5 Years	A-Car & C-Car	40 (per train)	2, 4, 5, 7, 9
<b>Task 10</b>	5 Years	A-Car & C-Car	40 (per train)	2, 7, 9
<b>Task 11</b>	6 Years	A-Car	16	2, 3, 4, 5, 6, 7, 9
<b>Task 12</b>	5 Years	C-Car	20 (per train)	5
<b>Task 13</b>	6 Years	A-Car & C-Car	32 (per train)	5, 8

In Table 3, the occasion of maintenance operation can be determined from the age of the train, which is counted from its first service year and month. For instance, Train ID 01 from Operator 01 needs task 01 every year in month 8.

Table 3. Trains of each operator.

Operator	# Train	Train ID	Year Start	Month Start
Operator 01	8	01, 02, ..., 08	2016	8
	12	09, 10, ..., 20	2018	9
	15	21, 22, ..., 35	2029	10
Operator 02	20	01, 02, ..., 20	2010	2
	15	21, 22, ..., 35	2013	5
Operator 03	20	01, 02, ..., 20	2017	11
	16	21, 22, ..., 36	2020	12
	4	37, 38, ..., 40	2022	9
Operator 04	10	01, 02, ..., 10	2012	4
	10	11, 12, ..., 20	2020	4
	15	21, 22, ..., 35	2025	3
Operator 05	20	01, 02, ..., 20	2015	6
	15	21, 22, ..., 35	2017	1
	14	37, 38, ..., 49	2024	7
	10	50, 51, ..., 59	2025	8

To optimize the number of machines required for the maintenance tasks, we formulate a model to simulate the situation with assumptions as follows:

1. Every car in each train must be undertaking maintenance together.
2. The maintenance service depends only on age of train.
3. All tasks must be completed in a specified time.
4. During the same period of time, we will try to minimize the number of train from the same operator.
5. Service time of each task is fixed and equal for all cars.

#### 4. Machine Estimation and Overhaul Maintenance Scheduling

This section describes the details of 2 mathematical models. First, the “Machine Estimation” model is used to determine the number of machines required for performing the maintenance tasks. The solutions from this model are: 1) the number of machines of each type in each month and 2) the monthly maintenance plan. Then, the solutions from the first model are used as inputs for the “Overhaul Maintenance Scheduling” model. This model is used to schedule maintenance tasks during a month and verify whether the number of machines from the first model is enough.

##### 4.1. A Mathematical Model for Machine Estimation

The main purpose of this model is to find the minimum number of maintenance machines. Since trains under maintenance cannot be used to provide services, maintenance plan must consider continuity of the service. Therefore, the second objective of this model is to minimize the maximum number of trains from the same operator that are maintained during the same time. Details of the model is following.

##### Notations and Parameters

I	Set of maintenance task groups, $i = \{1, 2, 3, \dots, n\}$
T	Set of trains, $t = \{1, 2, 3, \dots, o_t\}$

- O Set of train operators,  $o = \{1, 2, 3, \dots, p\}$
- M Set of machine types,  $m = \{1, 2, 3, \dots, q\}$
- H Set of periods,  $h = \{1, 2, 3, \dots, r\}$
- $D_{im}$  Service time for task group  $i$  of machine  $m$  (month)
- $Task_{ioth}$  1 if train  $t$  of operator  $o$  requires maintenance task  $i$  in period  $h$ , 0 otherwise
- $S_i$  Allowable maintenance period of task  $i$
- $machine_{m0}$  The beginning number of machine type  $m$ ; equal to 0 for every  $m$
- M big M

Decision Variables

- $machine_{mh}$  Number of machine type  $m$  in period  $h$
- $Plan_{ioth}$  1 if train  $t$  of operator  $o$  assigned maintenance task  $i$  in period  $h$ , 0 otherwise
- $Num\_Train_{oth}$  1 if train  $t$  of operator  $o$  maintained during period  $h$ , 0 otherwise
- $MaxTrain$  Maximum number of trains from the same operator maintained during the same period.

Mathematic Model

$$minimize z = (M \times \sum_{m \in M} \sum_{h \in H} machine_{mh}) + MaxTrain \tag{0}$$

st.

$$machine_{m(h-1)} \leq machine_{mh} \quad \forall m \in M, \forall h \in H \tag{1}$$

$$Task_{ioth} = \sum_h^{h-1+S_i} Plan_{ioth} \quad \forall i \in I, \forall k \in K, \forall j \in J, \forall t \in T \tag{2}$$

$$\sum_{i \in I} \sum_{t \in T} \sum_{o \in O} Plan_{ioth} \times D_{im} \leq machine_{mh} \quad \forall m \in M, \forall h \in H \tag{3}$$

$$\sum_{i \in I} Plan_{ioth} \leq M \times Num\_Train_{oth} \quad \forall o \in O, \forall t \in T, \forall h \in H \tag{4}$$

$$\sum_{t \in T} Num\_Train_{oth} \leq MaxTrain \quad \forall o \in O, \forall h \in H \tag{5}$$

$$Plan_{ioth} \in \{0,1\} \quad \forall i \in I, \forall o \in O, \forall t \in T, \forall h \in H \tag{6}$$

$$machine_{mh} \geq 0 \quad \forall m \in M, \forall h \in H \tag{7}$$

$$Num\_Train_{oth} \in \{0,1\} \quad \forall o \in O, \forall t \in T, \forall h \in H \tag{8}$$

$$MaxTrain \geq 0 \tag{9}$$

It is a multi-objective function (0). The first priority is to minimize the number of machines for overhaul maintenance, and the other is to minimize the number of trains from the same operator that are maintained during the same period. Constraints (1) specify that the number of machines is non-decreasing. Each task is allowed to be maintain within a given time period. For example, task  $i$  that is scheduled to maintain in period  $h$  is accepted to be maintained in period  $h, h+1, \dots, h+S_i-1$ . Constraints (2) choose one of allowable periods to assign tasks. Constraints (3) calculate the number of machines required which depends on workload in each period. Trains under maintenance cannot be normally operated, therefore, constraints (4) and (5) find the maximum number of train from the same operator that are maintained during the same period. Constraints (6) to (9) specify values of decision variables.

During maintenance, trains could not provide services. Scheduling maintenance by grouping tasks having the same interval is importance to reduce time to enter and exit a depot. Tasks in each group must be done during the same visit. For example, Table 4 groups 13 tasks in Table 2 into 4 task groups. There are 6 tasks that have 5-year interval. If tasks are not grouped, a train has to enter and leave a depot 6 times during the same period. Once we group those tasks, all these 6 tasks will be done during the same visit so entering and leaving will be reduced to only one. Therefore, model 1 assigns task groups instead of individual task.

Table 4. Groups of tasks.

Intervals	Tasks					
<b>1 Year</b>	Task 01	Task 05	-	-	-	-
<b>3 Years</b>	Task 02	Task 08	-	-	-	-
<b>5 Years</b>	Task 03	Task 04	Task 06	Task 09	Task 10	Task 12
<b>6 Years</b>	Task 07	Task 11	Task 13	-	-	-

Table 2 provides details of maintenance task groups including service time of each machine. Since every car in a train must be serviced during the same visit. In this paper, we assume that every train consists of 2 A-Car and 1 C-Car. Therefore, service times spent on a train ( $D_{im}$ ) are shown in Table 5. For example, to maintain every-year task, a train takes  $8 \times 2$  hours for Task 01, 16 hours for Task 05 using machines 2. Total service time of machine 2 for every year task becomes 32 hours.

Table 5. Service time to maintain a train ( $D_{im}$ ).

Tasks	Service Times of Machines (hours/train)									
	1	2	3	4	5	6	7	8	9	10
Every year		32			32		32		32	16
Every 3 years	56	56		56	56		56		56	
Every 5 years		108	44	60	132	44	108	24	124	
Every 6 years		112	112	112	144	112	112	32	112	

Parameters  $Task_{ioth}$  are based upon the ages of trains from Table 3 and maintenance requirements in Table 4. However, it is not possible to maintain every train exactly on the specified interval. Allowable maintenance period ( $S_i$ ) is specified as a duration of time that we allow a train to be maintained. For example, a train that is scheduled to have maintenance in month 13 if  $S=3$ , it can be maintained during months 13-15.

Since this model is a long-term plan for resource requirement. It is inevitable to have a large problem size due to the number of time periods. The model cannot be solved at once. Therefore, we separate the model into sub problems. Each sub problem has shorter time period. Then, each of them is solved sequentially. The optimal solution from the previous sub problem becomes input of the next sub problem. For example, the first sub problem starts from the “Beginning Period” (BP) and ends at the “End of Period” (EP); time periods include  $\{BP, BP+1, \dots, EP\}$  as seen in Fig. 2

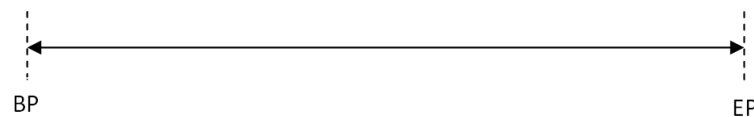


Fig. 2. Time horizon of the first sub problem.

After solving the first sub problem, maintenance plan during the first  $n$  periods is firmed and becomes inputs of the second sub problem. We develop a rolling plan every  $n$  periods. Therefore, the next sub problem starts from  $(BP+n)$  and ends at  $(EP+n)$ .



Fig. 3. Time horizon of the second sub problem.

If the model is solved at once, there are 480 periods, which is not possible to get the optimal solution. Using this method, each sub problem includes 60 periods and rolls every 12 periods. Therefore, there are 40 sub problems to be solved. After running all sub problems, we get the minimum number of machines in each period and also know monthly maintenance plan. These become input of the second mathematical model.

## 4.2. A Mathematical Model for Overhaul Maintenance Scheduling

From the first model, the minimum number of machines in each month are determined to satisfy the total maintenance time. However, when tasks are sequenced, some machines are required in many tasks. Tasks which share the same machine cannot perform at the same time. Therefore, the number of machines from model 1 may not be enough to perform all assigned tasks during a given time period. For example, assume that there are 7 maintenance tasks done within 20 hours.

- Task 1 uses machine 1 and 3 with 3 hours maintenance.
- Task 2 uses machine 1 and 4 with 2 hours maintenance.
- Task 3 uses machine 1 and 2 with 3 hours maintenance.
- Task 4 uses machine 1, 3 and 4 with 4 hours maintenance.
- Task 5 uses machine 2, 3 and 4 with 6 hours maintenance.
- Task 6 uses machine 1, 3 and 4 with 3 hours maintenance.
- Task 7 uses machine 1 and 3 with 4 hours maintenance.

It can be seen that machines 1, 2, 3, and 4 spend 19, 9, 19, and 15 hours for maintenance respectively. Since all machines spend less than 20 hours, model 1 shows that all maintenance tasks can be done by using only one machine. However, when tasks are scheduled as shown in Fig. 4. It takes at least 25 hours to complete all tasks or additional machines are needed in order to complete all tasks within 20 hours.

Machine	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
#1	Task 1		Task 2		Task 3		Task 4						Task 5			Task 6		Task 7							
#2				Task 3						Task 4			Task 5												
#3	Task 1								Task 4			Task 5			Task 6		Task 7								
#4				Task 2					Task 4			Task 5			Task 6										

Fig. 4. Gantt chart for the monthly scheduling.

Therefore, the second model is formulated to verify machine capacity by scheduling all tasks in each month. Maintenance tasks that are assigned to each train during a month and the number of machines from model 1 become inputs of model 2. This model aims to sequence maintenance tasks within each task group. The sequencing process works to minimize time that trains stay in a depot. For instance, Task01 and Task05 have the same interval so they are in the same task group in model 1. These two tasks are done during the same visit. Therefore, we try to arrange maintenance tasks to minimize gap between these tasks in order to reduce time that a train must spend in a depot. With this objective, operators have more trains to provide services and can reduce the number of required trains.

A planning horizon from the first model is 40 years. The second model is run with a shorter horizon of 1 month. Therefore, the outputs from the first model becomes inputs of 480 problems of model 2. Each time slot in the second model equals to 4 hours. Therefore, there are 180 time slots or periods in each model.

Details of the second model is following.

### Notations and Parameters

- S Set of periods,  $s = \{1, 2, \dots, u\}$
- J Set of maintenance tasks,  $j = \{1, 2, \dots, v\}$
- O Set of train operators,  $o = \{1, 2, \dots, p\}$
- T Set of trains,  $t = \{1, 2, \dots, o_t\}$
- M Set of machines,  $m = \{1, 2, \dots, q\}$
- $d_j$  Service time for maintenance task  $j$
- $\text{Plan}_{jot}$  1 if train  $t$  of operator  $o$  requires maintenance task  $j$ , 0 otherwise
- $\text{Num\_MC}_m$  Number of machine type  $m$
- $\text{TaskMC}_{jm}$  1 if maintenance task  $j$  requires machine  $m$ , 0 otherwise

### Decision Variables

- $\text{timetable}_{jots}$  1 if train  $t$  of operator  $o$  repairs task  $j$  in period  $s$ , 0 otherwise
- $\text{timestart}_{jots}$  1 from period  $s$  that train  $t$  of operator  $o$  starts task  $j$ , 0 otherwise 1



$timeend_{jots}$	0 from period $s$ that train $t$ of operator $o$ finishes task $j$ , 1 otherwise
$MCassign_{jotm}$	1 if train $t$ of operator $o$ uses machine $m$ for task $j$ , 0 otherwise
$MCuse_{jotms}$	1 if train $t$ of operator $o$ uses machine $m$ for task $j$ in period $s$ , 0 otherwise
$timetable\_train_{ots}$	1 if train $t$ of operator $o$ stays in depot in period $s$ , 0 otherwise
$timestart\_train_{ots}$	1 from period $s$ that train $t$ of operator $o$ enters depot, 0 otherwise
$timeend\_train_{ots}$	0 from period $s$ that train $t$ of operator $o$ leaves depot, 1 otherwise
$Max\_OperateTime$	the longest time that a train stays in depot
$OperateTime_{ot}$	time that train $t$ of operator $o$ stays in depot

### Mathematic Model

The purpose of this model is to schedule maintenance tasks to minimize the maximum time that each train stays in a depot and also minimize the total time that all trains are in the depot as shown in objective function (10).

$$z = M \times Max\_OperateTime + \sum_{o \in O} \sum_{t \in T} OperateTime_{ot} \quad (10)$$

Constraints (11) specify that if train  $t$  of operator  $o$  is scheduled for maintenance, the total time to repair task  $j$  is equal to the service time of that task.

$$\sum_{s \in S} timetable_{jots} = d_j \times Plan_{jot}, \quad \forall j \in J, \forall o \in O, \forall t \in T \quad (11)$$

Constraints (12) forces that in each period, a train must perform only one task.

$$\sum_{j \in J} timetable_{jots} \leq 1, \quad \forall s \in S, \forall o \in O, \forall t \in T \quad (12)$$

Constraints (13) - (17) specify that each task must be continuously performed. In period  $s$ , if both  $timestart_{jots}$  and  $timeend_{jots}$  are equal to 1,  $timetable_{jots}$  is equal to 1. However, if either  $timestart_{jots}$  or  $timeend_{jots}$  is equal to 0,  $timetable_{jots}$  is equal to 0. Furthermore,  $timetable_{jots}$  equal 1 means train  $t$  of operator  $o$  performs task  $j$  in period  $s$ . As in Table 6, a task performs on periods 7 to 13. Constraints 16 and 17 force tasks to be done continuously.

Table 6. Example for  $timestart_{jots}$ ,  $timeend_{jots}$  and  $timetable_{jots}$ .

S	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$timestart_{jots}$	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$timeend_{jots}$	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
$timetable_{jots}$	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0

$$timestart_{jots} \leq timestart_{jot(s+1)} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S - \{end\} \quad (13)$$

$$timeend_{jots} \geq timeend_{jot(s+1)} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S - \{end\} \quad (14)$$

$$timetable_{jots} \geq timestart_{jots} + timeend_{jots} - 1 \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (15)$$

$$timestart_{jots} \geq timetable_{jots} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (16)$$

$$timeend_{jots} \geq timetable_{jots} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (17)$$

Constraints (18) assign machines to tasks.

$$MCassign_{jotm} = TaskMC_{jm} \times Plan_{jot} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall m \in M \quad (18)$$

Constraints (19) - (20) represent limitation of the machines. In each period, occupied machines must not exceed the available number of machines. If there is a maintenance schedule, the machines must be reserved.

$$MCuse_{jotms} \geq MCassign_{jotm} + timetable_{jots} - 1 \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S, \forall m \in M \quad (19)$$

$$\sum_{j \in J} \sum_{o \in O} \sum_{t \in T} MCuse_{jotms} \leq Num\_MC_m \quad \forall s \in S, \forall m \in M \quad (20)$$

In constraints (21) specify that when a train is repaired, it must be in a depot.

$$timetable\_train_{ots} \geq \sum_{j \in J} timetable_{jots} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (21)$$

Constraints (22) – (26) are similar to constraints (13) - (17). They are used to determine the time that a train stays in a depot including service time and waiting time. When the train enters depot,  $timestart\_train_{jots}$  is equal to 1 and after the train leaves depot,  $timeend\_train_{jots}$  is equal to 0. Hence, if both  $timestart\_train_{jots}$  and  $timeend\_train_{jots}$  are equal to 1,  $timetable\_train_{jots}$  is equal to 1.

$$timestart\_train_{ots} \leq timestart\_train_{ot(s+1)} \quad \forall o \in O, \forall t \in T, \forall s \in S - \{end\} \quad (22)$$

$$timeend\_train_{ots} \geq timeend\_train_{ot(s+1)} \quad \forall o \in O, \forall t \in T, \forall s \in S - \{end\} \quad (23)$$

$$timetable\_train_{ots} \geq timestart\_train_{ots} + timeend\_train_{ots} - 1 \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (24)$$

$$timestart\_train_{ots} \geq timetable\_train_{ots} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (25)$$

$$timeend\_train_{ots} \geq timetable\_train_{ots} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (26)$$

Constraints (27) calculate time that a train stays in a depot

$$OperateTime_{ot} \geq \sum_{s \in S} timetable\_train_{ots} \quad \forall o \in O, \forall t \in T \quad (27)$$

Constraints (28) find the maximum time that each train stays in depot.

$$Max\_OperateTime \geq OperateTime_{ot} \quad \forall o \in O, \forall t \in T \quad (28)$$

Constraints (29) – (38) specify values of all decision variables.

$$timetable_{jots} \in \{0,1\} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (29)$$

$$timestart_{jots} \in \{0,1\} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (30)$$

$$timeend_{jots} \in \{0,1\} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall s \in S \quad (31)$$

$$timetable\_train_{ots} \in \{0,1\} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (32)$$

$$timestart\_train_{ots} \in \{0,1\} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (33)$$

$$timeend\_train_{ots} \in \{0,1\} \quad \forall o \in O, \forall t \in T, \forall s \in S \quad (34)$$

$$MCassign_{jotm} \in \{0,1\} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall m \in M, \forall s \in S \quad (35)$$

$$MCuse_{jotms} \in \{0,1\} \quad \forall j \in J, \forall o \in O, \forall t \in T, \forall m \in M, \forall s \in S \quad (36)$$

$$OperateTime_{ot} \geq 0 \quad \forall o \in O, \forall t \in T \quad (37)$$

$$Max\_OperateTime \geq 0 \quad (38)$$

Since inputs of this model come from model 1. If model 1 allows machines to be fully utilized, it may result in infeasible solution in model 2. Therefore, we experiment to find the maximum percentage of machine utilization in model 1 in order to get feasible solution in model 2 by changing constraint (3) of the first model to be (3-2).

$$\sum_{i \in I} \sum_{o \in O} \sum_{t \in T} Plan_{ioth} \times D_{im} \leq Limit \times MC_{mh} \quad \forall m \in M, \forall h \in H \quad (3-2)$$

Then, we run models 1 and 2 sequentially to find the maximum value of “Limit”. The analysis shows that 80% of the machine utilization is the most appropriate value.

## 5. Results

This paper uses an example of 5 operators, 204 trains, 10 machines, 4 task groups and a planning horizon of 40 years. Data used in the example is based on a railway system in Thailand. Two MIP models are used to quantify the minimum number of machines in each period and also assign maintenance tasks to different time periods. Fig. 5 shows the number of machines required in each period with 80% utilization.

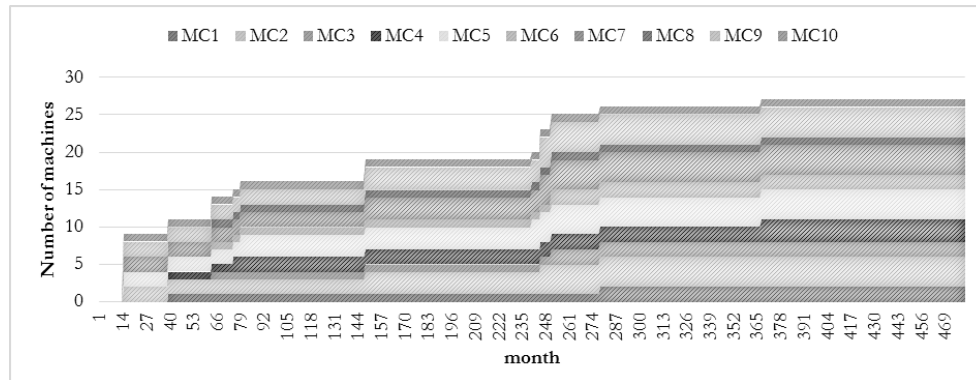


Fig. 5. Number of machines each period when allowable maintenance period is equal to 1/12 of interval.

We perform 3 main analysis as follows:

### 5.1. Allowable Maintenance Periods

As we mentioned in the previous section that sometimes it is not possible to provide maintenance exactly on time. In model 1 we use “ $S_i$ ” as an allowable maintenance period for task  $i$ . For example, if maintenance interval is 1 year and  $S_i$  equal 3 months. Mean that, maintenance can be done any time during months 13-15. For this section, we perform analysis of varying allowable ranges.

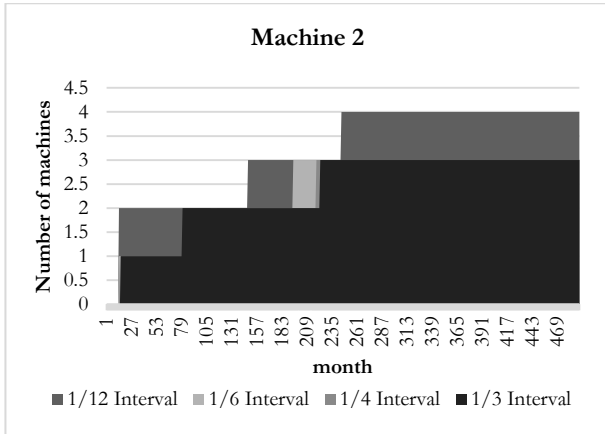
When changing the allowable maintenance periods to 1/12, 1/6, 1/4 and 1/3 of maintenance intervals. Examples of the number of machines (1, 2, 4 and 10) are shown in Fig. 6. In Figs. 6(a) and 6(b), it can be clearly seen that as the allowable maintenance periods increase, the number of required machines decrease. In Fig. 6(c), it is noticed that if the allowable maintenance period is expanded to a certain point, the number of machines might be not different (1/6 and 1/4 of intervals). However, if the machine has very low utilization, only one machine is enough in every case like machine 10 in Fig. 6(d).

From the result, when the allowable maintenance period is 1/12 of interval, the number of required machines is less than that without allowance. The number of machines required in case of 1/12 is only 37.5% of the number of machines required in case of no allowance. When the allowable maintenance periods are 1/6, 1/4 and 1/3 of intervals, the number of machines required are only 27%, 26% and 26% of the number of machines required in case of no allowance, respectively. Using allowable maintenance periods of 1/6, 1/4 and 1/3 of intervals requires approximately the same number of machines. Therefore, 1/6 of interval is recommended because, maintenance tasks can be done closer to the target time.

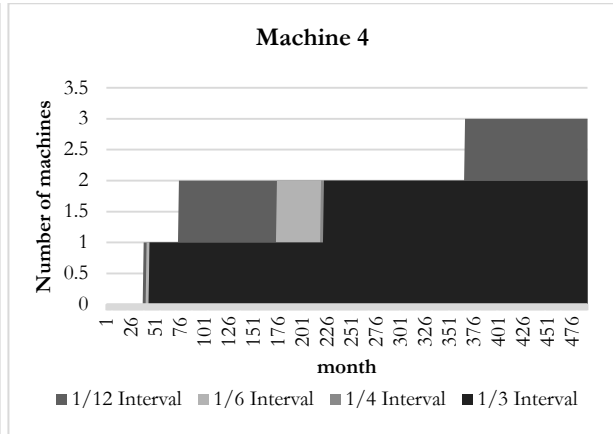
### 5.2. Maintenance Plans Using Different Lengths of Time Slots

Since the first model is a long term plan, the problem is big due to the number of time periods involved. This section studies the differences between using different lengths of time slots. We compare between using monthly and quarterly periods. For example, solving 40-year horizon, the number of time periods is 480 slots using monthly period; however, it reduces to 160 slots using quarterly period.

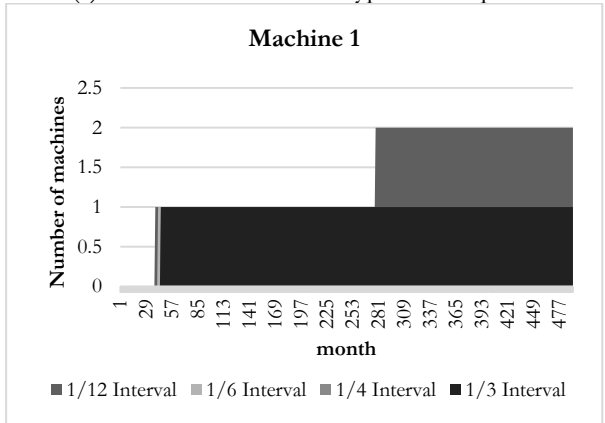
Figure 7 shows the difference between planning by quarterly period and monthly period with allowable maintenance period of 1/4 of interval. We found that in some periods of time, the number of machines required using quarterly period increases faster than the one using monthly period. However, the total number of machines required using quarterly and monthly periods are not significantly different (Fig.7 (c)). Therefore, quarterly periods can be used in a long term plan.



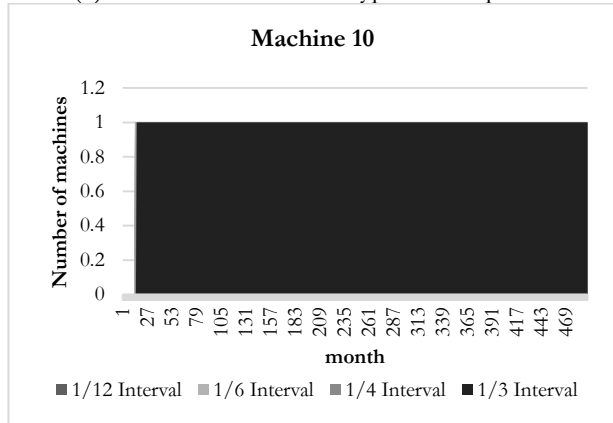
(a) Number of machine type 2 each period



(b) Number of machine type 4 each period

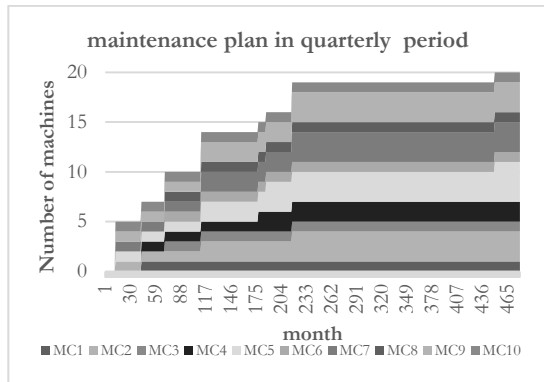


(c) Number of machine type 1 each period

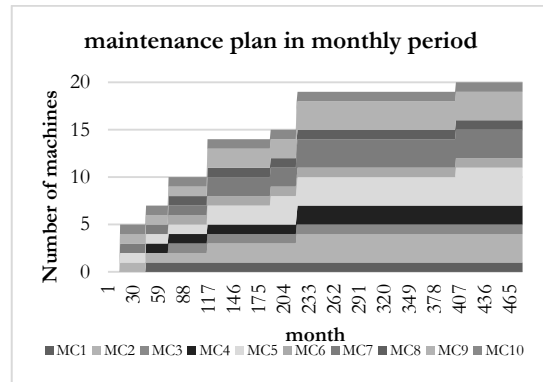


(d) Number of machine type 10 each period

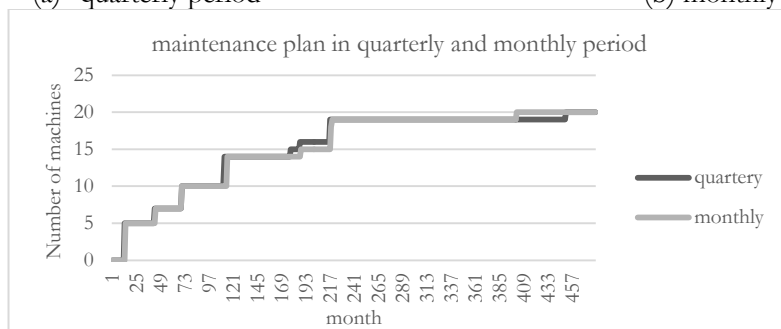
Fig. 6. Number of machine each period.



(a) quarterly period



(b) monthly period



(c) total machine for quarterly and monthly period

Fig. 7. The number of machines using quarterly period and monthly period.

### 5.3. Centralized Vs Decentralized Depots

This analysis is performed to determine the differences of the number of machines required when using centralized and decentralized depots.

From the above example having 5 operators, if each operator has its own maintenance center, it requires a lot of overhaul maintenance machines as in Fig. 8. Each maintenance center must have at least one machine of each type. Therefore, machine utilization is very low for a decentralized system.

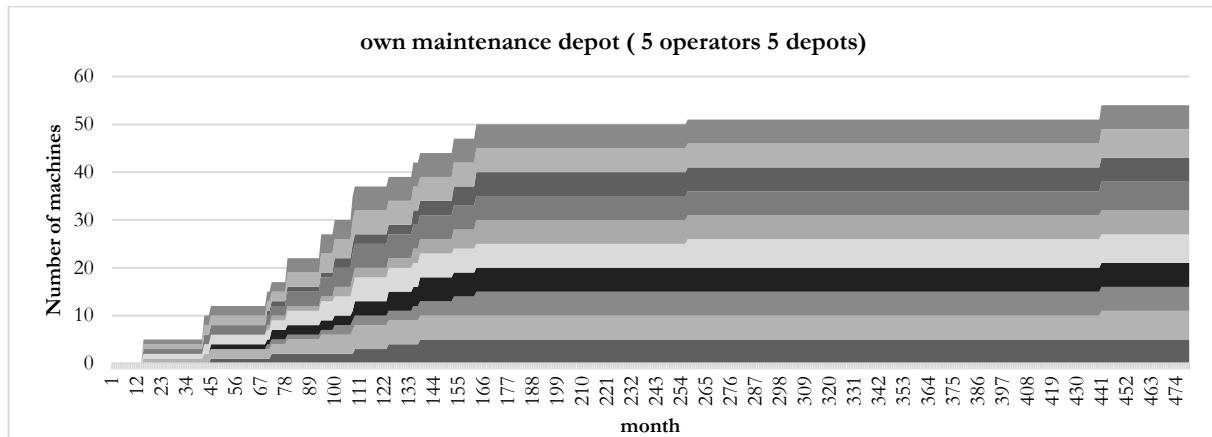


Fig. 8. Every operator has its own maintenance depot.

Using centralized maintenance depot and sharing resources, the number of machines are reduced. In Fig. 9(a), we group operators 1, 2, and 3 having 110 trains into one depot and operators 4 and 5 having 94 trains into another depot. In Fig. 9(b), all operators are assigned into one depot. It shows that once many operators share depot(s), the number of machine will decrease. However, in practice, we must consider other issues such as locations of depots and logistics of trains when design the centralized system.

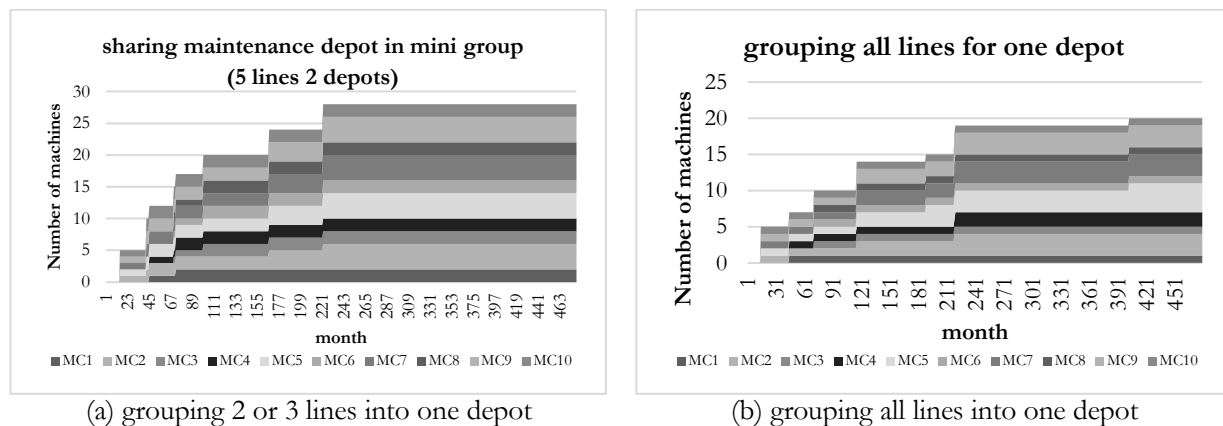


Fig. 9. Centralized maintenance depot.

## 6. Conclusions

Maintenance is costly and unprofitable but it is indispensable. Maintenance process is to repair or replace parts of the railway system to warrant that the risk of accident is limited. Reducing the maintenance costs without reducing maintenance is one of the critical factors. Fixed cost of maintenance mainly comes from machines. This paper proposes 2 mixed integer programming models to determine the minimum number of machines required for overhaul maintenance tasks of rolling stocks. The first model is developed to determine resource requirement to serve a long term plan. Due to a long planning horizon, the model cannot be solved at once. A rolling plan for sub problems are sequentially solved to find optimal solution.

The outputs from the previous sub problem become inputs of the following sub problem. The second model is used to verify feasibility of the long term plan by scheduling all maintenance tasks into time slots. This model has an objective to reduce the maximum and overall durations of time that trains spend in a depot since while trains are in depot, they cannot provide services. With the second objective, the number of required trains for each operator can be reduced.

Experiments are performed to study 3 different issues: 1) allowable maintenance periods, 2) units of time slots, 3) centralized and decentralized depots. The first analysis found that if the allowable maintenance period increases, the number of machines will decrease. The second analysis found that using different lengths of time slots does not result in significantly different solutions. With quarterly period, sometimes the number of machines increases faster than using monthly period; however, finally the number of every machine type becomes the same. And the last analysis found that a centralized system significantly decreases the number of machine required comparing to a decentralized system. However, there are other issues that must consider in design a centralized system.

## References

- [1] W. Tharmmaphornphilas, "Study and comparison of operations system for EMU maintenance depot," Report NSTDA, Bangkok, Thailand, 2016.
- [2] TEAM Consulting Engineering and Management Co., Ltd., TEAM Logistics and Transport Co., Ltd., and Daoreuk Communications Co., Ltd., "Mass rapid transit master plan in Bangkok metropolitan region: M-MAP," OTP, Bangkok, Thailand, 2010.
- [3] A. Canca, M. Sabido, and E. Barrena, "A rolling stock circulation model for railway rapid transit system," *Trans. Res. Proc.*, vol. 3, pp. 680-689, Jul. 2014.
- [4] R. Morris. (2000). *Skytrain Design Cross Section*. [Online]. Available: <http://2bangkok.com/2bangkok-skytrain-btsmain3.html> [Accessed: 1 July 2016].
- [5] N. Noomwongs, "Study and comparison of maintenance rolling stock," Report NSTDA, Bangkok, Thailand, 2016.
- [6] G. Budai, D. Huisman, and R. Dekker, "Scheduling preventive railway maintenance activities," *J. Oper. Res. Soc.*, vol. 57, pp. 1035-1044, Sept. 2006.
- [7] F. P. J. H. M. Ceng and L. A. M. van Dongen, "Application of remote condition monitoring in different rolling stock life cycle phases," in *the 2nd International Through-Life Engineering Conference.*, 2013, pp. 135-138.
- [8] C. Sriskandarajah, A. K. S. Jardine, and C. K. Chan, "Maintenance scheduling of rolling stock using a genetic algorithm," *J. Oper. Res. Soc.*, vol. 49, pp. 1130-1145, Nov. 1998.
- [9] P. Chutima, "Introduction," in *Operation Scheduling Techniques (in Thai)*, 2nd ed. Bangkok: Chulalongkorn University Press, 2012, pp. 1-21.
- [10] P. Chutima, "The scheduling of Job Shop," in *Application of the Scheduling Techniques in the Industry (in Thai)*. Bangkok: Chulalongkorn University Press, 2008, pp. 165-194.
- [11] M. Gopalakrishnana, S. Mohana, and Z. He, "A tabu search heuristic for preventive maintenance," *Comput. Ind. Eng.*, vol. 40, pp. 149-160, Jun. 2001.
- [12] H. Go, J. S. Kim, and D. H. Lee, "Operation and preventive maintenance scheduling for containerships: Mathematical model and solution algorithm," *Eur. J Oper. Res.*, vol. 229, pp. 626-636, Sept. 2013.
- [13] B. M. Worrall and B. Mert, "Application of dynamic scheduling rules in maintenance planning and scheduling," *Int. J. Prod. Res.*, vol. 18, pp. 57-71, Mar. 1980.
- [14] G. L. Giacco, A. D'Ariano, and D. Pacciarelli, "Rolling stock rostering optimization under maintenance constraints," *Int. J. Intell. Transp. Syst. Res.*, vol. 18, pp. 95-105, 2014.
- [15] T. C. E. Cheng, J. E. Diamond, and B. M. T. Lin, "Optimal scheduling in film production to minimize talent hold cost," *J. Optimiz. Theory App.*, vol. 79, Dec. 1993
- [16] S. Y. Wang, Y. T. Chuang, and B. M. T. Lin, "Minimizing talent cost and operating cost in firm production," *J. Ind. Prod. Eng.*, vol. 33, pp. 17-31, 2016.
- [17] H. Qin, Z. Zhang, A. Lim, and X. Liang, "An enhanced branch and bound algorithm for the talent scheduling problem," *Eur. J Oper. Res.*, vol. 250, pp. 412-426, Oct. 2015.
- [18] B. M. Smith, "Constraint programming in practice: Scheduling a rehearsal," Report APES-67-2003, 2003.

- [19] N. Sakulsom and W. Tharmmaphornphilas, "A multi-objective music rehearsal scheduling problem," in *12th Asia Pacific Industrial Engineering and Management Systems Conference*, 2011, pp. 25–29.
- [20] N. Sakulsom and W. Tharmmaphornphilas, "Scheduling a music rehearsal problem with unequal music piece length," *Comput. Ind. Eng.*, vol. 70, pp. 20-30, Apr. 2014.
- [21] J. I. V. Z. Fokkert, D. D. Hertog, F. J. V. D. Berg, and J. H. M. Verhoeven, "The Netherlands schedules track maintenance to improve track workers' safety," *Interface*, vol. 37, no. 2, pp. 133-142, Apr. 2007.
- [22] Y. C. Lai, D. C. Fan, and K. L. Huang, "Optimization rolling stock assignment and maintenance plan for passenger railway operations," *Comput. Ind. Eng.*, vol. 85, pp. 284-295, Apr. 2015.