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Article

Optimization Approach for Multi-Period Fuel Replenishment

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Abstract. This paper proposes mathematical models and solution approaches for solving the multi-period fuel replenishment planning problem. The model aims to search for a set of routes, determining the quantity of several petroleum products to be loaded on individual vehicle compartments, and specifying the quantity to be discharged to customer tanks over a given planning horizon in which multiple constraints are satisfied. The objective function is to minimize the transportation unit cost, equal to the total transportation cost divided by the sum of replenished quantity. As the model size grows exponentially when the number of customers, vehicles, and time period increases, an exact algorithm is not feasible. Hence, in this study, we propose two heuristic approaches: two-phase method (2PM) and three-phase method (3PM). The 2PM is primarily designed for solving small problems whereas the 3PM adopts a similar approach but has the ability to solve larger problems. The proposed solutions were tested using a real-life scenario and randomly generated test instance. The results showed that our solution outperforms the solution constructed by experienced planners and also proved that considering multiple periods when devising the fuel replenishment plan, gives superior results in comparison to single periods.

Keywords: Multi-period fuel replenishment planning problem, petroleum products, transportation unit cost, exact algorithm, heuristic approach.

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1. Introduction

Fuel replenishment planning from the depot, often called fuel terminal, to customers, in this case, petrol stations and industrial customers, is exceptionally complicated. The petrol station's inventory level and fuel replenishment plan are generally controlled by the supplier through the *vender management inventory* (VMI). In other words, the supplier decides the quantity of petroleum products to be delivered and time of delivery. Industrial customers manage the inventory level themselves and place orders to the supplier before the order cutoff time, specifying the quantity and delivery time window. Then, the supplier decides which vehicle to deliver the product and when the product should be delivered based on the vehicle availability, given that products must arrive at the customer location according to the requested delivery time window.

To describe further the fuel replenishment plan, one has to determine the set of routes, approximate delivery time (trip), vehicles to be used for delivering petroleum products for those routes, quantity of the product to be loaded on each vehicle compartment and specify how and when to discharge to each customer's storage tank following the safety guidelines and country regulations. Moreover, the planners need to ensure that the replenishment plan satisfies the following constraints:

- The inventory level of all petrol stations' storage tanks must be maintained above the minimum requirement (safety stock level) at any given planning horizon.
- Each vehicle cannot be operated over its allowable operating hours, and must not exceed the allowable number of trips.
- Every vehicle compartment must be loaded with at least 85% of its compartment size, otherwise be emptied.
- Each vehicle cannot be loaded over its regulated weight limit.

In addition to the above constraints, there are many other factors that further complicate fuel replenishment planning, including:

- **Multiple product grades:** There are many product specifications available in the market, but the common products are super unleaded, unleaded and diesel. The specific gravity of these products varies, reflecting the difference in weight per liter. Also, each vehicle compartment can only carry one product at a time and product contamination of the storage tank is strictly prohibited.
- **Heterogeneous fleet characteristic:** Several vehicle types are used, differing in terms of size, vehicle compartment configuration, number of compartments, compartment size and existence of other specialist equipment such as pump, evaporator etc. Planners have to match the vehicles to the customers' characteristics and requirements. Besides, vehicle compartments are not equipped with a flow meter therefore the product must be entirely emptied once the unloading has started.
- **Limited number of vehicles/drivers:** The petroleum industry usually subcontracts fuel delivery services to a private transport company, where the number of contracted vehicles/drivers is agreed and reviewed periodically.
- **Heterogeneous customer characteristics:** Customers are located in different locations, and have different vehicle size accessibility. Each customer has a different number of storage tanks, storage tank size, and set of petroleum products. Some customers require a vehicle equipped with a pump as their storage tanks are located above ground.
- **Multiple delivery time windows:** In the metropolitan area, heavy vehicles are prohibited during the daytime, especially on weekdays. Moreover, each customer has its own specified delivery time windows.
- **Limited terminal operating hours:** Some fuel terminals do not operate 24 hours, so the vehicle must load the products within the operating hours.
- **Limited route selection:** There are many cases where the vehicle visits multiple customers on the same trip. To select the route or customer combination, planners need to follow the routes that are pre-defined according to road safety guidelines.

Further complication may arise when the number of customers increase, consequently leading to a higher number of vehicles, storage tanks, routes, trips, etc.

As detailed above, fuel replenishment planning is considerably complicated, requiring many planners and lead time to complete the fuel replenishment plan. Another challenge is that planners need to submit a completed fuel replenishment plan to the transport company by the agreed timeline; otherwise, they will not have sufficient time for vehicles'/drivers' planning and preparation. Due to the time constraints, it is challenging for planners to devise a fuel replenishment plan that minimizes transportation costs while satisfying all the constraints and requirements.

Additionally, considering fuel replenishment in a single period does not necessarily guarantee the minimum cost in the long term. It may provide the best solution for the first day, but it could potentially give a poor solution on the following day because it is a sequential decision. The solution is to holistically consider multiple periods when planning for the fuel replenishment. Hence, in this study, we propose mathematical models and solution approach for solving the multi-period fuel replenishment planning problem. The objective is to minimize the transportation unit cost over a given planning horizon. The transportation unit cost refers to the total transportation cost divided by the sum of delivered quantity.

In this paper, the authors make the following contributions:

1. propose a mathematical formulation for solving the multi-period fuel replenishment planning problem,
2. propose two solution algorithms for solving small and larger problems,
3. implement and test the proposed solutions using a real-life scenario and randomly generated test instance, and
4. prove that considering multiple periods gives a better outcome in comparison to a single period.

2. Multi-Period Fuel Replenishment Problem

2.1. Problem Description

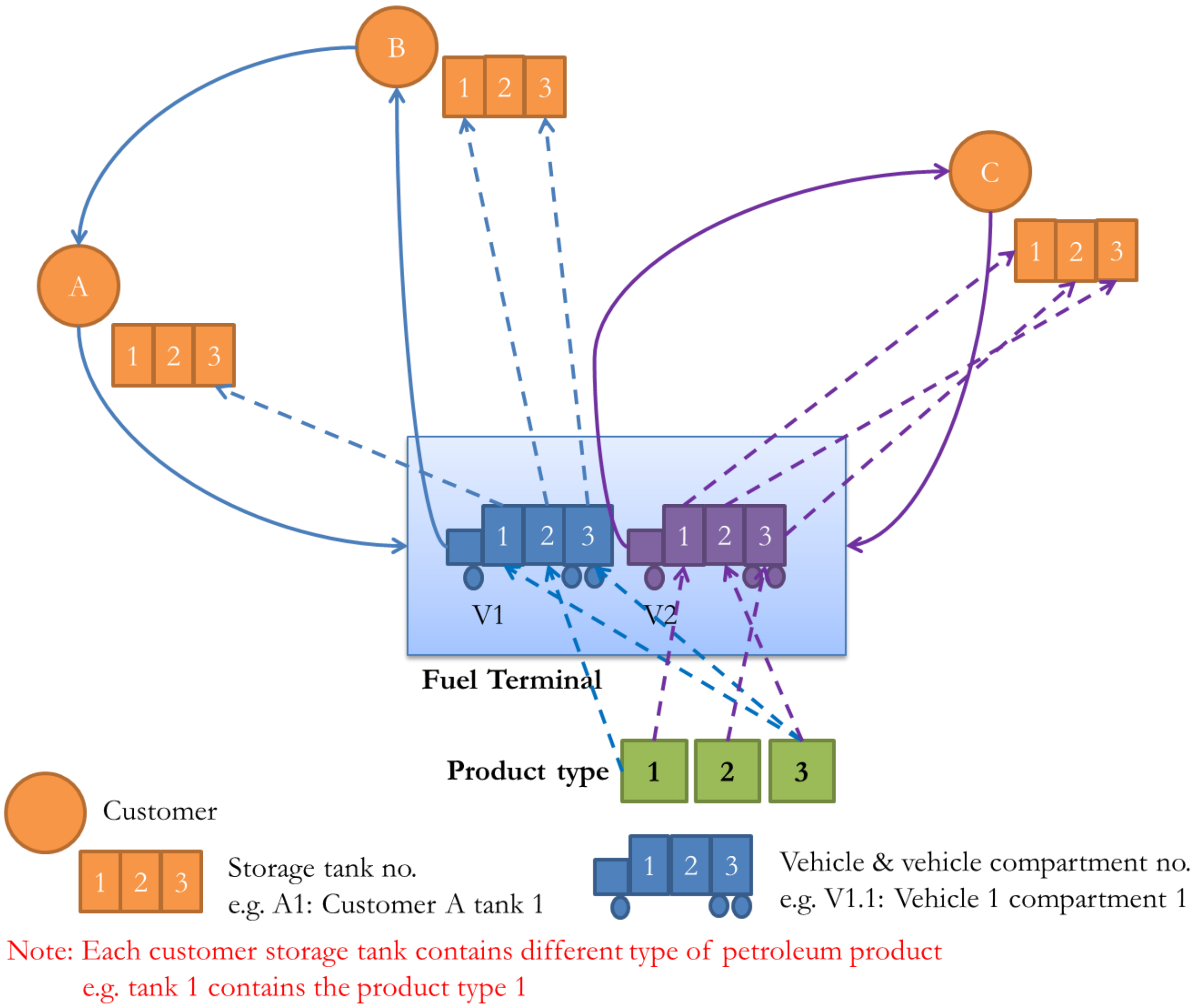


Fig. 1. Illustrative fuel replenishment diagram.

To understand the problem clearly, we prepared an illustrative fuel replenishment diagram as shown in Fig. 1, in which there are one fuel terminal, two vehicles with three vehicle compartments each (V1 and V2) and three customers with three storage tanks each (A, B and C). The solid arrows represent the route of each vehicle, and each dotted arrow represents the product movement from terminal to vehicle compartment, and to customer storage tank.

For ease of comprehension, let A1, A2, … represent customer A's tanks 1, 2, and so on. And B1, B2, … be customer B's tanks. Let V1.1, V1.2, … be vehicle V1's compartments 1, 2, and so on. And V2.1, V2.2, … be vehicle V2's compartments. Each storage tank contains different types of petroleum product: tank 1, 2, … contains product type 1, 2, and so on.

Fig. 1 shows that vehicle V1 is loaded with product type 3, 1 and 3 into vehicle compartment 1, 2 and 3 respectively. It departs to customer B and discharges the products individually from vehicle compartments 2 and 3 into customer B tank 1 and 3. Next, vehicle V1 visits customer A and discharges the remaining compartment (V1.1) into customer A tank 3, finally returning to the fuel terminal to prepare for the next delivery. Similarly, vehicle V2 is loaded with product type 1, 3 and 2 into vehicle compartments 1, 2 and 3 respectively. It departs to customer C and discharges compartment 1, 2 and 3 into customer C tank 1, 3 and 2. Lastly, it returned to the fuel terminal to prepare for the next delivery. For simplicity, the unloading patterns are summarized as follows:

1. V1.2 -> B1 (Product type 1)
2. V1.3 -> B3 (Product type 3)
3. V1.1 -> A3 (Product type 3)
4. V2.1 -> C1 (Product type 1)
5. V2.2 -> C3 (Product type 3)
6. V2.3 -> C2 (Product type 2)

In this example, it looks simple and straight forward, yet, in order to come up with the completed fuel replenishment plan, planners need to determine the set of routes, estimated time of arrival for each route, vehicles to be used for delivering petroleum products for those routes, quantity of the product to be loaded in each vehicle compartment and specify how and when to discharge to each customers' storage tank.

In general, the replenishment plan is usually created on a day-to-day basis (e.g., if today is 1st January, the planner creates the fuel replenishment plan for 2nd January). In the example below, let today = d and tomorrow = d+1, we describe how to determine the fuel replenishment quantity required at period d+1 for a petrol station.

Example:

Tank capacity	30,000 liters
Tank safety stock level	9,000 liters
Start inventory level at period d	15,000 liters
Estimated fuel consumption at period d	7,000 liters
Estimated fuel consumption at period d+1	8,000 liters
Planned replenishment quantity at period d	5,000 liters

Firstly, the estimated start inventory level at period d+1 is determined by subtracting the estimated fuel consumption at period d from the start inventory level at period d, and adding the planned replenishment quantity at period d.

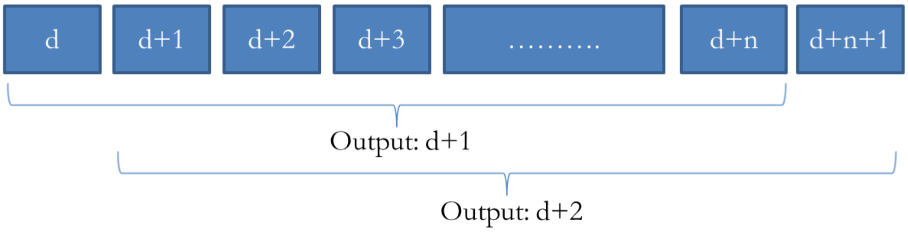
Estimated start inventory level at period d+1 = 13,000 liters (15,000 – 7,000 + 5,000)

By understanding both the estimated start inventory level at period d+1, as well as the estimated fuel consumption at period d+1, we can simply determine a feasible fuel replenishment quantity by subtracting the inventory level from the tank capacity, taking the estimated fuel consumption into account.

Feasible fuel replenishment quantity at the beginning of period d+1 = 17,000 liters (30,000 – 13,000)
Feasible fuel replenishment quantity at the end of period d+1 = 25,000 liters (30,000 – 13,000 + 8,000)

In this example, the feasible fuel replenishment quantity ranges between 17,000 and 25,000 liters. The maximum replenishment quantity at the beginning of period d+1 is 17,000 liters and it is 25,000 liters at the end of period d+1. However, it can be anticipated that the inventory level will reach the tank safety stock before the end of period d+1. Hence, the range of feasible replenishment quantity at period d+1 is adjusted to 17,000 and 21,000 liters and the storage tank must be replenished before it reaches the tank safety stock level.

The above example depicts the estimation of the feasible fuel replenishment quantity for only one storage tank in a single period, when in reality, we generally determine it for all tanks simultaneously and, routes, vehicles and vehicle compartments are also considered at the same time. Moreover, as stated earlier, considering the fuel replenishment in a single period cannot guarantee that it gives the minimum cost in the long term, thus, in this study we propose mathematical models and a solution approach for solving the multi-period fuel replenishment planning problem. The multi-period fuel replenishment planning approach is shown in Fig. 2.



Today: d
Tomorrow: d+1

Fig. 2. Multi-period fuel replenishment planning approach.

Let today = d and tomorrow = d+1. At period d, we take into consideration the replenishment plan from period d to period d+n, but the replenishment plan as of period d+1 (tomorrow) will be used and submitted to the transport company. Similarly, on the next day (period d+1), we consider the replenishment plan from period d+1 to period d+n+1, but the replenishment plan as of period d+2 will be used and submitted to the transport company on the following day. The rationale for using a single period fuel replenishment plan is that fuel consumption at every petrol station is stochastic in nature. Therefore, the more recent inventory and sales information obtained from individual petrol stations, the more accurate the replenishment plan.

2.2. Related Literature

There are many articles regarding the application of optimization methods for solving the fuel replenishment problem over the last three decades. Before the VMI idea was introduced to this area, Brown and Graves [1] proposed the optimization algorithm to assign orders to vehicles in order to minimize transportation costs, while honoring vehicle and driver working hour restrictions. In this study, customers place orders with the supplier, and the supplier assigns these orders to vehicles. This approach is often called the pull system. In a pull system, customers control their inventory level and place orders to the supplier, indicating their desired delivery time.

Brown and Graves [1] developed an integer programming model using the set partitioning concept. They first solved the problems with an exact algorithm but later came up with heuristics to solve real-time problems. Ronen [2] conducted a similar study and proposed 3 models to solve vehicle dispatching: *set partitioning* (SP), *elastics set partitioning* (ESP) and *set packing* (SPK) models. The objectives of the first 2 models are transportation cost minimization. The ESP model involves a penalty in the case of constraint violations in the model, while the SPK model aims for maximizing the overall profit. Profit, in this case, refers to the cost saving between using the contracted vehicle against the spotted vehicle.

Abdelaziz *et al.* [3] investigated the fuel replenishment problem by taking vehicles with multiple compartments into consideration. They developed a model in order to minimize the total transportation cost subject to several constraints. In comparison with the previously described articles, this model is more complex and realistic as it considers the vehicle compartment allocation. Moreover, in this study, they allow up to two customers in the same route. However, they found that solving the problem to optimality is considerably difficult due to the high number of constraints and variables. Therefore, they proposed the *variable neighborhood search* (VNS) heuristics in order to reach a near optimal solution.

Avella *et al.* [4] studied the exact and heuristic methods for solving the problem of satisfying customers' orders on the desired delivery date from one fuel terminal with a limited number of heterogeneous fleet vehicles. They proposed a Branch-and-Price algorithm based on a set partitioning concept. It is well understood that the performance of the Branch-and-Price algorithm strongly depends on the initial set of columns [5]. Therefore, they proposed the heuristic approach to obtain a good initial solution before applying Branch-and-Price algorithm.

Other studies have taken into account the VMI concept (sometimes referred to as the push system). In a push system, a supplier manages customer's inventory levels by placing the order quantity associated with the customer's inventory level. Not only is the order quantity for each customer managed by the supplier, but also the delivery time. A push system improves vehicle utilization, and the supplier has more control over the customers than a pull system. The following reviews explain how to deal with the fuel replenishment problem with the VMI concept.

Cornillier *et al.* [6] proposed two model formulations to solve the fuel replenishment problem, where these two models have totally different model's objectives and are solved sequentially. The first model is the SP problem which is used to select routes for all customers requiring a delivery. The objective of the SP model is to minimize overall transportation costs. The second model is the *tank truck loading problem* (TTLP) where it is used for determining order quantity for each tank corresponding to the customer's inventory level and tank truck compartments. Considering these two models, they have named this type of problem as *petrol station replenishment problem* (PSRP).

In contrast to the previous articles that merely consider fuel delivery in a single period, Cornillier *et al.* [7] proposed a heuristic algorithm to optimize the delivery of several petroleum products to a set of petrol stations over a given planning horizon. This problem is defined as *multi-period petrol station replenishment problem* (MPSRP). In this problem, the objectives are to determine the quantity of each product to be delivered to each station, how to fill these products into vehicle compartments, and how to plan vehicle routes that give the maximum total profit. The term profit refers to the revenue minus the sum of total transportation cost. This type of problem is slightly different in comparison to the problem addressed in this paper. The MPSRP merely involves the petrol stations where the multi-period fuel replenishment planning problem comprises petrol stations and industrial customers.

Later, Cornillier *et al.* [8] proposed a heuristic approach to solve the PSRP in a single period with specified time windows or *petrol station replenishment problem with time windows* (PSRPTW). The study aims to optimize the delivery of several petroleum products to a set of petrol stations using a limited heterogeneous fleet of vehicles in order to maximize the overall profit (the difference between revenue and routing costs), subject to the conditions that delivery is made within the specified time windows and the order quantity must be calculated associated with vehicle compartments. To solve the problem, they proposed two heuristic approaches.

Popovic *et al.* [9] and Hanczar [10] conducted a similar study to Cornillier *et al.* [7] but proposed different approaches. Popovic *et al.* [9] applied the heuristic approach to solve the MPSRP and used a simulation approach to analyze the results. Hanczar [10] proposed the heuristic approach consisting of 2 steps "first cluster – second route". Both studies considered these problems as the inventory routing problem since the supplier has full control of customer's inventory level, including determining order quantity as well as the delivery period.

Cornillier *et al.* [11] conducted a further study to solve the PSRPTW with multiple depots. This problem is called *multi-depot petrol station replenishment problem with time windows* (MPSRPTW). They proposed a new heuristic algorithm to solve the problem by using trips, not routes as in the previous studies. In this case, a route is referred to a tour that starts and ends at the same depot, whereas a trip is a combination of its route and the vehicle used. From this statement, it implies that multiple trips can have the same route.

Popovic *et al.* [12] proposed VNS heuristics to solve multi-product multi-period inventory routing problem in fuel delivery. Their proposed technique is based on a constructive heuristic or random feasible solution. Then, a shaking procedure and the randomized variable neighborhood descent (RVND) local search procedure are applied to improve the solution.

Recently, Vidovic *et al.* [13] also proposed a similar approach to Popovic *et al.* [12], but the way they obtained the initial solution, as well as improving the solution are different. Firstly, they partially solved the

MIP model (relaxed MIP model) to obtain the initial solution, and then improving the solution by using a variable neighborhood descent (VND) search.

Most of these studies proposed heuristic approaches to solve the fuel replenishment problem, due to the fact that the exact method might not be an appropriate approach to solve large scale problems [14]. To give a better understanding of the past studies, we have summarized their main characteristics in Table 1.

Table 1. Main characteristics of past studies vs this paper.

Authors	No. of vehicle	No. of customers per trip	No. of visit per customer per day	VMI Concept	Fuel Terminal	Period	Time windows
Brown and Graves [1]	Limited	One	One	No	Several	Single	No
Ronen [2]	Limited	One	One	No	One	Single	No
Abdelaziz *et al.* [3]	Limited	Up to two	One	No	One	Single	No
Avella *et al.* [4]	Limited	Several	One	No	One	Single	No
Cornillier *et al.* [6]	Unlimited	Up to two	One	Yes	One	Single	No
Cornillier *et al.* [7]	Limited	Up to two	One	Yes	One	Multi	No
Cornillier *et al.* [8]	Limited	Several	One	Yes	One	Single	Yes
Popovic et al. [9]	Limited	Up to two	One	Yes	One	Multi	No
Hanczar [10]	Limited	Several	One	Yes	One	Multi	No
Cornillier *et al.* [11]	Limited	Several	One	Yes	Several	Single	Yes
Popovic *et al.* [12]	Unlimited	Up to three	One	Yes	One	Multi	No
Vidovic *et al.* [13]	Unlimited	Up to four	One	Yes	One	Multi	No
This paper	Limited	Up to two	Several	Yes	One	Multi	Yes*

*use trip sequence to specify time windows in range (e.g. 1st trip: 8:00 - 12:00)

As shown in Table 1, it is obvious that approximately half of the previous studies have considered the fuel replenishment problem with multiple periods. However, we found none of them have proved that solving the model considering multiple periods outperforms those with a single period. In addition, to the best of the authors' knowledge, none of these studies allow customers to be visited more than once per day, which does not reflect the real-life situation where high demand customers could be served several times per day.

In this paper, we propose heuristic approaches to solve the multi-period fuel replenishment planning problem that allows customers to be visited several times per day. We use a real-life scenario and randomly generated test instance to demonstrate that considering multiple periods gives a better outcome in various aspects in comparison to a single period.

3. Mathematical Model

3.1. Problem Statement

The problem statement for the multi-period fuel replenishment planning problem is as follows:

"Given one fuel terminal with an unlimited supply of multiple grade products, two sets of customers, (i) petrol stations and (ii) industrial customers, a limited number of heterogeneous multi-compartment vehicles, consumption rate of each product at each petrol station, order quantities and delivery time windows at each industrial customer over the planning horizon, find the delivery plan minimizing transportation unit cost specifying vehicle routes and product-compartment-customer assignment and approximate delivery time (trip), such that all consumptions and demands are met within the storage allowance (safety stock level and tank capacity). The vehicle route is defined as a sequence of visits respecting specified time windows at customer sites. Each vehicle route must honor the vehicle's weight limit. Each vehicle operates within the driver's allowable working hours. The product-compartment-customer assignment specifies product quantity to be loaded in the vehicle's compartment and it's discharging destination tank."

To avoid any ambiguity, we made the following assumptions:

- Only one fuel terminal is considered.
- No product shortage at the fuel terminal.
- Fuel terminal operating hours is known.

- Product density is known.
- Limited heterogeneous fleet of vehicles.
- Weight limit, volume capacity, and compartment configuration for each vehicle are known.
- Maximum vehicle operating hours and maximum number of trips are known.
- Vehicle compartments are not equipped with flow meter. They must be completely emptied once the unloading has started.
- Each vehicle compartment must be loaded at least to 85% of its capacity due to the safety reasons, otherwise be emptied.
- Possible routes are pre-defined.
- Daily sales consumption for all petrol stations storage tanks is deterministic.
- Each customer tank contains only one product, and product crossover is not allowed.
- Customer tank capacity and safety stock level for individual customer tanks are known. Customer inventory level must not exceed the tank capacity and must not fall below the safety stock level.
- Individual customers can be visited several times on any given planning horizon.
- Several trips can be assigned to the same vehicle.
- Up to two customers can be visited in the same trip.
- Total time (including travel, loading and unloading time) for each trip is known.
- Transportation cost for all routes associated with each vehicle is known.

3.2. Notations

Sets

V	is the set of vehicles, indexed by v.
K	is the set of vehicle compartments, indexed by k.
R	is the set of possible routes, indexed by r.
I	is the set of customers, indexed by i.
J	is the set of customer tanks, indexed by j.
D	is the period (day), indexed by d.
T	is the set of trips, indexed by t.

Parameters

a_{ij}	is the safety stock level of tank j of customer i.
b_{ij}	is the capacity of tank j of customer i.
c^{vr}	is the total cost of route r using vehicle v.
g_{ij}	is the product density associated to product of tank j of customer i.
h^{vr}	is the total time for visiting route r using vehicle v.
o_{ijdt}	is the estimated inventory level of tank j of customer i during trip t at period d.
q^{vk}	is the compartment size of compartment k of vehicle v.
s_{ijdt}	is the estimated sales consumption of tank j of customer i during trip t at period d.
H_d^v	is the maximum allowable vehicle operating hour of vehicle v at period d.
N^v	is the total amount of compartment of vehicle v.
W^v	is the maximum weight limit of vehicle v.
M	is a scaling coefficient.

Variables

x_{ijdt}^{vkr}	is the quantity of product loaded into compartment k of vehicle v, to be discharged to tank j of customer i in route r during trip t at period d.
y_{ijdt}^{vkr}	equal to 1 if compartment k of vehicle v is assigned to tank j of customer i in route r during trip t at period d, otherwise 0.
z_{dt}^{vr}	equal to 1 if route r is delivered by vehicle v during trip t at period d, otherwise 0.

3.3. Model Formulation

From the notations, parameters and variables defined above, the *multi-period fuel replenishment planning problem* (MPFRP) is formulated as follows:

Objective function:

$$\text{Maximize} \quad \sum_{v\in V}\sum_{r\in R}\sum_{d\in D}\sum_{t\in T}\left[\sum_{k\in K}\sum_{i\in I}\sum_{j\in J} x_{ijdt}^{vkr} - Mc^{vr}z_{dt}^{vr}\right] \tag{1}$$

Subject to:

$$a_{ij} \le o_{ijdt} \qquad \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{2}$$

$$o_{ijdt} + \sum_{v\in V}\sum_{k\in K}\sum_{r\in R} x_{ijdt}^{vkr} \le b_{ij} \qquad \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{3}$$

$$o_{ijd,t+1} = o_{ijdt} - s_{ijdt} + \sum_{v\in V}\sum_{k\in K}\sum_{r\in R} x_{ijdt}^{vkr} \qquad \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{4}$$

$$-0.15q^{vk} \le x_{ijdt}^{vkr} - q^{vk}y_{ijdt}^{vkr} \le 0 \qquad \forall v\in V, \forall k\in K, \forall r\in R, \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{5}$$

$$\sum_{r\in R}\sum_{i\in I}\sum_{j\in J} y_{ijdt}^{vkr} \le 1 \qquad \forall v\in V, \forall k\in K, \forall d\in D, \forall t\in T \tag{6}$$

$$(1-N^v) \le \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} y_{ijdt}^{vkr} - N^v z_{dt}^{vr} \le 0 \qquad \forall v\in V, \forall r\in R, \forall d\in D, \forall t\in T \tag{7}$$

$$\sum_{r\in R}\sum_{t\in T} h^{vr}z_{dt}^{vr} \le H_d^v \qquad \forall v\in V, \forall d\in D \tag{8}$$

$$\sum_{r\in R} z_{dt}^{vr} \le 1 \qquad \forall v\in V, \forall d\in D, \forall t\in T \tag{9}$$

$$\sum_{k\in K}\sum_{r\in R}\sum_{i\in I}\sum_{j\in J} g^{ij}x_{ijdt}^{vkr} \le W^v \qquad \forall v\in V, \forall d\in D, \forall t\in T \tag{10}$$

$$x_{ijdt}^{vkr} \in \mathbb{R}^+ \qquad \forall v\in V, \forall k\in K, \forall r\in R, \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{11}$$

$$y_{ijdt}^{vkr} \in \{0,1\} \qquad \forall v\in V, \forall k\in K, \forall r\in R, \forall i\in I, \forall j\in J, \forall d\in D, \forall t\in T \tag{12}$$

$$z_{dt}^{vr} \in \{0,1\} \qquad \forall v\in V, \forall r\in R, \forall d\in D, \forall t\in T \tag{13}$$

Our objective is to minimize the transportation unit cost, which is equal to the total transportation cost divided by the sum of replenished quantity. This implies simultaneously maximizing the vehicle loaded quantity and minimizing the total transportation cost. To combine these two objectives in the objective function, we employed a scaling coefficient (M) to maintain the balance between the two objectives as shown in Eq. (1). The value of this coefficient can be determined from the interactive method described in this paper.

Eq. (2) ensures that the inventory level for individual tanks must not fall below the minimum requirement (safety stock level) in any given planning horizon. Eq. (3) guarantees that the inventory level after replenishment does not exceed the tank capacity. Eq. (4) ensures the stock equilibrium between connecting trips. Eq. (5) and (6) specify that one vehicle compartment can only be assigned to one customer tank for each trip. If a vehicle compartment is used, it must be loaded at least 85% of its compartment size. Eq. (7) ensures that vehicle v visit route r only if at least one compartment is assigned. Eq. (8) ensures that each vehicle must not be operated over the maximum allowable operating hour. Eq. (9) specifies that one route can only be assigned to one trip. Eq. (10) guarantees each vehicle is not carrying products over its weight limit.

4. The Two-Phase Method (2PM)

As the model size grows exponentially as the number of customers, vehicles, and time period increase, the exact algorithm is apparently not feasible to solve Eq. (1)-(13). In this study, we propose two heuristic approaches: *two-phase method* (2PM) and *three-phase method* (3PM). The 2PM is primarily designed for solving small problems whereas the 3PM offers a similar approach but has the ability to solve larger problems. The details of 2PM are outlined below and 3PM is outlined in the following section.

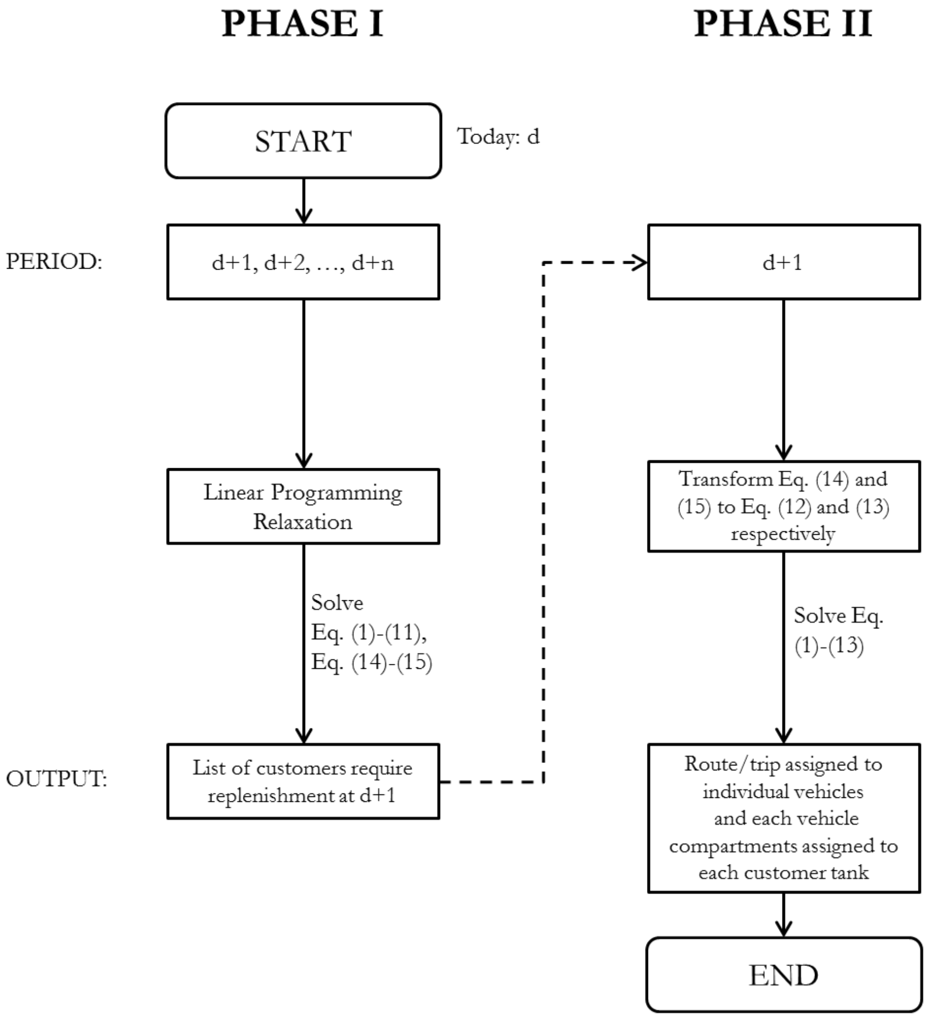


Fig. 3. A Flowchart of the 2PM.

Phase I

The 2PM is a heuristic approach that solves the multi-period fuel replenishment planning problem by decomposing the solution process into two phases. A flowchart of 2PM is shown in Fig. 3. In phase I, the linear programming relaxation technique is applied, transforming Eq. (12) and (13) into Eq. (14) and (15) respectively. Next, we solve the Eq. (1)-(11) and Eq. (14)-(15) to optimality.

$$0 \leq y_{ijdt}^{vkr} \leq 1 \qquad \forall v \in V, \forall k \in K, \forall r \in R, \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T \quad (14)$$
$$0 \leq z_{dt}^{vr} \leq 1 \qquad \forall v \in V, \forall r \in R, \forall d \in D, \forall t \in T \quad (15)$$

The linear programing relaxation technique was first introduced by Lovasz [15] for the set covering problem. This technique fundamentally transforms an integer programming problem into a linear programming problem which is solvable in a polynomial time. The solution obtained from the linear programming relaxation can be used as an initial step to further obtain the solution of the original problem. There are many studies applying the linear programming relaxation in solving various problems [16–22].

Despite the fact that the results obtained from phase I are not feasible in the real world, they indicate which customers should be serviced during the considered planning horizon, especially those customers that need to be replenished tomorrow. As described earlier in section 2.1, the replenishment plan is usually created on a day-to-day basis (e.g., if today is 1st January, the planner creates the fuel replenishment plan for 2nd January) since the fuel consumption is stochastic in nature. The more recent inventory and sales information obtained from individual petrol stations, the more accurate the replenishment plan.

Phase II

In phase II, only the customers requiring replenishment in period d+1 (tomorrow) as obtained from phase I are considered, all other customers are eliminated. By considering only the period d+1 and removing some customers from the problem, the size of the model is significantly reduced and solving the problem to optimality is now practical. Next, the Eq. (1)-(13) is solved to optimality using the branch-and-bound algorithm.

5. The Three-Phase Method (3PM)

The 3PM is fundamentally similar to 2PM, but composed of three phases as shown in Fig. 4.

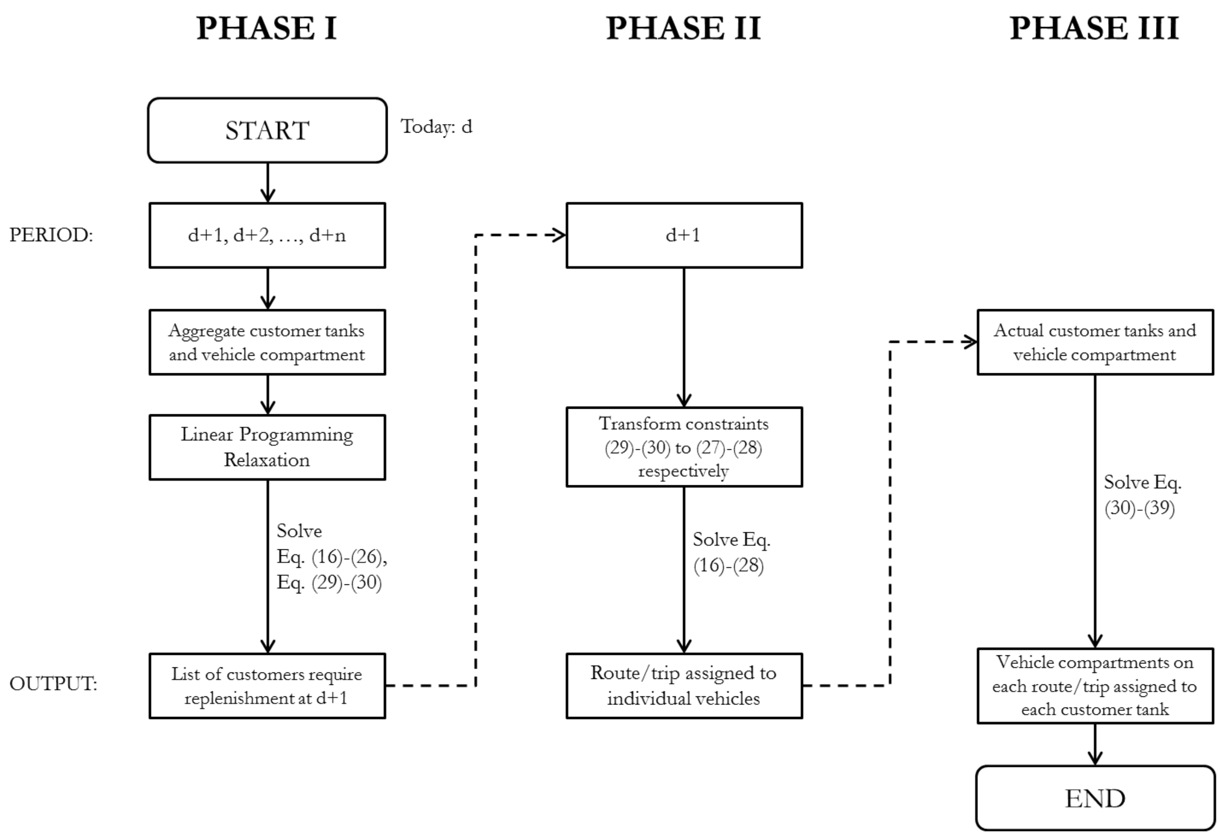


Fig. 4. A flowchart of the 3PM.

Phase I

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In phase I, the main objective is to reduce the model size and to obtain the list of customers requiring replenishment during the considered planning horizon. Firstly, we aggregate the customer storage tanks and vehicle compartments for individual customers and vehicles respectively. The phase I model is then formulated as follows:

Parameters

a_i	is the total safety stock level of customer i.
b_i	is the total capacity of customer i.
c^{vr}	is the total cost of route r using vehicle v.
g_i	is the average product density of customer i.
h^{vr}	is the total time for visiting route r using vehicle v.
o_{idt}	is the estimated total inventory level of customer i during trip t at period d.
q^v	is the maximum loaded volume of vehicle v.
s_{idt}	is the estimated total sales consumption of customer i during trip t at period d.
H_d^v	is the maximum allowable vehicle operating hour of vehicle v at period d.
W^v	is the maximum weight limit of vehicle v.
M	is a scaling coefficient.

Variables

x_{idt}^{vr}	is the quantity of total product loaded into vehicle v, to be delivered to customer i in route r during trip t at period d.
y_{dt}^{vr}	equal to 1 if vehicle v is assigned to route r during trip t at period d, otherwise 0.
z_{dt}^{vr}	equal to 1 if route r is delivered by vehicle v during trip t at period d, otherwise 0.

Phase I Model

Objective function:

$$\text{Maximize} \quad \sum_{v\in V}\sum_{r\in R}\sum_{d\in D}\sum_{t\in T}\left[\sum_{i\in I} x_{idt}^{vr} - Mc^{vr}z_{dt}^{vr}\right] \tag{16}$$

Subject to:

$$a_i \le o_{idt} \qquad \forall i\in I, \forall d\in D, \forall t\in T \tag{17}$$

$$o_{idt} + \sum_{v\in V}\sum_{r\in R} x_{idt}^{vr} \le b_i \qquad \forall i\in I, \forall d\in D, \forall t\in T \tag{18}$$

$$o_{id,t+1} = o_{idt} - s_{idt} + \sum_{v\in V}\sum_{r\in R} x_{idt}^{vr} \qquad \forall i\in I, \forall d\in D, \forall t\in T \tag{19}$$

$$-0.15q^v \le \sum_{i\in I} x_{idt}^{vr} - q^v y_{dt}^{vr} \le 0 \qquad \forall v\in V, \forall r\in R, \forall d\in D, \forall t\in T \tag{20}$$

$$\sum_{r\in R} y_{dt}^{vr} \le 1 \qquad \forall v\in V, \forall d\in D, \forall t\in T \tag{21}$$

$$y_{dt}^{vr} - z_{dt}^{vr} = 0 \qquad \forall v\in V, \forall r\in R, \forall d\in D, \forall t\in T \tag{22}$$

$$\sum_{r\in R}\sum_{t\in T} h^{vr} z_{dt}^{vr} \le H_d^v \qquad \forall v\in V, \forall d\in D \tag{23}$$

$$\sum_{r\in R}\sum_{i\in I} g^i x_{idt}^{vr} \le W^v \qquad \forall v\in V, \forall d\in D, \forall t\in T \tag{24}$$

$$\sum_{r\in R} z_{dt}^{vr} \le 1 \qquad \forall v\in V, \forall d\in D, \forall t\in T \tag{25}$$

$$x_{idt}^{vr} \in \mathbb{R}^+ \qquad \forall v\in V, \forall r\in R, \forall i\in I, \forall d\in D, \forall t\in T \tag{26}$$

$y_{dt}^{vr} \in \{0,1\}$ $\quad \forall v \in V, \forall r \in R, \forall d \in D, \forall t \in T$ (27)

$z_{dt}^{vr} \in \{0,1\}$ $\quad \forall v \in V, \forall r \in R, \forall d \in D, \forall t \in T$ (28)

Similar to 2PM, the linear programming relaxation technique is applied, relaxing Eq. (27) and (28) into (29) and (30) respectively. Next, Eq. (16)-(26) and Eq. (29)-(30) are solved to optimality to obtain the list of customers requiring replenishment in period d+1 (tomorrow).

$0 \le y_{dt}^{vr} \le 1$ $\quad \forall v \in V, \forall r \in R, \forall d \in D, \forall t \in T$ (29)

$0 \le z_{dt}^{vr} \le 1$ $\quad \forall v \in V, \forall r \in R, \forall d \in D, \forall t \in T$ (30)

Phase II

In phase II, the main purpose is to determine the vehicles to be used, trips and routes assigned to individual vehicles where each route indicates the customers to be visited (up to two customers can be visited in the same trip).

Next, only the period d+1 (tomorrow) is considered and the model constructed based on the list of customers requiring a delivery in period d+1 as obtained from phase I. Other customers not on the list are removed from the master problem. The customer tanks and vehicle compartments remain aggregated at this stage. Eq. (29) and (30) are reverted to Eq. (27) and (28) respectively. Next, Eq. (16)-(28) is solved using the branch-and-bound algorithm.

Phase III

Upon the completion of phase II, we understand the vehicles to be used and customers to be serviced in each trip and route. Next, we disaggregate the customer tanks and vehicle compartments back to the original, and solve the phase III model to determine how individual vehicle compartments are loaded and assigned to each customer tank in each trip and route. The phase III model is described as follows.

Phase III Model

Objective function:

$$\text{Maximize} \quad \sum_{v \in V} \sum_{k \in K} \sum_{r \in R} \sum_{i \in I} \sum_{j \in J} \sum_{d \in D} \sum_{t \in T} x_{ijdt}^{vkr} \tag{31}$$

Subject to:

$a_{ij} \le o_{ijdt}$ $\quad \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (32)

$o_{ijdt} + \sum_{v \in V} \sum_{k \in K} \sum_{r \in R} x_{ijdt}^{vkr} \le b_{ij}$ $\quad \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (33)

$o_{ijd,t+1} = o_{ijdt} - s_{ijdt} + \sum_{v \in V} \sum_{k \in K} \sum_{r \in R} x_{ijdt}^{vkr}$ $\quad \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (34)

$-0.15q^{vk} \le x_{ijdt}^{vkr} - q^{vk} y_{ijdt}^{vkr} \le 0$ $\quad \forall v \in V, \forall k \in K, \forall r \in R, \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (35)

$\sum_{r \in R} \sum_{i \in I} \sum_{j \in J} y_{ijdt}^{vkr} \le 1$ $\quad \forall v \in V, \forall k \in K, \forall d \in D, \forall t \in T$ (36)

$\sum_{k \in K} \sum_{r \in R} \sum_{i \in I} \sum_{j \in J} g^{ij} x_{ijdt}^{vkr} \le W^v$ $\quad \forall v \in V, \forall d \in D, \forall t \in T$ (37)

$x_{ijdt}^{vkr} \in \mathbb{R}^+$ $\quad \forall v \in V, \forall k \in K, \forall r \in R, \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (38)

$y_{ijdt}^{vkr} \in \{0,1\}$ $\quad \forall v \in V, \forall k \in K, \forall r \in R, \forall i \in I, \forall j \in J, \forall d \in D, \forall t \in T$ (39)

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6. Computational Results

6.1. Data

In this study, we used a real-life scenario and randomly generated instance to test the 2PM and 3PM respectively. Firstly, we describe the real-life data.

Real-life scenario

Table 2 shows the number of customers, the number of customers requiring a vehicle equipped with a pump when unloading the product to storage tanks, as well as the weekly consumption separated by customer type. As seen, although the number of the petrol stations is less than the number of industrial customers, if we look at the weekly consumption, approximately 89% of the demand is from the petrol stations.

Table 2. Customer profile (real-life scenario).

Customer type	No. of sites	No. of sites (pump required)	Weekly consumption (liters)
Petrol station	11	2	503,600 (89%)
Industrial customer	17	17	61,500 (11%)

Table 3 gives the number of petrol stations categorized by range of their daily consumption. Most of the petrol stations sell more than 3,000 liters a day considering all product types. There are 3 product types where the consumption rate for each product is completely different. As shown in Table 4, the product type 2 has the highest consumption (83% of the total consumption).

Table 3. Number of petrol stations categorized by range of daily consumption (real-life scenario).

Daily consumption (liters)	No. of sites
0-3,000	3
3,001-6,000	4
6,001-9,000	3
>9,000	1

Table 4. Percentage of demand consumption by product type (real-life scenario).

Product Type	Percentage (%)
1	6
2	83
3	11

Table 5 shows the tank capacity of all petrol stations categorized by range of tank size. Table 6 provides details of the vehicle profile. There are three vehicles in total; two of them are chartered as a contracted vehicle (one for each vehicle type), and one vehicle (type 2) is rented on a spot basis. The contracted vehicle involves the monthly fixed cost regardless of how many trips are used. For both contracted vehicle and spotted vehicle, if the vehicle is used, there are other charges including loading fee and distance fee. Loading fee is charged based on the number of trips made by each vehicle, and the loading rate varies depending on the vehicle type, as well as the contract type. Distance fee is charged based on the total distance in kilometers. Similar to loading fee, the distance fee for each vehicle is different. Apparently, loading and distance fee for the spotted vehicle are much higher than those of the contracted vehicle as no fixed cost is paid.

Table 5. Number of petrol station's storage tanks categorized by range of tank size (real-life scenario).

Tank size (liters)	No. of tanks
0-6,000	1
6,001-15,000	15
15,001-30,000	11
>30,000	5

Table 6. Vehicle profile (real-life scenario).

Vehicle type	Total capacity (liters)	Max. loaded weight (Kg)	Pump equipped (Yes/No)	No. of compartments	Capacities (liters)
1	22,700	19,295	Yes	3	11,400, 7,500, 3,800
2	19,000	16,150	No	3	7,600, 7,600, 3,800

To give more clarity on how the transportation cost is calculated, an example is provided below:

Example:

Loading fee per trip	\$50.00
Distance fee per kilometer	\$4.00
Fixed cost per month	\$3,000.00 (\$100.00 per day)
Days	2
Total number of trips	10
Total kilometer travelled	500

The variable transportation cost = (50*10) + (4*500) = \$2,500
The total transportation cost (incl. fixed cost) = 2,500 + (2*100) = \$2,700

Randomly generated test instance

To come up with the larger test instance, we randomly generated conditions similar in nature as the real-life scenario. Table 7 shows the number of customers, the number of customers requiring the vehicle equipped with a pump when unloading the product to storage tanks, as well as the weekly consumption. In this test instance, we use only one customer type (petrol station) but with a larger scale. As seen, the number of petrol stations, as well as the weekly consumption is much higher than that of the real-life scenario (approximately 5 times). Table 8 gives the number of petrol stations categorized by range of their daily consumption. Most of the petrol stations sell more than 6,000 liters a day considering all product types and the consumption rate for each product can be inferred from Table 4.

Table 7. Customer profile (randomly generated case).

Customer type	No. of sites	No. of sites (pump required)	Weekly consumption (liters)
Petrol station	50	25	3,077,306

Table 8. Number of petrol stations categorized by range of daily consumption (randomly generated case).

Daily consumption (liters)	No. of sites
0-3,000	5
3,001-6,000	9
6,001-9,000	10
>9,000	26

Table 9 shows the tank capacity of all petrol stations categorized by tank size. In this test, there are only two tank sizes: 10,000 and 30,000 liters. We assume a 30,000 liter tank size for all tanks containing product type 2 due to the highest consumption.

Table 9. Number of petrol station's storage tanks by tank size (randomly generated case).

Tank size (liters)	No. of tanks
10,000	100
30,000	50

Table 10 and 11 provide details of the physical vehicle profile as well as the vehicle cost profile. We based these physical vehicle profiles, vehicle cost profiles and configurations on the real-life scenario in one of the countries that is similar in terms of the characteristics. In this test, there are two vehicle types with six contracted vehicles in total (two type 1 vehicles and four type 2 vehicles).

Table 10. Vehicle profile (randomly generated case).

Vehicle type	Total capacity (liters)	Max. loaded weight (Kg)	Pump equipped (Yes/No)	No. of compartments	Capacities (liters)
1	24,000	20,000	No	5	6,000, 4,000, 4,000, 4,000, 6,000
2	18,000	15,000	Yes	5	4,000, 4,000, 2,000, 4,000, 4,000

Table 11. Vehicle cost profile (randomly generated case).

Vehicle type	Monthly fixed cost ($)	Loading fee per trip ($)	Distance fee per kilometer ($)
1	8,333	33	0.7
2	6,667	30	0.5

Referring to the real-life scenario, the daily consumption of each product fluctuates from the estimated consumption demand. The average and standard deviation of the daily consumption variability for each product are shown in Table 12. As seen, the demand consumption for product type 2 is the most stable, followed by product type 3 and 1.

Table 12. Daily consumption variability (randomly generated case).

Product	Mean	S.D.
1	26.8%	49.9%
2	2.7%	14.2%
3	17.0%	44.8%

In this test, the fuel terminal is located at coordinates (32, 103), where the petrol stations coordinates are randomly generated between coordinate (0, 0) and (200, 200) as illustrated in Fig. 5.

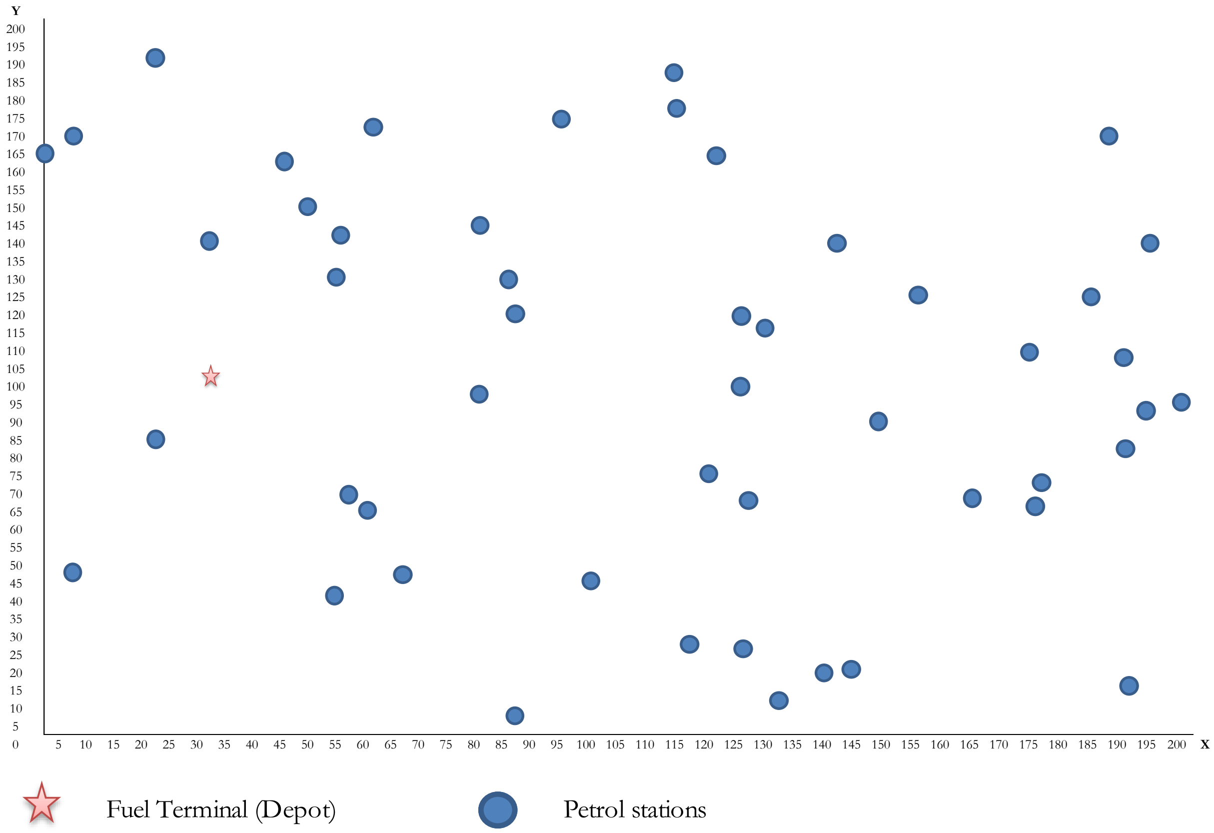

Fig. 5. Fuel terminal and petrol stations network.

6.2. Performance and Effectiveness of Proposed Algorithms

The model was implemented using the VB.NET programming language and the IBM ILOG CPLEX Version 12.6 where the data are kept in the MS Access database 2010. The model was run on an Intel Core i7 with 8GB of memory.

The Two-Phase Method (2PM)

As stated earlier, we tested this approach using real-life conditions. In this section, we provide the results of the 2PM run on several scenarios. Moreover, we compare the results against the replenishment plan created by experienced planners. Before describing the results, we explain how we obtain the coefficient (M).

We determined an appropriate coefficient (M) from the interactive method, solving the Eq. (1)-(13) in a single period with different coefficients (M). In this experiment, we tested this over a 9 day planning cycle. The coefficient (M) that gives the best result was selected to be used in the 2PM. Table 13 shows the operational results associated with each coefficient (M). In this case, we selected coefficient (M) = 70 as it gives the best result in terms of transportation unit cost. We also tested the sensitivity altering the coefficient (M) between 70 and 80 and found no difference, hence, we used coefficient (M) = 70 for all test scenarios.

Table 13. Results summary as a function of the coefficient (M) (2PM).

Coefficient (M)	Total loaded volume (liters)	Avg. loaded volume per trip (liters)	Transportation cost ($)*	Transportation unit cost ($/liter)
1	623,019	14,160	6,090.8	0.0098
20	613,212	17,034	5,394.8	0.0088
30	628,885	17,469	5,283.4	0.0084
40	625,392	18,394	5,119.8	0.0082
50	587,640	18,956	4,845.9	0.0082
60	579,745	19,325	4,729.6	0.0082
70	565,827	20,957	4,541.6	0.0080
80	571,773	20,420	4,615.2	0.0081

*including fixed cost component

The individual test scenarios are described as follows:

1. Consider 1 day planning horizon and solve the problem to optimality
2. Consider 2 days planning horizon and solve the problem to optimality
3. Consider 3 days planning horizon and solve the problem to optimality
4. Consider 3 days planning horizon and apply 2PM to solve the problem
5. Consider 4 days planning horizon and apply 2PM to solve the problem
6. Consider 5 days planning horizon and apply 2PM to solve the problem

Firstly, we solved all the above scenarios to optimality using the branch-and-bound technique. It was found that as the considered planning horizon was extended to 3 days, the exact method is no longer feasible due to the significant increase in the problem size. In this context, 3 days planning horizon means: if today is period d, we look 3 days ahead (period d+1, d+2 and d+3) in order to come up with the replenishment plan for period d+1.

Table 14 shows the growth in the number of rows, columns and non-zero elements when considering longer periods. These are the cumulative number of rows, columns and non-zero elements after 9 days planning cycle. The problem size grows exponentially as the number of considered planning horizon increases.

Table 14. Problem size of original problem (real-life scenario).

Planning horizon (days)	No. of rows	No. of columns	No. of non-zero
1	18,896	34,611	104,080
2	34,962	64,365	253,716
3	54,950	100,925	476,788
4	74,763	137,491	752,126
5	81,670	168,325	1,039,085

As previously mentioned, the exact method is no longer practical when the considered planning horizon reaches 3 days, thus, we applied the 2PM for 3, 4 and 5 days planning horizon. Now, the problem size decreased significantly as shown in Table 15.

Table 15. Problem size after applying 2PM.

Planning horizon (days)	No. of rows (2PM)	No. of columns (2PM)	No. of non-zero (2PM)
3	16,901	30,577	102,983
4	17,715	32,083	108,155
5	19,179	34,783	117,092

Next, we explain the results for each scenario as shown in Table 16 in terms of operational performance, transportation cost and solution times.

Table 16. Results summary of 2PM (real-life scenario).

Scenario	No. of potential product run out	Total loaded volume (liters)	Avg. loaded volume per trip (liters)	Transportation cost ($)*	Transportation unit cost ($/liter)	Solution times (sec)
1	18	582,424	20,801	4,599.6	0.0079	39
2	10	556,456	20,609	4,563.9	0.0082	866
3			*Unable to solve - Out of memory*			
4	6	583,227	20,830	4,675.1	0.0080	78
5	3	586,573	20,949	4,684.7	0.0080	74
6	2	604,188	20,834	4,766.1	0.0079	66

*including fixed cost component

According to these results, it is clear that scenario 6 (considering 5 days planning horizon and apply 2PM) is the best despite giving the same transportation unit cost as scenario 1. Scenario 6 is most effective in preventing product run out due to the fact that it explores 5 days ahead as opposed to scenario 1 that takes only one single period into account. Moreover, it can be seen that the longer period of considered planning horizon, the less number of potential product run out. Regarding the solution time, scenario 6 is slightly higher than scenario 1 but it is certainly acceptable, taking only 66 seconds to solve 9 days planning cycle where it takes approximately 50,400 seconds if performed by experienced planners. We also found that it is possible to visit customers more than once a day, ranging between 3–9% out of total number of customer visits.

Lastly, we implemented the 2PM on scenario 6 over a 16 day planning cycle and compared it against the actual results of the replenishment plans made by the experienced planners (more than 10 years of experience) over the same planning cycle. The results comparison is shown in Table 17. The results obtained from 2PM are much better than those created by the experienced planners in all dimensions: operational performance, transportation cost and the solving times. Moreover, the cost of human resources can also be saved due to the reduction in fuel replenishment planning time.

Table 17. Results comparison between 2PM and experienced planner.

Scenario	No. of potential product run out	Total loaded volume (liters)	Avg. loaded volume per trip (liters)	Transportation cost ($)*	Transportation unit cost ($/liter)	Solution times (sec)
0**	6	1,119,792	19,307	8,986.0	0.0080	93,600
6	4	1,122,642	20,412	8,779.7	0.0078	173

*including fixed cost component

**replenishment plan created by an experienced planner

The Three-Phase Method (3PM)

The 2PM approach is very effective and efficient in solving the problem of multi-period fuel replenishment, however, it has a limitation in the problem size. If the problem size is huge, the 2PM will become unpractical and unable to solve the problem. Therefore, we introduced the 3PM which offers a similar approach but has an ability to solve a larger problem. To test this approach, we used randomly generated test instance as described in section 6.1.

Similar to the 2PM, we determined an appropriate coefficient (M) from the interactive method over a 1 day planning cycle. Table 18 shows that coefficient (M) = 60 gives the lowest transportation unit cost, thus, we use this for all test scenarios.

Table 18. Results summary as a function of the coefficient (M) (3PM).

Coefficient (M)	Total loaded volume (liters)	Avg. loaded volume per trip (liters)	Transportation cost ($)*	Transportation unit cost ($/liter)
1	478,578	19,941	4,848.8	0.0101
10	474,477	19,770	5,047.2	0.0106
20	478,452	19,936	4,915.9	0.0103
30	479,307	19,971	4,874.6	0.0102
40	476,209	19,842	4,788.5	0.0101
50	480,000	20,000	4,842.0	0.0101
60	479,246	19,969	4,776.9	0.0100
70	475,960	19,832	4,793.9	0.0101
80	476,446	19,852	4,789.0	0.0101

*including fixed cost component

The individual test scenarios are described as follows:
1. Consider 1 day planning horizon and apply 3PM
2. Consider 2 days planning horizon and apply 3PM
3. Consider 3 days planning horizon and apply 3PM
4. Consider 4 days planning horizon and apply 3PM
5. Consider 5 days planning horizon and apply 3PM

Tables 19, 20 and 21 show the number of rows, columns and non-zero elements respectively at each stage of the 3PM approach for all test scenarios. These are the cumulative number of rows, columns and non-zero elements for a 7 day planning cycle. As seen, the problem size grows exponentially as the number of days increases, especially the non-zero element. Before applying the 3PM approach, it went beyond 60 million when we extended the planning horizon to 5 days. This cannot be solved to optimality with an exact algorithm nor the 2PM. Hence, decomposing such a problem in 3 phases, the problem size reduces significantly, and the problem is now solvable in a polynomial time.

Table 19. Comparison of the number of rows in each step of 3PM.

Planning horizon considered (days)	No. of rows (Original)	No. of rows (Phase I)	No. of rows (Phase II)	No. of rows (Phase III)
1	905,352	66,584	51,136	6,455
2	1,810,697	133,168	65,210	6,381
3	2,716,051	199,752	62,110	6,295
4	3,621,401	266,336	76,865	6,235
5	4,526,751	332,920	77,005	6,597

Table 20. Comparison of the number of columns in each step of 3PM.

Planning horizon considered (days)	No. of columns (Original)	No. of columns (Phase I)	No. of columns (Phase II)	No. of columns (Phase III)
1	1,767,864	122,696	93,280	9,450
2	3,535,728	245,392	118,910	9,300
3	5,303,592	368,088	113,080	9,150
4	7,071,456	490,784	141,180	9,060
5	8,839,320	613,480	141,470	9,660

Table 21. Comparison of the number of non-zero in each step of 3PM.

Planning horizon (days)	No. of non-zero (Original)	No. of non-zero (Phase I)	No. of non-zero (Phase II)	No. of non-zero (Phase III)
1	5,705,028	364,588	276,864	19,560
2	14,880,936	960,568	380,720	19,500
3	27,837,945	1,787,940	361,960	19,170
4	44,322,734	2,846,704	452,550	18,870
5	64,410,260	4,136,860	453,500	20,025

We tested the above scenarios over a 7 day planning cycle. In this case, we solved phase I and III to optimality but limited the solution time in phase II to 600 seconds as we found phase II typically takes time due to the problem size and containing integer variables.

Table 22. Results summary of 3PM.

Planning horizon (days)	No. of potential product run out	Total loaded volume (liters)	Avg. loaded volume per trip (liters)	Transportation cost ($)*	Transportation unit cost ($/liter)	Solution times (sec)
1	1	3,160,809	19,273	35,052.5	0.0111	982
2	0	3,174,344	19,356	35,712.0	0.0113	1842
3	0	3,157,988	19,256	35,302.8	0.0112	1958
4	0	3,141,775	19,157	35,221.5	0.0112	1686
5	0	3,176,172	19,367	35,342.4	0.0111	1994

*including fixed cost component

Table 22 shows the results summary for individual scenarios in terms of operational performance, transportation cost and solution times. The test result proves that the 3PM can give an outstanding performance within a reasonable solving time. The average loaded volume per trip is considerably high; in terms of the vehicle capacity utilization, it was higher than 95% for all test scenarios. The number of potential run out is also very impressive. Moreover, with the same problem size, it normally requires 4 hours to complete each day planning cycle, even when performed by an experienced planner, thus, for a 7 day planning cycle, it would take approximately 28 hours or 100,800 seconds. In this case, we found approximately 14–18% of customers are visited more than once a day.

By comparing the results among all test scenarios, it is obvious that scenario 5 (considering 5 days planning horizon and apply 3PM) gives the best result in all aspects. This is a similar finding as the 2PM results where we achieved a better result when the number of considered planning horizon increases.

7. Conclusion

In this paper, the authors purpose two heuristic approaches, the 2PM and 3PM, to solve the multi-period fuel replenishment planning problem. The 2PM is primarily designed for solving small problems, whereas the 3PM adopts a similar approach but has the ability to solve a larger problem. We used real-life data and randomly generated test instance to test the 2PM and 3PM, respectively. According to the results, both approaches prove that the solution obtained from the multi-period model is superior to single-period in many aspects. In addition, the proposed solution (2PM) outperforms the solution constructed by the planners who possess more than 10 years of fuel replenishment planning experience.

Furthermore, it is possible for customers to be visited more than once a day, which reflects the real-life situation where high demand customers could potentially be served several times per day. Additionally, by allowing customers to be visited more than once a day, it can give a better outcome in terms of vehicle utilization as there is more opportunity for delivery to other customers in the same route, maximizing the vehicle compartment usage.

Future studies could consider incorporating inventory holding cost and opportunity cost in the model in case a product runs out, better reflecting the real-life situation. The authors are also interested in finding new solution approaches that are able to solve a very large scale problem given that the replenishment plan must be constructed within a reasonable time frame. It would also be of interest is to find a solution approach that can recommend replenishment plan adjustment, which happens in real-time due to an uncertainty in fuel consumption at each customer site, as well as other unexpected issues such as vehicle breakdown, product outage at fuel terminal etc., while maintaining a low transportation unit cost. As fuel consumption is stochastic in nature and the fuel consumption forecast is the most crucial factor for fuel replenishment planning, investigation of a statistical model and new forecasting method that accurately predicts fuel consumption at each customer site would also be beneficial.

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