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Determination of Natural Frequency of Aerofoil Section Blades Using Finite Element Approach, Study of Effect of Aspect Ratio and Thickness on Natural Frequency

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Abstract. In this paper an attempt has been made to calculate the natural frequencies of aerofoil section blades using the finite element method. The effect of varying the aspect ratio and thickness on the frequencies has also been studied. It is observed that by increasing the value of aspect ratio, the natural frequency in all the modes is increased. It is also concluded that, there is some optimum value of thickness at which the natural frequencies in different modes of vibration are minimum.

Keywords: Aerofoil, finite element method.

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1. Introduction

With ever increase in demand for larger capacity steam turbines and more efficient rotary compressors, blade vibrations and the resulting fatigue have become more and more an important subject in the design of turbomachinaries. Fatigue failure of turbomachine blades is one of the most vexing problems of the turbomachine manufactures. Aflobi [1] gave reasons for failure of blades and concluded that resonance of blade vibration frequency with nozzle excitation frequency is the major cause of blade failure. Cranch [2] making use of bending mode determined the first three modes of vibration of rotating beam by the use of Rayleigh energy approach. Carneige [3] derived a theoretical expression for the work done due to centrifugal effects for small vibrations of rotating cantilever beams and established an equation for the fundamental frequency of vibration by the use of Rayleigh-Ritz method. Sisto [4] solved the equation of motion of a pretwisted cantilever blade using Ritz principle. Roastard [5] calculated the first five frequencies of twisted cantilever beam using Galkerine finite element method and concluded that the effect of twist is proportional to the width to thickness ratio. Klein [6] studied the free vibration of elastic beams with non-uniform characteristics by a new method that comprised the advantages of both finite element method and Rayleigh-Ritz analysis. Lag [7] worked on vibration characteristics of non-uniform blades and determined the natural frequency using Galkerin finite element method. Lee [8] studied vibration characteristics of a tapered beam and obtained formula for fundamental frequency. He further concluded that a decrease of cross sectional dimension in thickness with constant breadth increases the fundamental bending frequency whereas decreases higher modes of bending frequencies. Rao [9] concluded that a decrease of cross sectional dimension in breadth with constant thickness increase the fundamental bending frequency as well as the higher modes of bending frequencies. Downs [10] worked upon transverse vibration of cantilever beams of unequal breadth and taper. He obtained results by dynamic discretization technique and concluded that by increasing mode order the vibration becomes concentrated towards the tip. Dhar [11] studied the effect of non-linearities in material properties and stiffness on natural frequencies of turbine blade using non-linear finite element method and concluded that at transient stage there was a remarkable change in blade frequency.

Thomas [12] developed a model using finite element method for Timoshenko beams and concluded that the disc radius has predominant effect on flapwise mode of vibration. The frequency increases with increase in disc radius. Murthy [13] used Integration Matrix method to study the vibration characteristics with the effect of asymmetry of cross section and concluded that bending frequencies decrease with asymmetry and torsional frequencies increase with asymmetry. Shran [14] studied the free vibration of turbine blades with the temperature effect using finite element method and concluded that as the material is heated the frequencies start decreasing because the value of modulus of elasticity of material decreases.

The continuum approach for a freestanding blade requires a lot of analytical work before a numerical procedure can be adopted to perform frequency analysis. Discretizing the blade and using appropriate element relations is simpler than the analytical work that goes with continuum methods and thus several research workers favour the discrete methods.

1.1. Discrete Analysis of Blades

Other distinct advantages of discrete methods are their compatibility to complicated methods like laced and pocketed blades and the availability of well tested finite element programs to model any complicated blade group. The discrete analysis techniques can be broadly classified into the following methods as applied to turbine blade problems using beam theories.

1.1.1. Holzer method

Holzer method has been developed for torsional vibration calculations of a given system. The given system is discretized into several rigid inertials connected by massless elastic torsional shaft. The dynamic properties are transferred from station to station using field and point dynamic properties. The boundary conditions are used to set up a criterion and determine whether an assumed frequency satisfies this criterion for a natural frequency. The application of Holzer method to bending vibration problem is tedious as the bending problem involves four state quantities compared with only two state quantities in torsional problem.

1.1.2. Myklestad/Prohl method

To determine natural frequency of beams Myklestad/Prohl method is used. The beam is discretized into several masses connected by massless beam elements having the original flexural rigidity of the beam between the stations. Depending on the boundary conditions a suitable criterion can be set up and the natural frequency is determined by trial and error root search technique.

1.1.3. Matrix method

The matrix is a means of numerically integrating a function that is expressed in terms of the values of the function at equal increments of the independent variable. They can be derived by expressing a function as a polynomial in the form of Newton's forward interpolation formula. The integration matrix has been applied to obtain the natural frequency of a propeller blade for cross-coupled bending modes.

1.1.4. Finite difference method

The Finite difference method uses a series of regularly spaced grid points and approximates the partial differential equation at each point. The resulting algebraic equations are than solved at each grid point by applying boundary conditions. Rapid solutions are possible because the algebraic equations are usually linear and simple. Thus Finite difference methods are relatively inexpensive but they do not readily cope with complex problems unless a large number of grid points are used.

1.1.5. Finite element method

The Finite element method has become very popular in recent years. It can be used to determine the combined bending and torsional vibration modes taking into account the effect of root flexibility. The Finite element method seeks to approximate the solution of partial differential equation integrated over a series of arbitrarily shaped finite elements. The result is a system of simultaneous linear equations, which usually do not have regular structure. Therefore their solutions are more expensive and time consuming. However the number of nodes required in finite element method is usually much less than the number of grid points in finite difference method for comparable accuracy. Finite element method is advantageous for complex problems.

A survey of the available literature shows that a lot work has been carried out on the analysis of vibration characteristics of turbomachine blades using different methods. In the present work an attempt has been made to calculate the frequencies of turbomachine blades using finite element method, to analyze the different aerofoil sections having different chord length, length and thickness. The details of formulation are discussed below.

2. Finite Element Formulation

The geometry of the blade is essentially an aerofoil section, which is axisymmetric and has same taper along its length. This blade can be modelled by three-dimensional solid modelling method. The discretization of the blade is done with 8 noded isoparametric hexagonal solid elements in which each node of the element has three degrees of freedom i.e. displacements *u*, *v* and *w* in the *x*, *y* and *z* directions respectively. For an eight node hexagonal element we have considered the mapping onto a cube of 2-unit side placed symmetrically with *r*, *s* and *t* coordinates as shown in Fig. 1.

On the master cube element the Lagrange shape function is applied, which can be written as:

$$N_i = \left(\frac{1}{8}\right)(1 + r_i r)(1 + s_i s)(1 + t_i t) \tag{1}$$

where i=1 to 8, and *r*, *s*, and *t* represent the coordinates of the nodes *i*.

The element nodal displacements are represented by the vector

$$q = [q_1, q_2, q_3, \ldots, q_{24}]^T \tag{2}$$

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With the use of the shape function N_i, the displacements at any point inside the element in terms of its nodal values can be defined as:

$$\begin{cases} u = N_1 q_1 + N_2 q_2 + \ldots + N_8 q_{22} \\ v = N_1 q_2 + N_2 q_5 + \ldots + N_8 q_{23} \\ w = N_1 q_3 + N_2 q_6 + \ldots + N_8 q_{24} \end{cases}$$

or

$$[U] = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = [N][q]^T \tag{3}$$

where $[N]$ is the shape function matrix; $[q]$ is the nodal displacement vector.

Also the coordinate mapping is done with the same shape functions as given below:

$$\begin{cases} x = N_1 x_1 + N_2 x_2 + \ldots + N_8 x_8 \\ y = N_1 y_1 + N_2 y_2 + \ldots + N_8 y_8 \\ z = N_1 z_1 + N_2 z_2 + \ldots + N_8 z_8 \end{cases} \tag{4}$$

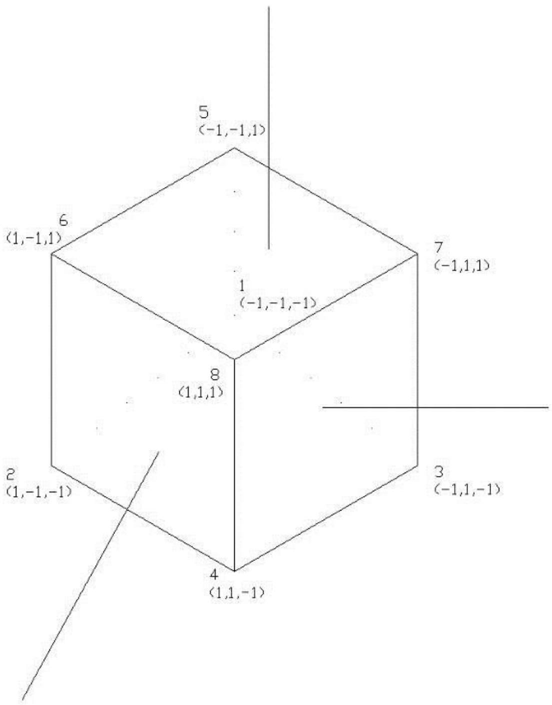

Fig. 1. Eight node hexagonal element.

The components of strains for a three dimensional stress element can be expressed as:

$$\begin{bmatrix} \xi_x \\ \xi_y \\ \xi_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{bmatrix} \tag{5}$$

In order to find the derivatives of the displacements in global coordinates, we have to first obtain the derivatives of the displacements in natural coordinates. Differentiating Eq. (5) with respect to coordinates *r*, *s* and *t*, we get

$$\begin{bmatrix} \frac{\partial u}{\partial r} & \frac{\partial v}{\partial r} & \frac{\partial w}{\partial r} \\ \frac{\partial u}{\partial s} & \frac{\partial v}{\partial s} & \frac{\partial w}{\partial s} \\ \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} & \frac{\partial w}{\partial t} \end{bmatrix} = \Sigma \begin{bmatrix} \frac{\partial N_i u_i}{\partial r} & \frac{\partial N_i v_i}{\partial r} & \frac{\partial N_i w_i}{\partial r} \\ \frac{\partial N_i u_i}{\partial s} & \frac{\partial N_i v_i}{\partial s} & \frac{\partial N_i w_i}{\partial s} \\ \frac{\partial N_i u_i}{\partial t} & \frac{\partial N_i v_i}{\partial t} & \frac{\partial N_i w_i}{\partial t} \end{bmatrix} \tag{6}$$

Now using Jacobian inverse, we get

$$\begin{bmatrix} \frac{\partial u}{\partial r} & \frac{\partial v}{\partial r} & \frac{\partial w}{\partial r} \\ \frac{\partial u}{\partial s} & \frac{\partial v}{\partial s} & \frac{\partial w}{\partial s} \\ \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} & \frac{\partial w}{\partial t} \end{bmatrix} = \begin{bmatrix} J^*_{11} & J^*_{12} & J^*_{13} \\ J^*_{21} & J^*_{22} & J^*_{23} \\ J^*_{31} & J^*_{32} & J^*_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial r} & \frac{\partial v}{\partial r} & \frac{\partial w}{\partial r} \\ \frac{\partial u}{\partial s} & \frac{\partial v}{\partial s} & \frac{\partial w}{\partial s} \\ \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} & \frac{\partial w}{\partial t} \end{bmatrix} \tag{7}$$

Expanding the above equations and substituting the values of *u*, *v* and *w* in that from Eq. (3), we get a sub matrix of strain displacement matrix.

$$[B_1] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \tag{8}$$

Similar to Eq. (8), we can obtain eleven sub matrices of size 6×3. Using these sub matrices we can formulate strain displacement matrix as given below:

$$[B]=[B_1][B_2][B_3]\ldots[B_{11}] \tag{9}$$

Thus the strain-displacement takes the form

$$\varepsilon=[B][q]^T \tag{10}$$

The element characteristics matrix i.e. stiffness matrix of 8 node isoparametric hexahedral element can be obtained by performing numerical interaction of the following equation by Gauss quadrature 2×2×2 method.

$$[K]_{33\times33}=\iiint[B]^T[D][B]\left|\det J\right|drdsdt \tag{11}$$

where [D] is element property matrix defined as

$$\frac{E}{(1+\mu)(1-2\mu)}=\begin{bmatrix} 1-\mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 1-\mu & \mu & 0 & 0 & 0 \\ \mu & \mu & 1-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\mu \end{bmatrix} \tag{12}$$

where E is the elastic modulus; μ is the Poisson's ratio.

3. Elemental Mass/Inertia Matrix

The kinetic energy of the blade is given by

$$T=\frac{1}{2}\int_v u^T u\rho dv \tag{13}$$

where ρ is the density of the blade material and u is the velocity vector of point at x. Substituting the value of U in Eq. (13), the kinetic energy expression can be written as

$$T=\frac{1}{2}q^T\left(\int_v \rho N^T N dv\right)q \tag{14}$$

where the bracketed expression is termed as element mass matrix.

$$[m]=\int_v \rho N^T N dv$$

In the discretized natural coordinates, the expression of element mass matrix can be written as:

$$[m]=\iiint \rho N^T N\left|\det J\right| drdsdt \tag{15}$$

The mass matrix of the 8 node hexahedral element can be computed by integrating Eq. (15) using Gauss quadrature 2×2×2 method.

4. Vibration Analysis

For performing the vibration analysis of a continuum, it is necessary that continuum system is discretized into small elements so that the stiffness and mass/inertia matrices of individual elements can be computed easily as described above. The assembled matrix of these elements represents properties of complete continuum. Thus the original equation of the motion of the continuum is discretized into a system of governing equations of motion for nodal displacement.

The general equation of motion for a discretized damped vibration structure is given by:

$$[M]\left\{\frac{d^2x}{dt^2}\right\}+[C]\left\{\frac{dx}{dt}\right\}+[K]\{x\}=[F] \tag{16}$$

where [*M*], [*C*] and [*K*] represent overall mass matrix, damping coefficient matrix and stiffness matrix respectively. $\{x\}$, $\left\{\frac{dx}{dt}\right\}$ and $\left\{\frac{d^2x}{dt^2}\right\}$ are displacements, velocity and acceleration vectors of the assemblage respectively. [*F*] is external excitation force vector.

In mechanical systems such as turbine blade, the damping contributed by the system is negligible, though material of the blade produces some amount of hysteresis losses. Neglecting damping and considering it as a free vibration problem, the equation of motion for discretized system is expressed as:

$$[M]\left\{\frac{d^2x}{dt^2}\right\}+[K]\{x\}=0 \tag{17}$$

The vibration analysis of the blade requires the solution of above differential equation of motion for natural frequencies the eigenvalue analysis i.e. determination of natural frequency of freely vibrating blade is essentially a characteristic or eigenvalue problem. The equation of motion of blade/wing is given by:

$$[K]\{\phi\}-\lambda[M]\{\phi\}=0 \tag{18}$$

When a blade/wing is discretized by finite element method, it yield symmetric, positive, definite and banded mass and stiffness matrices. In order to be able to solve above equation in most economical and efficient manner, it is desirable that related Eigen value routine does not destroy these properties in subsequent computation.

Ideally, an efficient eigenvalue estimation algorithm is expected to possess the following necessary features:

(1) The algorithm should be able to compute a few desired roots.
(2) It should exploit matrix sparsity.
(3) It should be numerically stable so that it yields a reliable and accurate solution.

In the present studies inverse iteration technique as discussed in the next section is used for finding the eigenvalues and eigenvectors.

4.1. Inverse Iteration Method

The inverse iteration technique is effectively used to calculate an eigenvector and at the same time the corresponding eigenvalue can also be evaluated. We assume that stiffness matrix *K* is positive definite, whereas mass matrix *M* is a symmetric banded mass matrix.

In this method the basic relation considered is

$$K\phi=\lambda M \tag{19}$$

in which we have to calculate the smallest eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_p$ and corresponding eigenvectors $\phi_1, \phi_2, \ldots, \phi_p$ by satisfying Eq. (19) by directly operating on it. We assume a vector for ϕ, say, x_1 and assume a value for $\lambda=1$. We can then evaluate the right hand side of Eq. (19). Hence we may calculate

$$R_1=(1)Mx_1 \tag{20}$$

Since x_1 is an arbitrarily assumed vector, we do not have, in general $Kx_1 = R_1$. If Kx_1 were equal to R_1 then x_1 would be an eigenvector except for trivial cases and our assumptions would have been very lucky. Instead, we have an equilibrium equation as:

$$Kx_2=R_1, \quad x_2\neq x_1 \tag{21}$$

where x_2 is the displacement solution corresponding to the applied forces R_1. Since we know that we have to use iteration to solve for an eigenvector we may assume that x_2 may be a better approximation to an eigenvector than x_1 was. By repeating the cycle we obtain an increasingly better approximation to an eigenvector. This procedure is the basis of inverse iteration technique.

In the following we first consider the basic equations used in inverse iteration and then present a more effective form of the technique. In the solution we assume a starting iteration vector x_1 and then evaluate in each iteration step $k = 1, 2, \ldots$.

$$K\overline{x}_{k+1}=Mx_k \tag{22}$$

and

$$x_{k+1}=\frac{\overline{x}_{k+1}}{\left(\overline{x}_{k+1}{}^{T}M\overline{x}_{k+1}\right)^{0.5}} \tag{23}$$

provided that x_1 is not M-orthogonal to ϕ_1; hence, $x_1 TM\phi_1 \neq 0$ and $x_k+1 \rightarrow \phi_1$ as $k \rightarrow \infty$.

The basic step in the iteration is the solution of the equation in Eq. (22) in which we evaluate a vector x_{k+1} with a direction closer to an eigenvector than had the previous iteration vector x_k. The calculation in Eq. (23) merely ensures that the M-weight length of the new iteration vector x_{k+1} is unity. That is we want x_{k+1} to satisfy the mass orthonormality relation i.e.

$$x_{k+1}^T M x_{k+1} = 1 \tag{24}$$

Substituting for x_{k+1} from Eq. (23) into Eq. (24) we find that Eq. (24) is indeed satisfied. If the scaling in Eq. (23) is not included in the iteration, the elements of the iteration vectors grow or decrease in each step and the iteration vectors do not converge to ϕ_1 but to a multiple of it.

The relation in Eq. (22) and Eq. (23) state the basic inverse iteration algorithm.

5. Problem Description

In the present study an aerofoil section blade of a turbine is chosen for frequency analysis. The blade is of uniform configuration along its length. It is held fixed to one end and hangs freely at the other. The blade is solid and material properties are constant and isotropic. The following data are used:

1.	Blade material	:	Nickel alloy
2.	Modulus of elasticity	:	2.11x1011 N/m^2
3.	Density	:	7850 Kg/m^3
4.	Poission's ratio	:	0.3
5.	Length of blade	:	300 mm
6.	Chord length	:	25 mm
7.	Aerofoil sections employed	:	(a) NACA-006
			(b) NACA-0010
			(c) NACA-16-021
			(d) NACA-0018
			(e) NACA-65-015

The aerofoil section blade is treated as a cantilever beam, therefore constraints are applied to all the nodes which are fixed to the base of the rotor. The displacements at all the fixed nodes are put to zero. Further the number of frequencies to be extracted is defined. In this study first five modes of frequency are extracted.

6. Frequency Analysis of Various Aerofoil Sections

The solution methodology developed in previous section is applied to various types of aerofoil sections. In the present study the geometry of different aerofoil sections has been taken from the research data of National Advisory Committee for Aeronautics (NACA). The NACA has classified different aerofoil sections and designated by NACA code [15]. Though hundreds of different sections have been developed, in the present study only six different types of sections have been chosen to determine the natural frequencies of vibration. The natural frequencies of these types of aerofoil sections are listed in Table 1 and variation is shown is Fig. 2.

6.1. Effect of Thickness of Aerofoil Section

The effect of thickness of the aerofoil section on the natural frequencies of five modes of vibration could easily be seen from the Table 1. If we move from NACA 006 to NACA 16-021 we find that with increase in thickness frequency first decreases and after some value of thickness it starts increasing (Fig. 3). It is interesting to note that there is some optimal value of thickness (NACA 65-015) for which the natural frequency in all the modes is minimum. The reason attributable for this phenomenon is from here onwards the blade starts behaving like a plate with cross-coupled mode shape.

Table 1. Natural frequency of various aerofoil sections.

Aerofoil Section	Natural Frequency (Hz) Modes				
	I	II	III	IV	V
NACA 006	76.596	177.982	489.479	607.47	1118
NACA 66-009	62.603	170.263	400.212	638.65	1070.88
NACA 0010	49.992	167.022	319.94	624.615	935.05
NACA 65-015	41.44	157.766	264.622	772.36	823.32
NACA 0018	44.406	166.156	284.38	832.303	907.059
NACA 16-021	42.473	164.154	272.222	798.48	1033.8

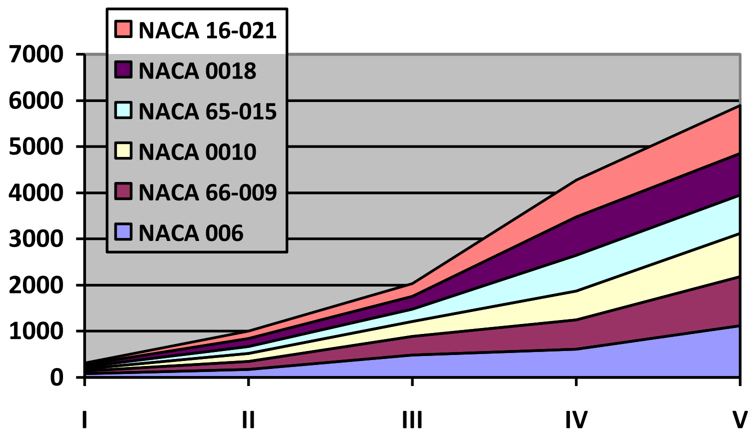

Fig. 2. Variation of vibration frequency with vibration modes.

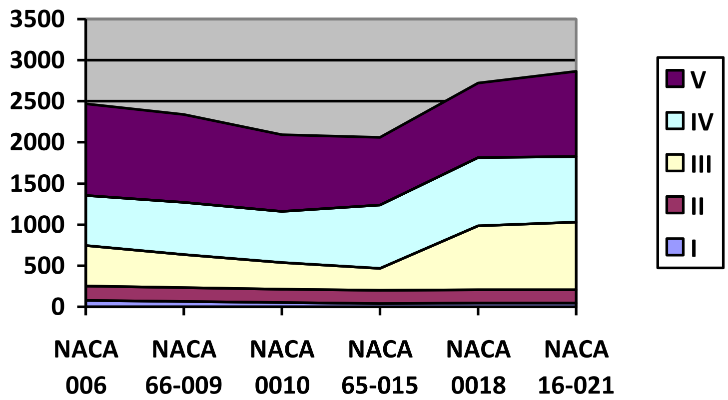

Fig. 3. Variation of vibration frequency with blade profile.

6.2. Effect of Aspect Ratio

The ratio of chord length to the length of blade is commonly referred as aspect ratio. In the present study the effect of aspect ratio on the natural frequency characteristics of various aerofoil sections has also been studied. The study is restricted to first two modes only as the turbine is expected to run in these frequency ranges. For this purpose the chord length has been fixed to 25 mm and the length has been taken as 100, 200, 300 and 400 mm.

The variation of natural frequency in fundamental mode for the different aspect ratios is listed in Table 2.

Table 2. Effect of aspect ratio natural frequency of fundamental mode.

Aerofoil Section	Frequency (Hz) in Fundamental Mode Aspect Ratio			
	0.0625	0.0835	0.125	0.25
NACA 0006	57.12	76.596	116.1	246.83
NACA 66-009	46.66	62.603	97.86	231.22
NACA 0010	36.663	49.992	80.564	211.433
NACA 65-015	28.82	41.423	74.46	206.232
NACA 0018	29.22	44.406	83.231	286.32
NACA 16-021	26.332	42.473	88.333	336.321

Close look to the Fig.4 reveals that the natural frequency in fundamental mode of aerofoil section is increased to approximately 5 times when aspect ratio is increased from 0.0625 to 0.25. This is due to the fact that when aspect ratio increases, aerofoil section becomes stiffer; thereby increase in frequency is observed. However in NACA 16-021 and other blades the natural frequency in fundamental mode raises to more than 10 times. This sounds that NACA 16-021 blade is behaving like a plate.

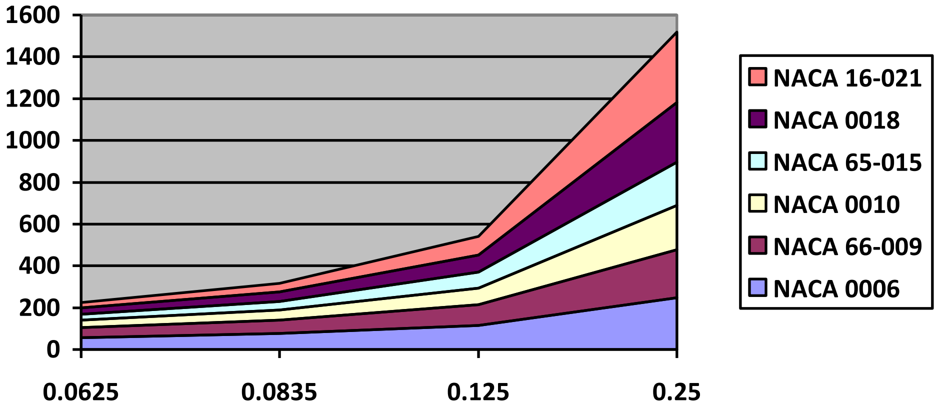

Fig. 4. Variation of vibration frequency in fundamental mode with aspect ratio.

The effect of aspect ratio on the natural frequency of second mode for various blade sections is listed in Table 3.

Table 3. Effect of aspect ratio natural frequency of second mode.

Aerofoil Section	Frequency (Hz) in Second Mode Aspect Ratio			
	0.0625	0.0835	0.125	0.25
NACA 006	107.12	177.982	378.76	1434.96
NACA 66-009	100.663	170.233	367.53	1408.54
NACA 0010	96.83	167.022	366.232	1351.23
NACA 65-015	90	157.726	348.52	1340.3
NACA 0018	95.43	166.156	368.53	1434.26
NACA 16-021	93	164.154	366.633	1685.633

It is observed from Fig. 5 that the natural frequencies of practically all types of blade sections are increased ten to twenty folds with increasing aspect ratio, which clearly shows that behaviour of blade is like a plate instead of a beam. Therefore from the present study it can be safely concluded that for NACA-006, NACA-0010 and a few other types of blade sections the beam modelling approach is good approximation for determining fundamental frequency however, when turbine operate at high speed i.e. near the second resonance condition, the blades starts behaving a plate which needs different modelling methodology.

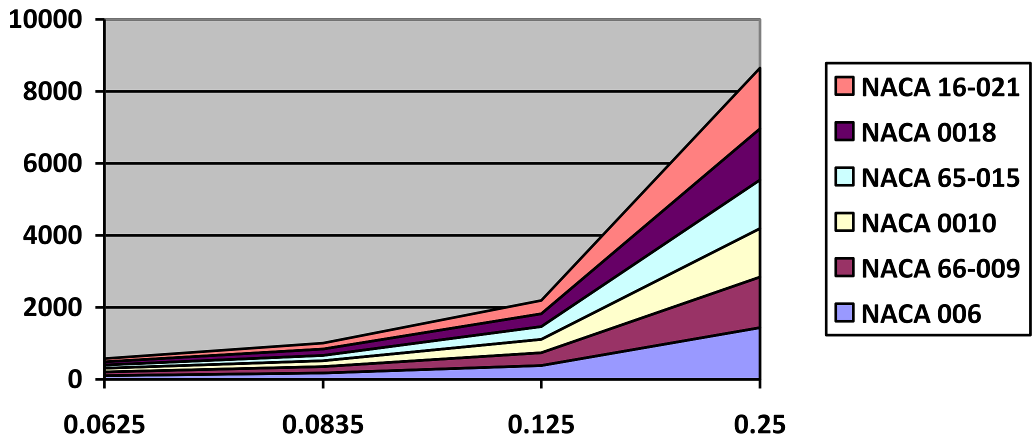

Fig. 5. Variation of vibration frequency in second mode with aspect ratio.

7. Conclusions and Future Work

7.1. Conclusions

Based upon the studies conducted and reported in previous chapters, following major conclusions can be drawn:

(1) It is observed that by increasing the value of aspect ratio, the natural frequency in all the modes is increased.
(2) At higher values of aspect ratio some of the blade sections such as NACA-16-021 start behaving like a plate instead of a beam as the stiffness in cross coupled mode of vibration is found to be increased.
(3) It is also concluded that, there is some optimum value of thickness at which the natural frequencies in fundamental and second mode of vibration are minimum.

7.2. Future Work

(1) In the present study vibration behaviour of a single blade has been studied. It can be extended for a group of blades.
(2) As observed from the present study that at some stage, aerofoil section blade starts behaving like a plate instead of a beam, therefore this requires developing a hybrid element, which can incorporate both types of behaviours of the blade.

References

[1] D. Afolbi, "The rogue failures of turbine blades," *ASME J. of Sound and Vibration*, vol. 122, pp. 158-165, 1988.
[2] E. Cranch and A. Adler, "Bending vibration of variable section beams," *ASME J. of Applied Mechanics*, vol.29, pp 103-108, 1956.
[3] W. Carnegie, "Vibration of pre twisted cantilever blades: An additional effect of torsion," *ASME J. of Applied Mechanics*, vol. 176, pp. 315-319, 1962.
[4] F. Sisto and A. T. Chang, "A finite element method for vibration analysis of twisted blades based on beam theory," *AIAA J.*, vol. 22, pp. 1646-1651, 1984.
[5] D. Rosard and P. Lester, "Natural frequency of twisted cantilever beam," *ASME J. of Applied Mechanics*, pp. 241-244, Jun. 1999.

[6] L. Klein, "Transverse vibration of non-uniform beams," *ASME J. of Sound and Vibration*, vol. 37, pp. 491-505, 1994.
[7] K. J. Lag and S. Reman, "An approach for estimating vibration characteristics of non-uniform blade," *AIAA J.*, vol. 17, pp. 995-1002, 1995.
[8] T. W. Lee, "Transverse vibration of a tapered beam carrying a concentrated mass," *ASME J. of Sound and Vibration*, vol. 43, pp. 366-370, 1996.
[9] J. S. Rao and S. Banerjee, "Coupled bending-torsion vibrations of rotating blades," *ASME Gas Turbine and Fluid Engineering Conference*, no. 76-GT-43, 1976.
[10] B. Downs, "Transverse vibration of cantilever beams having unequal breadth and depth taper," *ASME J. of Applied Mechanics*, vol. 44, no. 4, pp. 737-742, 1977.
[11] D. Dhar and A.M. Shran, "Free vibration analysis of turbine blades using non-linear finite element method,"*AIAA J.*, vol. 35, pp. 590-591, 2000.
[12] J. Thomas and B. Abbas, "Finite element model for dynamic analysis of Timoshenko beams," *ASME J. of Sound and Vibration,* vol. 41, pp. 291-299, 1976.
[13] V. Murthy, "Dynamic characteristics of rotor blades: Integration matrix method," *ASME J. of Turbomachinary*, vol. 115, pp. 595-597, 1997.
[14] A. M. Shran and R. Bahree, "A study of undamped rotor blade vibration frequency due to transient radiative heat flux using finite element analysis," *AIAA J.*, vol. 27, pp. 802-808, 1989.
[15] I. H. Abbott and A. E. Vondoenhoff, *Theory of Wing Sections: Including a Summary Of Aerofoil Data*. New York: Mcgraw-Hill, 1959.