Faculty of Manufacturing Engineering

PROPERTIES OF INDEFINITE MATRIX CONSTRAINTS FOR LINEAR PROGRAMMING IN OPTIMAL SOLUTION

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PROPERTIES OF INDEFINITE MATRIX CONSTRAINT FOR LINEAR PROGRAMMING IN OPTIMAL SOLUTION

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DECLARATION

I declare that this thesis entitled "Properties of Indefinite Matrix Constraint for Linear Programming in Optimal Solution" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ……………………………….

Name : ……………………………….

Date : ……………………………….

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Manufacturing Engineering.

Signature	:	……………………………….
Supervisor Name	:	……………………………….
Date	:	……………………………….

DEDICATION

Dedicated to my mother and family members for their support

Honourable supervisors and lecturers for their supervision

Faithful friends for their encouragement and assistance

ABSTRACT

Finding the optimum solution in engineering and science is a common problem where one wishes to get the objective under certain constraints. This situation is also a typical issue in manufacturing industries where maximum profit and minimum cost are a common objective under certain constraints on the available resources. One approach to solve optimization is to use formulation problem in linear form and subjects to linear constraints, the problem can be deliberated as linear programming problem. The linear constraints can be in a form of a matrix. There are limited researches that discuss the effect of the properties of matrix constraint to the solution. In fact, the matrix constraint has significant influence to the existent of the optimal solution to the optimization problem. This research focused on the investigation of characteristics of non-symmetric indefinite square matrices of linear programming problems which represent the constraints of linear programming problems. The non-symmetric indefinite square matrices are generated randomly by the MATLAB simulation software and its indefinite properties are verified through the principal minor test, quadratic form test and eigenvalues test. The solutions of the primal and dual linear programming problem are simulated and discussed. Optimization software, LINGO, is used to validate the solutions to assure the reliability of the simulated solutions in the MATLAB software. Based on the simulation results, some of the non-symmetric indefinite random matrices found duality gap and those matrices could not provide optimal solution to the problem. Whereas, some indefinite matrices with certain characteristics could achieve optimal solution and no duality gap presented. An indefinite random matrix with all positive off-diagonal entries and the determinant of leading principal minors with positive sign at odd orders and negative sign at even orders surely deliver the optimal solution to the linear programming problems. This research may contribute to the advancement of linear programming solution particularly when the constraints form an indefinite matrix.

ABSTRAK

Mencari penyelesaian optimum dalam bidang kejuruteraan dan sains adalah satu masalah umum apabila seseorang ingin mencapaikan objektif di bawah kekangan tertentu. Keadaan ini juga merupakan isu tipikal dalam industri pembuatan di mana keuntungan maksimum dan kos yang minimum adalah matlamat yang utama di bawah kekangan tertentu terhadap sumber yang sedia ada. Satu pendekatan untuk menyelesaikan pengoptimuman adalah melalui perumusan masalah dalam bentuk lelurus dan subjek kepada kekangan lelurus. Masalah ini boleh dikenalkan sebagai masalah pengaturcaraan lelurus. Kekangan lelurus boleh dibentukkan dalam bentuk matriks. Terdapat penyelidik yang terhad membincangkan pada kesan sifat kekangan matriks kepada penyelesaian. Sebenarnya, kekangan matriks mempunyai pengaruh yang signifikan terhadap penyelesaian optimum dalam masalah pengoptimuman. Oleh itu, penyelidikan ini memberi tumpuan kepada penyiasatan ciri-ciri matriks persegi tidak terbatas yang tidak simetri terhadap masalah pengaturcaraan lelurus yang mewakili kekangan masalah pengaturcaraan lelurus. Matriks persegi tidak terbatas yang tidak simetri dihasilkan secara rawak oleh perisian simulasi MATLAB dan sifat tidak terbatasnya disahkan melalui ujian principal minor, ujian "quadratic form" dan ujian nilai eigen. Penyelesaian masalah pengaturcaraan lelurus pertama dan kedua disimulasikan dan dibincangkan. Perisian pengoptimuman, LINGO, digunakan untuk mengesahkan kebolehpercayaan penyelesaian simulasi dalam perisian MATLAB. Berdasarkan keputusan simulasi, beberapa matriks persegi tidak terbatas yang tidak simetrididapati menjumpai jurang dualiti dan matriks tersebut tidak dapat memberikan penyelesaian yang optimum terhadap masalah tersebut. Sebaliknya, beberapa matriks tidak terbatas dengan ciri-ciri tertentu boleh mendapatkan penyelesaian yang optimum dan tiada jurang dualiti yang dibentangkan. Matriks rawak yang tidak terbatas dengan semua penyertaan luar pepenjuru yang positif dan penentu tanda positif pada perintah ganjil dan tanda negatif pada perintah ganda akan memberikan penyelesaian yang optimum kepada masalah pengaturcaraan lelurus. Kajian ini boleh menyumbang kepada perkembangan masalah pengaturcaraan lelurus terutamanya untuk kekangan yang dalam bentuk matriks tidak terbatas.

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LIST OF ABBREVIATIONS

FCHV	-	Fuel Cell Hybrid Vehicle
FLP	-	Fuzzy Linear Programming
ID	-	Indefinite
L	-	Lower Triangular Part
LP	-	Linear Programming
ND	-	Negative Definite
NSD	-	Negative Semidefinite
PD	-	Positive Definite
PSD	-	Positive Semidefinite
RHS	-	Right Hand Side
U	-	Upper Triangular Part

LIST OF PUBLICATIONS

Journal

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Researchers and decision makers in industries are often challenged to investigate the optimum solution to problems with constraints. In manufacturing industries, there is variety optimization problems found in production planning and scheduling. For example, in a production area, manager needs to decide a balance mix of product variety and quantity in order to maximize profit or minimize cost. In fields of research, scientists look at different levels of variables to get the optimum response.

There are numbers of optimization problems found in the real world situation; these problems include continuous optimization, constrained optimization, deterministic optimization and stochastic optimization. Continuous optimization problem is a model involves continuous variables. Constrained optimization problem arises from applications in which the variables have definite constraints, i.e. linear and non-linear. Deterministic optimization problem is a problem that formulated with known parameters while stochastic optimization problem involves random variables in the formulation of the optimization problem itself (Hannah, 2015).

Various types of optimization techniques are tailored for different optimization problems. Every technique has its own strengths and advantages. For example, heuristic approaches are implemented by Soylu (2015) to search for the feasible solution in a bi-objective mixed 0-1 integer linear programming problem, a constrained programming approach was suggested by Gao and Qin (2016) to settle the location problems in a p-hub centre by finding new locations for the p-hubs with the aim of minimizing the travel time between other nodes to the hubs nodes while Zhou (2017) proposed a hybrid stochastic and deterministic optimization approach which can increase the reliability and stability of simulation processes when solving problems with random initializations.

A large scale of applied mathematics is involved in the optimization theory and optimization techniques. The optimization techniques frequently used by decision makers in solving optimization problems are linear programming, quadratic programming, heuristic methods and dynamic programming (Vasant et al., 2016).

Basically, optimization problems can be categorized into linear and nonlinear types. Linear programming problem involves model with linear equation and it is always a polynomial of degree one (1). In two dimensions, they always form lines while in other dimensions; they might form planes, points or hyperplanes. A nonlinear programming problem consists of nonlinear objective function or some of the constraints are nonlinear. The nonlinear equations in nonlinear problem comprise higher degree polynomials, such as square roots and trigonometric functions.

Linear programming is assuredly to be the easiest and a natural mechanism for formulating enormous array of problems with modest work. The linear programming formulations are well-known than nonlinear programming for several reasons. Its mathematics is better, the theory is broader and it is easier to be computed and implemented when solving optimization problem (Cheney and Kincaid, 2012).

The foundation perceptions of linear programming were presented by Dantzig (1947) and it often abbreviated as LP. Although this model was initiated half a century ago, it is still an active field for research and an effective method for modelling and solving problems in the 21st century (Dowman and Wilson, 2002). The linear forms of constraints and objective functions in linear programming problem allow decision makers to easily define them. The formulation of LP model involves linear objective function and linear equalities (=) or inequalities constraints (≥). A decision or a set of decision variables is essential to achieve a state goal of objective under certain constraints (Luenberger and Ye, 2008).

Moreover, LP known as mathematical programming. For every mathematical programming, the problem is required to be transformed into mathematical model by comprising all necessary criteria of that problem. The purpose of applying LP is to search for optimal solution with the purpose of minimizing or maximizing the objective function by involving all the requirements (Hillier and Lieberman, 2001). However, LP solution may involve fractional value answer to the value of decision variables (Ma et al., 2012).

The implementation of LP approaches in solving optimization problems was revealed in different fields of study, such as business, economic and engineering. Besides, it can be a decision support tool for modelling various sorts of problems in scheduling, planning, assignment and routing. The original LP problem is known as primal problem and there is a reflection of that primal problem, which is known as dual problem. Dual problem can be used as original program solution and surpass advantageous knowledge of the optimal solution to the primal problem (Bazaraa et al. 2010).

In general, the LP mathematical formulation can be written as:

Objective function: Maximize/Minimize $f(x) = c^T x$ (1.0)

Subject to: $A_{mxn} \geq b$ (1.1)

$x \geq 0$ (1.2)

where $f(x)$ is the objective function, maximizing or minimizing; c^T is a coefficient vector of objective function; x is a vector variable; A_{mxn} is a matrix constraint and b is right hand side (RHS) vector.

1.2 Problem Statement

The application of LP formulation in solving optimization problem has been widely discussed and studied by researchers. One of the interesting part to be studied in LP formulation is the structure of the coefficient of matrix constraints, A_{mxn}. Various types of matrix are available in linear algebra and different characteristics of matrix can provide altered solutions to the problems.

The matrices in general categorized into five characteristics, which are Positive Definite (PD), Negative Definite (ND), Positive Semidefinite (PSD), Negative Semidefinite (NSD) and Indefinite (ID). Previous publications have discussed the characteristics of symmetric matrices and their relationship with the primal and dual concept in LP problems. The major problems associated with definiteness of the symmetric matrix and duality concept in LP have been summarized as follows:

i. Previous study has shown ND matrix, NSD matrix and some ID matrices do not provide optimum solution. Negative definite matrix is associated with concave function (Romli, 2013).

ii. Preceding studies use symmetric matrices to verify the LP duality to primal and dual solution (Dhagat and Tiwari, 2009; Nasseri et al., 2010; Lavaei and Low, 2012; Boley, 2013). However, it is hardly find in previous research on the implementation of non-symmetric matrices in solving the LP duality problem.

The unique characteristics of ID matrix make it unable to deliver a stable and definite solution to LP problems. There is a limited source of information about the ID matrix characteristics in relation to primal and dual solution of LP. Although there are number of publications on primal and dual problems, but the study on the properties of random non-symmetric square matrices in the primal and dual problem has not been performed yet. Besides, the effect of ID matrix to the solution has not been explored. So, this research aims to examine the effect of different characteristics of random non-symmetric ID square matrices to the primal and dual solution of LP problems.

1.3 Objectives

i. To simulate the LP problems with various numbers of variables and constraints based on the characteristics of non-symmetric indefinite square matrices.
ii. To investigate the effect of non-symmetric indefinite square matrices to the primal and dual solution of LP problems through MATLAB simulation.
iii. To study the duality gap in the solution of the non-symmetric indefinite square matrices.

1.4 Scopes

Previous studies have considered the properties and characteristics of different definiteness of symmetric matrices to the LP solution, i.e. PD, ND, PSD and NSD. The study on ID matrix to the LP problems is not widely discussed by those researchers and there is limited resources related to ID matrix.

To further advance the knowledge on the characteristics of ID matrix, this study narrowed down the study scope by focusing on the properties and characteristics of ID matrix and examining its effect to the primal and dual LP problem. Other four characteristics of definite properties of matrix were not considered, which is PD, ND, PSD and NSD.

Furthermore, this study also explored non-symmetric of ID matrix to broaden the knowledge on this field of research. The works in this study covers the following

i. Random number generator in MATLAB software is used to simulate the parameters in the LP problem formulation with sizes of n variables and m constraints.

ii. Number of problems of order sizes of $5x5$, $10x10$ and $20x20$ is simulated based on three characteristics of ID non-symmetric square matrices for validation purposes.

iii. Small sized problem is considered and simplex method is chosen to solve the LP problems in this study.

iv. MATLAB is used to solve the generated LP problems and LINGO software is used to validate the reliability of the achieved LP solutions provided by MATLAB Software.

1.5 Significant of Study

This research examines and investigates the effects of random ID non-symmetric matrices on the primal and dual solution of LP problems. Furthermore, this research studies on the correlation of different sizes and characteristics of ID non-symmetric matrices to the LP solutions to provide new sources of information in this field. Last but not least, this research explores random non-symmetric matrices in providing results as symmetric matrices in solving the primal and dual problems.

1.6 Organization of Thesis

The overview of the whole thesis was summarized and structured into five chapters. The general information of each chapter has been described as follows:

Chapter 1 introduces the background on optimization problems and the optimization algorithm used in this research with linear programming approach. The problem statements highlight the current issues in related research field. The objectives and scope are determined in order to achieve the target of this research. Significance of this research is deliberated to show the relationship between ID non-symmetric matrices to the solution of LP problems.