Modelling the Deceleration Rate in the Train Braking Profile

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Abstract **- This paper deals with the analysis of deceleration rate in the train braking profile for one of major transportations company in the Europe. The aim is to establish the relation between the deceleration rate and the factors preferred by the client. Of all the factors, the most preferred factor was an average gradient experienced by the train. The method used in this paper is hard technique of Operational Research. Mathematical calculation is used to generate the average gradient experienced by each train which will be used to match with the deceleration rate to establish the relation between these two variables using regression analysis. As a conclusion, there was a relation between deceleration rate and average gradient experienced by the train and it was noticeable that driver's actual braking performance of applying deceleration rate was affected by the varying gradient more than constant gradient. As an additional work, the relation between braking distance and deceleration rate is also established. The model can be used as an initial study to determine the distance when the driver should start to brake optimally in further study.**

Index Terms: **Regression, modeling, braking profile and train.**

I. INTRODUCTION

It is a major concern of customers to expect higher standard from any transportation company and cite reliable as a top concern. According to [1] journey time is a key performance factor in mass transit systems.

A research done by the problem owner [2] showed there is strong correlation between runtime and brake rate of which the deceleration rate had been applied by the driver. In detail, this paper aims to determine the relation between the deceleration rate and the infrastructure condition in the braking profiles for all the in-service run of trains.

The braking profile is the portion of the inter-station run from the point which the driver begin to brake as the train approaches the destination platform, to the point at which the wheels stop turning at the platform [3]. It must be noted that in this study, the brake rate is referred as the deceleration rate applied by the driver during the braking profile. Basically, every in-service train has a different braking profile [4],[5].

It can be noted that the average deceleration rate on the braking profile is influenced by the entry speed (the initial speed of when the train start to brake on braking profile), the distance (referred as braking distance in this project) or length of time for braking and finally the gradient.

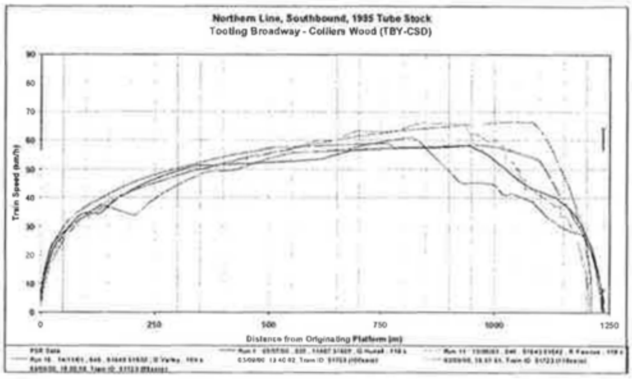


Fig. 1. One of the graph in Excel Multigraph file produced to study

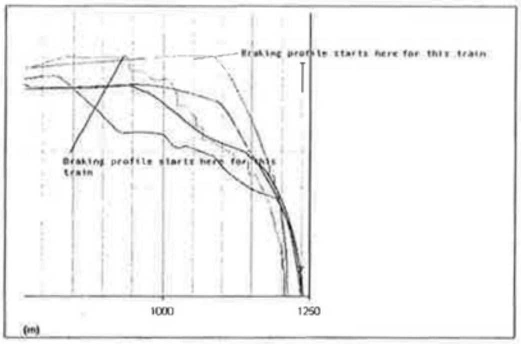


Fig. 2. Braking profile for each in-service train from Fig.1. In general, different trains have different braking profile as each train starts to brake at different distance.

Figure 1 shows the performance of the in-service train at a specific link in one of the lines in terms of speed against distance [6]. Run time for this link is the length of time taken from when the train departs the starting platform to the time the train stops at destination platform. There are 3 in-service trains in the multigraphs with date, time and train ID provided. However, the only part of the run that is being analysed in this study is the braking profile, which has been zoomed in Figure 2.

Currently the simulator [2],[3],[4] developed by the client assumes a fixed brake rate with deceleration of

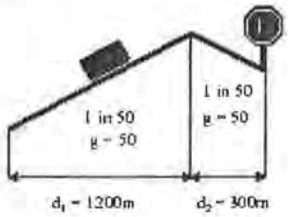
$0.6ms^{-2}$. Real drivers will brake with different deceleration rate. Therefore, it is necessary to establish how this changes when approaching the destination station and the impact they have on total run times. It is aimed to determine the deceleration rate when the driver start to brake and the infrastructure condition, narrowing down to the braking profiles for all the in-service run of trains only.

II. OBJECTIVES

It is of great interest of the problem owner to analyse the potential effects of factors on deceleration rate by identifying the relation between average deceleration rate and infrastructure conditions. Of this, only average gradient experienced by the trains will be considered since it is believed to be the most important one by the problem owner.

III. METHODOLOGY

The negative sign shows the train encountered an uphill gradient of the track while the positive sign shows the train encountered a downhill gradient of the track. A calculation from average gradient concept [7] is applied to get the value of average gradient, G, as follows:



$$G = \frac{D}{\left(\frac{d_1}{g_1} + \frac{d_2}{g_2} + \ldots + \frac{d_n}{g_n}\right)}$$

where $D = d_1 + d_2 + \ldots + d_n$

The mass of the train is uniformly distributed throughout the length of the train i.e. the centre of mass is longitudinally in the centre of the train. There has been some research [7] to support this assumption.

In general, average gradient experienced by each train might be different even though they are in the same link because the calculation of average gradient is based on the distance from the starting point of braking to the platform stop and this distance depends on when the driver started to brake.

To conduct statistical analysis in this project, the information needed are average deceleration rate, gradient profile to generate average gradient for all runs on the links and selected links to be studied together with detail of characteristics to support the analysis.

IV. RESULTS OF CASE STUDY

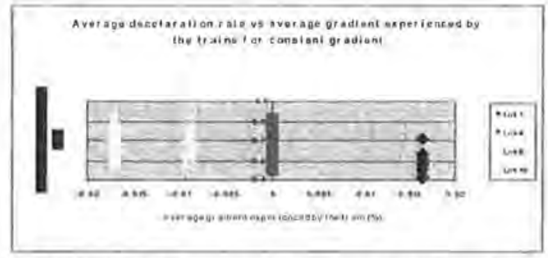


Fig. 3. Variability in average decelaration rate still present even though the trains experienced the same average gradient on the particular links.

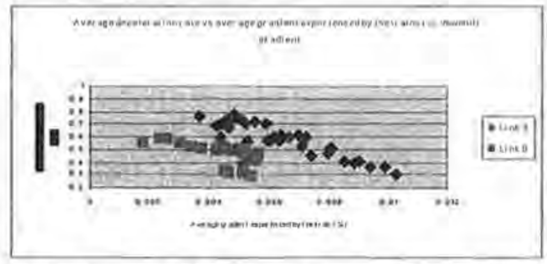


Fig. 4. Average deceleration rate against gradient for all the in-service trains that experienced a varying downhill average gradient.

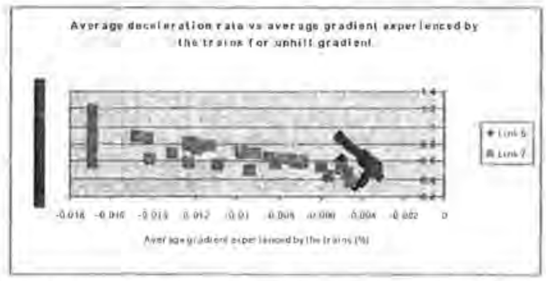


Fig. 5. Variability in the average deceleration rate for all the in-service train which experienced a varying uphill average gradient.

Based on Figure 3, the differences or variability of the average deceleration rate applied by the trains still exist even though the train experienced the same average gradient. Clearly the point at zero gradient measure the deceleration rate actually applied by the driver and so the variability shown is real. The reason for there being a similar degree of variability for other instances of constant gradient is probably the effect of different speed and braking distance. Based on Figure 4, it can be noted that for varying downhill gradient, the steeper the average gradient, the lower the deceleration rate. Based on Figure 5, It can be noted that for varying uphill gradient, the steeper the average gradient, the higher the deceleration rate.

A. Regression analysis for average deceleration against average gradient

Figure 6 shows all the actual observation of actual average deceleration rate for all selected links regress with the percentage of average gradient experienced by the trains

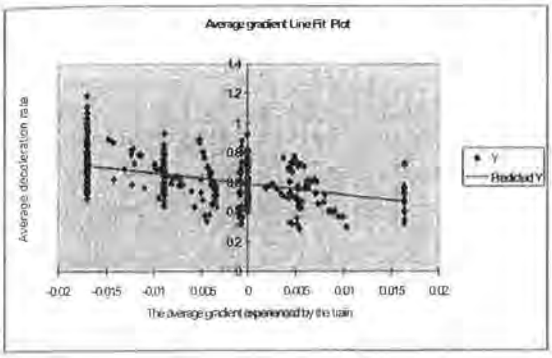


Fig. 6. The least square line fit the trend of the data.

for uphill, downhill and level gradient of the track on the braking profile.

Based on the summary output produce by Excel, the regression model for the analysis between average deceleration rate and average gradient experienced by the train is establish as follows:

$$a = 0.5905 - 7.3085(G) + \varepsilon$$

where a is average deceleration rate and G is average gradient rate.

Interpretation of the model for both uphill and downhill average gradient in percentage are as follows:

1. The estimated y-intercept, is 0.5905. This indicates that the estimated average deceleration rate on braking profile is equal to $0.5905 ms^{-2}$, when the percentage of average gradient experienced by the train is 0, which means the train move on a level track. The average gradient could be either negative (if the train experienced an uphill gradient on the braking profile) or positive value (if the train experienced a downhill gradient on the braking profile). However, the model parameter should be interpreted only within the sample range of average gradient between –0.99 to +0.99 only because the percentage of average gradient cannot be out of this range according to average gradient concept.

2. The slope, of the least squares line is calculated to be –7.3085. For this part, result for uphill and downhill gradient is interpreted differently. If the train experienced a downhill gradient (with positive value for the percentage of average gradient), the mean average deceleration rate is estimated to decrease by $7.3085 ms^{-2}$ for every unit increase of percentage of average gradient. It shows that the steeper the downhill gradient, the lower the average deceleration rate. If the train experienced an uphill gradient (with negative value for the percentage of average gradient), the mean average deceleration rate is estimated to increase by $7.3085 ms^{-2}$ for every unit increase of percentage of average gradient. It shows that the steeper the uphill gradient, the higher the average deceleration rate.

3. Variation of average deceleration rate in the random error distribution. There will almost certainly be some variation in average deceleration rate due strictly to random phenomena that cannot be anticipated or explained. Random error, ε is referred to all unexplained variations in average deceleration rate that care caused by important but unincluded variables or by unexplained random phenomena such as braking distance, entry speed and etc. Since we allow random error, this model is known as a probablistic model. In other words, we hyphothesizing a probabilistic relationship between average deceleration rate and percentage of average gradient on braking profile.

4. For random error distribution, ε, the true value of σ^2 is unknown. Using the data available, the best estimate of σ^2, denoted by s^2:

$$\text{Estimated } \sigma^2 = s^2 = 0.016525$$
$$\text{Estimated } \sigma = s = 0.1286$$

The variability of this model is small because the s^2 is small. Therefore, we can assume the error in the estimation of the model parameters of intercept and slope together with the error of prediction when Y is used to predict y for some value of X is also small. S measure the spread of distribution of y values about the least square line, so it is not surprised to find that most of the observations lie within 2s or 2(0.1286)= 0.2572 of the least square line.

5. Test for the usefulness of the hypothesized model:

To test the null hypothesis that the linear model contribute no information for the prodiction of average deceleration rate, Y, against the alternative hypothesis that linear model is useful for predicting average deceleration rate, the test are:

$$H_o: b = 0$$
$$H_1: b \neq 0$$

Test statistic is Student's t statistic and from the summary output,

$$T = -11.6579$$

It is two-tailed test, with $\alpha/2 = 0.05/2 = 0.025$. Ho is rejected because the p value is smaller than $\alpha/2$. Therefore, there is a linear relationship between average deceleration rate and average gradient experienced by the trains.

6. The coefficient of determination = $r^2 = 0.25455$

It implies that 25% of the sum of squares of deviations in the sample of Y value about Y is explained by the average gradient experienced by the trains, x. The higher the coefficient of determination of the model, the better the predicted model. Since it is only 25% of the variation of average deceleration rate is accounted for by the differences in average gradient and only one independent variable in the model, this relatively small value of r^2 should not be too surprising. If other variables related to average deceleration rate (such as braking distance, entry speed and curvature displayed in problem structuring before) were included in the model, they would probably account for a significant portion of the remaining 75% of the variation in average deceleration rate not explained by the average gradient experienced by the trains.

B. Additional work: Statistical analysis for braking distance experienced by each train

As explained above, even though the average gradient is constant, but there is still variability in the average deceleration rate. Taking braking distance as a predictor or independent variable, which might have an impact on the average deceleration rate, the plotted graph can be shown as in Figure 7 bellow:

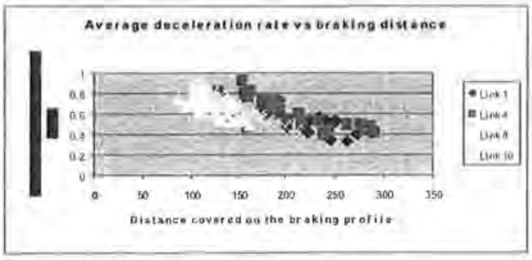


Fig. 7. Graph shows the average deceleration rate against the braking distance on the braking profile.

As discussed in the previous chapter there is also a relationship between the point before the station at which the driver starts to brake and the deceleration rate applied on the braking profile. Despite the condition to least square not being satisfied, it was nevertheless taught useful to regress deceleration rate versus braking distance for selection of the case where the gradient are constant.

From human eye, we could see that there is a correlation between average deceleration rate and the braking distance approaching the destination platform based on Figure 8. The natural hypothesis would be the larger the braking distance, the lower the deceleration rate applied by the train.

C. Regression analysis for average deceleration rate and the braking distance

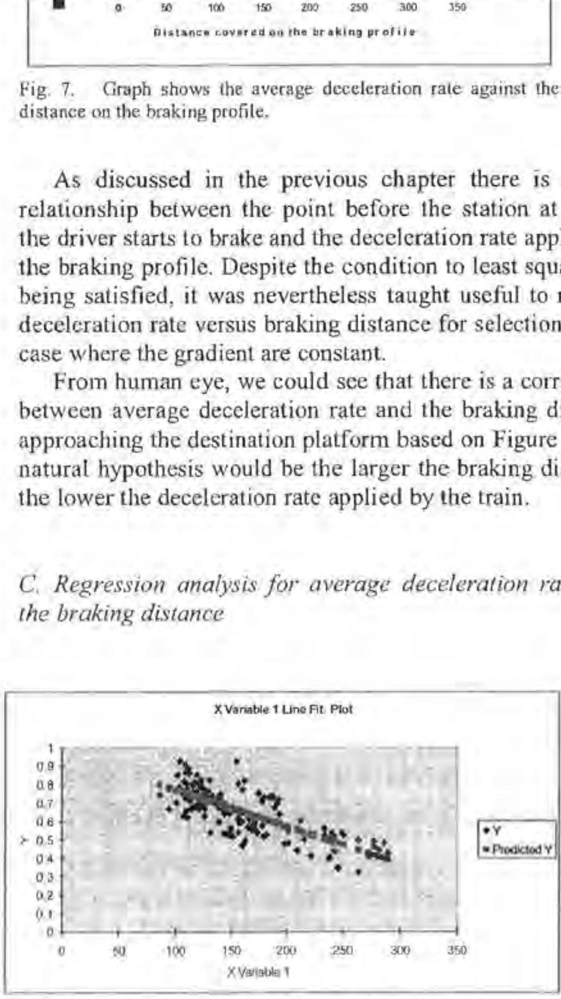


Fig. 8. The least square line fit the trend of the data.

The regression model for the analysis between average deceleration rate and average braking distance experienced by the train is establish as $a = 0.9686 - 0.00199(d) + \varepsilon$ where d is braking distance.

The coefficient of determination in the braking distance $r^2 = 0.585$. This means that the 58% of the variation in the average deceleration rate are accounted for by the differences in braking distance in this model. The r^2 for braking distance model is higher than r^2 for percentage of average gradient experienced by the train. This imply that the braking distance have an impact on average deceleration rate when the driver is braking approaching the platform on the braking profile more than average gradient experienced by the train.

The impact of the factors on the average deceleration rate while braking could be seen in Figure 9 below.

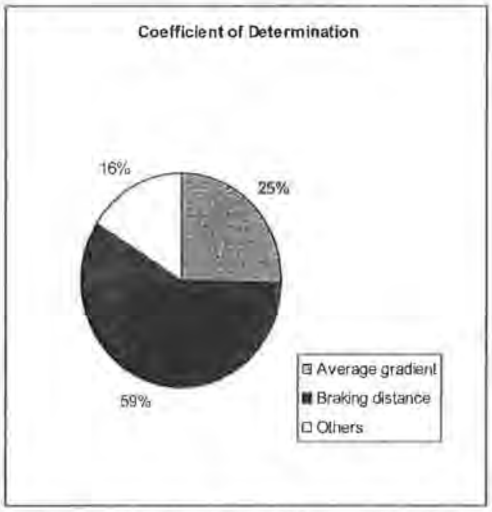


Fig. 9. The percentage of coefficient of determinations showing how much the variability of the average deceleration rate effected by the factors.

V. CONCLUSION AND FUTURE WORK

With strong correlation between deceleration rate while braking on the braking profile and run time figured out by previous research done by the client , it is important to identify what affect does the performance of brake rate has in the operation. This study concern on identifying which factors give an impact on deceleration rate while braking and how it effect the deceleration rate by establishing the relation between average deceleration rate and the selected factors preferred by the client.

There are various factors that have impacts on the average brake rate on the braking profile such as average gradient, braking distance, entry speed, curvature and etc [8]. In average, the deceleration rate for braking on all the links range form $0.5ms^{-2}$ to $0.75ms^{-2}$ and the current simulator average deceleration rate is $0.6ms^{-2}$. According to the client, the ideal deceleration rate the train can achieve is $0.7ms^{-2}$ but due to poor braking techniques, the deceleration rate becomes lower than that.

The main objective in the whole for the client is to get an optimum deceleration rate, so that the journey time could be optimised as well. In order to get an optimum deceleration rate, all the factors affecting the performance of average brake rate should be identified and evaluated as how it gives impact on deceleration rate on the braking profile. The only factor preferred by the client is average gradient experienced by the train presently. Obviously, the gradient of the track could not be changed since it is only possible to establish the relation between average deceleration rate and average gradient experienced by the train as follows:

Average deceleration rate = 0.5905 – 7.3085 (Average gradient) + ε

However, we could change the average gradient experienced by each train if we change the distance of where it starts on braking. More ever, after doing regression for braking distance, we could see the relation between average deceleration rate and braking distance which conclude that the braking distance is inversely proportionate to the deceleration rate.

The relation between the average deceleration rate and braking distance are as follows:

Average deceleration rate = 0.9686 – 0.00199 (Braking distance) + ε

From the analysis, one could conclude that the data shows that driver actual braking performance is affected by varying gradient more than constant gradient. More ever, it is also important for the client to determine the optimum braking distance since by having an optimum braking distance, train can overcome the force of gravitational optimally whenever the train experience whatever average gradient.

In the light of the conclusion above, it is possible to make a few recommendations for future work to the client. They should consider placing a marker board for the driver to start applying brake when approaching the destination station. This can only be done if the braking distance is calculated. The braking distance for every link will be different so that the drivers know the specific distance where they need to start applying brake. The braking distance can be generated based on one of equations from literature of Calculation for Train Braking Distance[7]. More ever, the deceleration can be calculated by using equation in literature [10].

Finally, it would be a better model for further research if all relevant factors could be combined together to determine the effect of the factors alone and when it interact with each other as a multivariate analysis.

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VII. BIOGRAPHY

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