



**Faculty of Electrical Engineering**

**ELECTRICITY DISTRIBUTION NETWORK FOR LOW AND  
MEDIUM VOLTAGES BASED ON EVOLUTIONARY APPROACH  
OPTIMIZATION**

**Ihsan Jabbar Hasan**

**Doctor of Philosophy**

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**ELECTRICITY DISTRIBUTION NETWORK FOR LOW AND MEDIUM  
VOLTAGES BASED ON EVOLUTIONARY APPROACH OPTIMIZATION**

**IHSAN JABBAR HASAN**

**A thesis submitted  
in fulfillment of the requirements for the degree of Doctor of Philosophy**

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**2015**

## DECLARATION

I declare that this thesis entitled “ELECTRICITY DISTRIBUTION NETWORK FOR LOW AND MEDIUM VOLTAGES BASED ON EVOLUTIONARY APPROACH OPTIMIZATION” is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in the candidature of any other degree.

Signature :

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Date : / /

## APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is significant in terms of scope and quality for the award of Doctor of Philosophy.

Signature :

Supervisor Name: **Datuk Professor Dr. Mohd Ruddin Bin Ab Ghani**

Date : / /

## **DEDICATION**

This thesis dedicated to my lovely family.

## ABSTRACT

The optimum planning of distribution systems consists of the optimum placement and size of new substations, feeders, capacitors, distributed generation and other distribution components in order to satisfy the future power demand with minimum investment and operational costs and an acceptable level of reliability. This thesis deals with the optimization of distribution network planning to find the most affordable network design in terms of total power losses minimization and voltage profiles improvement. The planning and operation of distribution networks are driven by several important factors of network designing. The optimum placement and sizing of the capacitor banks into existing distribution networks is one of the major issues. The optimum placement and sizing of the new substations and distribution transformers with adequate feeder connections with minimum length and maximum functionality are vital for power system as well as optimum placement and sizing of the distributed generators into the existing grid. This thesis commonly investigated the impacts of these factors on voltage profile and total power losses of the networks and aims to reduce the capital cost and operational costs of the distribution networks in both LV and MV levels. Optimum capacitor installation has been utilised in terms of reactive power compensation to achieve power loss reduction, voltage regulation, and system capacity release. The Particle Swarm Optimization (PSO) is utilized to find the best possible capacitor placement and size. The OpenDSS engine is utilized to solve the power flow through MATLAB coding interface. To validate the functionality of the proposed method, the IEEE 13 node and IEEE 123 node test systems are implemented. The result shows that the proposed algorithm is more cost effective and has lower power losses compare to the IEEE standard case. In addition, the voltage profile has been improved. Optimum placement of distribution substations and determination of their sizing and feeder routing is another major issue of distribution network planning. This thesis proposes an algorithm to find the optimum distribution substation placement and sizing by utilizing the PSO algorithm and optimum feeder routing using modified Minimum Spanning Tree (MST). The proposed algorithm has been evaluated on the two types of distribution network models which are the distribution network model with 500 customers that includes LV residential and commercial loads as well as MV distribution network, and 164 nodes in MV level. The test network is generated by fractal based distribution network generation model software tool. The results indicate that proposed algorithm has succeeded in finding a reasonable placement and sizing of distributed generation with adequate feeder path. Another sector of power system that is taken into account in this work is Distributed Generators (DGs). In power system, more especially in distribution networks, DGs are able to mitigate the total losses of the network which effectively has significant effects on environmental pollution. This thesis aims to investigate the best solution for an optimal operation of distribution networks by taking into consideration the DG. The PSO method has been used to solve the DG placement and

sizing on the IEEE 34 and 123 nodes test systems, respectively. It has been utilized to demonstrate the effectiveness of the PSO method to improve the voltage profile and minimize the cost by mitigating the total losses of the network.

## ABSTRAK

*Perancang optimum dalam sistem pengagihan mengandungi penempatan optimum dan pemilihan saiz yang optimum tentang pencawang baru, penyuiap, kapasitor, penjana teragih komponen lain-lain dalam system pengagihan. Matlamat utama perancang optimum ada untuk memuaskan permintaan kuasa masa depan dengan pelaburan dan operasi kos yang minimum serta tahap kebolehpercayaan yang boleh dipercayai. Tesis ini berurusan dengan pengoptimuman rancangan sistem pengagihan untuk mencari reka bentuk rangkaian yang paling berpatutan dari segi pengurangan jumlah kehilangan kuasa serta peningkatan voltan profil. Perancangan dan operasi rangkaian pengagihan didorong oleh beberapa faktor penting dalam mereka rangkaian. Penempatan dan saiz bank kapasitor yang optimum ke dalam rangkaian pengagihan merupakan salah satu isu utama. Penempatan dan saiz pencawang baru yang optimum dan transformer pengagihan dengan sambungan penyuiap yang mencukupi dengan panjang yang minimum dan fungsi maksimum adalah penting bagi sistem kuasa serta penempatan dan saiz penjana teragih kepada grid yang sedia ada. Tesis ini menyiasat kesan faktor-faktor ini terhadap profil voltan dan jumlah kerugian kuasa rangkaian bertujuan untuk mengurangkan kos modal dan kos operasi rangkaian pengagihan di kedua-dua peringkat LV dan MV. Pemasangan kapasitor yang optimum telah digunakan dari segi pampasan kuasa reaktif bertujuan untuk mencapai pengurangan kuasa kerugian, pengatur voltan, dan pelepasan kapasiti sistem. Particle Swarm Optimization (PSO) digunakan untuk mencari penempatan kapasitor dan saiz yang terbaik. OpenDSS digunakan sebagai enjin untuk menyelesaikan aliran kuasa melalui antara muka system coding MATLAB. Untuk mengesahkan fungsi kaedah yang dicadangkan, IEEE 13 bus dan IEEE 123 bus telah digunakan. Hasil kajian menunjukkan bahawa algoritma yang dicadangkan adalah lebih kos efektif dan mempunyai kehilangan kuasa yang lebih rendah berbanding dengan kes piawai IEEE. Di samping itu, profil voltan yang telah bertambah baik. Penempatan pencawang pengagihan yang optimum dan penentuan saiz pencawang pengagihan serta penyuiap adalah satu lagi isu utama dalam perancangan rangkaian pengagihan. Tesis ini mencadangkan algoritma untuk mencari penempatan pengedaran pencawang dan saiz yang optimum dengan menggunakan algoritma PSO serta penyuiap yang optimum dengan menggunakan Minimum Spanning Tree yang diubahsuai. Algoritma yang dicadangkan itu telah dinilai pada kedua-dua jenis model rangkaian pengagihan yang merupakan rangkaian pengagihan dimodel dengan mempunyai 500 pelanggan termasuk LV beban kediaman dan komersil serta rangkaian pengagihan MV, dimana 164 nod terdapat pada tahap MV. Rangkaian ujian dihasilkan dengan menggunakan alat perisian model generasi rangkaian pengagihan yang berasaskan fraktal. Keputusan menunjukkan algoritma yang dicadangkan itu telah berjaya dalam mencari penempatan dan saiz penjana teragih yang munasabah dengan penyuiap yang mencukupi. Penjana teragih (DGs) merupakan satu lagi sektor sistem kuasa yang diambil kira dalam tesis ini. Dalam sistem kuasa, terutamanya rangkaian pengagihan,*



*DGs dapat mengurangkan jumlah kerugian rangkaian dengan berkesan. Tesis ini bertujuan untuk mengkaji penyelesaian terbaik bagi operasi optimum rangkaian pengagihan dengan mengambil kira DG. Kaedah PSO digunakan untuk menyelesaikan kedudukan dan saiz DG pada system ujian IEEE 34 dan 123 nod. Ianya telah digunakan untuk menunjukkan keberkesanan kaedah PSO untuk meningkatkan profil voltan dan meminimumka kos dengan mengurangkan jumlah kehilangan rangkaian.*

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## LIST OF ABBREVIATIONS

ACO	Ant Colony Optimization
CSV	Comma-Separated Value
DG	Distributed Generation
DP	Dynamic Programming
DSP	Distribution System Planning
EPRI	Electric Power Research Institute
GA	Genetic Algorithm
Gbest	Best Global Position
GIC	Geomagnetically-Induced Currents
GIS	Geographical Information Systems
IHS	Improved Harmony Search
LV	Low Voltage
MILP	Mixed-integer linear programming
MINLP	Maximum Integer Non-Linear Programming
MST	Minimum Spanning Tree
MV	Medium Voltage
NFE	Number of Function Evaluation
NLP	Non-Linear Programming
OpenDSS	Open Distribution System Simulator

PSO	Particle Swarm Optimization
Pbest	Best Personal Position
SA	Simulated Annealing
SAIDI	System Average Interruption Duration Index
SAIFI	System Average Interruption Frequency Index
TS	Tabu Search

## LIST OF SYMBOLS

$X_i^t$	Current position of Particle
$X^{Pbest}$	Best personal position of Particle
$X^{Gbest}$	Best global position of Particle
$V_i^t$	Velocity of particle $i$ towards previous vector
$V_i^{t+1}$	Velocity of particle $i$ towards next position
$w$	Inertia weight factor
$\alpha$ & $\beta$	Acceleration coefficient
OF	Objective Function
$C_1$	Personal learning coefficient
$C_2$	Global learning coefficient
$r_1$ & $r_2$	Uniformly distributed random numbers
Z	Total cost function
$C_L$	Cost of real power losses
$P_{Loss}^i$	Active power losses on bus $i$
$C_{Cost}^j$	Cost of capacitor $j$
$C_C^j$	Capacity of capacitor $j$

$nb$	Number of buses
$nc$	Number of selected capacitor
$V_i$	Voltage magnitude at bus $i$
$\delta_i$	Voltage angles of bus $i$
$Y_{ij}, \phi_{ij}$	Line admittance magnitude and angle between buses $i$ and $j$
$Q_{\max}$	Demanded reactive power
$Q_C^j$	Capacitor size at bus $j$
$CL$	Total losses cost for a study year
$VC_s$	Total substation variable cost
$C_{var}(j)$	Cost of substation $j$
$d_{lb}(j,i)$	Consumer demand $i$ which connected to substation $j$
$FC_s$	Total fixed cost of substations
$C_{fix}(j)$	Fixed cost of substation $j$
$IC$	Investment Cost
$C_S$	Total substation installation and operation costs
$C_l$	Total cost of the lines
$CC$	Annuitized capital cost
$D$	Discount rate
$T$	Number of operation years
$PF$	Penalty factor
$N_v$	Number of violations
$\beta$	Fine rate
$D_j^i$	Distance between substation $j$ to consumer $i$

$R_{\max}^j$	Maximum acceptable radius of substation $j$
$\Delta V$	Line voltage drop
$Q$	Reactive power
$L_{real\ power}$	Real Power Losses
$APQ_i$	Active power from output bus $i$
$APD_i$	Active power on demand bus $i$
$VS_i$	Voltage from sending bus $i$
$VR_i$	Voltage on receiving bus $i$
$Ysr_i$	Admittance of sending and receiving bus $i$
$\delta s_i$	Phase angle of sending bus $i$
$\delta r_i$	Phase angle of receiving bus $i$
$\theta y_i$	Phase angle of $Y_i < \theta_i$
$V_P$	Voltage profile objective function
$V_{rate}$	Rated voltage
$\gamma$	Violation coefficient