

### UNIVERSITY OF STRATHCLYDE

( Department of Electronic and Electrical Engineering )

## 19900: MSc Individual **Project**

## **Final Report: Improving Frequency and ROCOF Accuracy During Faults, for P Class Phasor Measurement Units**

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Submission Date: 31/10/2014

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## Acknowledgments

A million thanks to Dr Andrew Roscoe as my MSc Project Supervisor for his concern, generous and continuous support in order for me to get the project completed within some period of time.

I also would like to thank the following person for providing me with academic advise, technical advice and suggesting valuable and utmost information towards the completion of this MSc Project:

- 1. Prof. John Soraghan ( Post Graduate Director )
- 2. Dr. Stephen Weiss (Post Graduate Deputy Director)
- 3. Dr. Lina Stankovic (MSc Project Co-ordinator)
- 4. Dr. Victoria Catterson (Academic Advisor)

Also sweet and special thanks are extended to my beloved wife Ruzana Mohd Monil and my kids ( Along, Angah, Aiman, Aina, Alya, Aisya and Amsyar ) and not forgotten my relatives in my hometown who have been fully supporting me during my studies with their supporting, helping, encouragements and patience from the beginning of the course till the end.

#### **Abstract**

The report is basically improving or reducing the peak excursions of frequency and ROCOF (Rate of Change of Frequency) measurements from P class Phasor Measurement Unit (PMU) during unbalanced faults. It begins with some literature reviews from the paper which has shown a significant improvement on improving frequency and ROCOF accuracy during faults particularly in modification of algorithms relative to the reference algorithm under IEEE Standard C37.118. The report then goes to the practical approaches of the project. It starts with understanding the whole simulation model which was carried out using software Matlab/Simulink (SimPowerSystem) previously. Reviewing some fundamental operations, functions and commands for the software are taken place to ensure getting better understanding on simulation operation and then start to focus on related part of simulation for improvement as project objective was defined.

The algorithm of weight average factor is chosen to be part of project improvement factor since the previous paper reviewed proofed that weighted average was given a significant reduction of peak excursion then weight amplitude of voltage signals is focused for further evaluation. Increasing the amplitude weight has resulted frequency and ROCOF unwanted peak excursion improved by 69.2% and 66.54% relative to previous paper set as baseline. The results are achieved through simulation of phase B-C fault simulation. The report then finally ends with some conclusions based from the analysis that has been done and some suggestion on further recommendation to apply the new proposed algorithm to real fault simulation and also might need to be tested under EMC ( electromagnetic Compatibility ) standard compliance such as flickering issue .

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### 1. Motivation of Synchronization Measurement

Increasing load demand and market pressure in electric power supply has caused the operation become more complex and the occurrence of power system disturbance or faults around the world has led to the significantly important of synchronization measurement whose measures power system phasor synchronously, precisely and accurately. In power systems, Wide Area Measurements (WAM) have been used in Electrical Measurement System (EMS) functions for a long time in the economic dispatch, tie line bias control and others which require wide area measurements [1].

Sensing, communication, computerization, visualization and control are important factors in synchronized measurement system which has becomes a power system operational paradigm nowadays. Figure 1 shows a typical process of how signals are initially sensed by measurement equipment from a power system unit ( power plant, transmission lines, high voltage transmission lines and power substation ) which transmitted to Supervisory Control and Data Acquisition (SCADA) which consists of computer or control signal center through communication medium ( modem and GPS-Global Positioning System ) [2].

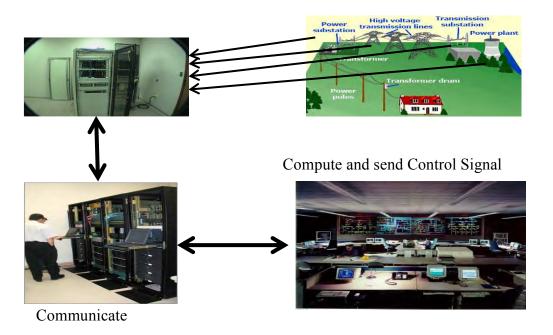


Figure 1, Power System Operational Paradigm, from [2]

Power systems always exposed to various type of disturbance which lead to a significant evaluation of measurement parameters by any proposed phasor estimation algorithm during power system faults and disturbance (dynamic conditions). Faults could cause critical changes of values to parameter of measurement unit. Furthermore, it might also cause appearing of unwanted decaying DC component in current waveforms.

Application of synchrophasors in power system protection seems as evolving concept since its performance of estimation algorithm during disturbance is critical for protection application and the quality of estimated synchrophasor also might affects other applications. This explain why the performance of synchrophasor estimation algorithm during disturbance or fault is critical and important to protection applications [3].

In synchronized measurements high resolution of synchronized data is needed since the data from different locations are not captured precisely at the same time. Voltage does not change suddenly unless there is a large disturbance nearby and system monitoring is more critical during disturbance and transients. For that reason faster synchronized data of Phasor Measurement Unit is needed to capture the dynamics [2].

#### 2. Introduction to Phasor Measurement Units

#### ( **PMU** )

A Phasor Measurement Unit (PMU) is a transducer or device which plays roles to converts three phase analog signal of voltage or current into synchrophasors and it also been defined by the IEEE as "device that produces Synchronized Phasor, Frequency and ROCOF (Rate of Change of Frequency) estimates from voltage and/or current signals and a time synchronizing signal". In other words, it means the synchrophasor and frequency values must comply the general definition and minimum accuracy required in standard IEEE C37.118.1 and the device also must provide a real time data output which confronts to standard requirements [2].

Phasor measurement unit (PMU) is also a device which mostly as microprocessor based and play roles to report the magnitude and phase angle of an analog and/or derived phasor with respect to the global time reference. The hardware of phasor measurement unit is the same as digital fault recorder or digital relay except for synchronization part as shown in figure 2 and figure 3 shows a typical common location of PMU in a power system unit. The priority of PMU or any other instrument installed and connected to the power system grid is to monitor power system parameters as well as the power system performance.

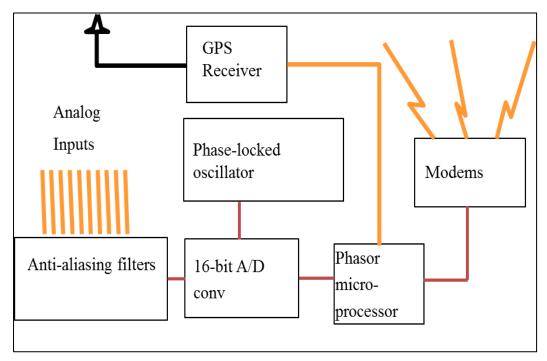


Figure 2, Synchronization interaction, from [1]

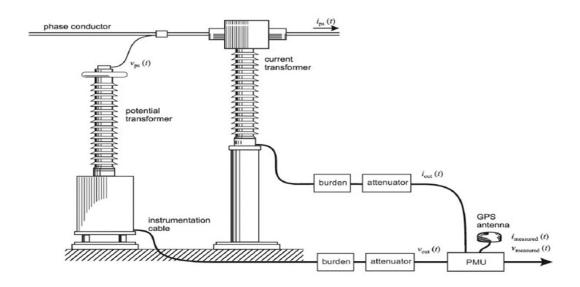


Figure 3, Phasor Measurement Unit ( PMU ) location in power system , from [2]

#### 2.2 Basic PMU Applications

Global behavior PMU could be understood from local measurement and phasor measurement data which can be used to enhance existing control center functionalities. Phasor measurement data with GPS signal could provide synchronized voltage and current phasor measurement across wide area. PMU can be used to validate system performance, model parameters and control equipment settings as to become more reliable protection systems [2]. This PMU instrument extracts the parameters of magnitude, phase angle, frequency and ROCOF (Rate of Change of Frequency) from analog input signal (continuous signal).

#### 2.3 PMU Standards

Since the intention of PMU connected to the power grid is to monitor power system parameters, the standard of PMU is provided to describe and quantify the performance as to monitor the power grid parameters performance. In review history, Phasor Measurement Units standard was revised from C37.118-2005 to C37.118.1-2001 due to the previous standard does not regulate dynamic performance and does not clarify time stamping. The revised standard clarifies time stamping and latency requirements rigorously and specifies dynamic test condition (e.g.  $\pm$  2Hz operating range and  $\pm$  1Hz/s ROCOF) [8].

The new Standard IEEE C37.118.1-2001 defines synchrophasors, frequency and rate of change of frequency (ROCOF) measurement at all operating condition in power system. It describes time tag and synchronization requirements for all of them above and it also specifies the methods for evaluating these measurement and requirements for compliance with standard at steady state and dynamic conditions. The standard also defines PMU either under stand alone measurement unit or a functional unit

within another measurement unit [8].

In PMU, amplitude and phase is the key measured parameters but the frequency and ROCOF are seems likes less interest by some academic literature reviewer even though for some proposed PMU applications, the measurements of frequency and ROCOF could be very important for power system control and protection actions. One aspect which not stressed and considered by the standard IEEE C37.118.1 is the influence of disturbances factor which may give effect to PMU measurement especially during power system faults. Faster reporting and information of system performance through PMU report such as P class device is highly deemed by utility customers for the case of fault occurrence [4].

The standard IEEE C37.118.1 provides strict requirements for the required response to harmonic / inter harmonic signal content and dynamic events. For P Class algorithm of the standard, it specifies accuracy requirements for frequency and ROCOF measurements since both of them giving excessive errors for the off nominal frequency and harmonic contamination [5].

This PMU result also could visualize both the steady state and dynamic conditions of the whole power system network. As to ensure uniformity and accuracy of the PMU operation the standard C37.118-2005 which mainly describes about the steady state performance requirements for synchrophasor was revised to standard IEEE C37.118.1-2011 whose clarifies for the both steady state and dynamic conditions [6]. The latest standard includes time stamping and latency requirement rigorously which old standard did not have it.

Discrete Fourier Transform ( DFT ) is among the common techniques for synchrophasor estimation algorithm which based on the steady state concept of phasor by assuming phasor parameters to be constant within the computational windows.

This technique is eventually deteriorates at off-nominal frequencies and in the presence of decaying DC and it does not perform satisfactorily during power system in dynamic states. Modification of the algorithm is needed to give more accurate measurement particularly during transient state or dynamic state. Since power system is often exposed to several types of faults or disturbance, it is very important also to evaluate the performance of proposed algorithm in dynamic state as well as to ensure it compliant with standard IEEE C37.118.1.2011 [6].

Improving functionality of System Integrity Protection Schemes (SIPS) and control of FACTS (Flexible AC transmission system) devices for system stability requires accurate transient information from PMU. Wide area protection algorithm whose based on ROCOF used to perform load shedding is also rely on measurement accuracy and timely especially related to frequency deviations after a disturbance [7]. Apart of that, protection system requires fast and shorter response time and accurate PMU measurements during faults.

## 3. Synchrophasor Fundamentals

At presents, synchrophasors are widely and mainly applied for wide area monitoring and increasingly being considered for protection application in smart grids as well. At one stage it becomes a situational awareness method and decision support to the power system especially related to accuracy of measurement in protection [6].

In reality, the power system seldom operates exactly at the nominal frequency. As such, actual frequency of the system at the time of measurement needs to take account during the phase angle calculation. In one example, if the actual frequency is 59.5Hz and the nominal frequency is 60Hz, the period of the waveforms will be 16.694ms

instead of 16.666ms (with a difference of 0.167%). Since the accuracy of measurement is important, the captured phasors are to be time tagged based on the time of the Coordinated Universal Time (UTC) [2].

#### 3.1 Synchrophasor Definition

Synchrophasor is defined as a precision time tagged positive sequence phasor which measured at different location. The Phasor Measurement Unit (PMU) as illustrated in figure 4 is eventually converts three phase analog signal of voltage or current into synchrophasors and also been defined by the IEEE as "a device that produces Synchronized Phasor, Frequency and ROCOF (Rate of Change of Frequency) estimates from voltage and/or current signal and a time synchronizing signal" [2].





Figure 4, Phasor Measurement Unit, from [2]

The synchrophasor representation of the signal  $x(t) = A_m \cos(\omega t + \emptyset)$  is the value of  $\tilde{A} = Ae^{j\phi}$  where  $\emptyset$  is the instantaneous phase angle relative to a cosine function at nominal frequency synchronized to UTC.  $\emptyset$  is the offset from a cosine function at nominal frequency synchronized to UTC and a cosine has a maximum at t equal to zero (0).

#### 3.2 Definition of Phasor

A phasor is defined as a sinusoidal signal which represented by a cosine function with magnitude A (rms value of voltage/signal), frequency  $\omega$ , and phase angle  $\phi$ . The starting time defines the phase angle of phasor ( $\phi$ ). However, the differences between phase angles are independent of starting time. The value of  $\emptyset$  depends on the time scale, particularly t=0 [2]. Above description is shown in figure 5.

$$x(t) = A_m \cos(\omega t + \emptyset) \tag{1}$$

Phasor is a rotating vector which can be represented by two formats:

Polar coordinates:

$$\tilde{A} = Ae^{j\phi} = (A, \phi)$$
Where  $A = \frac{A_m}{\sqrt{2}}$  is root mean square (rms) value

Rectangular coordinates:

$$\tilde{A} = A_{re} + jA_{im} = (A_{re}, A_{im})$$
 (3)

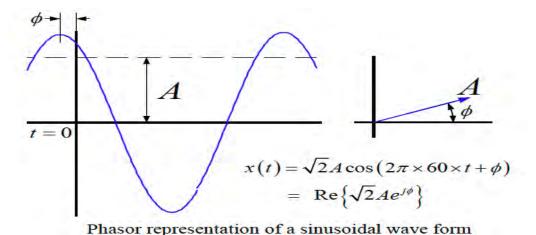


Figure 5, Phasor representation of a sinusoidal waveform, from [2]

The measurement parameters are taken from current transformer (CT) and voltage

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transformer (VT) of three phase sinusoidal AC voltages and currents at a frequency of 50 Hz (depend on country specification). Figure 6 shows phase "a" quantities (voltages and currents) lead phase "b"quantities by  $120^{0}$ , which leads phase "c" by  $120^{0}$  and the polarity is depend on phasor rotation practice [2].

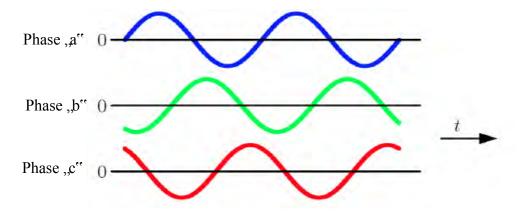


Figure 6, Three Phase Sinusoidal Waveforem, from [2]

As explained above, seldom the system operates at exactly the normal frequency and the actual frequency of the system during measurement period always used for phase angle calculation. The captured phasors are to be time-tagged based on the time of the UTC and the accuracy of measurement is very important. PMUs measure synchronously the positive sequence voltages and currents, phase voltages and currents and local frequency and rate of change of frequency. The evaluation with other phasors must be operated with the same time scale and frequency [8].

#### 3.3 Phasor Estimation

According to standard IEEE C37.118, there is no standard phasor algorithm for estimation used by different PMU manufacturers but most phasor calculation in commercial PMUs uses a 1 cycle window (centering in window). In order to reduce noise, several manufacturers use the average value over an even number of windows [2].

In synchrophasor continuous voltage or current signal is sampled and figure 7 shows typical sampling process on how 12 points per cycle is sampled (example of sampling rate of 12 x 60 Hz = 720Hz). Discrete Fourier Series (DFT) method used to compute the magnitude and phasor of each phase of 3 input signals. It also using one period of data to reduce the effect of measurement noise. By synchronizing the sampling processes different signals which could be hundreds or thousands of miles apart, it is possible to put the phasors on the same phasor diagram. [1].

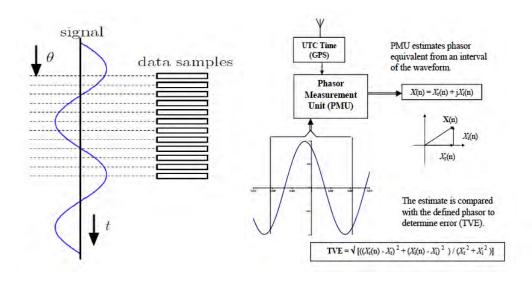


Figure 7, Phasor Sampling and Estimation Technique, from [2]

# 3.4 Frequency and Rate of Change of Frequency Estimation

In the IEEE Standard C37.118.1, to comply with the frequency and ROCOF requirements is much more problematic compared to comply with the Total Vector Error ( TVE ) specification which is relatively much easier particularly for the case off nominal frequency due to the influence of harmonic contamination. Any unwanted

signals emerging from the filters, even tiny amounts of ripple on the measurement of phase angle could impart large ripple or noise onto the measured signal phase especially on the measurement of frequency [5].

With the reliable and accurate source such as Global Positioning System (GPS) which can provide time traceable for sufficient accuracy, the PMU shall capable to keep the Total Vector Error (TVE), Frequency Error and the ROCOF Frequency Error within the required limits. A PMU shall calculate and be capable of reporting frequency and ROCOF from a given a sinusoidal signal as shown in equation below [8].

$$x(t) = A_m \cos[\psi(t)]$$

Frequency is defined as:

$$f(t) = \frac{1}{2\pi} \frac{\mathrm{d}\psi(t)}{\mathrm{d}t} \tag{4}$$

The ripple or noise also will further compounded the measurement of ROCOF which is defined as:

$$ROCOF(t) = \frac{\mathrm{d}f(t)}{\mathrm{d}t} = \frac{1}{2\pi} \frac{\mathrm{d}^2 \psi(t)}{\mathrm{d}t^2}$$
 (5)

Synchrophasors are always calculated relatively to the system nominal frequency  $f_o$ . If the cosine argument is represented as :

$$\psi(t) = \omega_0 t + \varphi(t)$$

$$= 2\pi f_0 t + \varphi(t)$$

$$= 2\pi [f_0 t + \frac{\varphi(t)}{2\pi}]$$
(6)

Then the formula for frequency becomes as below:

$$f(t) = f_o + \frac{\mathrm{d}\left[\frac{\varphi(t)}{2\pi}\right]}{\mathrm{d}t} = f_o + \Delta f(t)$$

$$16$$
(7)

Where  $\Delta f(t)$  is the deviation of frequency from nominal, f(t) is the actual frequency and then the formula for ROCOF is:

$$ROCOF(t) = \frac{d^{2}\left[\frac{\varphi(t)}{2\pi}\right]}{dt^{2}}$$
$$= \frac{d(\Delta f(t))}{dt}$$
(8)

Based on formulas of frequency and ROCOF shown above, the frequency is computed as the first derivative of the synchrophasor phase angle and ROCOF is then computed based on the second derivative of the same phase angle [8].

The PMU measurements are only useful when frequency and ROCOF compared with each other at phasor data concentrator (PDC). Since many PMU works with different classes, reporting rates (measurement windows) and variable data transfer, the timestamps of PMU also will vary. For a P class PMU, a device might report at a rate of  $F_{s=}10$ Hz (every 20ms) with timestamps around 20ms prior to the reporting instants due to the window length is 2 cycles [5].

### 3.5 Synchrophasor Measurement Evaluation

There could be differences in both amplitude and phase between PMU measurement value and the theoretical values of synchrophasor representation of a sinusoid. In the IEEE Standard C37.118.1.2001, the amplitude and phase differences are considered together in the quantity called as Total Vector Error (TVE). The TVE express the difference between a perfect sample of theoretical synchrophasor and the estimation at the same instant of time. TVE which expressed as per unit of the theoretical phasor defined as below [8].

$$TVE(n) = \sqrt{\frac{(\ddot{X}_r(n) - X_r(n))^2 + (\ddot{X}_i(n) - X_i(n))^2}{(X_r(n) - X_i(n))^2}}$$
(9)

Where  $\ddot{X}_r(n)$  and  $\ddot{X}_i(n)$  are the sequence of estimation and  $X_r(n)$  and  $X_i(n)$  are the sequences of theoretical value of input signals at the instances of time (n).

For frequency and ROCOF measurement, its errors are the absolute value of the difference between estimation values and theoretical values given in Hz and Hz/sec respectively. The formulas for both as below.

Frequency measurement error; 
$$FE = |f_{true} - f_{measured}|$$
  
=  $|\Delta f_{true} - \Delta f_{measured}|$  (10)

ROCOF measurement error; RFE = 
$$\left| \left( \frac{\mathrm{d}f}{\mathrm{d}t} \right)_{\mathrm{true}} - \left( \frac{\mathrm{d}f}{\mathrm{d}t} \right)_{\mathrm{measured}} \right|$$
 (11)

The time tag of the estimated values for the same instant of time will be given for both measured and true values. Synchrophasor, frequency and ROCOF estimation measurements shall be made and reported for the same time of reporting [8].

## **4 P-Class Phasor Measurement Unit (PMU)**

The IEEE Standard defines two classes of performance of measurement evaluation, which are P class and M Class. The letter of "P"from P class is referred to protection applications which requires fast response. For M class ( letter "M"for measurement ) referred to analytic measurements which often require greater precision but do not require minimal reporting delay. This project and final report is referred to P class of

PMU in term of compliance with the requirement of standard. The requirements particularly for synchrophasor, frequency and ROCOF shall be met at all times either the PMU function is a stand alone unit or as part of other measurement unit [8].

The parameters that extracted from the signals appearing at its input terminals by PMU are magnitude, phase angle, frequency and ROCOF. The signals could be corrupted by harmonics and others noises which might complicate the measurement accuracy towards system fundamental frequency. To reject undesirable signal components, the filtering associated with the computation of PMU within the limits provided by the filter attenuation [8].

The frequency is computed from the first derivative of synchrophasor phase angle and then ROCOF is computed as the second derivative of the same phase angle. Frequency and ROCOF are seem less reliable measurement because they are more sensitive to undesirable components of harmonics, off nominal frequency or noise. The standard needs to cover synchronized phasor measurement as to ensure instruments unit will comply and perform similarly as standard requirement for test signals [8].

## **4.1** Reference Model of PMU Algorithms ( P Class )

The reference signal processing models in the PMU standard used to develop and verify performance requirements. The reference is just for information and does not imply to the only method of synchrophasors estimation. It eventually can be used to establish common fundamental to understanding performance requirement and confirming the performance. The reference model is a simple derivation based on a fixed frequency 2 cycle estimator designed to remove second harmonics for the

purpose to meet frequency deviation requirements. For the P class verification, since the filter length is constant for all outputs rates, the PMU output generated at lower rates might contain additional aliasing components. [8]

The standard IEEE C37.118 describes the tests which a PMU should compliant and the standard also presents an algorithm for the purpose to demonstrate that the standard can be complied with as a minimum level. The standard still allows further improvement of algorithm for the purpose of better performance in some areas compared to the existing standard. Since certain frequency and ROCOF results in standard IEEE C37.118 unable to achieve successfully by using existing algorithm, modification of specifications or algorithm is needed for further improvement and same time must ensure to comply the standard [4].

#### 4.2 Basic Algorithm of Synchrophasor Estimation Model

Figure 8 shows basic processing steps within PMU which assumes fixed frequency sampling synchronized to time reference and followed by complex multiplication with the nominal frequency carrier.

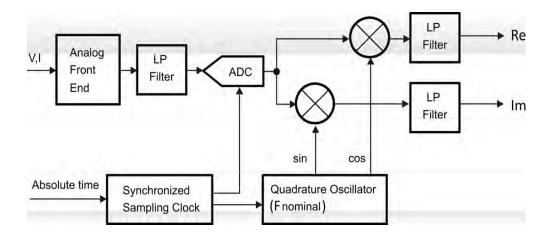


Figure 8, Single phase section of PMU signal processing unit, from [8]

In the basic architecture of P class PMU, the inputs signals are correlated with quadrature waveforms at nominal frequency fo and the output of each phase is a single fundamental phasor which the magnitude is proportional to the voltage. The phase angles of each phase rotate at  $2\pi(f - fo)$  which f is the actual system frequency. This standard uses core single phase Fourier analysis section, using fixed frequency for the quadrature oscillator and a fixed weight triangular filter of length  $(2/f_o)$  in seconds [7].

Given a set of samples of a single phase of the power signal  $\{x_i\}$ , the synchrophasor estimates X(i) at the *i*th sample time is :

$$X(i) = \frac{\sqrt{2}}{Gain} \times \sum_{k=N/2}^{N/2} x_{(i+k)} \times W_{(k)} \times \exp(-j(i+k)\Delta t\omega_0)$$
 (12)

Gain = 
$$\sum_{k=N/2}^{N/2} W_{(k)}$$
 (13)

Where:

 $\omega_o = 2\pi f_o$  where  $f_o$  is nominal power system frequency

N = FIR filter order ( number of filter taps is equal to N+1 )

 $\Delta t = 1/\text{sampling frequency}$ 

 $x_i$  = sample of the waveform at time  $t=i\Delta t$ 

 $W_{(k)} = low pass filter coefficients$ 

The equation above, X(i) represents complex demodulation and low pass filtering as shown in figure above. The low pass filtering  $(W_{(k)})$  can be applied individually to the real and imaginary outputs of the complex demodulator [8].

## 4.3 P class Reference Model for Phasor and Frequency Derivation

The P class phasor estimation algorithm uses fixed length two cycle triangular weighted FIR filter. The algorithm uses an odd number of samples (filter taps) to simplfy time stamp generation and phase compensation. Through this simplification, it allows conversion and filtering to use sample time stamp at the center of the window without adjustment. The reference algorithm uses a sample rate of 15 samples/cycle, which result  $60 \times 15$  samples = 900 samples/second (60Hz system) or  $50 \times 15$  samples = 750 samples//second (50 Hz system) depends on every manufacturers of PMU equipment [8].

The algorithm could be implemented using a two cycle FIR filter with triangular window coefficient as in figure below. It is a fixed length triangular weighted symmetric filter of length 2 cycles. It was designed to work optimally at the nominal frequency fo. The filter produces notches with high attenuation to attenuate contamination due to harmonics at every multiple of nominal frequency [5].

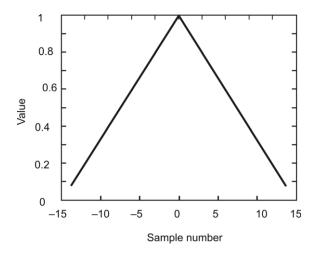


Figure 9, P class filter coefficient [ $N = 2 \times (15-1) = 28$ ], from [8]

The example of filter as shown in figure 9 shows the filter order N being equal to  $N = 2 \times (15-1) = 28$ . As long as the sample times are compensated for input delays, the time stamp at the center of window could produces an estimate whose phase follows the actual power system frequency. So, it does not need any phase or delay correction [8].

The example of filter weights coefficients  $(W_k)$  determination for P class are defined as below:

$$W_k = \left(1 - \frac{2}{(N+2)}|k|\right) \tag{14}$$

Where;

$$K = -\frac{N}{2}, -\frac{N}{2} + 1, \dots, \frac{N}{2}$$

N = filter order where N = 2(S-1); S is no. of samples per cycle at  $f_0$ .

Since the FIR filter is symmetrical, the timestamp of measurement can be positioned to a point of halfway filter time window.

Before extracting the frequency and ROCOF information, the Reference algorithm combine the phasors from all phases into a positive sequence phasor. During normal balanced operation,  $V_A$ ,  $V_B$  and  $V_C$  phasors are at  $120^0$  to each other, then the overall positive sequence ( $V_1$ ) phasor can be calculated as below:

$$V_1 = V_A + \alpha V_B + \alpha^2 V_C \tag{15}$$

$$V_1 = V_A + V_B e^{\frac{2\pi}{3}j} + V_C e^{-\frac{2\pi}{3}j}$$
 (16)

By assuming the single phase sections are effective at filtering out harmonics, any noise and unwanted harmonic signals and also the input signal magnitudes and