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A Hybrid Model for Improving Malaysian Gold Forecast Accuracy

Maizah Hura Ahmad[1], Pung Yean Ping[2], Siti Roslindar Yazir[3] and Nor Hamizah Miswan[4]

[1,2]Department of Mathematical Sciences, Faculty of Science
Universiti Teknologi Malaysia,
81310 UTM Skudai, Johor, Malaysia

[3]Faculty of Industrial Sciences & Technology,
Universiti Malaysia Pahang, Malaysia

[4]Fakulti Teknologi Kejuruteraan,
Universiti Teknikal Malaysia Melaka, Malaysia

Abstract

A hybrid model has been considered an effective way to improve forecast accuracy. This paper proposes the hybrid model of the linear autoregressive moving average (ARIMA) and the non-linear generalized autoregressive conditional heteroscedasticity (GARCH) in modeling and forecasting. Malaysian gold price is used to present the development of the hybrid model. The goodness of fit of the model is measured using Akaike information criteria (AIC) while the forecasting performance is assessed using bias, variance proportion, covariance proportion and mean absolute percentage error (MAPE).

Keywords: ARIMA-GARCH, hybrid model, heteroscedasticity, volatility clustering

1 Introduction

A popular precious metal for investment is gold. In Malaysia, one of the highest gold investment demand is for its own gold bullion coins called *Kijang Emas*. The coins which come in three different sizes of 1 oz, ½ oz and ¼ oz are minted by the Royal Mint of Malaysia. The daily selling and buying prices of these coins are important to investors in order to make an investment decision.

Autoregressive integrated moving average (ARIMA) models have been used for forecasting different types of time series to capture the long term trend. In the case of financial time series that have been shown to have volatility clustering where large changes in the data tend to cluster together and resulting in persistence of the amplitudes of the changes, ARCH based models have been used. In the context of Malaysian gold, the selling price of the 1 oz coins was modelled and forecast using ARIMA and GARCH models [1] [2]. While the models produced a good fit of the data with the GARCH being more superior, a hybrid of those two models is proposed to be able to improve forecasting accuracy [3].

In the current study, a selected series of Malaysian gold is modelled and forecast using the hybrid of ARIMA-GARCH. Akaike information criterion (AIC) is used to assess the goodness of fit. Bias, variance proportion, covariance proportion and mean absolute percentage error (MAPE) are used to evaluate the forecasting performances. All analyses are carried out using a software called E-views.

The paper is organized into 4 sections. Section 2 presents the methodology of the study. Section 3 presents the data analysis. The study is concluded in Section 4.

2 Methodology

Hybrid ARIMA-GARCH Models

ARIMA models are the most general class of models for forecasting a time series, applied in cases where data show evidence of non-stationarity [4]. Non-stationarity in mean can be removed by transformations such as differencing, while non-stationary in variance can be removed by a proper variance stabilizing transformation introduced by Box and Cox [3]. The ARIMA(*p*,*d*,*q*) can be written as

$$\phi_p(B)(1-B)^d y_t = \delta + \theta_q(B)\varepsilon_t$$

where $\phi_p(B) = 1 - \phi_1 B - ... - \phi_p B^p$ is the autoregressive operator of order *p*;

$\theta_q(B) = 1 - \theta_1 B - ... - \theta_q B^q$ is the moving average operator of order q; $(1-B)^d$ is the d^{th} difference; B is backward shift operator; and ε_t is the error term at time t. The orders are identified through the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the sample data. The error terms are generally assumed to be independent identically distributed random variables (i.i.d.) sampled from a normal distribution with zero mean, that is $\varepsilon_t \sim N(0,\sigma^2)$ where σ^2 is the variance. At this point, the model can be used for forecasting.

However, some time series errors do not satisfy the assumption of common variance. The variances are time-varying and conditional. The autoregressive conditional heteroskedasticity (ARCH) class of models pioneered by Engle in 1982 and generalized by Bollerslev in 1986 are popular class of econometric models for describing a series with time-varying conditional variance [5]. The generalized autoregressive conditional heteroskedasticity (GARCH) family models were developed to capture volatility clustering or the periods of fluctuations, and predict volatilities in the future [6]. Past variances and past variance forecasts are used to forecast future variances. The GARCH (p, q) model is

$$y_t = \mu + \varepsilon_t,$$

where $u_t = \varepsilon_t \sigma_t^2 = \varepsilon_t \sqrt{h_t}\,, \qquad \varepsilon_t \sim N(0,1)$

$$h_t = \delta + (\alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots) + (\beta_1 h_{t-1} + \beta_2 h_{t-2} + \cdots)$$

$$= \delta + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

where $\delta = \alpha_0(1-\beta_1),\ h_t = \sigma_t^2,\ \alpha_1 + \beta_1 < 1$ for stationarity; $\alpha_i,\ \beta_i > 0$

p is the order of the GARCH terms σ^2, which is the last period forecast variance. q is the order of the ARCH terms ε^2, which is the information about volatility from the previous period measured as the lag of squared residual from the mean equation.

Augmented Dickey-Fuller (ADF)

ADF is one of the widely used unit-root tests to determine stationarity. The testing procedure is applied to the model

$$\Delta y_t = \alpha_0 + \beta t + \theta y_{t-1} + \sum_{i=1}^{k} \alpha_i \Delta y_{t-1} + \varepsilon_t$$

where y_t is the tested time series, Δ indicates the first difference, k is the lag order of the autoregressive process. Rejection of the null hypothesis implies that the series is stationary.

Breusch-Godfrey Lagrange Multiplier Test (BG-LM)

BG-LM is a test for autocorrelation. The null hypothesis states that there is no serial correlation of any order up to a certain order lag.

ARCH Lagrange Multiplier Test (ARCH-LM)

ARCH-LM is used to test the presence of heterocedasticity. Let $\varepsilon_t = y_t - \mu_t$ be the residual series. The squared series, ε_t^2 is used to check the presence of ARCH effects where it is defined as follows,

$$\varepsilon_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2$$

where p is the length of ARCH lags and ε_t is the residual of the series. Test statistic for LM test is the usual F statistics for the squared residuals regression. Rejection of the null hypothesis implies that ARCH effect exists.

Akaike Information Criterion (AIC)

AIC is used to assess the goodness of fit of a model. It is defined as

$$\text{AIC} = 2k - 2\ln(L)$$

where L is the maximized value of the likelihood function for the estimated model and k is the number of free and independent parameters in the model.

Mean Absolute Percentage Error (MAPE)

MAPE measures the accuracy of forecast in terms of percentage. The formula is as follows:

$$\text{MAPE} = \left(\left(\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right|\right)/n\right)\times 100\%$$

where y_t is the actual value; $\hat{y}_t$ is the forecast value; n is the number of periods.

3 Data Analysis and Results

The data used in the study are daily selling prices of the 1 oz Malaysian gold recorded from 18th July 2001 until 15th April 2014 as plotted in Figure 1. A total of 2875 observations from 18th July 2001 until 25th Sept 2012 which account for 90% of the data are used for modelling. Out-sample forecasts are produced for observations in the period from 26th Sept 2012 until 15th April 2014.

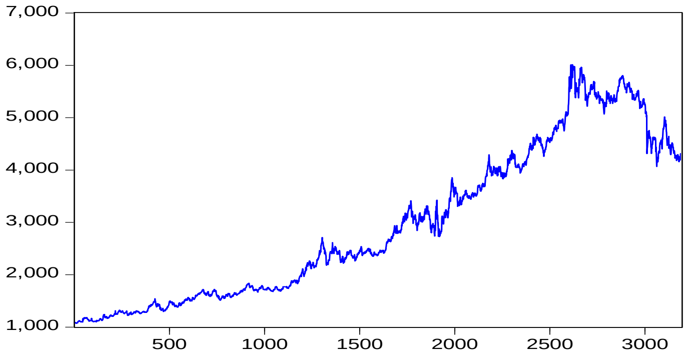


Figure 1: Daily 1 oz Malaysian Gold Prices from 18th Jul 2001 to 15th Sept 2014

An upward trend exists in the gold price data. Let $\{y_t\}$ be the time series of the daily gold price. The return on the t^{th} day is defined as $r_t = \ln(y_t) - \ln(y_{t-1})$. Figure 2 shows the plot of the returns which appears to be stationary. Most of the data are located around the mean of zero.

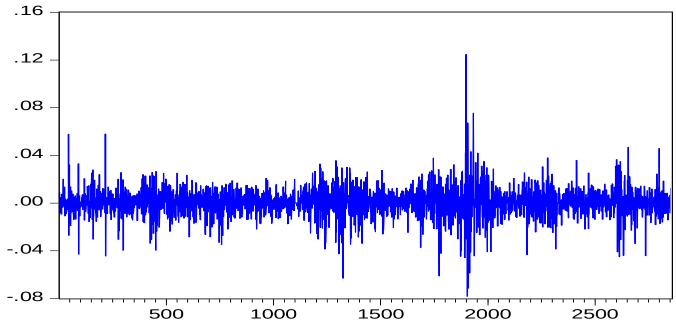


Figure 2: Plot of the Returns

The stationarity of the returns is confirmed by the ADF unit-root test as illustrated in Table 1. Based on the table, the null hypothesis that the returns are non-stationary is rejected.

Table 1: Unit Root Test of the Returns

Null Hypothesis: DLN has a unit root
Exogenous: None
Lag Length: 0 (Automatic based on SIC, MAXLAG=27)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-56.48802	0.0001
Test critical values:	1% level	-2.565767	
	5% level	-1.940934	
	10% level	-1.616625	

*MacKinnon (1996) one-sided p-values.

Using ordinary least squares method to estimate the parameters, the most appropriate ARIMA model for this series is ARIMA(1,1,1) with an AIC value of 10.08545 and MAPE value for in-sample forecast of 0.812356 [7]. The model is checked for serial correlation using Breusch-Godfrey Serial Correlation LM Test. The results are shown in Table 2, which indicate that with significance level of 5%, the developed model does not suffer from serial correlation up to lag 5.

Table 2: Breusch-Godfrey Serial Correlation LM Test

F-statistic	2.918448	Prob. F(5,2865)	0.0124
Obs*R-squared	14.55872	Prob. Chi-Square(5)	0.0124

The descriptive statistics and plot of the residuals of the model are presented in Figure 3 and Figure 4 respectively. As presented in Figure 3, the residuals have excess kurtosis and a mean which is very close to zero. From the Jarque-Bera statistic, the null hypothesis of residuals following the normal distribution is rejected.

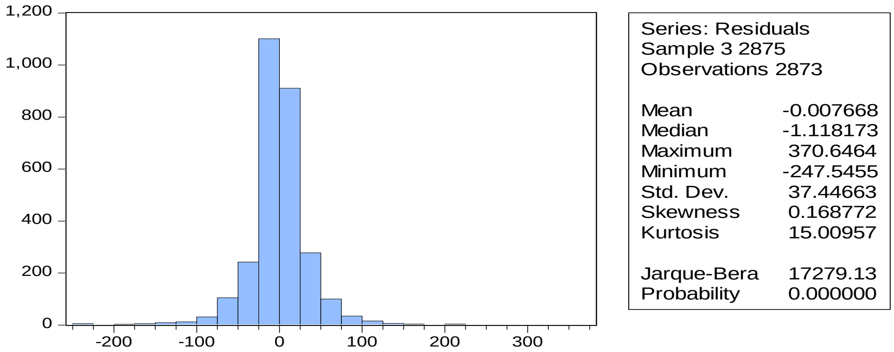


Figure 3: Descriptive Statistics of the Residuals for ARIMA(1, 1, 1)

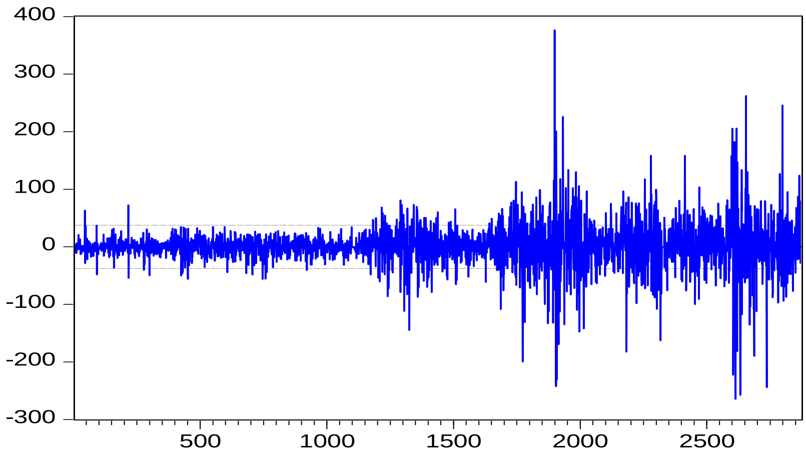


Figure 4: Volatility Clusterings in the Residuals for ARIMA(1, 1, 1)

As plotted in Figure 4, there are volatility clustering in the residuals. The residuals of the ARIMA are tested for ARCH effects using the ARCH- LM test. The results are presented in Table 3. From the table, with significance level of 5%, the null hypothesis of ARCH effects do not exist is rejected.

Table 3: Heteroskedasticity Test for ARIMA(1, 1, 1)

F-statistic	120.8169	Prob. F(1,2870)	0.0000
Obs*R-squared	116.0172	Prob. Chi-Square(1)	0.0000

Thus, although the hypothesis of no serial correlation in the model is not rejected, the presence of volatility clustering in the residuals and the results of the ARCH LM test show that the model is not a good fit. Hence, it is necessary to develop a better model for Malaysian gold price. A GARCH model is proposed to handle heteroscedasticity in the series. Table 4 presents the estimation results for the hybrid ARIMA (1, 1, 1)-GARCH(2, 1) model as applied to the Malaysian gold price.

Table 4: Estimation Result for ARIMA(1, 1, 1)-GARCH (2, 1)

Variance Equation

C	1.744212	0.487440	3.578313	0.0003
RESID(-1)^2	0.104766	0.022469	4.662635	0.0000
RESID(-2)^2	0.048091	0.023100	-2.081865	0.0374
GARCH(-1)	0.940387	0.006248	150.5098	0.0000

R-squared	0.003390	Mean dependent var	1.624782
Adjusted R-squared	0.002695	S.D. dependent var	37.52772
S.E. of regression	37.47711	Akaike info criterion	9.285385
Sum squared resid	4031012.	Schwarz criterion	9.299914
Log likelihood	-13331.46	Hannan-Quinn criter.	9.290622
F-statistic	1.627001	Durbin-Watson stat	1.995463
Prob(F-statistic)	0.135481		

In Table 4, with significance level of 5%, both the ARCH and GARCH effects are significant. They are the internal causes of volatility in the residuals. The AIC value of the hybrid model is 9.299914 with MAPE value for in-sample forecast of 0.808684. The residuals of the ARIMA-GARCH are tested for ARCH effects using the ARCH- LM test. The results are presented in Table 5.

Table 5: Heteroskedasticity Test for ARIMA (1, 1, 1)-GARCH (2, 1)

F-statistic	0.005485	Prob. F(1,2870)	0.9410
Obs*R-squared	0.005488	Prob. Chi-Square(1)	0.9409

The results in Table 5 indicate that at significance level of 5%, the null hypothesis of no ARCH effects cannot be rejected. The hybrid model is then tested for serial correlation as presented in Table 6.

Table 6: Ljung-Box Q-statistics on squared residuals for ARIMA(1,1,1)-GARCH(2,1)

lags	AC	PAC	Q-Stat	Prob	lags	AC	PAC	Q-Stat	Prob
1	0.001	0.001	0.0055		19	-0.002	-0.002	9.8829	0.908
2	-0.019	-0.019	1.0609		20	0.010	0.010	10.195	0.925
3	0.019	0.020	2.1524	0.142	21	-0.007	-0.007	10.349	0.944
4	-0.007	-0.008	2.3092	0.315	22	0.003	0.003	10.372	0.961
5	0.000	0.001	2.3099	0.511	23	0.011	0.010	10.733	0.968
6	0.026	0.026	4.3112	0.366	24	0.011	0.011	11.058	0.974
7	-0.019	-0.019	5.3550	0.374	25	-0.029	-0.030	13.484	0.941
8	-0.009	-0.008	5.6073	0.469	26	0.042	0.041	18.554	0.775
9	0.009	0.007	5.8215	0.561	27	-0.010	-0.012	18.868	0.803
10	-0.017	-0.016	6.6147	0.579	28	-0.010	-0.009	19.186	0.828
11	-0.004	-0.004	6.6683	0.672	29	-0.018	-0.021	20.089	0.827
12	0.003	0.001	6.6919	0.754	30	-0.016	-0.016	20.845	0.832
13	-0.017	-0.016	7.5387	0.754	31	-0.017	-0.017	21.711	0.832
14	-0.009	-0.009	7.7595	0.804	32	-0.007	-0.011	21.841	0.860
15	-0.023	-0.024	9.2705	0.752	33	0.008	0.009	22.030	0.882
16	-0.006	-0.004	9.3642	0.807	34	-0.015	-0.014	22.675	0.888
17	-0.008	-0.009	9.5398	0.848	35	0.003	0.003	22.707	0.911
18	-0.011	-0.011	9.8662	0.874	36	-0.019	-0.019	23.724	0.906

Based on the results in Table 6, the null hypothesis of no serial correlation cannot be rejected. The descriptive statistics of the residuals from the hybrid model are presented in Figure 5.

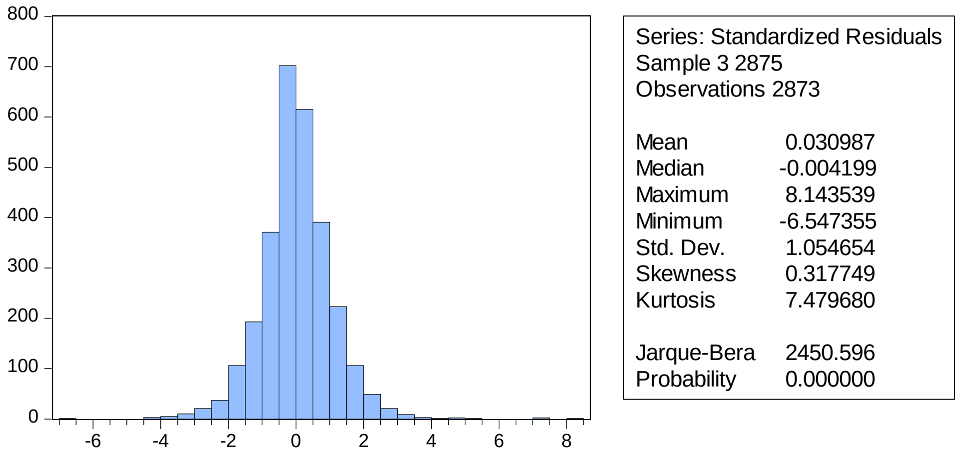
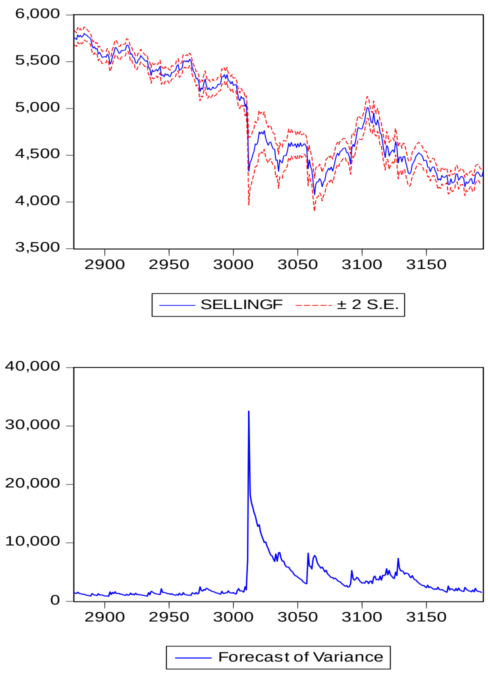


Figure 5: Descriptive Statistics of the Residuals for ARIMA(1, 1, 1)-GARCH(2, 1)

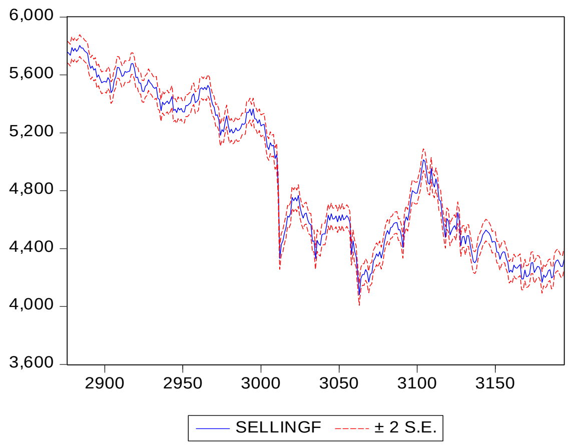
From the Jarque-Bera statistic in Figure 5, the residuals are not normally distributed. However, the hybrid model is used for forecasting. The results of out-sample forecasting are presented in Figure 6.



Forecast: SELLINGF	
Actual: SELLING	
Forecast sample: 2876 3194	
Included observations: 319	
Root Mean Squared Error	59.18209
Mean Absolute Error	39.06320
Mean Abs. Percent Error	0.827670
Theil Inequality Coefficient	0.006023
Bias Proportion	0.020026
Variance Proportion	0.000061
Covariance Proportion	0.979913

Figure 6: Forecasting Results of ARIMA (1, 1, 1)-GARCH (2, 1)

For comparison purposes, the out-sample forecasts for ARIMA(1, 1, 1) are plotted in Figure 7.



Forecast: SELLINGF	
Actual: SELLING	
Forecast sample: 2876 3194	
Included observations: 319	
Root Mean Squared Error	59.59525
Mean Absolute Error	39.19383
Mean Abs. Percent Error	0.830665
Theil Inequality Coefficient	0.006063
Bias Proportion	0.034607
Variance Proportion	0.000627
Covariance Proportion	0.964766

Figure 7: Out-sample Forecasts of ARIMA (1, 1, 1)

Conclusion

Table 7 presents some results of modelling and forecasting of the daily prices of 1 oz Malaysian gold recorded from 18th July 2001 until 15th April 2014. Two models were used, namely ARIMA and ARIMA-GARCH.

Table 7: Modelling and Forecasting Results

Models	ARIMA	ARIMA-GARCH
AIC	10.08545	9.299914
MAPE of in-sample	0.812356	0.808684
MAPE of out-sample	0.830665	0.827670
Bias Proportion	0.034607	0.020026
Variance Proportion	0.000627	0.000061
Covariance Proportion	0.964766	0.979913

Some information will always be lost due to using one of the candidate models. Based on the AIC values, the model that minimizes the estimated information loss more is ARIMA-GARCH. ARIMA is 0.46 times as probable as ARIMA-GARCH to minimize the information loss. The bias proportion, the variance proportion, and the covariance proportion sum up to 1. While the bias proportion measures how far the mean of the forecast is from the mean of the actual series, the variance proportion measures how far the variation of the forecast is from the variation of the actual series. The remaining unsystematic forecasting errors are measured by the covariance proportion measures. Based on Table 7, the forecasts produced by ARIMA-GARCH are better since the bias and variance proportions are lower than those produced by ARIMA. Furthermore, MAPE values for in-sample and out-sample forecasts for ARIMA-GARCH are lower than those for ARIMA. It can be concluded that in the case of the selling prices of 1 oz Malaysian gold, the hybrid model of ARIMA-GARCH can be an effective way to improve forecasting accuracy achieved by using ARIMA only.

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