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The Impact of Simplified UNBab Mapping Function On GPS Tropospheric Delay

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Abstract. The atmospheric delay issue is widely investigated in order to minimize the positioning error due to tropospheric and ionospheric delay. The mathematical modeling on the tropospheric model mapping functions should be revised and also simplified to represent simpler mapping function models. The zenith tropospheric delay can be amplified by a coefficient factor called mapping function to form total tropospheric delay. The simplified UNBab mapping function models for both hydrostatics and non-hydrostatics can provide better understanding due to its simpler models compared to the established models. The simplified mapping functions for UNBab models for hydrostatic and non hydrostatic components are given in a form of hyperbolic rather than continued fraction form for the established models. By using linear, hyperbolic, logarithm and also regression method, the mapping function models can be simplified and at the same time can produce similar result with the original models. The calculation of tropospheric delay by using simplified UNBab models for both components does not give significant difference from the original models.

INTRODUCTION

Global Positioning System (GPS) tropospheric delay refers to the refraction of the GPS signal as it passes through the neutral atmosphere from the satellite to the earth, which causes longer distance traveled by the signal. The issue of atmospheric delay is investigated to minimize the positioning error due to tropospheric and ionospheric delay. The mathematical modeling on the tropospheric model should be revised and also modified to improve the delay. The zenith tropospheric delay can be amplified by a coefficient factor called mapping function to form total tropospheric delay. Recently, the established tropospheric delay models use mapping functions in the form of continued fractions. Most of the modern models have separated mapping functions for the hydrostatic and the wet component, in a form of continued fraction [1].

The selected mapping function for this study is UNBab mapping function [2]. There is a need to simplify the mapping function models to allow faster calculation and also better understanding of the models. The UNBab models for hydrostatic and non-hydrostatic components are given in a form of continued fraction. These mapping function models are very tedious in calculating the value of mapping function, due to its continued fraction form [3]. As a coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function scale factor value plays an important role for getting the total tropospheric delay value.

The simplified UNBab for both components, either hydrostatic or non- hydrostatics are selected to show the impact of its mapping function scale factors on the tropospheric delay values.

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TROPOSPHERIC DELAYS

The tropospheric delays (TD) at arbitrary elevation angles can be expressed in terms of the zenith delays and mapping functions. This representation allows the use of separate mapping functions for the hydrostatic and wet delay components. The mapping function depends on the elevation angles, whereby at 90 degree of elevation angle, the mapping function scale factor value is 1. So, this value will give minimum value for the tropospheric delay (TD) as given in equation (1) below [4]:

$$TD = ZHD.m_h(\varepsilon) + ZWD.m_w(\varepsilon) \tag{1}$$

where ZHD is zenith hydrostatic delay (m), ZWD is zenith wet delay (m), $m_h(\varepsilon)$ is the hydrostatic mapping function (-), and $m_{nh}(\varepsilon)$ is the non-hydrostatic mapping function (-).

UNBab mapping function models for hydrostatic and non-hydrostatic (wet) components will be used to be modified to a simpler equation. This model is selected due to its have many operations and also its ability to achieve mapping function scale factor value down to 2 degree of elevation angle. This simpler equation can be used to calculate the mapping function scale factor by varying the elevation angles in the model.

Many mapping functions have been established from Marini [5] until UNB model which established in 2003. The formation of new mapping functions still in progress until now, in order to improve the established models. Some types of mapping functions are in a shape of a hyperbola. However, the graphs can also be obtained by using another form of equation which is simpler than the original equations. The simpler form of model will be discussed in detail.

UNBab Mapping Function Model

UNBab mapping function model has been established by Guo [2] from the University of New Brunswick has 7 operations in a form of continued fraction as given in equation (2) below. *UNBab* mapping function model has been separated into hydrostatic and non- hydrostatic components. The difference between the two components are their constants values a and b . The general *UNBab* mapping function form is given below:

$$UNBab_i(E) = \frac{1 + \dfrac{a_i}{1 + b_i}}{\sin E + \dfrac{a_i}{\sin E + b_i}} \tag{2}$$

where $i = h$ or nh, E is elevation angle (90 – z), $UNBabc_h$ is hydrostatic mapping function, $UNBabc_{nh}$ is non hydrostatic mapping function.

From the ray tracing delay values, (Guo, 2003) has introduced *UNBab* which has a 2-terms continued fraction form. He considers both a and b as a function of the height (H) and latitude (ϕ) of the station as given below.

Parameters for the hydrostatic function:

$$a_h = (1.53804 - 0.039491\text{H} + 0.17020\cos\phi)/1000,$$
$$b_h = (50.0724 - 0.814759\text{H} + 2.35232\cos\phi)/1000$$

and parameters for non-hydrostatic function:

$$a_{nh} = (0.73537 - 0.041172\text{H} - 0.00202\cos\phi)/1000,$$
$$b_{nh} = (32.5627 - 0.670636\text{H} - 0.15502\cos\phi)/1000$$

where H and ϕ are kept constant.

Regression method has been selected to be used to find the same type of graph for the original mapping function models.

Simplification of hydrostatic UNBab ($UNBab_h$) mapping function

Referring to equation (2), the hydrostatic component can be written as:

$$UNBab_h = \frac{1 + \dfrac{a_h}{1 + b_h}}{\sin E + \dfrac{a_h}{\sin E + b_h}} \tag{3}$$

where, E : elevation angle (radian).

The parameters a_h and b_h for the hydrostatic function is:

$$a_h = (1.53804 - 0.039491\text{H} + 0.17020\cos\phi)/1000.$$
$$b_h = (50.0724 - 0.814759\text{H} + 2.35232\cos\phi)/1000.$$

where, H = 0.1km and ϕ is 45 degrees.

By using the elevation angle from 2 to 90 degrees, the mapping function for the original. $UNBab_h$, has been named as P can be shown in Figure 1 below.

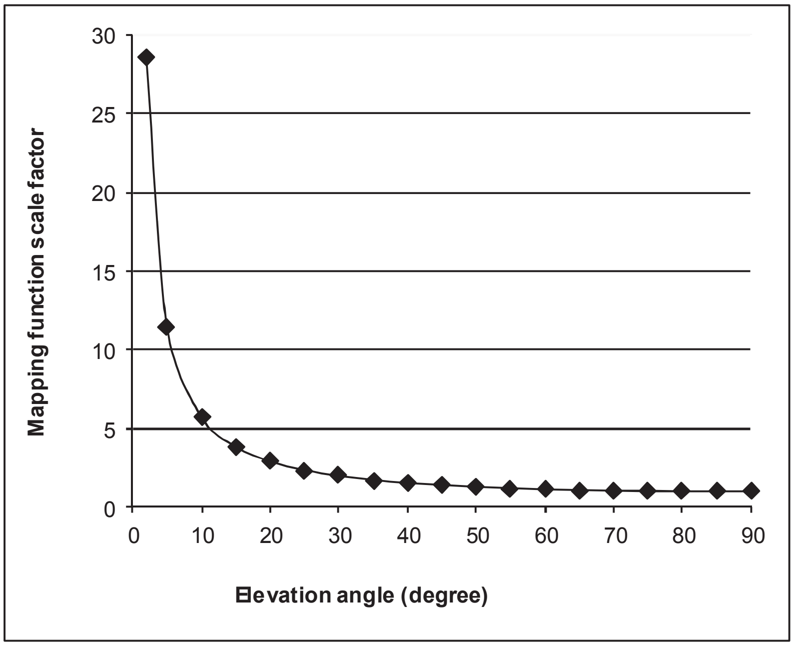

FIGURE 1. Graph of $UNBab_h(E)$ or P mapping function

The $UNBab_h$ mapping function or P gives a shape of hyperbola as given in Figure 1. Due to the shape of hyperbola and also using linear regression method, equation (3) can be simplified as:

$$P1 = AE^B \tag{4}$$

where $P1$: simplified $UNBab_h$, A, B are constants, E is elevation angle (degree).

By changing into logarithm scale, equation (4) can be written as:

$$\log_{10} P1 = B \log_{10} E + \log_{10} A \tag{5}$$

Therefore, equation (4) has changed it shape from hyperbolic graph to linear graph as derived in equation (5) above.

By linear regression method, equation (5) becomes:

$$\log_{10} P1 = -0.8924 \log_{10} E + 1.6604 \tag{6}$$

where B is -0.8924 and $\log_{10} A$ is 1.6604 which gives $A = 45.751$.

Therefore, equation (4) becomes:

$$P1(E) = 45.751E^{-0.8924} \tag{7}$$

The graphs of models P and P1 can be shown in Figure 2 below.

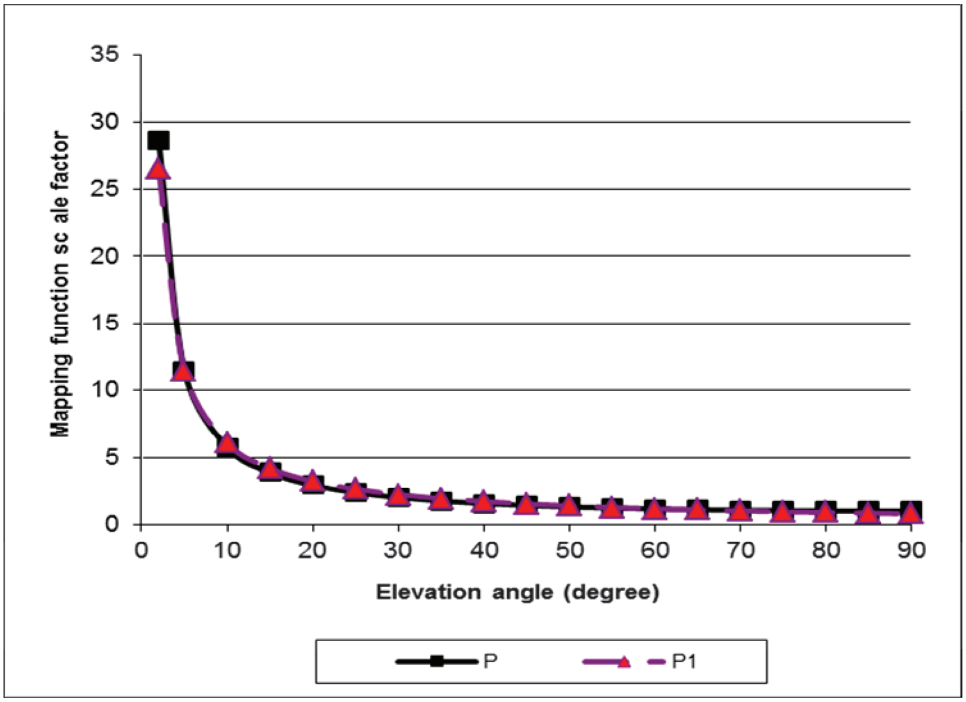

FIGURE 2. Graphs of P and P1 for $UNBab_h(E)$ mapping function

Simplification of non-hydrostatic ($UNBab_{nh}$) mapping function

Referring to equation (2), non-hydrostatic component can be written as:

$$UNBab_{nh} = \frac{1 + \dfrac{a_{nh}}{1 + b_{nh}}}{\sin E + \dfrac{a_{nh}}{\sin E + b_{nh}}} \tag{8}$$

where E is elevation angle (radian).

The parameters a_{nh} and b_{nh} for the nonhydrostatic function is:

$$a_{nh} = (0.73537 - 0.041172\text{H} - 0.00202\cos\phi)/1000,$$
$$b_{nh} = (32.5627 - 0.670636\text{H} - 0.15502\cos\phi)/1000.$$

and H = 0.1km and ϕ is 45 degrees.

By using the elevation angle from 2 to 90 degrees, the mapping function for the original $UNBab_{nh}$, has been named as Q can be shown in Figure 3 below.

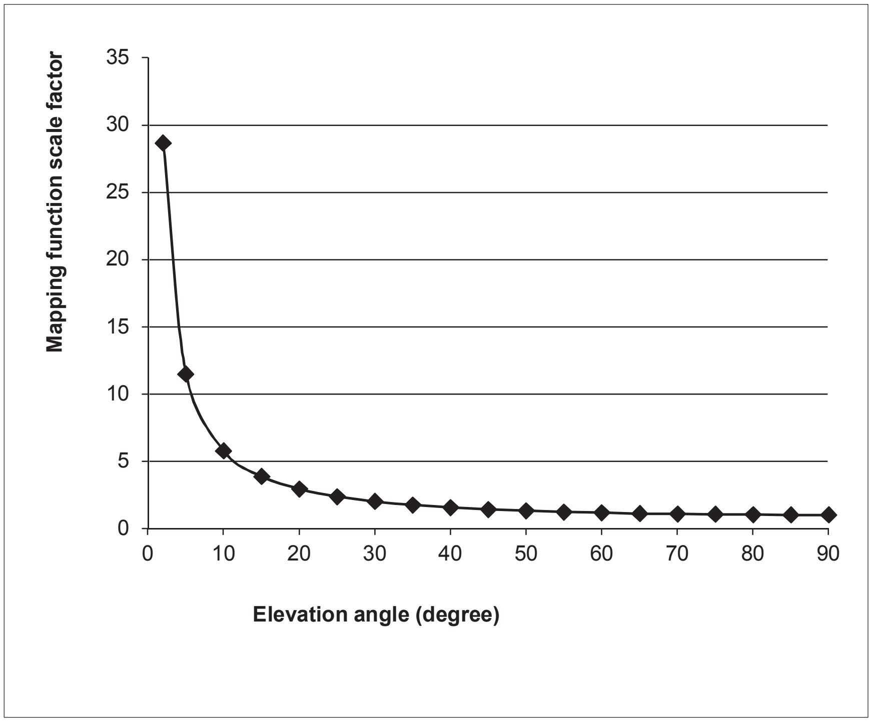

FIGURE 3. Graph of $UNBab_{nh}$ or Q mapping function

The $UNBab_{nh}$ mapping function, Q gives a shape of hyperbola as given in Figure 3. Due to the shape of hyperbola and also using regression method, equation (8) can be simplified as:

$$Q1 = AE^{B} \tag{9}$$

where $Q1$ is simplified $UNBab_{nh}(E)$, A, B are constants and E is elevation angle (degree). By changing into logarithm scale, equation (9) can be written as:

$$\log_{10} Q1 = B\log_{10} E + \log_{10} A \tag{10}$$

Therefore, equation (9) has changed the shape of hyperbolic graph into linear graph as derived in equation (10) above.

By using linear regression method the equation (10) can be written as:

$$\log_{10} Q1 = -0.8924\log_{10} E + 1.6605 \tag{11}$$

where B = -0.8924 and $\log_{10} A$ = 1.6605 which gives $A = 45.761$

Therefore, equation (9) becomes:

$$Q1(E) = 45.761E^{-0.8924} \tag{12}$$

The graphs of models Q and Q1 can be shown in Figure 4 below.

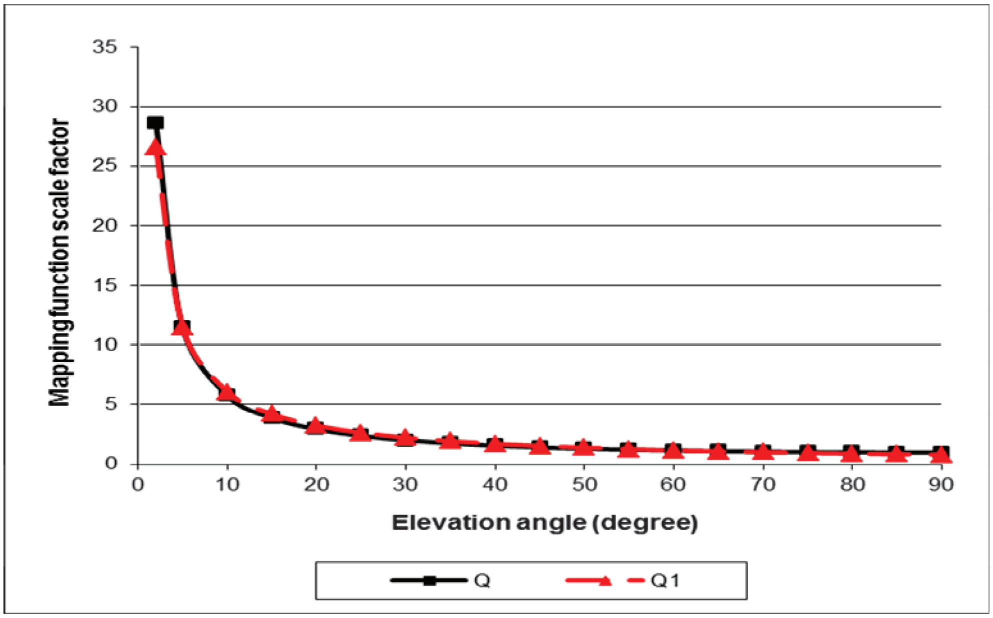

FIGURE 4. Graph of Q and Q1 for $UNBab_{nh}$ mapping function

Tropospheric Delay using the current UNBab mapping function

The $UNBab_h$ value should be multiplied by the zenith hydrostatic delay, ZHD value (2.31m) to obtain the hydrostatic component (A), while for obtaining non hydrostatic component (B), $UNBab_{nh}$ should be multiplied by the zenith wet delay, ZWD value (0.25m) as shown in Table 1.

TABLE 1. Tropospheric Delay (TD) using the current UNBab mapping function

Elev. angle, E (°)	A = $UNBab_h$ x ZHD	B = $UNBab_{nh}$ x ZWD	Current TD=A+B (meter)
2	66.13	7.16	73.29
5	26,50	2.87	29.37
10	13.30	1.44	14.74
15	8.92	0.97	9.89
30	4.62	0.50	5.12
45	3.27	0.35	3.62
60	2.67	0.29	2.96
75	2.39	0.26	2.65
90	2.31	0.25	2.56

Tropospheric Delay Using Simplified UNBab Mapping Function

The new trophospheric delay for hydrostatic component (C) can be obtained when the simplified UNBab, $SUNBabc_h$ from equation (7) multiplies the ZHD value (2.31m) and the non hydrostatic component (D) can be obtained when $SUNBabc_{nh}$ from equation (12) multiplies the ZWD value (0.25m) as shown in Table 2.

TABLE 2. New Tropospheric Delay (TD) using Simplified UNBab mapping function

Elev. angle, E (°)	C = $SUNBab_h$ x ZHD	D = $SUNBab_{nh}$ x ZWD	New TD = C + D (meter)
2	61.31	6.65	67.96
5	26.47	2.87	29.34
10	14.03	1.52	15.55
15	9.67	1.05	10.72
30	5.12	0.56	5.68
45	3.53	0.38	3.91
60	2.71	0.29	3.00
75	2.36	0.26	2.62
90	2.31	0.25	2.56

Comparison of Tropospheric Delay (TD)

The difference of tropospheric delay can be shown by comparing between simplified UNBab mapping function and the current UNBab mapping function. The current TD is calculated using UNBab mapping function and the new TD is calculated using simplified UNBab mapping function. The comparison of the tropospheric delay can be shown in Table 3 below:

TABLE 3. Percentage difference for the tropospheric delay (TD)

Elev. angle, E (^{0})	Current TD (meter)	New TD (meter)	Absolute Difference (%)
2	73.29	67.96	7.3
5	29.37	29.34	0.001
10	14.74	15.55	2.8
15	9.89	10.72	8.4
30	5.12	5.68	10.9
45	3.62	3.91	8.0
60	2.96	3.00	1.4
75	2.65	2.62	1.1
90	2.56	2.56	0.0

DISCUSSION

The current *UNBab* mapping function as given in equation (2) in a form of continued fraction has 7 operations. By using regression method, the *UNBab* mapping function either for hydrostatic and also non-hydrostatic components can be simplified to a simpler form. For hydrostatic component, $UNBab_h$ and for non-hydrostatic component $UNBab_{nh}$, these equations have been simplified to hyperbolic equations, which have only 2 operations.

For *UNBab*, by using logarithm, the hyperbolic equations can be changed to a linear graph. By using logarithm scale and regression method, the linear equations (log P1 & log Q1) can be generated. The difference percentage of the TD for the current model and the simplified model is very small and not significance. The simplified models P1 & Q1 can give simpler equation model and represent the original equation for both hydrostatic and non-hydrostatics components for *UNBab* mapping function.

CONCLUSION

The study shows that the simpler UNBab Mapping Function is a suitable choice to be an alternative for the mapping function due to there is no significant difference for the values of tropospheric delay between the current TD and the new TD values.

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REFERENCES

1. F. Kleijer, "Troposphere modeling and filtering for precise GPS leveling," Ph. D. thesis, Delft University, Netherlands, 2004.

2. J. Guo and R. B. Langley. "A New Tropospheric Propagation Delay Mapping Function For Elevation Angles Down To 2°" in Proceedings of the *16th International Technical Meeting of the Satellite Division of The Institute of Navigation* (ION GPS/GNSS), (GRL Publications, Portland, 2003), pp 368 – 376.

3. H. Sakidin and F. Miskon. "Simpler Wet Neill Mapping Function for Global Positioning System Tropospheric Delay" in Prosidang Simposium Kebangsaan Sains Matematik edited by Syakila et al. (Penerbit USM, Penang, 2013). pp 197 – 202.

4. T. Schuler, "On Ground-Based GPS Tropospheric Delay Estimation," Ph. D. thesis, University of Munchen, Germany, 2001.

5. J. W. Marini, Radio Science **7**, 223-231 (1972).