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Psychovisual Model on Discrete Orthonormal Transform

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Abstract. Discrete Orthonormal Transform has been a basis for digital image processing. The lesser coefficients of a Discrete Orthonormal Transform to reconstruct an image is the more compact support the Discrete Orthonormal Transform provides to an image. Tchebychev Moment Transform has been shown to provide a more compact support to an image than the popular Discrete Cosine Transform. This paper will investigate the contribution of each coefficient of the Discrete Orthonormal Transform to the image reconstruction. The error threshold in image reconstruction will be the primitive of Psychovisual Model to an image. An experimental result shall show that the Psychovisual Model will provide a statistically efficient error threshold for image reconstruction.

Keywords: Discrete Orthonormal Transform, Tchebychev Moment Transform and Psychovisual Model.
PACS: 07.05.Pj;

INTRODUCTION

Discrete Orthonormal Transforms have better image representation capability than the continuous orthogonal moments. Discrete Orthonormal Transforms are widely used in image processing applications such as image texture characterization [1], image reconstruction [2], image dithering [3] and image compression [4], [5], [6]. Recently, Tchebychev Moment Transform (TMT) has been shown to provide a more compact support to image compression [6] than the popular Discrete Cosine Transform. Tchebychev moments have its own advantage in image reconstruction error which has not been fully explored. The Tchebychev moments are capable of performing image reconstruction exactly without any numerical errors [2]. They involve only algebraic expressions and can be computed easily using a set of recurrence relation.

The visual details of image information are embedded into the amount of signal moment coefficients. In order to reduce the quantity of the irrelevant information to present visual image, the amount of moment coefficients to be encoded shall be determined by the psychovisual threshold of the human visual system (HVS) for each moment order. In order to estimate an ideal amount, each moment coefficient shall incremented one by one to analyze its effect on the error reconstruction. The sensitivity The DCT and TMT basis function are investigated in order to measure an optimal image representation.

The sensitivity of the moment coefficient on each moment order gives significant effect on the quality image reconstruction. An ideal error reconstruction threshold will be the primitive of psychovisual threshold to better image reconstruction performance.

DISCRETE ORTHONORMAL TCHEBYCHEV TRANSFORM

Discrete orthonormal Tchebychev transform is an efficient transform based on discrete Tchebychev polynomials. Mukundan [4], [5], [6] originally explores the possibility of using discrete orthonormal versions of Tchebichef polynomials for image compression. For a given set $\{t_n(x)\}$ and image intensity $f(x, y)$, the forward orthonormal Tchebychev transform of moment order $m + n$ is given as follows [7]:

$$T_{mn} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} t_m(x) t_n(y) f(x,y) \tag{1}$$

for m = 0, 1, 2, ..., M-1, n = 0, 1, 2, ..., N-1. $f(x, y)$ denotes the intensity value at the pixel position (x, y). The Discrete orthonormal Tchebychev polynomials $t_n(x)$ are defined using the following n recursive relation:

$$t_n(x) = \alpha_1 x t_{n-1}(x) + \alpha_2 t_{n-1}(x) + \alpha_3 t_{n-2}(x), \tag{2}$$

for x=0, 1, ..., M-1 and n = 2, 3, ..., N−1, where

$$\alpha_1 = \frac{2}{n}\sqrt{\frac{4n^2-1}{N^2-n^2}}, \quad \alpha_2 = \frac{(1-N)}{n}\sqrt{\frac{4n^2-1}{N^2-n^2}} \quad \text{and}$$

$$\alpha_3 = \frac{(1-n)}{n}\sqrt{\frac{2n+1}{2n-3}}\sqrt{\frac{N^2-(n-1)^2}{N^2-n^2}}. \tag{3}$$

The starting values for the above recursion can be obtained from the following equations:

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$$t_0(x) = \frac{1}{\sqrt{N}},\ t_1(x) = (2x+1-N)\sqrt{\frac{3}{N(N^2-1)}}. \quad (4)$$

The n recursion is illustrated by arrows with solid lines in Fig. 1.

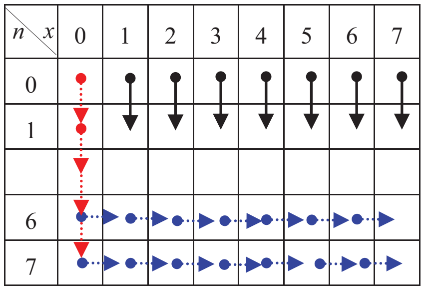

FIGURE 1. The 8×8 matrix representation of orthonormal Tchebychev polynomials, the solid arrows denote the n recursion and the dotted arrows denote x recursion.

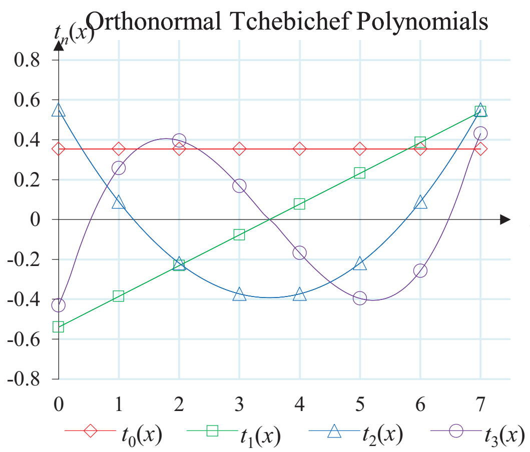

FIGURE 2. The First four Discrete Orthonormal Tchebychev Polynomials $t_n(x)$ for x = 0, 1, 2 and 3.

For a small image block such as N=8, coefficients α_1, α_2, and α_3 are small. The n recursion given in (2) is practically useful having pre-computed polynomials $t_n(x)$ for n=0 and 1. The first four discrete orthonormal Tchebychev polynomials are shown in Fig. 2.

The process of image reconstruction from its moments, the inverse TMT is given as follows:

$$\tilde{f}(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} T_{mn} t_m(x) t_n(y) \quad (5)$$

for m = 0, 1, 2, ..., M-1, n = 0, 1, 2, ..., N-1, where $\tilde{f}(x,y)$ denotes the reconstructed intensity value and M denotes the maximum order of moments used.

DISCRETE COSINE TRANSFORM

Discrete Cosine Transform (DCT) is widely used in the area of signal processing, particularly for transform coding of image compression. The two dimensional DCT is the basis of the JPEG image compression standard. The basis vectors of the DCT can be derived from the class of discrete Tchebychev polynomials [9]. In addition, DCT polynomial set $C_n(x)$ of size N=8 can be generated iteratively as follows:

$$C_0(x) = \frac{1}{\sqrt{N}},\ C_1(x) = \sqrt{\frac{2}{N}}\cos\frac{(2x+1)1\pi}{2N},$$

$$C_2(x) = \sqrt{\frac{2}{N}}\cos\frac{(2x+1)2\pi}{2N},\ C_3(x) = \sqrt{\frac{2}{N}}\cos\frac{(2x+1)3\pi}{2N}. \quad (6)$$

for x = 0, 1, 2, …, N-1. The first four one-dimensional DCT polynomials $C_n(x)$ of size N=8 above are shown in Fig. 3 for visual purposes.

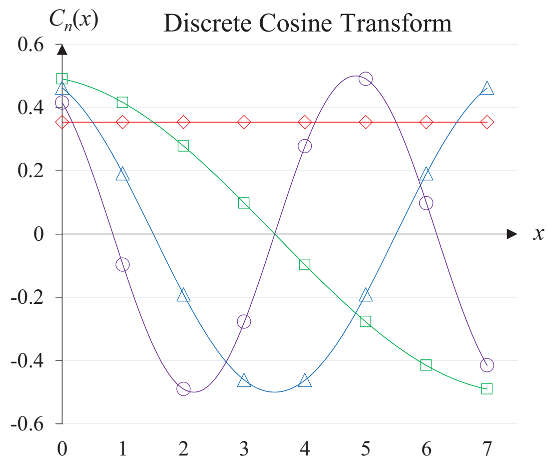

FIGURE 3. One-dimensional Discrete Cosine Transform of set $C_n(x)$ for n = 0, 1, 2, 3.

The definition of two-dimensional DCT for an input image A and output image B is given as follows [9]:

$$B_{pq} = \alpha_p \beta_q \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} A_{mn} \cos\frac{\pi(2m+1)p}{2M}\cos\frac{\pi(2n+1)q}{2N} \quad (7)$$

for p = 0, 1, 2, …, M-1 and q = 0, 1, 2, …, N-1, where

$$\alpha_p = \begin{cases} \frac{1}{\sqrt{M}}, p = 0 \\ \sqrt{\frac{2}{M}}, p > 0 \end{cases} \text{ and } \beta_q = \begin{cases} \frac{1}{\sqrt{N}}, q = 0 \\ \sqrt{\frac{2}{N}}, q > 0 \end{cases} \quad (8)$$

The inverse of two-dimensional DCT is given as follows:

$$\tilde{A}_{pq} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \alpha_p \beta_q B_{mn} \cos\frac{\pi(2m+1)p}{2M}\cos\frac{\pi(2n+1)q}{2N} \quad (9)$$

for p = 0, 1, 2, …, M-1 and q = 0, 1, 2, …, N-1.

EXPERIMENTAL DESIGN

A psychovisual model design will be conducted via quantitative experiment. The 80 images (24-bit RGB with 512×512 pixels) are chosen to be the input images. They are 40 natural images and 40 graphical images. They are transformed by TMT and DCT, quantized and reconstructed back to approximate the original image.The image reconstruction error shall be calculated by obtaining the differences between image reconstruction $g(i, j, k)$ and original image $f(i, j, k)$ which defined as follows:

$$E(s) = \frac{1}{3MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\sum_{k=0}^{2}\left|g(i,j,k) - f(i,j,k)\right| \tag{10}$$

where the original image size is $M\times N$ and the third index refers to the value of three color components. In addition the mean square error (MSE) and Peak Signal to Noise Ratio (PSNR) are also chosen here calculated to obtain to measure the quality of image reconstruction.

MOMENT ORDERS

This section provides a compact representation of the moment coefficient and the inverse moment coefficient. The block size S is taken to be 8. Based on the discrete orthonormal Tchebychev moments as defined in (1)-(5), a kernel matrix $K_{(S\times S)}$ is given as follows:

$$K = \begin{bmatrix} t_0(0) & t_1(0) & \cdots & t_{S-1}(0) \\ t_0(1) & t_1(1) & \cdots & t_{S-1}(1) \\ t_0(2) & t_1(2) & \cdots & t_{S-1}(2) \\ \vdots & \vdots & \ddots & \vdots \\ t_0(S-1) & t_1(S-1) & \cdots & t_{S-1}(S-1) \end{bmatrix} \tag{11}$$

Given an image block $F_{(S\times S)}$ with $f(x, y)$ denotes the intensity value of the image pixels for each colour component, the moment matrix $T_{(S\times S)}$ is defined in (1) above as follows:

$$T_{(S\times S)} = K^T_{(S\times S)} F_{(S\times S)} K_{(S\times S)} \tag{12}$$

This process is repeated for every block in the original image to generate Tchebychev moments. The inverse moment relation used to reconstruct the image block from the above moments is as follows:

$$G_{(S\times S)} = K_{(S\times S)} T_{(S\times S)} K^T_{(S\times S)} \tag{13}$$

where $G_{(S\times S)}$ denotes the matrix image of the reconstructed intensity value. This process is repeated for every $S\times S$ block of an image.

In general, moment order describes the numerical quantities at some distance from a reference point or axis [10]. Each 8×8 block image is arranged in a linear order of the moment coefficient. The implementation of moment by $M_{(S\times S)}$ where S=8 for TMT is as provided below:

$$M = \begin{bmatrix} m_{(0,0)} & m_{(0,1)} & \cdots & m_{(0,S-1)} \\ m_{(1,0)} & m_{(1,1)} & \cdots & m_{(1,S-1)} \\ m_{(2,0)} & m_{(2,1)} & \cdots & m_{(2,S-1)} \\ \vdots & \vdots & \ddots & \vdots \\ m_{(S-1,0)} & m_{(S-1,1)} & \cdots & m_{(S-1,S-1)} \end{bmatrix} \tag{14}$$

The moment of order zero $m_{(0,0)}$ represents the total intensity of an image [11]. The first order moment is represented as symbols $m_{(1,0)}$ and $m_{(0,1)}$. The second order moment is represented as symbols $m_{(2,0)}$, $m_{(1,1)}$ and $m_{(0,2)}$ and so on. The moment coefficients of each order are incremented one by one up to a maximum quantization value from an order zero to order fourteen. Recently, the first author proposed the quantization table for TMT image compression [6].

The quantization value is to determine the amount of moment coefficient for the visual quality image representation. The quantization is used as a threshold of the visibility HVS tolerance to reduce the quantity of moment coefficients.

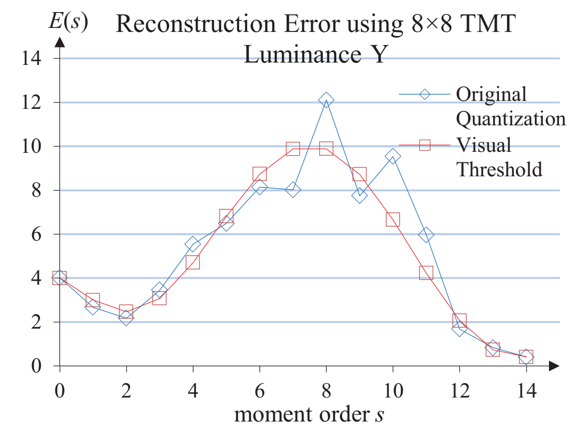

FIGURE 4. Average reconstruction error of an increment on Tchebychev moment coefficients on the luminance for 40 natural color images.

PSYCHOVISUAL MODEL ON TMT

The reconstruction error scores of an increment based on the quantization table [6] from an order zero to the order fourteen produces a curve. In order to produce a psychovisual threshold, the new a smooth transitional curve is needed which results in an ideal curve of average error scores. The average reconstruction error of an increment Tchebychev moment coefficients on luminance (Y) and Chrominance (U) for 40 natural images are shown in Figs. 2 and 3.

The blue line as depicted in Figs. 4 and 5 presents image reconstruction error for each moment order based on a quantization table value in [6] respectively. An ideal psychovisual threshold for luminance and chrominance is represented by a red curve. The authors propose a function as depicted by a red line in Figs. 4 and 5 for psychovisual error thresholds of Tchebychev basis function for luminance f_{VL} and chrominance f_{VR} which are defined as follows:

$$f_{ML}(x) = -0.00009895x^6 + 0.0045x^5 - 0.07129x^4 + 0.4354x^3 - 0.6352x^2 - 0.737x + 4. \quad (15)$$

$$f_{MR}(x) = -0.00008837x^6 + 0.0041x^5 - 0.0661x^4 + 0.4111x^3 - 0.6368x^2 - 0.4389x + 3. \quad (16)$$

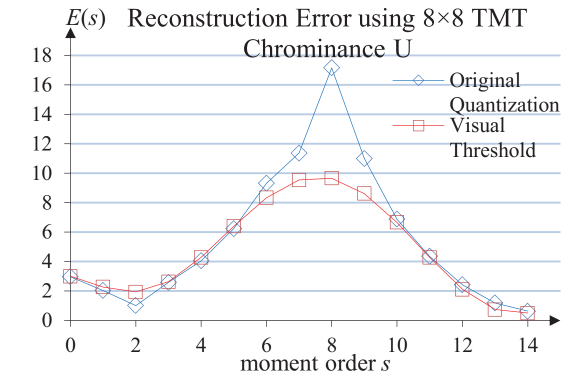

FIGURE 5. Average reconstruction error of an increment on Tchebychev moment coefficients on the chrominance for 40 natural color images.

The ideal error reconstruction for each moments order is used to determine the tolerance on image representation to the HVS. These functions are used as thresholds for each block 8×8 moment coefficients to reduce the amount of codes on moment coefficients.

PSYCHOVISUAL MODEL ON DCT

The effects of incrementing DCT coefficients based on from minimum value to the maximum JPEG quantization tables on a given order are measured by image reconstruction error to get a threshold function. The average full error score of an increment DCT coefficient on luminance (Y) and chrominance (U) for 40 natural images are shown in Figs**.** 6 and 7.

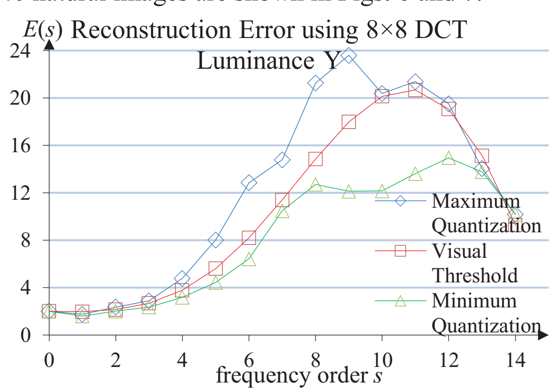

FIGURE 6. Average reconstruction error of an increment on DCT coefficient on the luminance for 40 natural color images.

The green and blue lines represent image reconstruction error from the minimum to maximum values on the JPEG quantization table. The curve from order zero to fourteen of average reconstruction error is analysed to get a smooth transition to produce an ideal curve of average error scores. An ideal psychovisual threshold for luminance and chrominance is represented by a red curve.

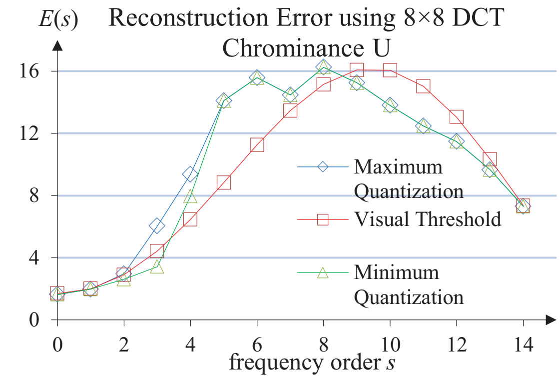

FIGURE 7. Average reconstruction error of an increment on DCT coefficient on the chrominance for 40 natural color images.

With reference to Figs. 6 and 7, the authors propose a psychovisual threshold for DCT basis function for luminance f_{VL} and chrominance f_{VR} of the quantization tables which are defined as follows:

$$f_{VL}(x) = 0.00005715x^6 - 0.002x^5 + 0.0202x^4 - 0.0561x^3 + 0.1683x^2 - 0.1743x + 2 \quad (17)$$

$$f_{VR}(x) = 0.0002785x^5 - 0.0082x^4 + 0.0471x^3 - 0.2082x^2 + 0.0588x + 1.7 \quad (18)$$

for $x = 0, 1, 2, ..., 14$.

TABLE 1. Reconstruction error score between 8×8 DCT and 8×8 TMT for 40 real images

Image	Default Quantization		Psychovisual threshold	
Measure	8×8 DCT	8×8 TMT	8×8 DCT	8×8 TMT
Full Error	5.5348	5.2584	5.4987	5.2456
MSE	70.9635	58.1587	69.5199	57.4476
PSNR	31.1903	31.3721	31.2516	31.3790

TABLE 2. Reconstruction error score between 8×8 DCT and 8×8 TMT for 40 graphical images.

Image	Default Quantization		Psychovisual threshold	
Measure	8×8 DCT	8×8 TMT	8×8 DCT	8×8 TMT
Full Error	6.1479	4.71429	5.8087	4.6034
MSE	113.8332	68.20336	100.0520	62.5664
PSNR	29.7903	31.4483	30.2278	31.6477

The statistical reconstruction error of psychovisual model for Tchebychev moments for 40 real and 40 graphical images respectively are shown in Table 1 and Table 2. The psychovisual threshold on TMT gives significantly better performance than DCT especially on graphical images. The experimental results show that psychovisual threshold performs better on both DCT and TMT by giving lower reconstruction error. In order to observe the effectiveness of a psychovisual threshold for 8×8 Tchebychev basis function, the reconstruction image is zoomed in 400%.

FIGURE 8. Original baboon image and its zoomed left eye.

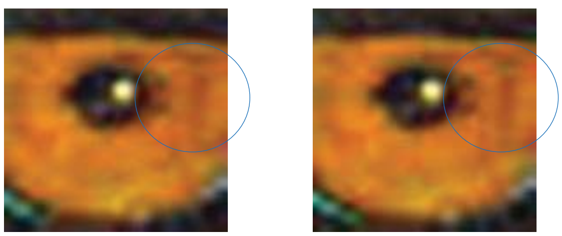

FIGURE 9. Output Images from quantization table (left) from standard JPEG against psychovisual threshold.

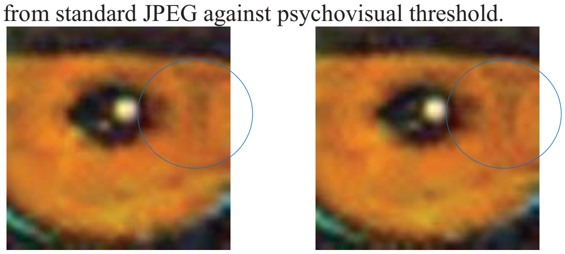

FIGURE 10. Output Images from quantization table (left) from original TMT against psychovisual threshold.

The experimental results of image reconstruction from TMT using psychovisual threshold as depicted on the right of Fig. 10 is closer toward to the original image.

CONCLUSSION

Moment functions based on discrete orthonormal Tchebichef polynomials have been used recently in image compression. This paper has introduced psychovisual model based on image reconstruction error. These threshold functions represent the contribution of each moment coefficient to reconstruct the compressed image. This psychovisual threshold is then used to determine the amount of moments to represent the visual details of image information. The experimental results show that the psychovisual model provides an efficient reconstruction error for a better image quality. Psychovisual model provides an optimal compact image representation from a minimum representation of moment coefficients. Image reconstruction using psychovisual model based on orthonormal Tchebychev moments has been used as an example to illustrate the efficient image compression based on the proposed psychovisual threshold. The psychovisual model can be suitability modified for an adaptive image compression to generate custom quantization tables. The proposed psychovisual model can be used to do high image compression rate and still get high quality image reconstruction.

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