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SLIDING MODE CONTROL OF A CLASS OF MISMATCHED UNCERTAIN SYSTEMS

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Abstract

A proportional-integral sliding mode control is proposed for a system with mismatch uncertainties. The proposed controller gives robust stability for system in the presence of parameters variations, uncertainties and disturbances. A simulation study for a numerical example is given to illustrate the effectiveness of this control design.

1 Introduction

The control of dynamical systems, whose mathematical models contain uncertainties, has occupied the attention of researchers in recent times and has been extensively studied. These uncertainties could be due to parameters, constant or varying, which are unknown or perfectly known, or due to unknown or imperfectly known inputs into the system [1]. Sliding mode control (SMC) has been widely applied to system with uncertainties since it was introduced about three decades ago. A salient future of this control is that it is completely robust to systems with matched uncertainties [2]. It is certainly true that many systems can be classified under this category. However, there are some systems which unfortunately are affected by uncertainties which do not satisfy the matching condition. A sliding mode control scheme for mismatched uncertain systems has been recently developed by [3,4,5,6]. All these researches used the traditional method to design the sliding surface. However, [7] has developed a sliding mode control scheme used an integral-type sliding surface.

In this paper we will consider a class of uncertain dynamical systems in mismatched condition and design the sliding surface based on proportional-integral (PI) sliding mode control. We also proposed a new control scheme to control such a system with mismatched uncertainties. The proposed control scheme differ from the one developed by [7] in the sense that the proposed sliding gain is applied for both matched and mismatched uncertainties.

2 System Design

Consider the following uncertain systems [4]:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \qquad (1)$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input, and the continuous function $f(x,t)$ represents the uncertainties with the matched and mismatched parts. Note that $f(x,t)$ is uniformly bounded with respect to time t, and locally uniformly bounded with respect to state x. The following assumptions are taken as standard:

Assumption i: There exists a known non-negative function such that $\|f(x,t)\| \le \beta(x,t)$, where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption ii: The pair (A,B) is controllable and the input matrix B has full rank.

In this study, we utilized the PI sliding surface define as follows:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau)d\tau \qquad (2)$$

where $C \in \Re^{mxn}$ and $K \in \Re^{mxn}$ are constant matrices. The matrix K satisfies $\lambda_{\max}(A + BK) < 0$ and C is chosen so that CB is nonsingular. It is well known that if the system is able to enter the sliding mode, $\sigma(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ [8] i.e,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0 \qquad (3)$$

If the matrix C is chosen such that CB is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(x,t) \qquad (4)$$

Substituting equation (4) into system (1) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C)\}f(x,t) \qquad (5)$$

Theorem 1: If

$$\left\|\tilde{F}(x,t)\right\| \leq \beta_1(x,t) = \left\|I_n - B(CB)^{-1}C\right\|\beta(x,t)$$

the uncertain system in equation (5) is boundedly stable on the sliding surface $\sigma(t) = 0$.

Proof:

For simplicity, we let

$$\tilde{A} = (A + BK) \qquad (5a)$$

$$\tilde{F}(x,t) = \{I_n - B(CB)^{-1}C\}f(x,t) \qquad (5b)$$

and rewrite (5) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(x,t) \qquad (6)$$

Let the Lyapunov function candidate for the system is chosen as

$$V(t) = x^T(t)Px(t) \qquad (7)$$

Taking the derivative of $V(t)$ and substituting equation (5) into it, gives

$$\begin{aligned}\dot{V}(t) &= x^T(t)[\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(x,t)Px(t) + x^T(t)P\tilde{F}(x,t) \qquad (8)\\ &= -x^T(t)Qx(t) + \tilde{F}^T(x,t)Px(t) + x^T(t)P\tilde{F}^T(x,t)\end{aligned}$$

where P is the solution of $\tilde{A}^T P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q. It can be shown that equation (8) can be reduced to:

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1(x,t)\|P\|\|x(t)\| \qquad (9)$$

Since $\lambda_{\min}(Q) > 0$, consequently $\dot{V}(t) < 0$ for all t and $x \in \mathrm{B}^c(\eta)$, where $\mathrm{B}^c(\eta)$ is the complement of the closed ball $\mathrm{B}(\eta)$, centered at $x = 0$ with radius $\eta = \dfrac{2\beta_1(x,t)\|P\|}{\lambda_{\min}(Q)}$. Hence, the system is boundedly stable.□

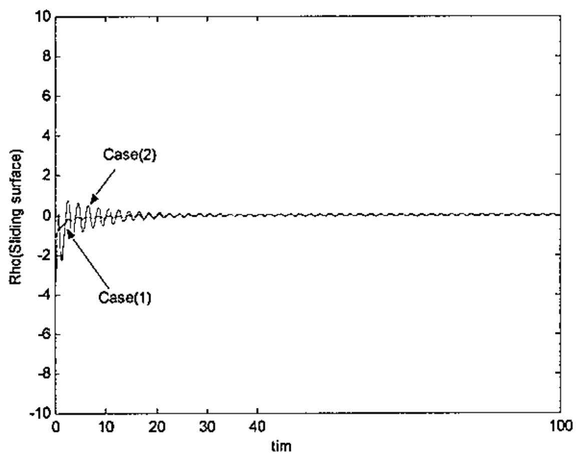

Figure 1: Sliding surface for matched and mismatched conditions.

Remark: For the system with uncertainties satisfy the matching condition, i.e, $rank[B \mid f(x,t)] = rank[B]$, then equation (5) can be reduced to $\dot{x}(t) = (A + BK)x(t)$ [9]. Thus asymptotic stability of the system during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system in equation (1) onto the sliding surface $\sigma(t)=0$ and the system remains in it thereafter.

3 Variable Structure Controller Design

For the uncertain system in equation (1) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t)=-(CB)^{-1}[CAx(t)+\phi\sigma(t)] - k(CB)^{-1}\frac{\sigma(t)}{\|\sigma(t)\|+\delta} \quad (10)$$

where $\phi \in \Re^{m\times m}$ is a positive symmetric design matrix, k and δ are positive constants.

Theorem 2: The hitting condition of the sliding surface (2) is satisfied if

$$\|A+BK\|\|x(t)\| \geq \|f(x,t)\| \quad (11)$$

Proof:

In the hitting phase $\sigma^T(t)\sigma(t)>0$; using the Lyapunov function candidate $V(t)=\frac{1}{2}\sigma^T(t)\sigma(t)$, we obtain

$$\begin{aligned}\dot{V}(t) &= \sigma^T(t)\dot{\sigma}(t)\\ &= \sigma^T(t)[-(CA+CBK)x(t)-\phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\|+\delta}+Cf(x,t)] \quad (12)\\ &\leq -[\{\|\phi\|+\left\|\frac{k}{\|\sigma(t)\|+\delta}\right\|\}\|\sigma(t)\|^2 + \{\|C\|\|A+BK\|\|x(t)\|-\|C\|\|f(x,t)\|\}\|\sigma(t)\|]\end{aligned}$$

It follows that $\dot{V}(t)<0$ if condition (11) is satisfied. Thus, the hitting condition is satisfied.□

4 Example

To illustrate the performance of the proposed controller, consider the third order single input system as given in reference [2,3,4], i.e

$\dot{x}(t)=Ax(t)+Bu(t)+f(x,t)$, where

$$A=\begin{bmatrix}-0.03 & 0.01 & 0.01\\ -0.05 & -0.15 & 0.05\\ -0.09 & 0.03 & -0.17\end{bmatrix} \quad B=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

and $f(x,t)=\Delta A(t)x(t)+f_1(t)$. In order to show the robustness of the proposed controller with respect to the mismatched condition we consider the following cases for the function $f(x,t)$:

Case I: Both the matrices $\Delta A(t)$ and $f_1(t)$ satisfy the matching condition; i.e. we consider the case where $\Delta A(t)$ and $f_1(t)$ as given below:

$$\Delta A(t)=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0.5\sin(3.14t) & 0 & 0\end{bmatrix} \text{ and}$$

$$f_1(t)=\begin{bmatrix}0\\0\\5\sin(3.14t)\end{bmatrix}$$

Case II: The matrix $f_1(t)$ satisfy the matching condition as given as in Case I while the matrix $\Delta A(t)$ does not satisfy the mismatched condition and is given as follows:

$$\Delta A(t)=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

where

$a_{11}=0.01+0.44\sin(3.14t)$,
$a_{12}=0.01+0.004\cos(3.14t)$,
$a_{13}=0.008+0.002\sin(6.28t)$,
$a_{21}=0.55+0.220\sin(3.14t)$,
$a_{22}=0.05+0.020\cos(3.14t)$,
$a_{23}=0.04+0.010\sin(6.28t)$,
$a_{31}=0.5\sin(3.14t), a_{32}=0, a_{33}=0$.

In both cases, we choose $K=[-4.03\ -0.67\ -0.45]$ such that $\lambda(A+BK)=\{-0.1,-0.3,-0.4\}$, $C=[450\ 50\ 10]$, $\phi=1000$, $k=10$ and $\delta=0.0001$. Figure 1 displays the sliding surface for both cases with respect to time. Figures 2(a), 2(b) and 2(c) show the state responses for both cases subjected to the initial condition $x(0)=[10\ 0\ 0]^T$. The corresponding control input for both cases is shown in Figure 3. The results also show that the state trajectory hit and slide on the sliding surface as intended for both cases. This clearly demonstrates that the nonlinear system with mismatch uncertainties utilizing the proposed controller is practically stable.

5 Conclusions

In this paper, the PI sliding mode control technique is proposed for controlling uncertain system where the uncertainties does not satisfy the matching condition. It has been shown mathematically and through computer simulations that the proposed control scheme is capable of controlling the uncertain system and is practically stable with respect to the mismatched uncertainty condition.

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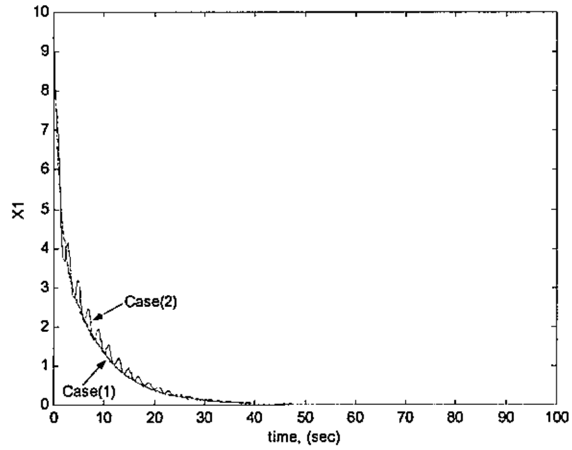
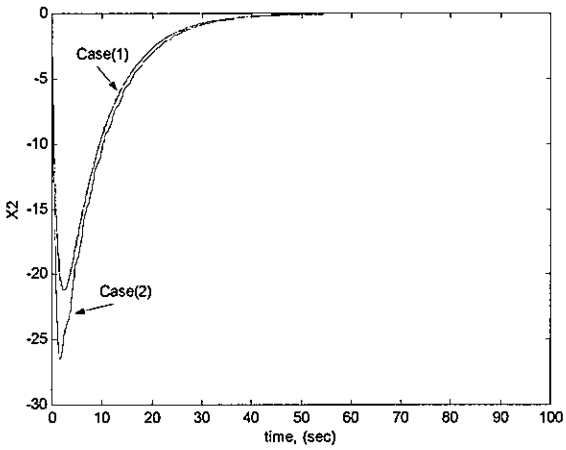
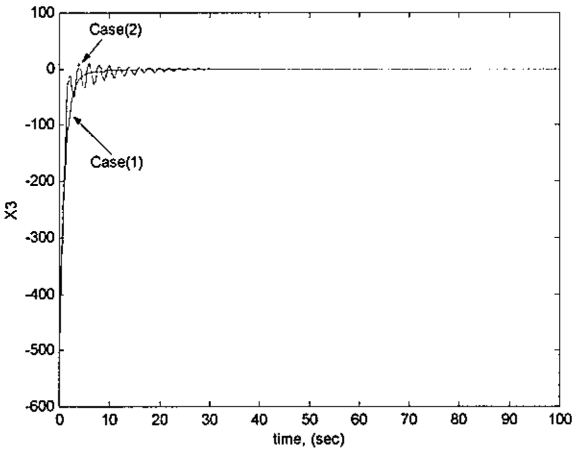


2(a)



2(b)



2(c)

Figures 2(a), 2(b) and 2(c): State responses for matched and mismatched conditions.

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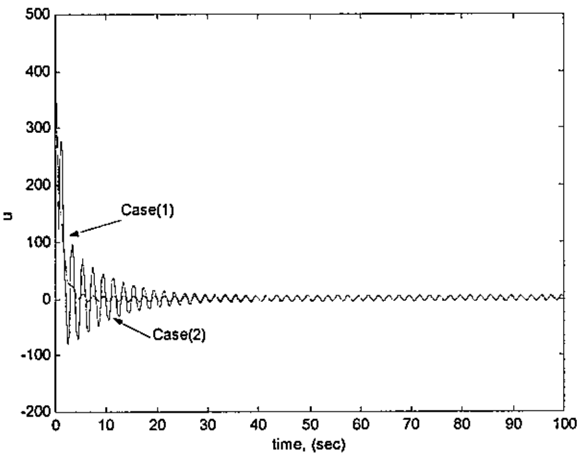

Fig. 3: Controlled input for matched and mismatched conditions.