Transmissibility of a Laminated Rubber-Metal Spring: A Preliminary Study

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Abstract. This paper discusses the isolation performance of a laminated rubber-metal spring. A multi-degree-of-freedom-system model using mass-damper-spring components is developed to calculate its transmissibility due to only axial vibration in the spring. The results show the embedded layers of metal plate can improve the transmissibility at high frequency.

Introduction

Development of vibration isolators is still progressing, in particular those made from rubber material. Vibration isolators have been applied for vibration isolation to reduce vibration transmission and are widely used in buildings, vehicles and for sensitive instruments including for earthquake protection [1-4]. For the latter, the rubber material requires very high damping to ensure sufficient dissipation of vibration energy from the seismic waves. Mordini and Strauss [5] showed that the strength of a high damping isolator can be improved by adding some glass fibre fabrics into the rubber blocks.

A laminated rubber-metal spring has also been developed for this application. The embedded metal plates are intended to prevent the rubber blocks of the conventional isolator from excessive bulging effect which otherwise reduces the performance of the isolator as well as its durability [3-4]. The idea first to develop the reinforcing rubber blocks by thin steel plates came from a French engineer named Eugene Freyssinet [6]. This invention was continued in the 1960's at the Malaysian Rubber Producer's Research Association (MRPRA, now known as Tun Abdul Razak Research Centre, TARRC) in United Kingdom. This type of spring is known as a laminated rubber metal spring. Fig. 1 shows the illustration. Fuller *et al.* [7] later discussed the use of natural rubber compounds in the laminated rubber-bearing which was shown to have long-term stability of stiffness after being used for forty years of service as a seismic isolator.

As it is designed to work well for earthquake protection, the bearing is predominantly intended to provide isolation when subjected to a shear force at very low frequency. For other applications such as for engine mountings, isolation of buildings from ground-borne vibration near railway lines or train vehicle suspensions, where the input force comes from vertical direction and can extend up to higher frequencies, there appears a lack of discussion for this situation. Investigation on the possibility of using the laminated spring for these applications is therefore of interest. The next section presents a simple model as the preliminary step to study the effect of the axial force on the laminated spring.

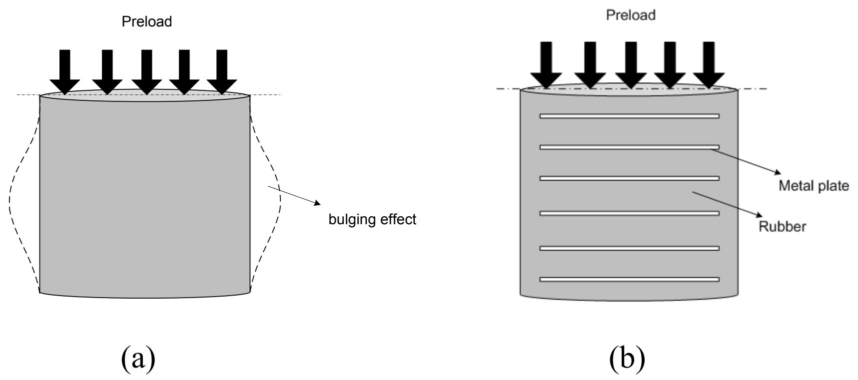

Figure 1: (a) A bulging effect in a rubber isolator due to a large preload and (b) a laminated rubber-metal spring.

Lumped Parameter Model

In this paper, the laminated spring system is modelled using a lumped parameter model consisting of mass, damper and spring components. To assess the vibration isolation performance, the spring is loaded with a lumped mass *M* excited with a harmonic force F_e. The rubber is ideally modelled as a massless component having a constant stiffness *k* and damping coefficient *c*. The embedded plate is treated as a rigid solid mass *m* without damping. The laminated spring is attached on a structure and the transmissibility is derived. Only vertical motion is taken into account. The rotational motion is neglected. Fig. 2 shows the schematic diagram of the model of laminated spring with one layer of metal plate which creates a two-degree-of-freedom system.

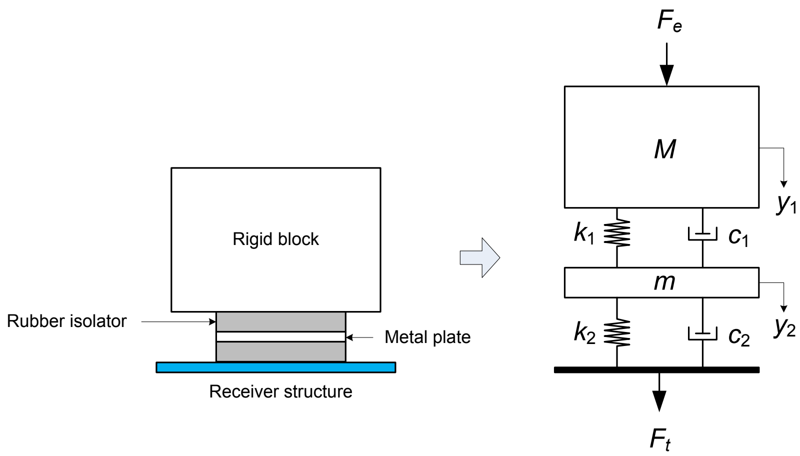

Figure 2: Mass-damper-spring model of a laminated-rubber metal spring.

Two equations of motion can be derived as follows

$$M\ddot{y}_1 + c_1(\dot{y}_1 - \dot{y}_2) + k_1(y_1 - y_2) = F_e \qquad (1)$$

$$m\ddot{y}_2 + c_1(\dot{y}_2 - \dot{y}_1) + c_2\dot{y}_2 + k_1(y_2 - y_1) + k_2 y_2 = 0 \qquad (2)$$

Due to a harmonic force, the resulting motion of the system is also harmonic. Substituting $y = Ye^{j\omega t}$ in Eqs. (1) and (2) with Y the complex amplitude and ω the frequency, the equations of motion can be expressed in matrix form as

$$\left(-\omega^2\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} + j\omega\begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1+c_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}\right)\begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} = \begin{Bmatrix} F_e \\ 0 \end{Bmatrix} \tag{3}$$

or in general form

$$[-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}]\tilde{\mathbf{Y}} = \tilde{\mathbf{F}} \tag{4}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, $\tilde{\mathbf{Y}}$ and $\tilde{\mathbf{F}}$ are the vectors of complex displacement amplitude and force, respectively. The displacements Y_1 and Y_2 at each frequency ω can therefore be obtained by

$$\tilde{\mathbf{Y}} = [-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}]^{-1}\tilde{\mathbf{F}} \tag{5}$$

where $\mathbf{A}^{-1}$ indicates the inverse of matrix **A**. For the case of the system in Fig. 1, the force transmitted to the receiver structure can be written as a function of the displacement of the bottom metal layer. For an excitation force of unit amplitude $F_e = 1$, the transmissibility, i.e. the amplitude ratio of transmitted force to excitation force, is given by

$$T = \left|\frac{F_t}{F_e}\right| = |F_t| = Y_2\sqrt{(k_2^2 + \omega^2 c_2^2)} \tag{6}$$

For two layers of metal plates inside the rubber (three-degree-of-freedom system), the mass, damping and stiffness matrices in Eq. (4) are given by

$$\mathbf{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1+c_2 & -c_2 \\ 0 & -c_2 & c_2+c_3 \end{bmatrix} \text{ and } \mathbf{K} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix} \tag{7}$$

By observing the structural pattern of the matrices in Eqs. (3) and (7), for N layers of metal plates, the matrices thus have dimensions of $(N+1)\times(N+1)$ and are expressed as

$$\mathbf{M} = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & m_N \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_1 & -c_1 & 0 & 0 & \cdots & 0 \\ -c_1 & c_1+c_2 & -c_2 & 0 & \cdots & 0 \\ 0 & -c_2 & c_2+c_3 & -c_3 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & c_N \\ 0 & 0 & \cdots & -c_{N-1} & & c_{N-1}+c_N \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & \cdots & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & \cdots & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & & \vdots \\ \vdots & \vdots & & & \ddots & k_N \\ 0 & \cdots & \cdots & 0 & -k_{N-1} & k_{N-1} + k_N \end{bmatrix} \tag{8}$$

Results and Discussion

Fig. 3 plots the transmissibility for different numbers of layers N of embedded metal plates. The calculation is made assuming the mass of each plate is the same as the loaded mass. The damping is also assumed very small. Peaks indicating amplification of the injected force to the receiver structure can be seen at low frequencies, for example at 9 Hz and 23 Hz in Fig. 3(a), which appear at the natural frequencies of the system. The more plates are embedded in the rubber, the higher the number of degrees-of-freedom and the more natural frequencies the system has. The results also show that the resonant peaks shift towards lower frequencies as the number of intermediate plates increases.

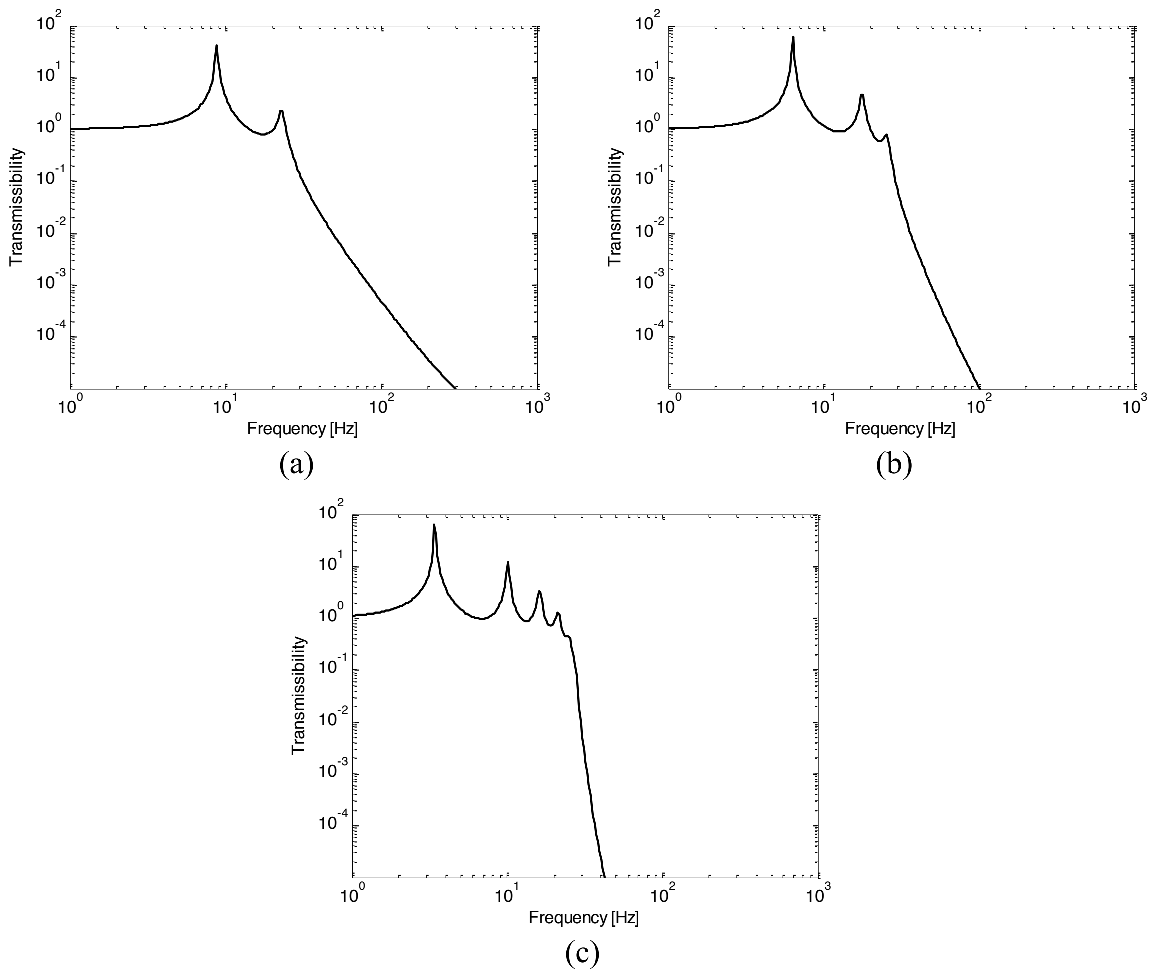

Figure 3. Transmissibility of laminated rubber-metal spring from lumped parameter model with different numbers of layers of plate: (a) $N = 1$, (b) $N = 2$ and (c) $N = 5$.

The isolation region (where the transmissibility is less than unity) can be seen roughly above 25 Hz. It is interesting to observe that the transmissibility curve rapidly decreases with the increasing of the number of embedded plates in the rubber. This indicates that the laminated rubber-metal spring improves the vibration isolation at high frequency. However, this lumped parameter model ignores the mass of the rubber which can contribute to more internal resonances. The width of the rubber layer is neglected as at high frequencies when the wavelength is much smaller than the rubber layer thickness, wave effects from various directions will affect the isolation performance. This wave effect has been highlighted in the classical rubber rod isolator [2], [8]. Torsional degrees of freedom can also be important.

Conclusion

Transmissibility of a laminated rubber-metal spring has been modelled using a lumped parameter system assuming only transverse vibration in the spring. This simple model shows improvement of isolation performance of the spring when more layers of plates are included, but creates more resonances towards low frequency. However, an extended model is required to take into account the mass of the rubber, the layer width and various directions of wave propagation inside the spring including the rotational motion for comprehensive analysis.

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