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Determination of Vibration Strength of a Small Structure-Borne Source Using a High Mobility Reception Plate

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Abstract – *This paper presents the characterization of vibration strength obtained from reception plate method by applying the mobility concepts. It describes a laboratory-based measurement procedure, which determines the strength of a vibration source in terms of the total squared free velocity of the source. The source used in the experiment is the small electric fan motor installed on high mobility aluminum panel in order to neglect the influence of the source mobility. The complexity of the mobilities at the contact points are reduced using the single value of effective mobility. The aim is to validate the data obtained from the reception plate method with one from the direct measurement. A good agreement is found between the two results.*

Keywords: *effective mobility, reception plate method, squared free velocity, structure-borne source, source-reciever.*

I. Introduction

Annoyance due to noise in a building is still one of major problems in engineering. This noise is often caused by vibration from rotating machines which are channeled to the building structure. This then carries the vibrational waves and radiates noise into the air. The most risky structure is the one of industrial or factory building which contains many vibrating machineries, such as the iron and steel industries, foundries, saw mills, textile mills and crushing mills among many others. Those machines are capable of injecting high level of vibration which not only causes noise but also hazardous to the structure where the machines are installed. Such machines which causes propagation of vibrational waves into the neighboring structure is called structure-borne sources.

The vibration effect to a structure is rather vital. The symptom before the structural damage is sometimes not visible. With information of the vibration input power from the sources; preliminary control measure can be planned. For example while planning to install a huge vibrating machine, the supplied information of the machine's vibration input power allows a structural engineer to take preventive action, such as to ensure the support structure is strong enough to absorb the potential vibration power. This will give time to reinforce the structure or install some damper at the certain locations e.g. at the contact points between structure and source [1].

The treatment of structure-borne sound sources remains a challenging problem due to many uncertainties and difficulties. For example, measurement or determination of the force excitation to a building floor, by active components like pumps, compressors, fans and motors, which is an important mechanism of vibration and noise generation and also an important parameter to obtain the potential vibration input power [2].

Determining the force between an installation and a building structure directly is a rather cumbersome task. Force sensors would have to be inserted between installation (source) and building element (receiver), which is difficult or even impossible for large and heavy installations. Even if it is possible, one must be careful not to alter the vibro-mechanical behavior of the source-receiver system in the frequency range of interest [3].

Therefore, rather than to predict the vibration input power *in situ*, prediction before installation is of interest. This paper presents a laboratory-based method using a reception plate. A small fan motor was used as the vibration source and its input power 'strength' was measured. This is represented by the free velocity which is one of parameters to characterize the structure-borne source. The using of effective mobility of the reception plate is also discussed.

II. Mathematical Formulation

II.1. *Vibration input power*

Figure 1(a) shows a diagram of a source having impedance Z_S and free velocity v_f.

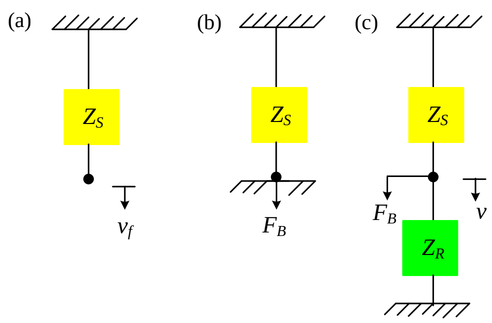


Fig. 1:Mathematical diagram of a vibration source and a receiver.

If the source is then attached rigidly on a rigid surface (Figure 1(b)), the resulted force F_B is called the blocked force. From definition

$$F_B = Z_S v_f = \frac{v_f}{Y_S} \tag{1}$$

where Z_S and Y_S are the impedance and mobility of the source, respectively.

If the source is now connected to a receiver with impedance Z_R (Figure 1(c)) and assuming both the source and receiver move in the same velocity v, the blocked force is the sum of the force from the source F_S and force at the receiver F_R. The blocked force can thus be written as

$$F_B = F_S + F_R = (Z_S + Z_R)v \tag{2}$$

By re-arranging Eq. (2), the velocity of the source-receiver system can be obtained in terms of the properties of the source and receiver

$$v = \frac{F_B}{Z_S + Z_R} = \frac{F_B Z_R^{-1}}{\frac{Z_S}{Z_R} + 1} = \frac{F_B Z_R^{-1} Z_S^{-1}}{\frac{1}{Z_R} + \frac{1}{Z_S}} \tag{3}$$

Eq. (3) can also be written as the function of the mobility $Y = 1/Z$ and the free velocity as

$$v = Y_R (Y_S + Y_R)^{-1} v_f \tag{4}$$

Assume the source is attached to the receiver through a single contact point, the power injected to the receiver, P_{in} is defined by

$$P_{in} = \frac{1}{2}\mathrm{Re}\{Z_R\} v^2 \tag{5}$$

By substituting Eq. (4) into Eq. (5), the input power can be expressed as

$$\begin{aligned} P_{in} &= \frac{1}{2}\mathrm{Re}\{Z_R\}\left(\frac{Y_R}{Y_S + Y_R}\right)^2 v_f^{\,2} \\ &= \frac{1}{2}\mathrm{Re}\left\{\frac{1}{Y_R}\right\}\left(\frac{Y_R}{Y_S + Y_R}\right)^2 v_f^{\,2} \end{aligned} \tag{6}$$

Eq. (6) can then be simplified and be written as

$$P_{in} = \frac{1}{2}\frac{\mathrm{Re}(Y_R)}{(Y_S + Y_R)^2} v_f^{\,2} \tag{7}$$

For N contact points, Eq. (7) is expressed in terms of matrices and vectors

$$\begin{aligned} P_{in} &= \frac{1}{2}\{v_f\}^T \left[([Y_S] + [Y_R])^T\right]^{-1} \times \\ &\times \mathrm{Re}([Y_R])\left[[Y_S^*] + [Y_R^*]\right]^{-1}\{v_f^*\} \end{aligned} \tag{8}$$

where $\{v_f\}$ is the free velocity vector and $[Y_{S,R}]$ is the mobility matrices. For six component of excitations (3 translational and 3 rotational), the required matrix size is $6N \times 6N$. However, in this paper, only translational force perpendicular to the receiver plane is taken into account. The matrix size then reduces to $N \times N$ given by

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{12} & Y_{22} & \cdots & \cdots \\ \vdots & & \ddots & \vdots \\ Y_{N1} & & \cdots & Y_{NN} \end{bmatrix} \tag{9}$$

where Y_{ij} is the point mobility for $i = j$ or transfer mobility for $i \neq j$.

II.2. Reception Plate Method

In this section, the reception plate equation is introduced. Using the reception plate method, a machine under test is attached on a plate under its normal operating conditions [4,5]. The total structure-borne power transmitted is obtained from the measured spatial average of the mean square plate velocity

$$P_{in} = \eta S \ddot{m} \omega \left\langle v_R{}^2 \right\rangle \tag{10}$$

where η is the damping loss factor of the plate, S is the plate area, $\ddot{m}$ is the mass per unit area, $\left\langle v_R{}^2 \right\rangle$ is the spatially average of mean-squared velocity and ω is the operating frequency. The damping loss factor can be obtained by

$$\eta = \frac{\mathrm{Re}\{Y_P\}}{\omega M \left\langle |Y_t|^2 \right\rangle} \tag{11}$$

for Y_P is the point mobility of the reception plate, M the total mass and $\left\langle |Y_t|^2 \right\rangle$ is the spatially average squared transfer mobilities.

From Eq. (8) and (10), by using high mobility reception plate where $Y_R >> Y_S$, this gives

$$\eta S \ddot{m} \omega \left\langle v_R{}^2 \right\rangle = \frac{1}{2} \{v_f\}^T \left[[Y_R]^T\right]^{-1} \times \times \mathrm{Re}([Y_R])[Y_R^*]^{-1} \{v_f^*\} \tag{12}$$

To obtain the free velocity v_f of the source under test, Eq. (11) is difficult to solve as the velocity at each source feet itself carries phase information.

However, this can be treated assuming in-phase velocity for all the feet and small variation between the point and transfer mobilities (Eq. (9)) at the contact points. For this purpose, the concept of effective mobility is introduced.

II.3. Effective Mobility

The effective mobility sums the point and transfer mobilities for each contact point to be a "single mobility" [6,7]. For zero phase assumption, it is expressed as

$$Y_i^{eff} = Y_{ii} + \sum_{\substack{i=1 \\ i \neq j}}^{N} Y_{ij} \tag{13}$$

For random phase assumption, it is given by

$$\left| Y_i^{eff} \right|^2 = |Y_i|^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} |Y_{ij}|^2 \tag{14}$$

Hence by using the effective mobility, Eq. (12) can be written as

$$\eta S \ddot{m} \omega \left\langle v_R{}^2 \right\rangle = \frac{1}{2} \sum_{i}^{N} \mathrm{Re}\left(\frac{1}{Y_{R,i}^{eff}} \right) |v_f|^2 \tag{15}$$

By also assuming small variation of $Y_{R,i}^{eff}$ at each contact point on the reception plate, Eq. (15) can be further simplified as

$$\eta S \ddot{m} \omega \left\langle v_R{}^2 \right\rangle = \frac{1}{2} \mathrm{Re}\left(\frac{1}{Y_R^{eff}} \right) \sum_{i}^{N} |v_f|^2 \tag{16}$$

where Y_R^{eff} is the average effective mobility for all contact points. From Eq. (16), the total squared free velocity $\sum_{i}^{N}\left|v_f\right|^2$ of the tested structure-borne source can now be obtained.

III. Experiment and Results

The experiment was conducted using a table fan motor as the source and an aluminium plate of 1 mm thick and dimensions 1.4×0.8 m as the receiver. To use Eq. (16), the mobility of the receiver must be much larger than that of the source. Fig. 1 shows the comparison between the measured mobilities of the motor and the receiver using the instrumented impact hammer and accelerometer. It can be seen that on average the plate mobility is 20 dB larger than that of the motor indicating that the source mobility can be neglected in Eq. (12). The measured data are the point mobility of the plate at the contact point location and the mobility of the source at the feet of the motor.

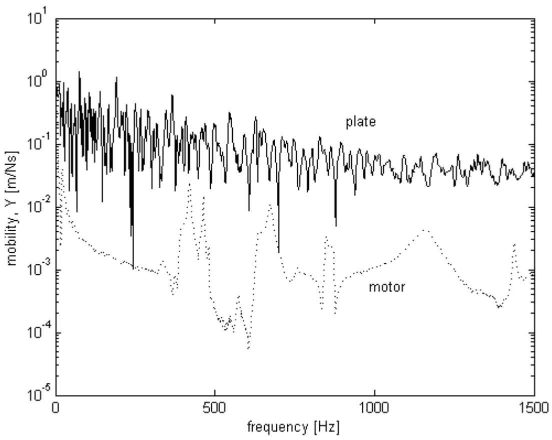
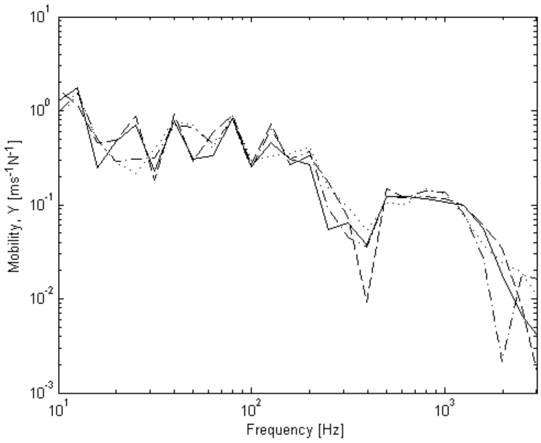
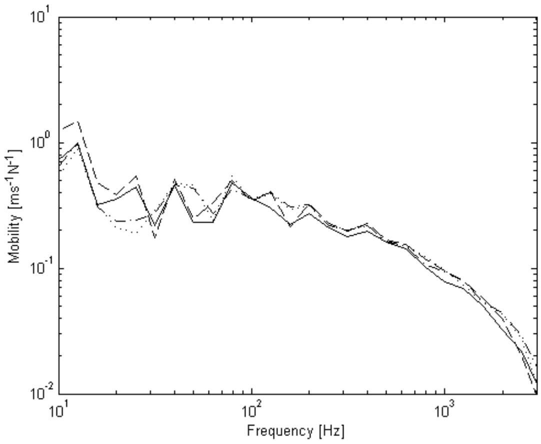


Fig. 1: Comparison between the source and receiver mobilities using in the experiment.

The variations of the effective mobilities are shown in Fig. 2 for zero and random phase assumptions in one-third octave bands.



(a)



(b)

Fig. 2: Effective mobility of the reception plate assuming: (a) zero phase and (b) random phase.

The results show that in general, the variation is within 5 dB for each contact point location at the reception plate.

To obtain the spatially average of mean-squared velocity $\left\langle v_R^{\ 2}\right\rangle$ in Eq. (16), the motor was attached on the receiver plate through four contact points and was run at normal condition. See Fig. 3. Eight locations were chosen as the measurement points scattered on the plate to represent the spatial average. The result is shown in Fig. 4.

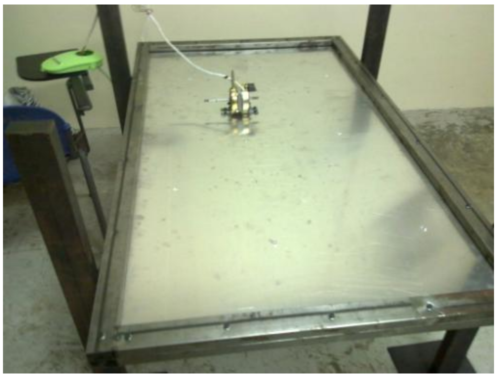


Fig. 3: The motor attached on high mobility reception plate.

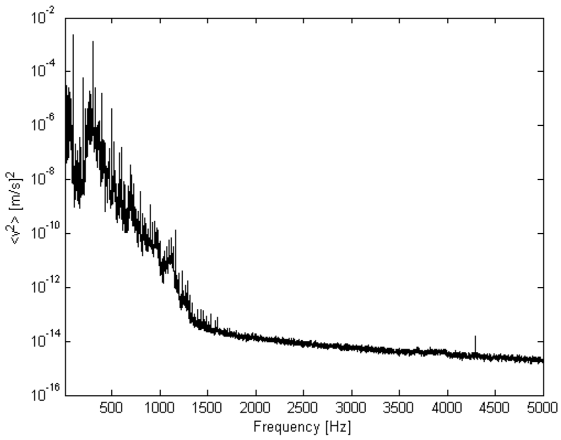


Fig. 4: Measured spatially average of mean-squared velocity of the reception plate.

It can be seen from Fig. 5 that the response dies off above 1.5 kHz. This shows the effective frequency range of excitation given by the motor.

The damping loss factor of the plate (calculated from Eq. (11)) is plotted in Fig. 5 up to 1.5 kHz in one-third octave bands.

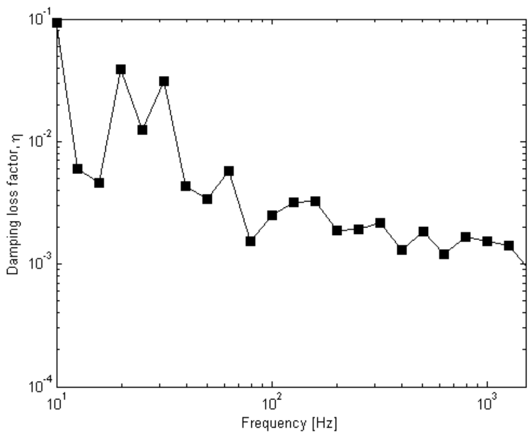
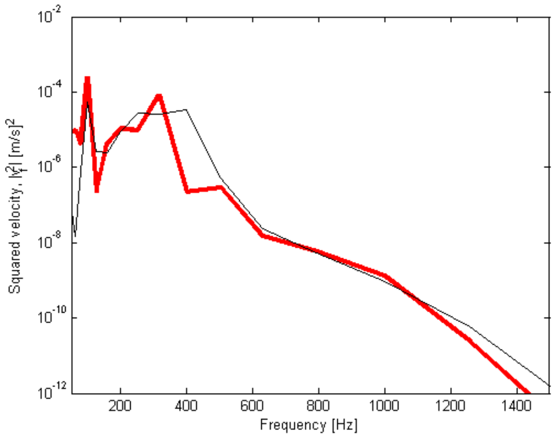
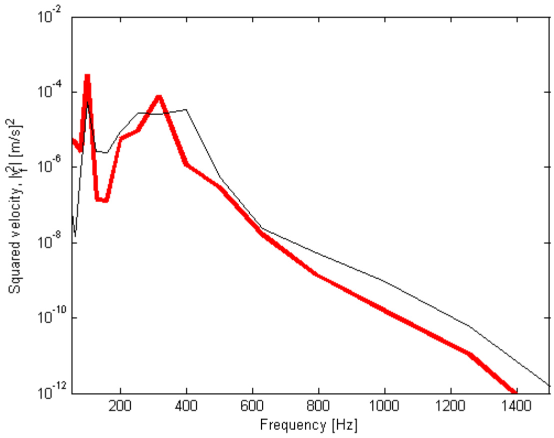


Fig. 5: Measured damping loss factor from the reception plate.

Finally, the free velocity from the reception plate method as in Eq. (16) can be obtained. Fig. 6 presents the estimated squared free velocity compared with that from the direct measurement. The latter was conducted by hanging the motor and the velocity was measured at each feet using accelerometer.



(a)



(b)

Fig. 6: Comparison of squared free velocity between that obtained using reception plate method (thick line) and that from direct measurement (thin line): (a). zero phase effective mobility and (b). random phase effective mobility.

A good agreement between the estimated squared velocity and that from the measurement can be seen for the zero phase assumption as in Fig. 6(a). A discrepancy between 300-500 Hz might be because of the interference due to the small spatial range between the contact points. The same phenomenon can also be seen for the random phase assumption in Fig. 6(b). However, in random phase the two results differ by roughly

10 dB above 600 Hz. This indicates that the excitation phase of the motor might be still in-phase up to 1.5 kHz.

IV. Conclusion

Determination of the vibration strength of a structure-borne source represented by the free velocity has been conducted using a reception plate method with high mobility panel. This is proposed for a mechanical installation to a plate-like structure. A good agreement is achieved for the squared free velocity of the source between the result from the method and that from direct measurement.

Although the method has been applied successfully, there are several factors which might be considered to improve the method. Among many is to investigate the effect of phase excitation and also effect of excitation locations which contribute to the result of plate mobility. Instead of the exact result, the possible range of the vibration strength in terms of its statistical variation across the frequency is of interest.

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