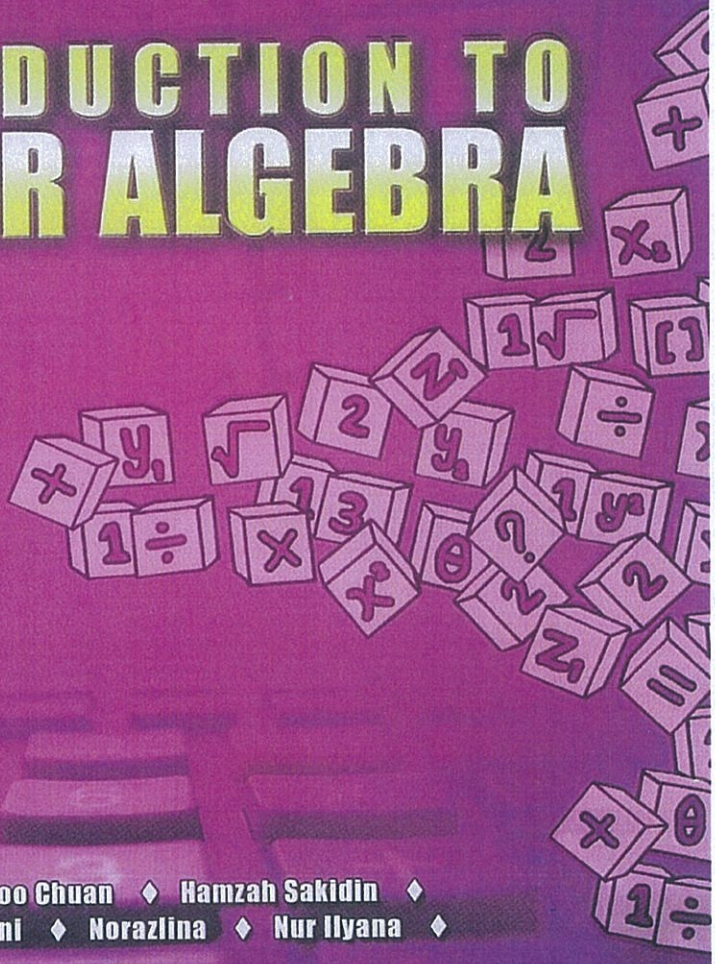
INTRODUCTION TO LINEAR ALGEBRA



♦ Tay Choo Chuan ♦ Hamzah Sakidin ♦
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Preface

One of the most challenging aspects of mathematics learning is to give students suitable examples and exercises which can improve their understanding.

This book is designed to familiarize the student with the form of questions asked in Linear Algebra. The topics are based on the syllabus of Linear Algebra teaching in UTeM.

The material in this book will cover questions and answers for:

- Functions and Graphs
- Matrices and System of Linear Equations
- Trigonometry
- Analytic Geometry
- Complex Numbers

An excellent student must have an initiative to learn before being taught by lecturers. By using this book, students can be more prepared before attending a tutorial session. The examples are presented in a sequence of steps with full details so that students can follow systematically.

In preparing this book, we would like to thank all the individuals involved. Such comments and feedback are always welcome.

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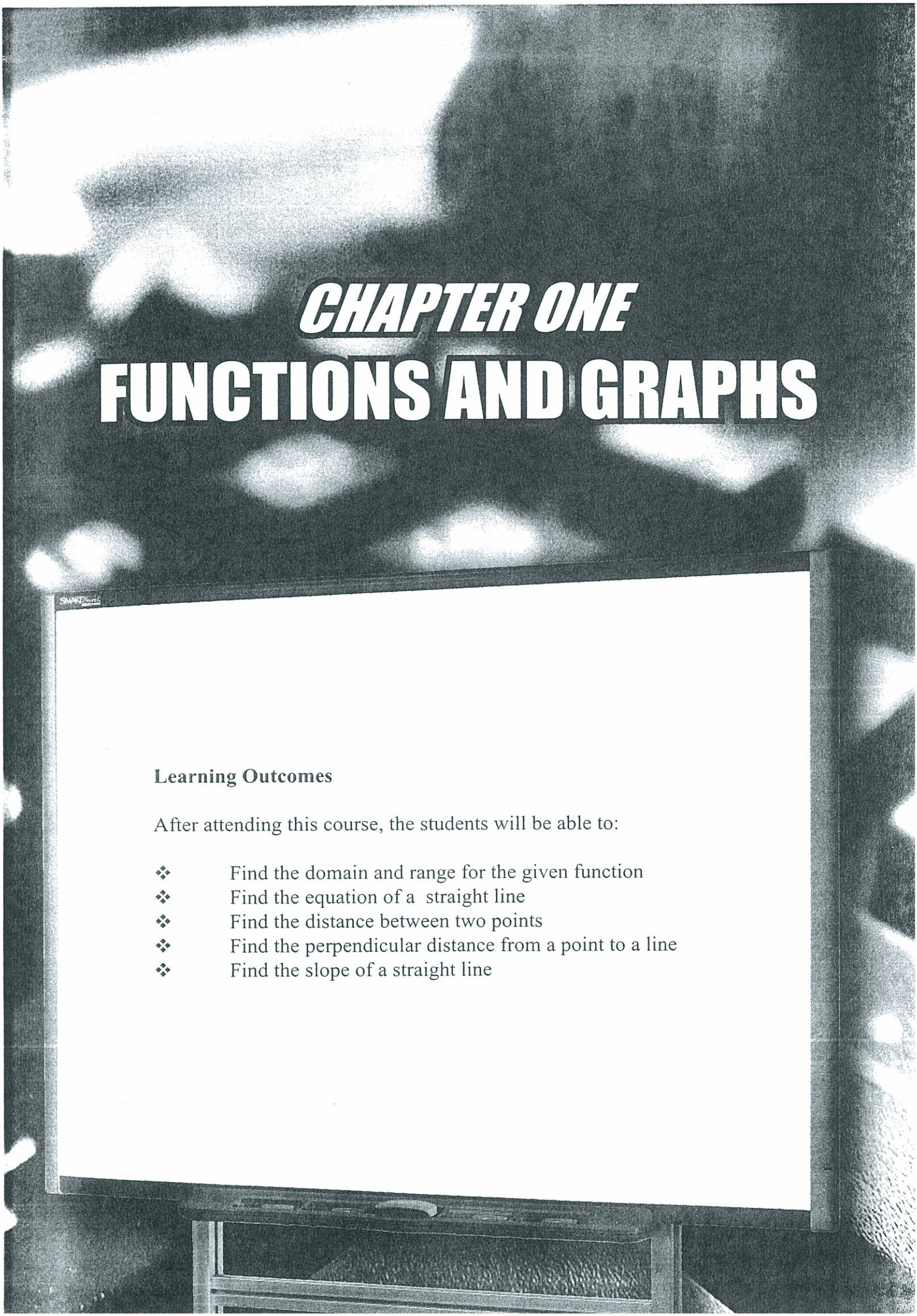

CHAPTER ONE
FUNCTIONS AND GRAPHS

Learning Outcomes

After attending this course, the students will be able to:

- Find the domain and range for the given function
- Find the equation of a straight line
- Find the distance between two points
- Find the perpendicular distance from a point to a line
- Find the slope of a straight line

1.0 Introduction to Functions

In everyday life, many quantities depend on one or more changing variables eg:

(a) plant growth depends on sunlight and rainfall

(b) speed depends on distance travelled and time taken

(c) voltage depends on current and resistance

(d) test marks depend on attitude, listening in lectures and doing tutorials (among many other variables!!)

1.1 Functions

A function is a rule that relates how one quantity depends on other quantities eg.

(a) $\boldsymbol{V = IR}$ where

V = voltage (V) I = current (A) R = resistance (Ω)

(b) $\boldsymbol{s = \frac{d}{t}}$ where

s = speed (m / s) d = distance (m) t = time taken (s)

1.2 Definition of a Function

Whenever a relationship exists between two variables (or quantities) such that for every value of the first, there is only *one* corresponding value of the second, then we say:

the second variable is a ***function*** *of the first variable.*

The first variable is the *independent* variable (usually x), and the second variable is the *dependent* variable (usually y).

The independent variable and the dependent variable are *real numbers*.

Example 1:

We know the equation for the area of a circle learned during primary school:

$$A = \pi r^2$$

This is a *function* as each value of the independent variable r gives us *one* value of the dependent variable A.

Review Exercises

1. Find the roots for the equation $x^2+2x+5=0$

2. Find the three roots of $z^3=-1$ in the form of $a+ib$. Plot the roots in Argand diagram.

3. Set $w=\dfrac{z^2+2\bar{z}}{2-z}$ with $z=1-2i$. Find $\mathrm{Re}(w)$ and $\mathrm{Im}(w)$.

4. Solve the equation $z^3-7=0$.

5. Find all the complex roots of $z^3+8=0$.

6. Solve the equation $z^5+32i=0$.

7. Show that $11+2i=-(1+2i)^3$.

8. Express $(1+i)^{-3}-(1-i)^{-3}$ in the form $a+ib$.

9. Write $\left(\dfrac{-\sqrt{3}-i}{2}\right)^{10}$ in the form of $a+ib$.

10. Given $z_1=1-\sqrt{3}i$ and $z_2=1+\sqrt{3}i$. Express z_1 and z_2 in polar form for $-\pi<\theta\le\pi$. Hence using De Moivre's Theorem, find the value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ in the form of $a+ib$.

11. Draw and shade the locus of points of z that satisfy $1\le|z|<2$.

12. Prove that $\dfrac{1+i\tan\theta}{1-i\tan\theta}=\cos 2\theta+i\sin 2\theta$ and hence or otherwise without using calculator, obtain the square roots of $\dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}$, and give your answer in the form of $a+ib$.

13. Express $\dfrac{1}{\cos\theta-i\sin\theta}$ in $a+ib$ form and hence show that $\dfrac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta}=\cos 2\theta+i\sin 2\theta$.

14. Given $z=\cos\theta+i\sin\theta$, show that $z+z^{-1}=2\cos\theta$ and $z^2+z^{-2}=2\cos 2\theta$.

15. Given $z_1=2(\cos 75^\circ+i\sin 75^\circ)$ and $z_2=3(\cos 25^\circ+i\sin 25^\circ)$.

    (i) Find the modulus of z_1z_2 and the argument of $\dfrac{z_1}{z_2}$.

    (ii) Find z_1^{10} and give your answer in the form of $a+ib$.

Authors' Profiles

Tay Choo Chuan is a graduate with Bachelor of Science (Hons) degree in Mathematics, Master of Science degree (Quality and Productivity Improvement) and PhD in Mathematics from University Kebangsaan Malaysia (UKM). The author has over 20 years of experience teaching in two level of education: secondary and tertiary. He is currently attached to Universiti Teknikal Malaysia Melaka (UTeM) as a senior lecturer in the Faculty of Electrical Engineering. He is also the author of several Mathematics books.

Hamzah Sakidin received the Bachelor of Science, Diploma of Education and Master of Science in mathematical modeling from University Kebangsaan Malaysia (UKM), Bangi Selangor, Malaysia in 1993, 1994 and 2001 respectively. He received his PhD in mathematical modeling from University Putra Malaysia in 2008. He joined University Teknikal Malaysia Melaka (UTeM) as a mathematics lecturer in August 2003. Recently, he is a Senior lecturer in the Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka (UTeM). His research interests are mathematical modeling and applied mathematics.

Norazlina Abd Razak is a graduate with Bachelor of Science (Hons) degree in Mathematics and Computing, Curtin University of Technology, Australia and Master of Engineering (Electrical) degree from University Technology Malaysia. Recently, she is a Senoir Lecturer in the Faculty of Electronic and Computer Engineering, Universiti Teknikal Malaysia Melaka (UTeM).

Irma Wani Jamaludin received the Bachelor of Industrial Science in Mathematics and Master of Science in Mathematics from Universiti Teknologi Malaysia (UTM), Skudai, Johor in 2004 and 2005 respectively. She joined Universiti Teknikal Malaysia Melaka (UTeM) as a Mathematics lecturer in 2006 and now as an academic staff at Faculty of Electrical Engineering, UTeM. Her research interests are algebra and applied mathematics.

Nur Ilyana Anwar Apandi begin her academic career in Department of Mathematics , Universiti Teknologi Malaysia (UTM) in year 2003. She migrated to Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka (UTeM) end of 2008, where she is now serving as mathematics lecturer. Graduate with Bachelor of Science (Hons) degree in Industrial Mathematics, UTM and obtained her Master of Science (Modelling in Applied Mathematics) degree from University of East Anglia, Norwich UK.

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