Mathematical Function Optimization using AIS Antibody Remainder method

David F. W. Yap, S. P. Koh, S.K.Tiong

Abstract—**Artificial immune system (AIS) is one of the metaheuristics used for solving combinatorial optimization problems. In AIS, clonal selection algorithm (CSA) has good global searching capability. However, the CSA convergence and accuracy can be improved further because the hypermutation in CSA itself cannot always guarantee a better solution. Alternatively, Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO) have been used efficiently in solving complex optimization problems, but they have a tendency to converge prematurely. In this study, the CSA is modified using the best solutions for each exposure (iteration) namely Single Best Remainder (SBR) - CSA. The results show that the proposed algorithm is able to improve the conventional CSA in terms of accuracy and stability for single and multi objective functions.**

Index Terms— **clonal selection, antibody, antigen, affinity maturation, mutation.**

I. INTRODUCTION

Optimization problem has been a challenge to many researchers in order to find the best local searching method. This problem also leads to a branch of knowledge which is the evolutionary computing and is greatly influenced by nature. Few decades ago, many methods have been developed, for instance, GA, PSO, Ant Colony or Artificial Immune System (AIS). In this study, the improved CSA is evaluated in comparison to conventional CSA and other evolutionary algorithms such as PSO and GA. These algorithms are described in the following paragraphs.

PSO was originally proposed by Kennedy and Eberhart [1] and is inspired from social behavior of individual organisms living together in groups [2]. Each individual in a group imitates other groups that are better, in order to improve its own group.

GA is inspired from a set of chromosome where each chromosome represents an individual solution (genes). The GA uses a search technique where genes in the population are improved across generation through a set of operation. During each generation, the genes go through the process of

selection, cross-over and mutation [3].

AIS is greatly reinforced by the immune system of a living organism such as human and animal. In humans, the immune system is responsible in maintaining stability of the physiological system such as protection from pathogens. In AIS, CSA is able to improve global searching ability as it uses the principle of clonal expansion and affinity maturation as the main forces of the evolutionary process [4].

Two algorithms based on CSA are proposed in this work to improve the performance of diversity and convergence over the standard CSA, that are responsible in finding the global solution of single objective function. They are half best insertion (HBI) CSA and single best remainder (SBR) CSA. Similar to CSA, the ease of implementation is sustained in the proposed algorithms.

II. PSO, GA AND AIS ALGORITHM

A. Particle Swarm Optimization

The PSO algorithm starts with a group of random particles that searches for optimum value for each updated generation. The i_{th} particle is denoted as $X_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$. During generation updating, each particle is updated by ensuing two best values. These values are the best solution (*mbest*) and the global best value (*gbest*) that has been obtained by particles in the population at particular generation. With the inclusion and inertia factor ω, the velocity equations are shown in Eqs. (1) and (2).

$$v_{i+1} = v_i \omega + \alpha_1 \cdot rnd() \cdot (mbest_i - x_i) + \alpha_2 \cdot rnd() \cdot (gbest_i - x_i) \quad (1)$$

$$x_{i+1} = x_i + v_i \quad (2)$$

Where *rnd()* is a random number between 0 and 1, α_1 and α_2 are learning factors to control the knowledge and the neighbourhood of each individual respectively. The PSO algorithm is described in the following steps.

Step	Process
1	Generate initial random particle swarms assigned with its random position and velocity
2	Compute the fittest value of N particles according to fitness function
3	Update values of the best position of each particle and the swarm
4	Update the position and velocity for each particle according to equation 1 and 2.
5	Repeat steps 3 and 4 until pre-defined stopping condition is achieved

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B. Genetic Algorithm

As described earlier, GA uses three main processes i.e. selection, crossover and mutation to improve genes through each generation. The selection process uses the objective function to assess the quality of the solution. Then, the fittest solutions from each generation are kept. Subsequently, the function of crossover generates new solutions given a set of selected members of the current population. In the crossover process, genetic material between two single chromosome parents is exchanged. Then, mutation triggers sudden change in the chromosomes unexpectedly. However, the mutation process is expected to avoid genes from trapping in local minima by adding random variables. The GA algorithm is described in the following.

Step	Process
1	Generate initial random population of individuals
2	Compute the fittest value of each individual in the current population
3	Select individuals for reproduction
4	Apply crossover and mutation operators
5	Compute the fittest value of each individual
6	Select the best individuals to generate new population
7	Repeat steps 3 to 6 until pre-defined stopping condition is achieved

C. Artificial Immune System

In AIS, CSA was inspired from the biological immune system, where antibodies (Abs) that are able to recognize antigens (Ags) are selected to proliferate. The selected Abs then enters the affinity maturation process. The algorithm was verified to be able to solve complex problem such as multi-modal and combinatorial optimization [5].
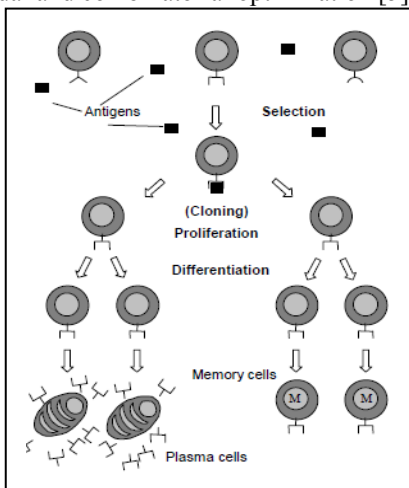

Figure 1. Clonal Selection Principle (de Castro & Von Zuben, 2001a)

The clonal selection theory describes how Ab detects the Ag and proliferates by cloning. As shown in Fig. 1, the immune cells will reproduce against the Ags. The new cloned cells are then differentiated into plasma cells and memory cells. The plasma cells produce Abs and go through mutation process to promote genetic variation. The memory cells are responsible for future Ags invasion. Finally, the selection mechanism keeps the Abs with the best affinity to the Ags in the next population [4]. The CSA pseudocode is described in the following.

Step	Process
1	Generate an initial random population of antibodies, Abs
2	Compute the fittest value of each Ab according to fitness function
3	Generate clones by clonning all cells in the Ab population
4	Mutate the clone population to produce a mature clone population
5	Evaluate the affinitiy value for each clone population
6	Select the best Ab to compose the new Ab population
7	Repeat steps 2 to 6 until a pre-defined stopping condition is achieved

D. Artificial Immune System and Particle Swarm Optimization Hybrid

AIS have the advantage to prevent the population from being trapped into local optimum. Besides, PSO has the ability to improve itself but tend to converge prematurely [6]. Therefore, the combination between AIS and PSO (AIS-PSO) is expected to improve the global search ability and avoid being trapped in local minima even though the population size is relatively small [7]. Hence, The AIS-PSO pseudocode is described in the following steps.

Step	Process
1	Select the best particles from PSO to be half of AIS initial population, N_1
2	Generate randomly other half of initial population of Abs, N_2
3	Combine N_1 and N_2 and compute fittest values of each Ab
4	Generate clones by cloning all cells in Ab population
5	Mutate the clone population to produce a mature clone population
6	Evaluate the affinity value for each clone in the population
7	Select the best Ab to compose the new Ab population
8	Repeat steps 4 to 7 until pre-defined stopping condition is achieved

E. Half Best Insertion Artificial Immune System

In AIS, clonal selection adapt B-cells (and T-cells) to kill the invader through affinity maturation by hypermutation. However, the adaptation requires B-cells to be cloned many times [8, 9], and the hypermutation process cannot always guarantee that the next generation will provide better solution. The stochastic factor (randomization) at times can even produce worse result from previous solution. Therefore, N number of the best Abs from the previous generation can be combined with the initial random Abs of the next generation to compose a new population for that next generation. This method known as Half Best Insertion (HBI) is expected to improve the convergence of the CSA algorithm. In HBI, half of the best antibodies from the previous generation are used in the next.

The N number of best Abs can be summarized as

$$\alpha \leq Abs_{best} / 2 \qquad (3)$$

where α number of best Abs.

The best Abs selection, A_S, of sth antibodies is

$$A_S \begin{cases} 0 \\ A_{S+1}, & \text{otherwise} \end{cases} \qquad (4)$$

Then, the new antibody population Ab' is

$$Ab' = Ab \cup A_S \qquad (5)$$

The HBI algorithm is described in the following steps.

Step	Process
1	Generate an initial random population of antibodies, Abs
2	Compute the fittest value of each Ab according to fitness function
3	Generate clones by cloning all cells in the Ab population
4	Mutate the clone population to produce a mature clone population
5	Evaluate the affinity value for each clone in the population and select number of best Abs, α
6	Generate next generation of initial random Abs and include α
7	Repeat steps 2 to 6 until pre-defined stopping condition is achieved

F. Figures

Hypermutation of good Abs in HBI algorithm would tend to produce bad solution. Thus, the Single Best Remainder (SBR) algorithm tries to avoid hypermutation process on the selected good Abs that produce worse solution due to stochastic factor. Therefore, the best Abs from previous generation is kept in global memory as single best antibody which is not affected by the next affinity maturation and hypermutation processes. The global single best antibody will be updated through generation and used in the next generation if the hypermutation result converges prematurely in the search space. Therefore, SBR is proposed in order to improve the convergence and accuracy of the CSA algorithm.

In SBR, the best antibody obtained for the clonal selection process is recorded as global solution, A_m. During each generation process, the randomize antibodies, A_r, is replaced by the best solution. The clone cell result after maturation, F_m, is evaluated based on the test function. Then, F_m is compared with the result of randomize antibodies (A_r) after the test function based evaluation, F_m. If F_m is larger or equal to F_t, the clone cell, C_{bp}, is replaced by A_r. Otherwise, the C_{bp} is maintained.

$$C_{bp} = A_r, \text{ if } F_m \geq F_t \qquad (6)$$

where $F_t = \text{testFunction}(A_r)$

and $F_m = \text{testFunction}(C_{bp})$

The SBR algorithm is described in the following steps.

Step	Process
1	Generate an initial random population of Abs
2	Compute the fittest value of each Ab according to fitness function
3	Generate clones by cloning all cells in the Ab population
4	Mutate the clone population to produce a mature clone population
5	Evaluate the affinity value for each clone in the population
6	Select best Ab, A_m, in 5 as global memory and repeat steps 1 to 5
7	Repeat steps 1 to 5 and compare the best Ab obtained with A_m
8	The best Ab from 7 is updated as the global memory, A_m
9	Repeat steps 1 to 9 until pre-defined stopping condition is achieved

All methods described above are evaluated using nine mathematical test functions. The termination criteria for all methods will be met if minimum error value is achieved or maximum number of evaluation allowed is exceeded.

III. Experiments on Test Function

The computing platform used for the experiment is AMD Phenom 9600B Quad-Core CPU running at 2.30 GHz, 2GB of RAM and Windows Vista Enterprise operating system. Each algorithm is evaluated based on 500 iterations, 10 dimensions and the mean of best fitness is obtained after 10 runs. The minimum error is set as 1e-25, while the population size P_0 is set to 20.

In the HBI, antibodies and memory size of 50% are maintained. At first iteration, CSA is used to obtain the first solution. Then, for the next iteration, half of the population is composed of the half best antibodies after hypermutation and the other half is given by randomized Abs. The new population then goes through the affinity maturation process similar to CSA.

Then, in SBR, similar to HBI, CSA is used to obtain the first solution. Then, for the next iteration, the best antibody, A_m, is kept as global memory. This A_m will never go through affinity maturation process, but will be assigned as a reference (memory) in case the hypermutation process produces worse solution.

The nine benchmark functions (objective functions) are described as follows.

1. Rastrigin's Function:

Rastrigin's function is mathematically defined as follows.

$$f_1(x) = \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i) + 10 \right) \qquad (7)$$

where $-5.12 \leq x_i \leq 5.12$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

2. De Jong's Function:

De Jong's function is mathematically defined as follows.

$$f_2(x) = \sum_{i=1}^{n} x_i^2 \qquad (8)$$

where $-5.12 \leq x_i \leq 5.12$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

3. Axis Parallel Hyper-ellipsoid Function:

Axis parallel hyper-ellipsoid function is mathematically defined as follows.

$$f_3(x) = \sum_{i=1}^{n} i \square x_i^2 \qquad (9)$$

where $-5.12 \le x_i \le 5.12$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

4. Rosenbrock's Function:

Rosenbrock's function is mathematically defined as follows.

$$f_4(x) = \sum_{i=1}^{n-1} 100 \square \left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2 \qquad (10)$$

where $-2.048 \le x_i \le 2.048$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

5. Sum of Different Power Function:

Sum of different power function is mathematically defined as follows.

$$f_5(x) = \sum_{i=1}^{n} |x_i|^{(i+1)} \qquad (11)$$

where $-1 \le x_i \le 1$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

6. Rotated Hyper-ellipsoid Function:

Rotated hyper-ellipsoid function is mathematically defined as follows.

$$f_6(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2 \qquad (12)$$

where $-65.536 \le x_i \le 65.536$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

7. Moved Axis Parallel Hyper-ellipsoid Function:

Moved axis parallel hyper-ellipsoid function is mathematically defined as follows.

$$f_7(x) = \sum_{i=1}^{n} 5i \square x_i^2 \qquad (13)$$

where $-5.12 \le x_i \le 5.12$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

8. Griewangk Function:

Griewangk's function is mathematically defined as follows.

$$f_8(x) = \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \qquad (14)$$

where $-600 \le x_i \le 600$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

9. Ackley Function:

Ackley's function is mathematically defined as follows.

$$f_9(x) = 20 + e - 20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} \qquad (15)$$

where $-32.768 \le x_i \le 32.768$, $i = 1 \ldots, n$

and global minimum is located at the origin and its function value is zero.

IV. RESULT AND DISCUSSION

The results for the test functions are shown in Fig. 2 to 10 and Table I. For Rastrigin's function, Fig. 2 shows that the PSO suffers from premature convergence while PSO-AIS is less accurate in giving the fitness value. On the other hand, SBR gives the best fitness value followed by HBI, CSA and GA.
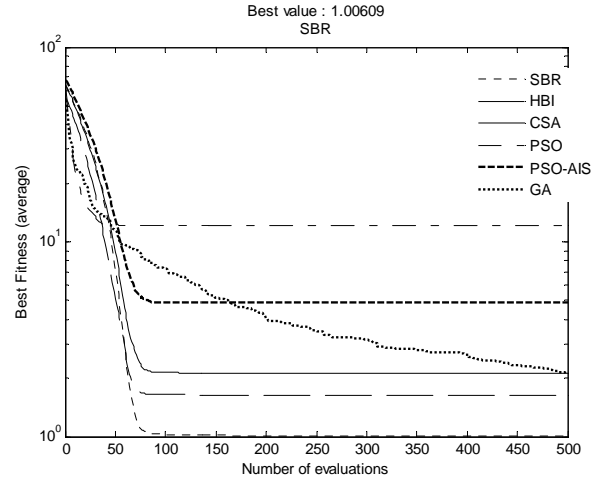

Figure 2. Algorithms evaluation on Rastrigin's function

However, for Dejong's function in Fig. 3, the GA fitness value is very close to CSA. The PSO converges rapidly up to 50 generations but perform no significant improvement beyond this which is also similar to Rastrigin's function. In contrast, GA converges very slow but seems to be able to perform even after 500 generations since there is no breaking point after that. The SBR achieved the best performance and is comparable to PSO-AIS and CSA.
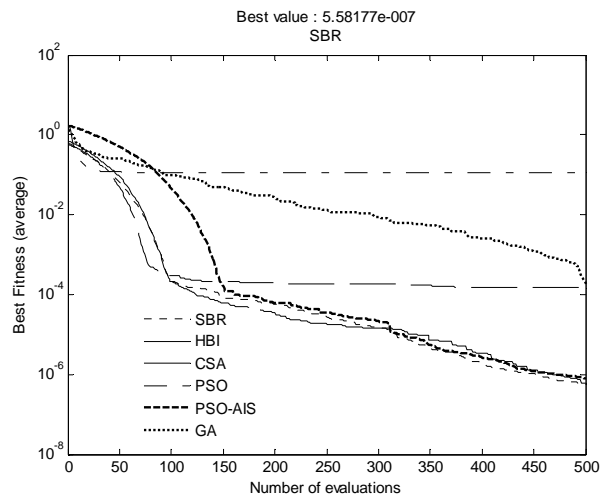

Figure 3. Algorithms evaluation on Dejong's function

The Axis Parallel Hyper-ellipsoid function in Fig. 4 shows that the CSA achieved the best fitness value which is comparable to SBR and PSO-AIS. In contrast, there is no significant improvement in PSO after 50 generations.

Figure 5 shows that PSO and GA perform badly among all algorithms. In contrast to Rastrigin's, Dejong's and Axis Parallel Hyper Ellipsoid, the PSO-AIS outperformed other algorithms. However, the fitness value of CSA and SBR are comparable.

Fig. 6 shows similarities to Fig. 3 and Fig. 4. The fitness value of CSA, SBR and PSO-AIS are comparable. Alternatively, PSO shows the worst performance with a bad fitness value.
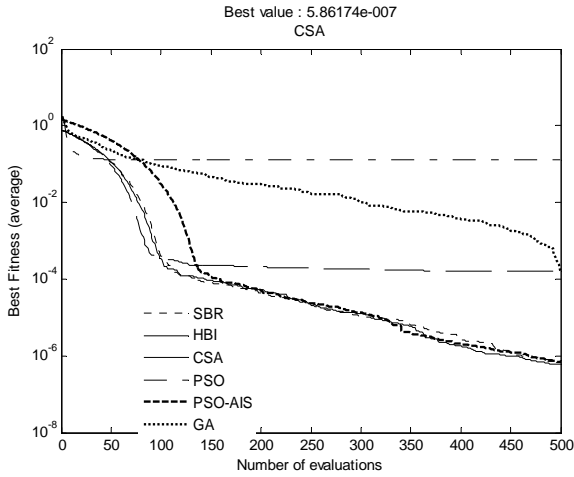

Figure 4.   Algorithms evaluation on Axis Parallel Hyper-ellipsoid function
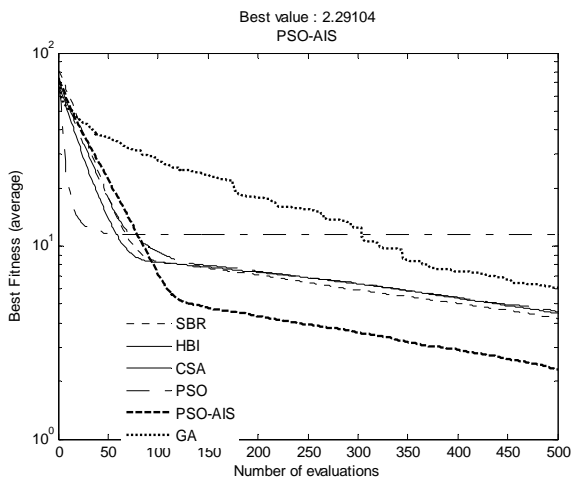

Figure 5.   Algorithms evaluation on Rosenbruck function
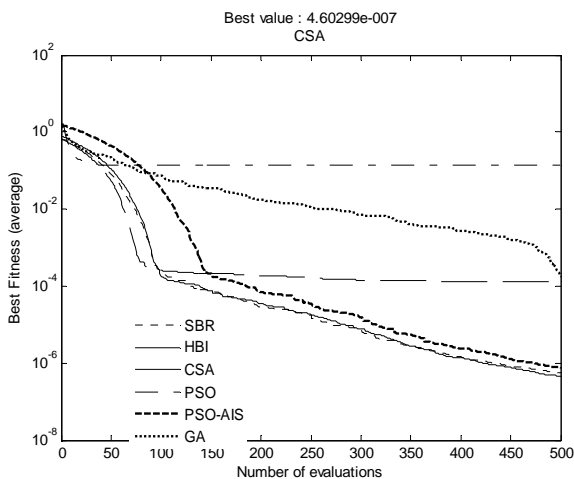

Figure 6.   Algorithms evaluation on Sum Differential Power function

The Rotated Hyper-ellipsoid function in Fig. 7 is similar in result to the Sum of Differential Power function. However, SBR is slightly better in performance than CSA and PSO-AIS.
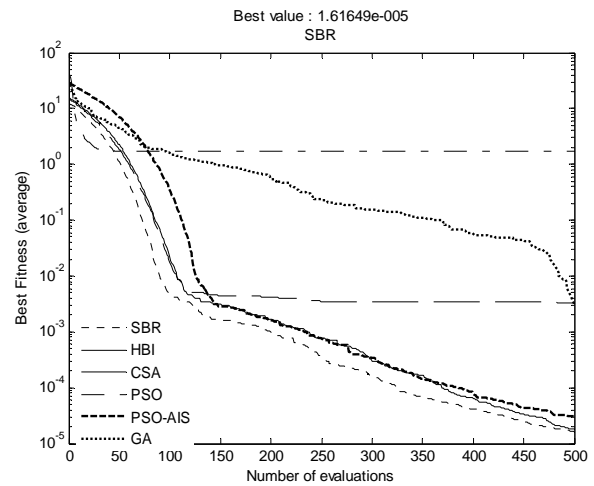

Figure 7.   Algorithms evaluation on Rotated Hyper-ellipsoid function

The Moved Axis Parallel Hyper-ellipsoid in Fig. 8 show similarities to Rotated Hyper-ellipsoid Function. The performance of SBR is also slightly better than CSA, followed by PSO-AIS.
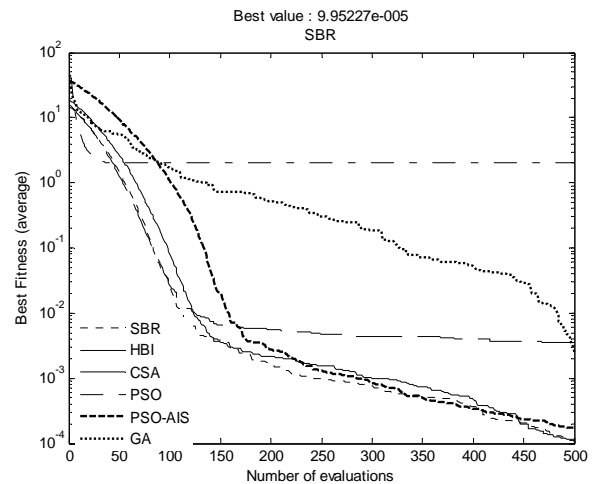

Figure 8.   Algorithms evaluation on Moved Axis Parallel Hyper-ellipsoid function

The results of Griewangk's function in Fig. 9 show similarities to the Moved Axis Parallel Hyper-ellipsoid result. Here, the CSA is slightly better than SBR and PSO-AIS.

The Ackley's function in Fig. 10 shows that the GA performance outperformed other algorithms followed by CSA and HBI. The SBR seems to suffer from premature convergence. In contrast to previous results, the SBR is worse than HBI. However, the PSO-AIS have the worse performance and this is followed by PSO. Both of the algorithms have no significant improvement after 100 generations.
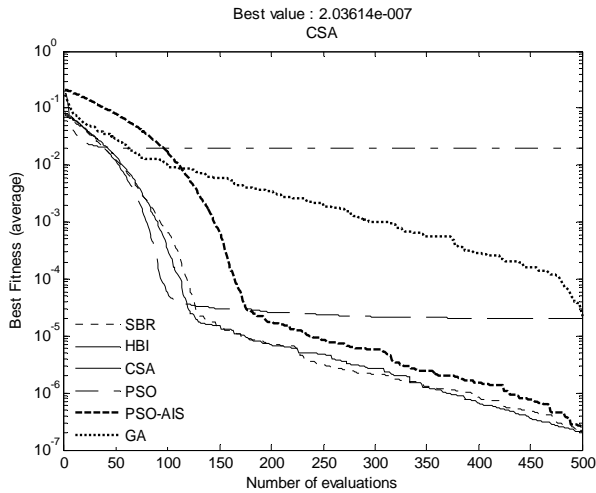
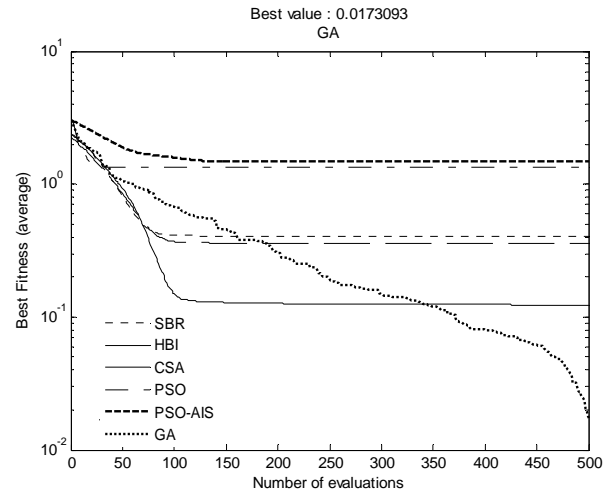Figure 9.   Algorithms evaluation on Griewangk function

Figure 10. Algorithms evaluation on Ackley function

Table I shows the mean and standard deviation value for nine test functions used to evaluate the algorithms performance. SBR method outperformed other algorithms for test function number 1, 2, 6 and 7. The CSA is the best for test function number 6, 8 and 11. The PSO-AIS achieved the best test function for number 7 while GA performed the best for test function number 9.

The most deemed stable algorithms are given by SBR, CSA and PSO-AIS, most probably due to the very small standard deviation value in between 1e-7 and 1e-6 for most of the test functions. The PSO performed badly since most of the standard deviation value is large.

TABLE I. MEAN AND STANDARD DEVIATION FOR EACH OF THE ALGORITHM BASED ON THE GIVEN TRANSFER FUNCTION

Function	SBR		HBI		CSA		PSO		PSO-AIS		GA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Eq. 7	**1.00609**	**1.15581**	1.61916	0.83718	2.10917	1.51617	12.1057	5.61802	4.8883	1.09404	2.1186	1.18767
Eq. 8	**5.6E-07**	**3.3E-07**	0.00014	3.7E-05	6.1E-07	4.8E-07	0.11565	0.07709	8.2E-07	6.3E-07	0.00019	0.00014
Eq. 9	7.6E-07	7.2E-07	0.00016	3E-05	**5.9E-07**	**3.9E-07**	0.12653	0.0811	6.9E-07	4.6E-07	0.00015	7.1E-05
Eq. 10	4.21392	0.71981	4.58547	1.8584	4.48752	1.38984	11.414	3.75693	**2.29104**	**1.304**	6.05602	2.18913
Eq. 11	5.6E-07	4.5E-07	0.00013	2.7E-05	**4.6E-07**	**1.5E-07**	0.13601	0.08611	7.4E-07	5.4E-07	0.00017	6.6E-05
Eq. 12	**1.6E-05**	**6.3E-06**	0.00333	0.00062	1.8E-05	1.3E-05	1.7223	2.46831	3E-05	3.3E-05	0.00293	0.00165
Eq. 13	**1E-04**	**0.00012**	0.00355	0.00118	0.00011	0.00011	1.9966	1.08895	0.00017	0.00033	0.00239	0.00075
Eq. 14	2.2E-07	2.4E-07	2E-05	7E-06	**2E-07**	**1.3E-07**	0.0194	0.02263	2.6E-07	2.1E-07	2.2E-05	1.6E-05
Eq. 15	0.40134	0.64744	0.35728	0.55102	0.12283	0.36299	1.33007	0.8346	1.47366	0.85525	**0.01731**	**0.00304**

V.   CONCLUSION

In this paper, we proposed two memory-based clonal selection AIS strategy using the local memory. They are known as SBR and HBI. While PSO is fast in obtaining the fitness value, it suffers from premature convergence. Alternatively, GA converges slowly to achieve the best fitness value. The preliminary simulation work clearly showed that the best result is given by SBR. More work would be done in tweaking certain parameters in SBR such as the memory allocation factor, best memory selection criteria or the number of best memory to be taken into consideration, in order to improve the performance of the algorithm.

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