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Statistical appraisal of economic design strategies of LV distribution networks

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A B S T R A C T

This paper presents a statistical approach for assessing general LV distribution network design strategies based on a large set of realistic test networks and optimal economic circuit design. The test networks are generated using a fractal-based algorithm that allows creation of generic networks with various topological features (e.g., typical of rural/urban/mixed areas) and characterised by different numbers of substations, numbers of customers, load densities, and so forth. In comparison to standards derived from a traditional approach, that is, case studies on a small number of specific real or test networks, the proposed approach facilitates the derivation of more robust conclusions on optimal network design policies and can thus be used as a valuable tool for decision support. The methodology is exemplified through numerical applications for both urban and rural areas.

1. Introduction

One of the main challenges for distribution network planners and operators is to develop optimal network design strategies, which involves evaluation of a wide range of options such as cable types, cable sizes, locations of substations, and types and sizes of transformers. Without general guidance as to how these choices should be made, network planners can only rely on their experience. This may lead to suboptimal decisions and to inconsistent strategies that eventually will increase the network cost. However, development of general network design guides is a complex task, especially for LV systems, which have millions of different samples. In addition, due to the sheer volume of network data, relevant information for LV systems is often unavailable or inadequate, and is generally insufficient for large scale network design. Thus, several works have addressed network design of specific networks [1,2] or idealised networks [3–5], incorporating existing design policies or standards and with specified load point positions and available substation sites. More recently, a distribution network model has been developed for large-scale distribution planning, which divides the network zone into mini-zones that are optimised independently [6,7]. In addition to that, a reference network model which uses real location coordinates of final customer to automatically generate the corresponding street map for simultaneous planning of high-, medium- and low voltage networks is reported in [8]. Regarding circuit design, life cycle cost (LCC) analysis for circuits [1,3,9] and transformers [10–12] is a consolidated methodology adopted in several countries for network design. However, the adoption of LCC analysis for strategic design and assessment of different types of networks has not been previously explored.

This paper introduces a statistical approach intended to support the decision making process of network planners and operators in the identification of the best design strategy for given LV network types. The approach is based on synthetic input information, but could potentially be applied to a large variety of networks, thus moving beyond the current state of the art. In particular, the paper suggests an approach for strategic assessment of LV distribution networks using the LCC methodology of optimal network design. The LCC model illustrated here intrinsically highlights the prominent role of losses in circuit design, thus resulting into promotion of socially efficient investment policies and taking into account environmental concerns. The main outcomes from the analysis serve to indicate optimal network design strategies for given areas (for instance, rural or urban, depending on the specific topological features) with different load densities and characteristics, and identify the optimal number of substations, circuit cost breakdown (investment, maintenance and losses), and so forth.

The methodology developed here is based on the generation and analysis of a large number of statistically similar networks, with some common topological parameters. This enables decision makers to draw much more robust conclusions than those reached through the study of specific case study networks. A key

Abbreviations: ADMD, after diversity maximum demand; DSO, distribution system operator; LCC, life cycle cost; OHL, overhead line; PDF, probability density function; PMT, pole mounted transformer; UGC, underground cable.

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point of the proposed methodology is its ability to reproduce realistic network topologies and lengths, as confirmed by practical collaborations with DSOs [13,14], with no simplifications regarding the consumers' positions. Hence, network-related metrics such as losses and voltage drops can be estimated with higher accuracy than by adopting simplified geometric approaches [4]. In this respect, the generated networks can be better adjusted to resemble the synthetic features of given real regions with specific characteristics (load density and consumer breakdown, for instance), for which an optimal design strategy is sought. Typical network design criteria such as fault levels, voltage drops, and circuit thermal ratings have also been considered. The holistic approach to strategic network design and assessment illustrated here has not been seen previously in the literature.

Numerical studies are presented to illustrate the applications of the proposed methodology to both urban and rural areas, highlighting the main features of each case and the differences between the two. In particular, the optimal number of 11/0.4 kV substations to supply areas with different load densities and a breakdown of the average costs of these networks are presented and discussed in a typical UK context.

The paper is structured as follows. Section 2 illustrates the characteristics of the tool for statistical network generation and substation placement. Section 3 describes the minimum LCC-based methodology used to design optimal economic circuits. Section 4 presents numerical applications of the developed methodology within typical urban and rural configurations. Finally, Section 5 contains concluding remarks and suggests potential future work.

2. A fractal model for statistical network creation

2.1. Fractal network generation

Consumers' locations and network branch connections play critical roles in network design, as these affect the length of the network as well as, together with the specific demand load patterns, the circuit sizing. Subsequently, simple geometric models [4] or network trees specifically generated for the lowest overall cost [5] may in general not be adequate to reproduce realistic network features and consumer distribution. It has also been shown that fractal models are more suitable to represent low load densities as compared to geometric models owing to their ability to generate realistic spatial consumer settlement with non-uniform load and supply points [15], resulting in spatial distributions of fractional dimension [16]. Building on these findings, the fractal network generation algorithm used in [17–19] is adopted in this paper to generate large sets of weakly meshed networks. However, as with the typical operation of the great majority of LV networks, further adjustments are carried out (see below) to transform the network into a number of radial ones.

2.2. Substation siting

The number N_s of MV/LV (11/0.4 kV, in typical UK cases) distribution substations is an input to the model. In the methodology discussed here, substations are placed so as to be indicatively at the centre of load clusters, consistent with the general design procedures in order to minimise the amount of equipment installed, losses and voltage drops [4]. A major advantage of the approach illustrated below is that the number of substations can be changed in a relatively straightforward manner, without the need for network reconfiguration, which allows for a better understanding of the cost of equipment, installation, maintenance, and network losses as a function of the number of substations for a given network configuration.

More specifically, the substation siting algorithm is based on the following steps:

- The total network area is divided into N_s regions with a radius heuristically defined as a function of the network area size and of the number of substations. Such regions are defined as circular areas (discs). The preliminary location of a substation is placed at the estimated local load centre within each defined disc. An example of identical networks with one and six substations is shown in Fig. 1a and b, respectively.
- Starting from the consumers' average loads and the preliminary substation location in each disc, an AC load flow calculation is performed on the generated network to determine the locations of the normally open points (NOPs) that would minimise losses and voltage drops. Consequently, the entire weakly meshed network is broken down into N_s radial networks (islands), with one substation serving each island.
- The substation position is then iteratively and heuristically readjusted across the points available in the specific island taking into account the geometric characteristics of the feeders emanated from the substation. More specifically, with reference to one island and one substation, the final substation location is selected so as to minimise the standard deviation σ of the overall feeder lengths, defined as:

$$\sigma = \sqrt{\frac{\sum_{f=1}^{N}(B^f - \mu)^2}{N-1}}, \quad B^f = \sum_{k=1}^{M^f} L^{fk} \quad , \mu = \frac{\sum_{f=1}^{N} B^f}{N} \tag{1}$$

In (1), N is the number of feeders from the substation, B^f is the overall length of feeder f, μ is the mean value of B^f over the N feeders, M^f is the number of branches along the f-th feeder, and L^{fk} is the length of branch k at feeder f.

It is important to mention that the trade-off between feeder length and supplied power has been preliminarily considered at the relevant centre of load. In fact, it has been observed that readjustment of the final transformer location within the radial networks by considering the "balanced feeder" criterion gives satisfactory results. This is necessary to avoid small loads being located too far from the substation; which potentially leads to voltage problems. In this regards, rather than co-optimising number and location of substations for a given specific network as for instance in [20], the proposed approach aims to analyse the impact of different network design strategies (and in particular number of substations) for a given area based on a large number of statistically similar networks. In this light, while detailed substation optimisation can be performed for real networks, it is much more useful for network planners to have statistical information on alternative design policies. In reality, social, geographic and cost constraints would restrict the final selection of the substation location.

2.3. Statistical network creation algorithm

The final network topology information is saved and exported as an output file that becomes the input data to the network design module described in Section 3. The complete network creation algorithm is synthesized in Fig. 2.

A number of statistically similar networks can be generated by changing the *seed* input parameter. More specifically, with different seed numbers new sets of random consumer load points can be generated which have an intrinsically similar network topology (driven by the same fractal control parameters [19] and characteristics such as consumer distribution, load density, substation density, and so on). An example of two statistically similar networks for

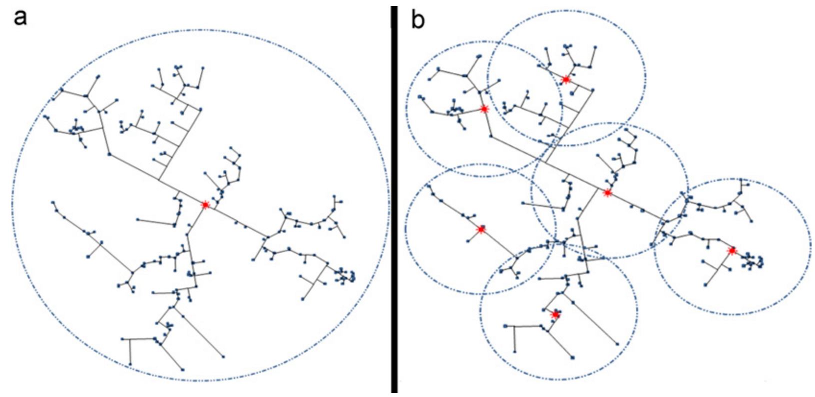

Fig. 1. Examples of a network supplied by (a) one substation and (b) six substations.

typical urban and rural networks with different seeds is shown in Figs. 3 and 4, respectively.

The main topological difference between urban and rural networks is driven by the consumer settlement patterns. Consumers in urban areas tend to be scattered more evenly, with relatively higher load densities. On the other hand, consumers in rural areas tend to aggregate in a more clustered fashion (villages), with large open areas dedicated to farms, green spaces, natural reservoirs, and so on, and consequently have relatively lower load densities.

Previous collaborations with several distribution design engineers had already shown that the networks generated by the fractal

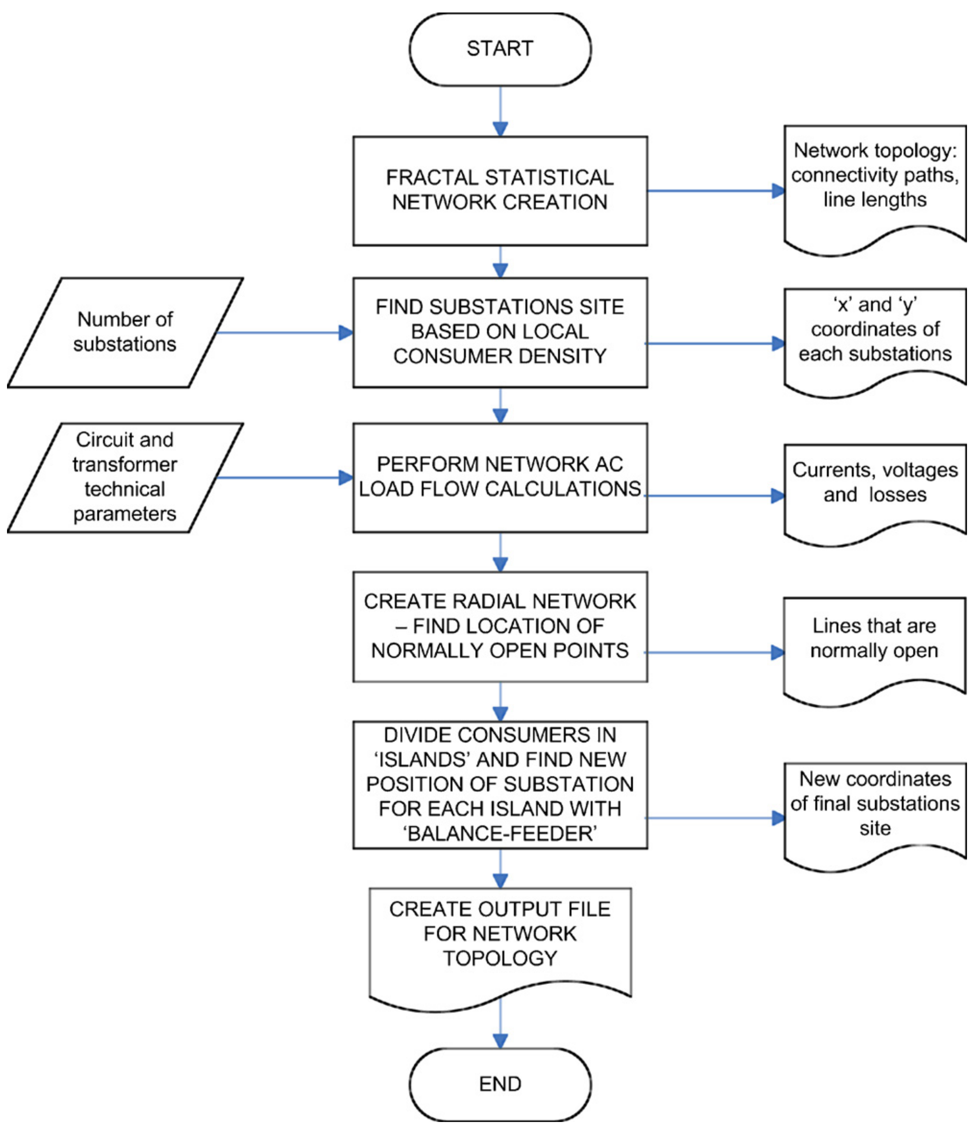

Fig. 2. Network creation flow chart.

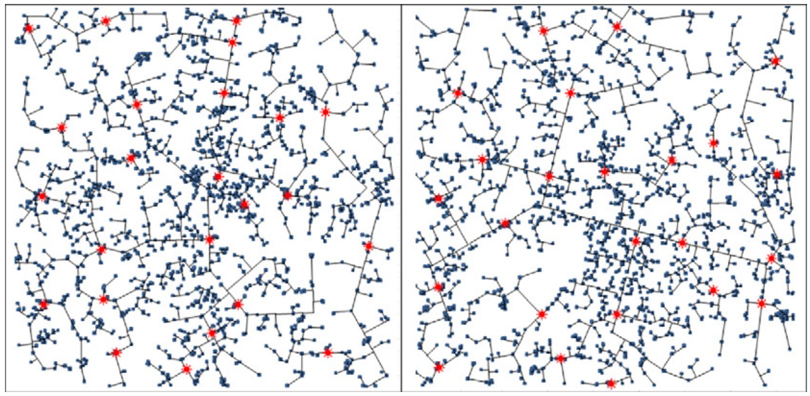

Fig. 3. Statistically similar urban networks with two different seeds.

algorithm resemble real networks [18]. More recent project collaborations with a number of DSOs in the UK have confirmed that the key statistical characteristics of the generated networks are comparable with those of real distribution networks of similar topologies, particularly in terms of the associated network lengths. More specifically, from the network data provided by DSOs, the average network length associated with a substation ground-mounted transformer (urban/semi-urban networks) in the UK system is about 1400 m, while the aggregated average network length of the distribution system modelled using the statistical network tool described here is about 1300 m [14]. For rural networks, the average network length associated with a PMT is about 200 m, which is again in very good match with the average real figure of about 209 m.

3. Optimal network design methodology

3.1. Load models and load flow analyses

In order to design a cost-effective network, a minimum LCC methodology is adopted here. More specifically, the annuitised plant investment cost is traded off against the relevant operational costs (namely, costs due to maintenance and losses). In addition, the equipment is selected to meet specific constraints such as voltage limits [21] and security standards [22] in order to meet optimality conditions. The overall algorithm flow chart is illustrated in Fig. 5, and is detailed in the sequel.

The network design and assessment carried out here begins with the analysis of time-varying demand patterns and relevant load flows, allowing more accurate results than analyses based on "average" loss factors (see e.g., [23]) or peak loads [17] only. In particular, for economic analysis it may be crucial to address the correlation between loads (and thus losses) and the variable cost of electricity in the time domain. Four typical consumer types commonly found in UK networks have been considered. Given the consumer types are domestic dominated with relatively small commercial and industrial buildings, the consumer points are randomly allocated across the network (see Section 4 for details). However, for larger commercial and industrial consumers which correspond to business district or industrial estates, it will be necessary to consider the type of neighbourhoods while allocating these consumer points. Then, for each consumer point and for each hour, a random variation of demand around the mean value of the after-diversity profiles is applied, according to typical statistical models estimated for UK loads [24]. Hence, it is possible to model "peaky" phenomena occurring in the network for better appraisal of losses and voltage drops. For this, a classic AC load flow is performed over a one-year time span. The power factor is assumed to be constant and equal to 0.9.

3.2. Optimal circuit design: formulation of the continuous optimisation problem

For a specific circuit, once the relevant values of current $I(t)$ are known on an hourly basis (the time step considered in the analysis), the optimisation problem can be stated as

$$\min_{I_c}(TC_c) = \min_{I_c}\left\{CC_c + CM_c + CL_c(I(t))\right\} \quad (2)$$

s.t. $\hat{I} < \bar{I}_c$.

In (2), the objective function to be minimised is the circuit annual total cost TC_c (that is, the levelised annual cash flows [25] corresponding to the circuit LCC over the considered life span), and

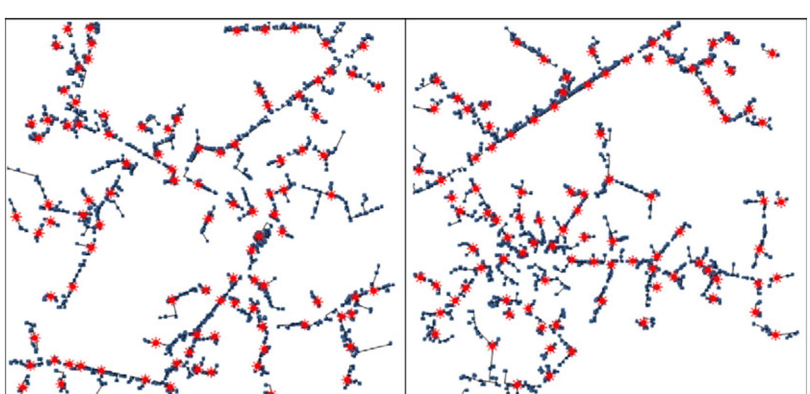

Fig. 4. Statistically similar rural network with two different seeds.

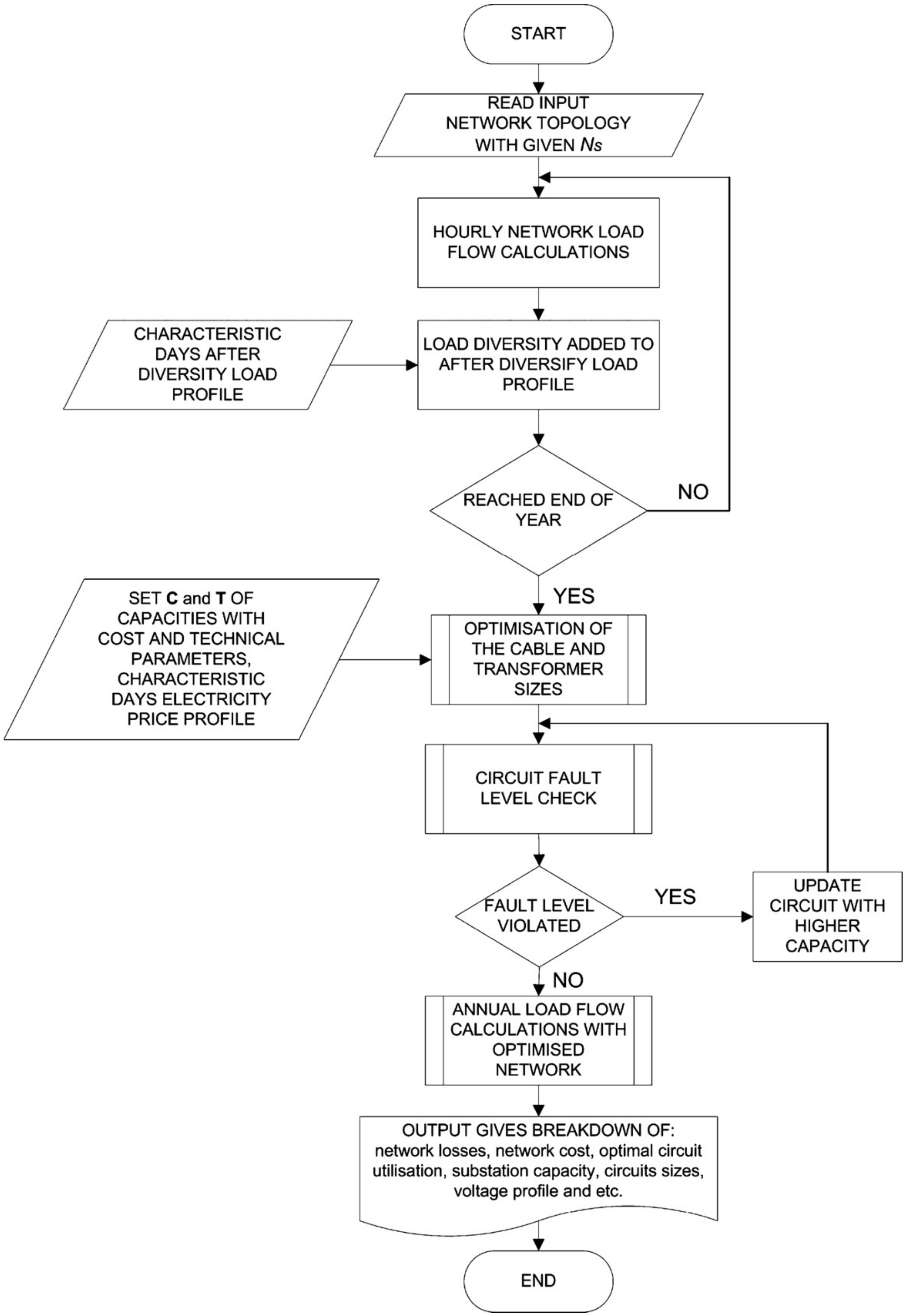

Fig. 5. Algorithm for network optimal economic design for given number of substations.

the optimisation variable is the circuit current-carrying capacity (or *ampacity*) I_c.

The annual total cost TC_c is composed of:

- *Annuitised cost of capital* CC_c [£/year]

$$CC_c = A \cdot IC_c = \frac{d \cdot (1+d)^n}{(1+d)^n - 1} \cdot IC_c \tag{3}$$

  where IC_c is the circuit investment cost [£], A is the annuity present worth factor [26], d is the discount rate, and n is the number of years of the network technical/economic operation.
- *Annual maintenance cost* CM_c [£/year], in general expressed through models developed *ad hoc*, for instance in terms of specific cost per circuit length.
- *Total annual cost of losses* CL_c [£/year], which is a function of the average current $I(t)$ circulating in the circuit at the hour t (for three-phase circuits, a balanced system is assumed) according to

$$CL_c = l \cdot R_c \cdot \sum_{t=1}^{8760} I^2(t) \cdot \pi(t) \tag{4}$$

  where l is a loss-related coefficient equal to 2 for single-phase circuits and to 3 for three-phase (balanced) circuits, R_c is the circuit resistance for each phase (an average value is assumed for all phases and over the time), and π is the estimated specific cost of losses [£/MW h] at the hour t.

The constraint in (2) refers to the thermal condition whereby the peak current $\hat{I}$ must be lower than the derived optimal circuit capacity $\bar{I}_c$.

One reference year is considered in this paper, with a focus on comparative assessment of different network characteristics and design strategies. Inclusion in the model of further parameters more relevant to network planning issues, such as load or energy cost increase/decrease across multiple years, will be object of future investigations.

Given a particular family of circuits (e.g., "Wavecon" cables [27]) the total cost (2) can be rewritten as a function of the circuit characteristics, and a closed-form expression for the continuous optimal circuit capacity $\bar{I}_c$ can be derived, as illustrated in [28]. The resulting values of circuit utilisation (defined as the ratio of circuit peak current to circuit optimal ampacity) for both underground cables (UGC) and overhead lines (OHL) for LV networks are typically quite low, in the range 15–25% for UGC and 10–15% for OHL. Hence, circuit design thermal constraints are generally non-binding, in line with findings that highlight the prominent role of losses in network economic [29,30] as well as environmental design [31,32].

3.3. Optimal network design: discrete optimisation

While the model (2) is valid for the design of a specific circuit, for network assessments the LCC optimisation problem as stated above needs to be extended to the network under consideration and can be written as

$$\min(TC_N) = \min_{I_c^i} \left(\sum_{i=1}^{N_N} (CC_c^i + CM_c^i + CL_c^i) \right) \tag{5}$$

s.t. *network constraints*, $\bar{I}_c^i \in \mathbf{C}$.

In (5), TC_N is the annual total cost of all the circuits in the network. The sum is over the overall number of circuits N_N in the network, and for each circuit i the optimal capacity $\bar{I}_c^i$ must be selected from the set $\mathbf{C}$ of available capacities for the considered circuit type. Typical network constraints include voltage and thermal limits and fault level requirements, as illustrated below.

In practice, the optimisation problem (5) can be approximated by solving problem (2) *separately* for each individual circuit i, which corresponds to

$$\min(TC_N) = \sum_{i=1}^{N_N} \min_{I_c^i}(CC_c^i + CM_c^i + CL_c^i) \tag{6}$$

s.t. *network constraints*, $\bar{I}_c^i \in \mathbf{C}$.

In fact, once the load patterns are given, in radial networks the load flows are primarily driven by topology, and changes in the single circuit impedances would affect the branch current distribution only marginally. Hence, solution of problem (6) would lead in practice to good results [28].

Once the optimal continuous circuit capacity has been calculated for each circuit in (6) through the model (2), the adjacent upper and lower capacity values from the set $\mathbf{C}$ are analysed. More specifically, for both these capacities the total cost (6) is calculated on the basis of the known branch current and the other input data, and the capacity yielding the overall minimum cost is selected as the optimal one (discrete optimisation). For larger capacity values, the capacities in $\mathbf{C}$ refer to several circuits in parallel.

3.4. Optimal transformer design

An equivalent problem (6) is also solved for substation transformers with the objective of minimising the annual total cost TC_T for all network transformers, namely,

$$\min(TC_T) = \sum_{j=1}^{N_s} \min_{S_T^j}(CC_T^j + CM_T^j + CL_T^j) \tag{7}$$

s.t. $\hat{S}_T^j < \bar{S}_T^j$, $\hat{S}_T^j \in \mathbf{T}$

In (7), $\bar{S}_T^j$ is the optimal transformer capacity [kV A] selected for each substation j from a set of available capacities $\mathbf{T}$, with the constraint that this capacity has to be higher than the power peak $\hat{S}_T^j$. The transformer capacities can refer to several units in parallel. The components (all expressed in [£/year]) of the total cost TC_T^j in (8) are as follows for a given transformer j:

- *Annuitised capital cost* $CC_T^j = A \cdot IC_T^j$, with IC_T^j being the transformer investment cost.
- *Annual cost of maintenance* CM_T^j (again to be defined case by case given the specific problem).
- *Annual cost of losses* CL_T^j, in turn expressed as

$$CL_T^j = LCL_T^j + NLCL_T^j = \sum_{t=1}^{8760} \left[3 \cdot R_T^j \cdot (I_T^j(t))^2 \cdot \pi(t) + P_{Fe}^j \cdot \pi(t) \right] \tag{8}$$

  In (8), the term LCL_T^j represents the cost of transformer load losses over one year, with R_T^j being the transformer phase resistance and $I_T^j(t)$ the hourly phase current. The term $NLCL_T^j$ represents the annual cost of no-load losses, given by the core iron losses P_{Fe}^j [W], assumed to be constant, weighted by the specific hourly cost of losses $\pi(t)$.

The optimisation problem in (7) is solved in discrete form by heuristically selecting, from the set $\mathbf{T}$, the optimal transformer minimising TC_T^j (once the hourly currents $I_T^j(t)$ are known from network load flow analysis) and with capacity greater than the annual peak demand.

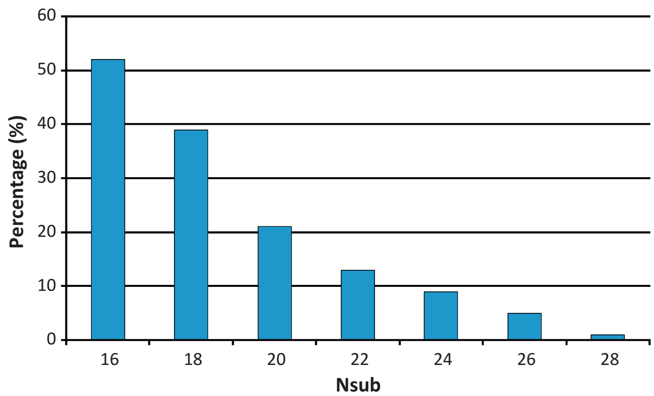

Fig. 6. Percentage of violated cases for the 5 MV A/km^2 case.

3.5. Fault level checks

After determining the optimal circuit capacities throughout the network, fault studies are carried out to determine the maximum through-fault current for every line, in line with standard procedures [33]. The maximum short-circuit current is calculated for each node starting from the 0.4 kV busbar of each substation (infeed node). The value of this current must be lower than the permitted short-circuit current of any of the lines connected to that node. If this is not the case, the line is replaced by one with a higher cross-section, which also exhibits a higher permitted short-circuit current. Moving downstream through the network, the check is repeated until all lines are examined and all fault current constraints are successfully met.

3.6. Methodology for the identification of the optimal number of substations

In order to identify the optimal number of substations minimising the overall network cost for a given load configuration, parametric analyses are run by changing the number of substations N_s across a suitable range. For each specific number of substations considered, the design algorithm illustrated in Fig. 5 is applied. The increment in N_s is set to approximately 10% of the heuristically estimated (based on experience and preliminary studies) optimal number of substations. For each of the identified candidate sets, 1000 seeds are simulated so as to build a statistical representation of the results. At this stage, the maximum/minimum voltages up to the service cut-out point (consumer point) are checked to give an indication of the suitability of given configurations to operate within statutory limits (+10%/−6% of the nominal voltage [21]). Given the high-level nature of the strategic studies conducted here (voltage control mechanisms are not modelled), the impact of voltage constraints is considered from a statistical point of view. More specifically, for a given load density and number of substations, the alternative under examination is considered statistically strong enough in terms of voltage profiles based on a certain *non-exceeding probability* of cases (corresponding to different seeds) for which the constraints are not met. A threshold of 10% has been selected, a figure that is generally accepted as reasonable for such strategic studies. Hence, for a given load density, in order to consider a particular number of substations as a potential candidate for the optimal network design, no more than 10% of the simulated seeds should exhibit voltage violations. For instance, Fig. 6 shows a typical voltage violation probability profile for a given load density (namely, 5 MV A/km^2) in urban areas, as a function of the number of substations. The number of violations decreases as the number of substations rises, with the decrease becoming less evident when the substation number becomes large enough to avoid voltage problems.

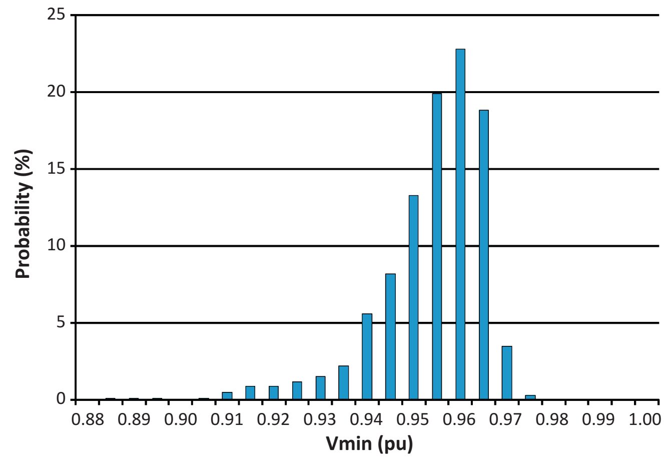

Fig. 7. Distribution of minimum voltages for the case 5 MV A/km^2, 24 substations, 1000 seeds.

Figs. 7 and 8 show the probability density function (PDF) of the minimum voltage and of the total network cost for the case of 24 substations supplying a specific load of 5 MV A/km^2. Here, only about 8.5% of the seeds generate a network with violation of the minimum voltage limit, making this configuration a candidate for optimality.

Once the subset of potential numbers of substations meeting the network design constraints for given load characteristics is established, the configuration exhibiting the minimum LCC is chosen as optimal (the *mean value* of the network LCC is considered here as the metric to compare different alternatives).

4. Numerical applications

4.1. Description of the case studies

A number of case studies have been carried out with reference to typical UK urban and rural networks. Each case uses a settlement of 2000 consumers have been considered and an increase in the value of load density has been achieved by decreasing the size of the relevant area. Then, for each load density case, the number of substations has been modified so as to identify the relevant impact on cost and other indicators. For each configuration analysed (load density and number of substations), simulations have been run for

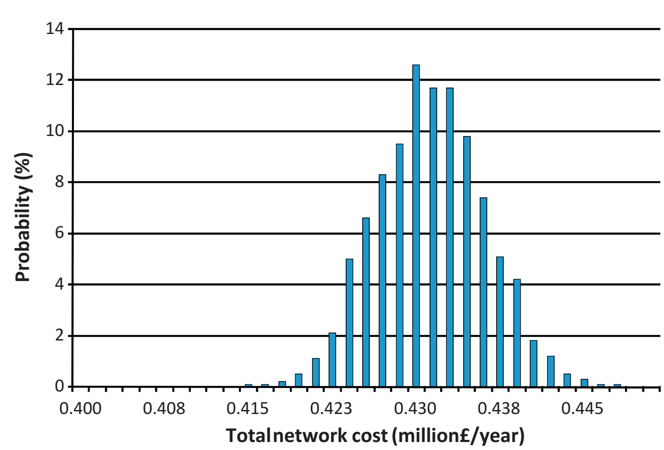

Fig. 8. PDF of the total network cost for the case 5 MV A/km^2, 24 substations, 1000 seeds.

Table 1
Case study consumers' distribution breakdown and ADMD.

Consumer types	Distribution breakdown urban/rural (%)	ADMD (kW)	Load factor (%)
Residential without electrical heating	80/90	1	35
Residential with electrical heating	12/8	5	60
Commercial	5/1	10	40
Industrial	3/1	30	31

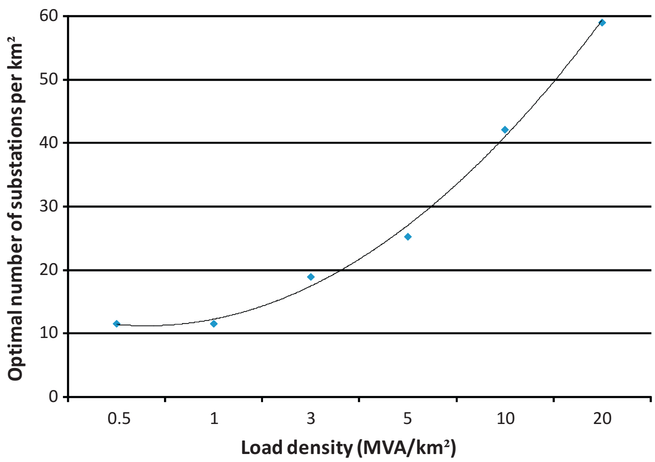

Fig. 9. Optimal number of substations density as a function of load density for urban areas.

one thousand seeds, corresponding to one thousand statistically similar networks (see for instance the two examples in Fig. 2).

Four characteristic user types have been used, as mentioned in Section 3. Table 1 shows the user distribution breakdown and their relevant *ADMD* and load factor. Urban networks are assumed to be supplied only by UGC and indoor substations, whereas rural networks use OHL and pole mounted transformers (PMT).

4.2. Results for urban areas

The optimal number of substations per km^2 for given load density values for urban areas is presented in Fig. 9. The overall trend can be approximated well with a second order function. It should be noted that for the lower load densities the optimal number of substations per km^2 saturates and does not change significantly, owing to arising voltage drop constraints. For higher load densities, voltage drops are not an issue as consumers are sited relatively close to the feeding substation, but an increasing number of substations per km^2 is needed to economically meet the network demand. A synthetic summary of the main results for optimal configurations for the load densities considered in the analysis is given in Table 2. Typical average urban load densities in the UK are in the range 3–8 MV A/km^2. Here, the load densities considered span a wide set of scenarios, from

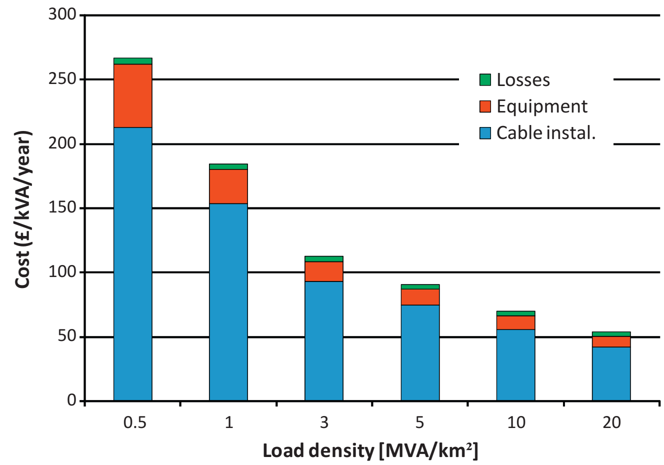

Fig. 10. Breakdown of equipment and losses average costs, including cable installation costs, as a function of load density.

suburban areas (1 MV A/km^2, with 0.5 being an extreme case closer to the situation in rural settlements) to city centre areas (20 MV A/km^2).

Fig. 10 shows the breakdown of the average cost of equipment and losses, including cable installation costs (in £/kV A/year), for the optimal number of substations identified for different load densities. The overall network cost is mainly determined by the extremely high UGC installation cost in UK urban areas, followed by the cost of indoor substations, while losses do not have a major influence. The difference in the cost per kV A increases with decreasing load density, mainly due to longer network lengths, which brings about higher UGC installation costs, and to voltage drop issues, which on average call for the use of more substations per area unit. From the more detailed equipment cost breakdown in Fig. 11, it can be seen that substation cost dominates UGC cost and rises exponentially with a decrease in the load density, as the specific number of substations increases. The overall cost of cables is relatively low compared to the other costs, so that investment in relatively losses-driven high cross-sectional cables is even more justified.

A breakdown of the average losses (as a percentage of the overall energy demand) for different load densities in the case of the optimal number of substations is shown in Fig. 12. Overall, losses are relatively low owing to use of the optimal design strategy. Transformer losses make up about 60% of the overall losses, with a substantial share due to core losses, higher for decreasing load density (due to the higher number of substations). This highlights the additional potential to decrease losses by adopting high-efficiency transformers.

With an increase in load density, voltage drop constraints are less binding, and this leads to saturation in the needed number of substations. Therefore, the number of consumers supplied from a given substation increases (Table 2). However, at the same time, the average transformer size increases as well. Indeed, it should

Table 2
Optimal number of substations, total costs, average transformer sizes and losses in urban areas.

Load density (MV A/km^2)	Area supplied (km^2)	Optimal number of substations	Optimal number of subst/km^2	Number of consumers per subst.	Total cost of the network (£/kW/year)	Average transformer size used (kV A)	Losses in the network (%)
0.5	9.5	108	11.4	18	267	79	2.7
1	4.7	55	11.6	36	184	140	2.5
3	1.6	30	19.0	67	112	230	2.2
5	0.9	24	25.3	83	91	281	2.2
10	0.5	20	42.2	100	70	332	2.0
20	0.2	14	59.1	143	54	465	2.0

Table 3
Optimal number of substations, total costs, average transformer sizes and losses in rural areas.

Load density (MV A/km^2)	Area supplied (km^2)	Optimal number of substations	Optimal number of subst/km^2	Number of consumers per subst.	Total cost of the network (£/kW/year)	Average transformer size used (kV A)	Losses in the network (%)
0.1	28.8	130	4.5	15	80	63	2.9
0.3	9.6	70	7.3	29	49	89	2.4
0.5	5.8	55	9.5	36	40	106	2.2
1	2.9	38	13.2	53	31	151	2.1

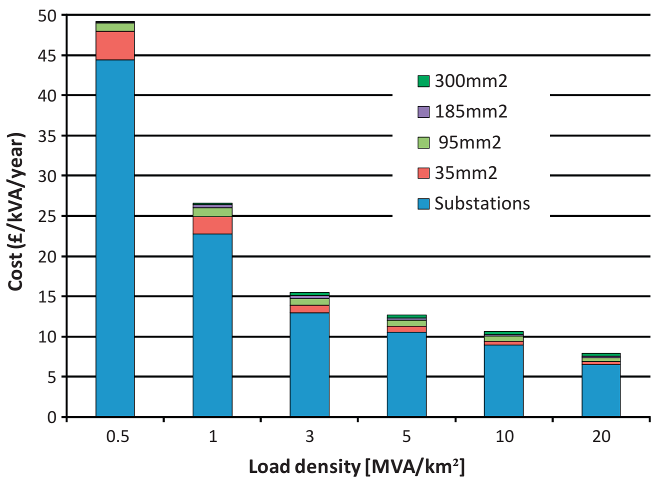

Fig. 11. Breakdown of the average equipment cost for different load densities.

be highlighted that the main constraint for high load densities in urban areas is the maximum available size of transformer, typically equal to 1 MV A.

4.3. Results for rural areas

The average load density for UK rural areas is approximately 0.17 MV A/km^2 [18]. However, load densities vary depending on the type and geographic characteristics of the area considered, and so rural load densities ranging from 0.1 to 1 MV A/km^2 have been analysed here. A similar network design procedure and analysis to the urban case have been performed for the rural case. Due to space limitations, only a few synthetic results for the optimal configurations found for various load densities are presented in Table 3. As in the urban case, the cost of supplying consumers decreases substantially with an increase in the load density. However, for a given load density (for instance 0.5 and 1 MV A/km^2) the total network cost to supply urban areas is about six times higher than for rural areas, owing to higher costs of indoor substations and, above all, the extremely high cable excavation cost in urban areas.

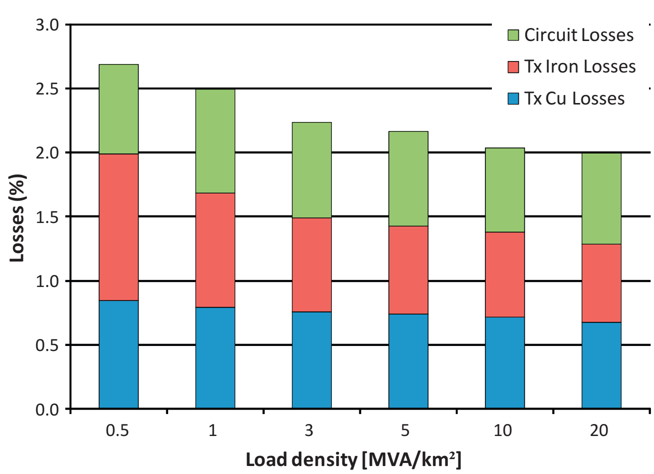

Fig. 12. Breakdown of the average losses for different load densities.

The binding network design constraint in rural areas appears to be voltage-related, as opposed to urban areas for which thermal constraints are generally more binding (as feeders are relatively shorter and more heavily loaded). In fact, the results suggest that in rural areas it is necessary to increase N_s, which essentially reduces the length of the OHL feeders associated with each transformer, so as to alleviate voltage drop issues. Hence, in order to supply a given number of consumers, rural areas would need many more transformers than urban areas.

5. Conclusions

This paper has introduced a statistical methodology for the generic appraisal of different LV distribution network design strategies that is suitable for decision making on large scale applications. In contrast to a traditional approach where the assessment is carried out based only on a small number of specific networks, the methodology developed here allows for the analysis of a large number of test networks generated through a fractal-based algorithm. The circuit design methodology adopts a minimum LCC approach for selecting the optimal size of conductors and transformers, and implicitly takes into account losses as a key driving factor for network investment, especially at LV where the great majority of losses occur. Numerical applications for urban and rural areas have been presented in order to illustrate a number of potential applications. These include investigation of the optimal number of substations for different load densities and topologies, and identification of typical cost breakdown trends for different network characteristics, so as to highlight the key drivers of improved network economic performance. The proposed model thus not only represents a valuable tool for decision support in the development of LV network design strategies, but could also be used to identify generic "*reference networks*", for instance for policy development purposes.

Work in progress includes extension of the model to the analysis of upstream voltage levels, and assessment of the impact of distributed energy resources on optimal network design practices.

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Appendix A. Nomenclature

A.1. List of symbols

A	annuity present worth factor
B	overall length of considered feeders [km]

C	set of available capacities for a given circuit type
CC_c	circuit annuatised cost of capital [£/year]
CC_T	transformer annuatised capital cost [£/year]
CM	circuit annual maintenance cost [£/year]
CM_T	transformer annual cost of maintenance [£/year]
CL_c	circuit total annual cost of losses [£/year]
CL_T	transformer annual cost of losses [£/year]
d	discount rate [%]
f	(superscript) feeder index
i	(superscript) circuit index
$I(t)$	circuit current in time period, t [A]
I_c	circuit current-carrying capacity [A]
$\hat{I}$	circuit annual hourly peak current [A]
$\bar{I}_c$	circuit optimal current-carrying capacity [A]
I_T	transformer hourly phase current [A]
IC_c	circuit investment cost [£]
IC_T	transformer investment cost [£]
j	(superscript) transformer index
k	(superscript) branch index
l	loss-related coefficient
L	length of branch at considered feeder [km]
LCL_T	cost of transformer load losses [£/year]
n	network technical/economic life [years]
N	number of feeders from a substation
N_S	number of substations
N_N	number of circuits in the network
N_T	number of transformers in the network
$NLCL_T$	annual cost of transformer no-load losses [£/year]
P_{Fe}	transformer core iron losses [W]
R_c	circuit resistance [Ω]
R_T	transformer phase resistance [Ω]
$\bar{S}_T$	optimal transformer capacity [kV A]
$\hat{S}_T$	transformer peak capacity [kV A]
T	set of available capacities for a given transformer type
TC_c	circuit annual total cost [£/year]
TC_N	annual total cost of all network circuits [£/year]
TC_T	transformer annual total cost [£/year]
μ	mean value of B over the considered feeders
$\pi(t)$	cost of losses at hour t [£/MW h]
σ	standard deviation of the overall feeder lengths

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