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MATHEMATICAL MODELING OF STRUCTURAL INTEGRITY ON HIGH RISE BUILDING USING PARALLEL AXIS THEOREM AND FINITE ELEMENT ANALYSIS

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Abstract

This study provides an introduction to the concepts and principles of seismic design, including strategies for designing earthquake-resistant buildings to ensure the health, safety, and security of building occupants and assets. There are three main factors that contribute to the successful seismic design. Firstly, the design team must consider a multi-hazard approach in designing the structure that accounts for the potential impacts of seismic forces as well as all the major hazards to which an area is vulnerable. Secondly, considerations need to be made pertaining to the performance-based requirements. This requirement may exceed the minimum life safety requirements of current seismic codes. Therefore, the safety factor must be established to respond appropriately to the threats and risks posed by natural hazards on the building's mission and occupants. Thirdly, since earthquake forces are dynamic and each building responds according to its own design complexity, it is essential that the design team work collaboratively and have a common understanding of the terms and methods used in the seismic design process. In addition to that, as a general rule, building that is designed to resist earthquakes should also resist blast (terrorism) or wind, suffering less damage.

Key words: Modeling, high rise building, seismic, earthquake

Introduction

Structure is a necessary part of life which occurs at any level, ranging from the molecular structure of material to the laws of the universe. Everything has structure, even if it has not yet been recognized. There are three main purposes of structure in buildings. Firstly, it functions as a spatial and dimensional organizer, besides identifying construction system. Support structure holds the building up and prevents it from collapse or deforms excessively. Building and structure are very closely related to each other. Support structures have the required strength and stiffness to resist the vertical loads that are caused by gravity. Apart from that, horizontal loads which due to wind and earthquakes are also safely absorbed by the support structures [1]. Even though there is no precise cutoff height of a high-rise building, different definitions are used for it. High-rise building can be classified as a building of 10 to 100 stories and more [1].

Building structures have become more flexible and taller thanks to the technological advancement like using high-strength materials and advanced construction techniques. However, the increasing height of modern tall buildings posed a series of challenges for structural engineers. In designing such a tall building, the structural system must meet three major requirements: strength, rigidity, and stability [2]. It is a well known fact that the strength requirement is the dominant factor in the design of low-rise structures. However, as building height increases, the rigidity and stability requirements become more important, and they are often the dominant factors in the structural design. Under lateral loads, interior forces are quite variable and increase rapidly with increases in height. Lateral deflection may vary as the fourth power of the height of a building [3], and structural dynamic behavior is thus one of the most important design considerations in the design of a modern tall building.

Numerous investigations on seismic behavior of tall buildings have been carried out in the past; in particular shaking table tests play an important role in earthquake-resistant design of structures, analysis of seismic responses and failure mechanisms [4- 6]. The finite element method (FEM) is a powerful tool for structural analysis of tall buildings. Fan and Long [7] adopted splines elements in the analysis of tall buildings. In their method, the element displacements are interpolated with splines functions and accurate results could be achieved with lower-order functions and a few degrees of freedom. In structural engineering, beam analysis is crucial for building safety. There are many different types of beam designs to be selected. It cans sometimes challenging for even the most experienced engineer to decide on the best beam for each architectural structure. The selected beam must have the finest structural integrity as possible. Different shapes, sizes, techniques and materials support different structural loads. The I-beam, flitch, cantilever and hip are some of the most commonly used types of beam design. The hip type of beam is used mostly in the construction of residential roofs. Hip beams support the angled beam used in many house roof designs. The hip beam is designed to support a triangular-shaped load, such as that of a sloped or pitched roof frame. Cantilever beams are used to suspend structures such as balconies. Most of the weight is distributed onto the foundation beams.

This weight distribution permits a building extension such as a balcony to be safely supported. Cantilever beams are also sometimes referred to as an end load beam type since the loads are always supported mainly on one side. Some bridges are designed with cantilever beams in their construction. Flitch beam design types are made from layers of wood and steel since they're designed to be strong as well as lightweight. Since flitch beams aren't made of solid steel, they're less expensive than pure metal varieties. A flitch beam type is used to nail into place on wood structures to provide extra support. Pure steel beams cannot be nailed onto wood, so flitch beams have a distinct advantage over solid metal on wooden building exteriors. Flitch beams are designed to support vertical loads. I-beams are by far the most common type of beam design used in construction; they're known as the universal beam. I-beams are columns that are straight in shape. They may be arranged into different support patterns that can form L, W, H and V shapes, among others. Rounded I-beams called C-channels may also be used in some specialty construction applications. I-beams may be used to create long spans of support in floors, walls and roofs. Beam design software is an analytical tool used to help in the selection of appropriate beams for a particular structure. A beam calculator determines what load beams of a certain shape and size will support. The material and building technique information are inputted before being calculated by beam design software.

1. Research Methodology

It is possible to have a situation where the stiffness force and the inertia force are exactly equal and opposite. When the stiffness and inertia forces are self-canceling the system is said to be in resonance. The amplitude of vibration is then controlled by the level of damping within the structure. At resonance the external forces are balanced by the damping forces. The frequency at which resonance occurs which is when the stiffness and the inertia forces cancel are called the natural frequency or the resonance frequency of the system. If an undamped system is disturbed from its equilibrium position and no external forces are applied then it will oscillate at this natural frequency. If an undamped system is excited at its natural frequency then the amplitude of oscillations will grow linearly with time so that the response can become very large. If damping is present then the amplitude of vibration will be limited by the damping so that they will not grow to infinity. Damping causes the peak amplitude to occur at a slightly lower frequency so that the damped resonance frequency is slightly lower than the natural frequency but for typical values of structural damping this change is so small it can be neglected. If the system is excited at some frequency other than resonance then the amplitude of the response is largely controlled by the stiffness and inertia forces.

The time for a complete cycle of oscillation of a Single Degree of Freedom (SDOF) system is known as the fundamental or natural period, T, usually expressed in seconds. The reciprocal of natural period is the linear natural frequency, f , usually called natural frequency, fundamental frequency, or just frequency, and expressed in Hz (i.e., cycles per second). It is important to distinguish between the natural frequency of a system (building, oscillator, etc.) and the frequency of an applied force. The natural frequency, f, in Equation 1.1 has nothing to do with an external force.

$$f = \frac{1}{T} \qquad (1.1)$$

The natural frequency can also be expressed in radians per second (rad/sec), in which case it is known as the circular frequency, angular natural (fundamental) frequency, or just angular frequency, ω.

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad (1.2)$$

It is easy to derive the natural frequency for the case of a simple harmonic oscillator. For a mass on a spring,

$$\omega = \frac{\sqrt{kg_c}}{m} = \frac{\sqrt{kg}}{w} \qquad (1.3a)$$

Or

$$\omega = \frac{\sqrt{k}}{m} \qquad (1.3b)$$

Substituting k from Hooke's law, $(F = kx)$ and recognizing that the mass, m can be calculated from the weight, W, an expression is derived for the natural frequency in terms of the static defection, x_{st}.

$$\omega = \frac{2\pi}{T} = \frac{\sqrt{Fg_c}}{x_{st}m} = \frac{\sqrt{Fg}}{x_{st}m} \qquad (1.4a)$$

Or

$$\omega = \frac{2\pi}{T} = \frac{\sqrt{F}}{x_{st}m} \qquad (1.4b)$$

Since equation 1.4 can be used to calculate the natural period, it is tempting to substitute the maximum allowable code drift (for example 2.5% of the total building height) for the static defection in order to calculate the natural building period. Such a substitution would require no structural analysis at all, but implies that the building will have maximum flexibility permitted by the code and will remain elastic when this drift is achieved. One problem with this approach is that it assumes the maximum allowable drift to be the same for all geographic regions, although the flexibility actually depends on the location since flexibility is affected by the building's seismic resistance. Thus, while the lateral forces on the building differ, the maximum drift and, thus, the period, do not. Obviously, the building period cannot be calculated in this way.

2. Development of Structure Design

Structural designs are created in order analyze the most suitable structural that can overcome earthquake effect on high-rise building. The research considers factor pertaining to the shape and possible factors available that affecting the structural integrity of a building when earthquake occurs. In this project, the design concept need to be first determined. This is due to consideration is about the structure of the building. In order to fit it on to the shape, the designed concept is the most important thing to be fixed first as it would determine whether the design is capable or not in sustaining tremors. This design is about 8 storey buildings which are using beam, frame, column, and level. It is about 80 cm tall and 40 x 40 cm cross section area.

In order to determine the design, the understanding about the application is very important. The principles and strategies of seismic design and construction are applied in a systematic approach that matches an appropriate response to specific conditions through the following major steps:

i. Analyze Site Conditions
  - The location and physical properties of the site are the primary influences the entire design process.

ii. Establish Seismic Design Objectives
  - A performance-based approach to establishing seismic design objectives is recommended. This determines a level of predictable building behavior by responding to the maximum considered earthquake. A threat/vulnerability assessment and risk analysis can be used to define the level of performance desired for the building project.

iii. Design Selection Appropriate Structural Systems
  - Seismic design objectives can greatly influence the selection of the most appropriate structural system and related building systems for the project.

3. Development of Structure Design

Three suitable design been selected because of certain reason on this designing solution. From 3 structural designs; the most suitable design being illustrated in solid work and the best design will go further on the analysis of the structural.

i. Design 1 using I-Beam

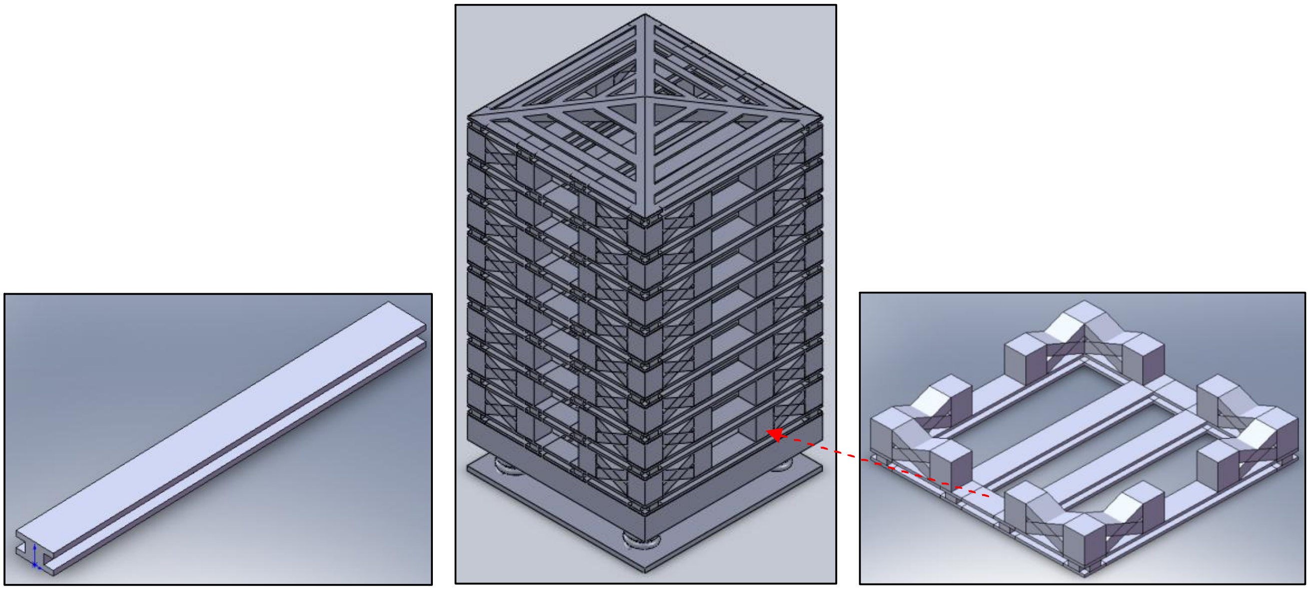

Figure 1: Design 1 of I-beam for high rise building

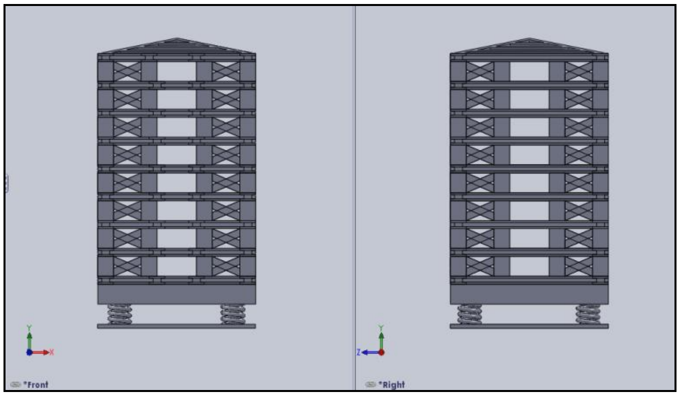

Figure 2: Front and right view of design 1

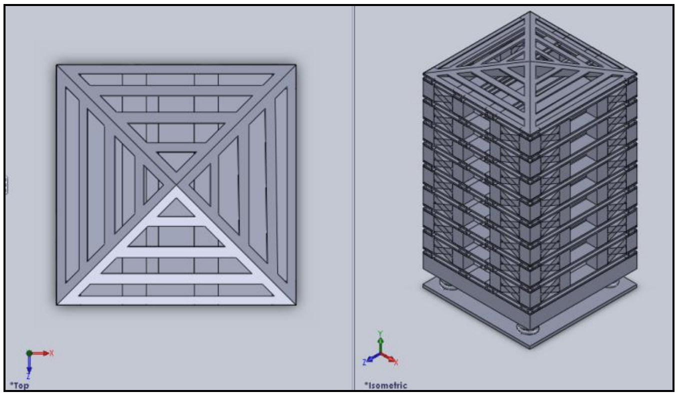

Figure 3: Top and isometric view of design 1

ii. Design 2 using T-Beam

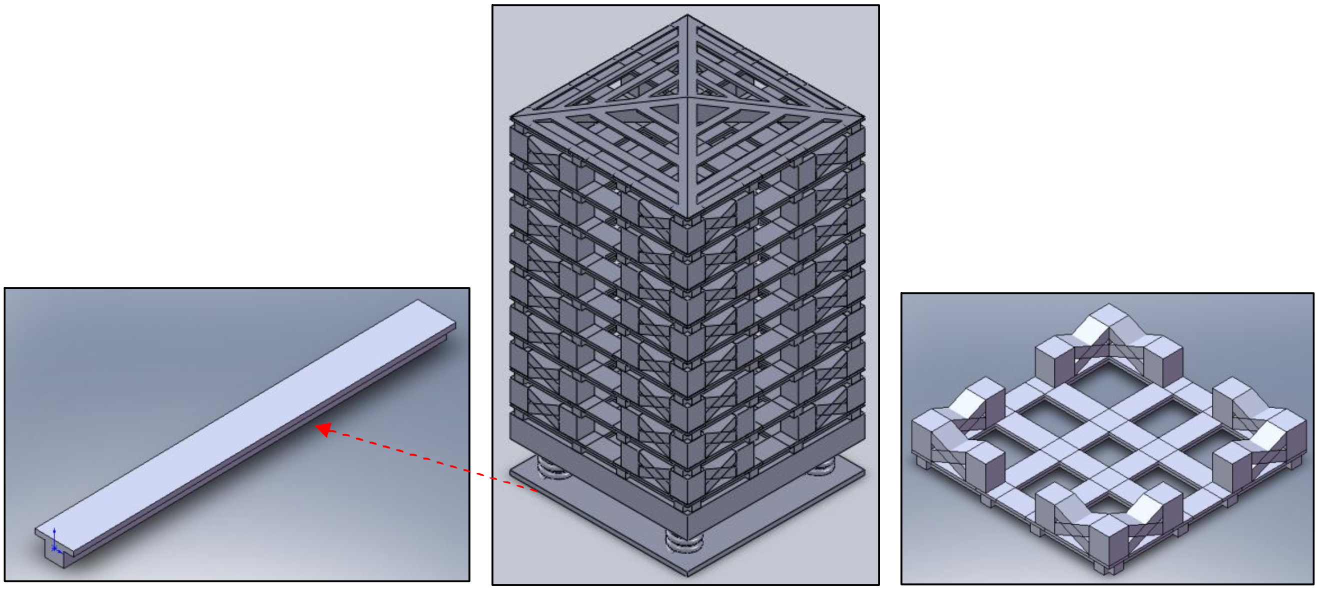

Figure 4: Design 2 of T-beam for high rise building

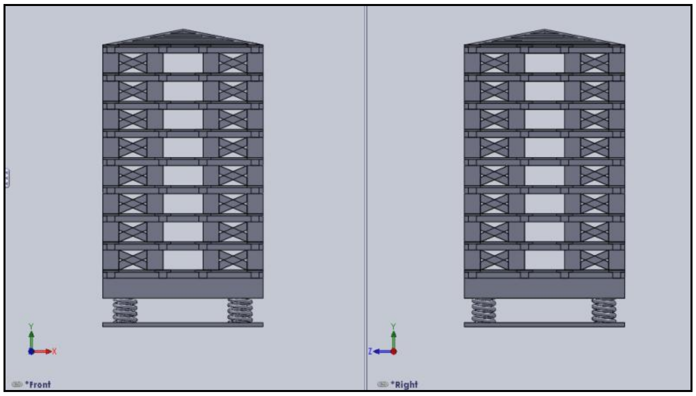

Figure 5: Front and right view of design 2

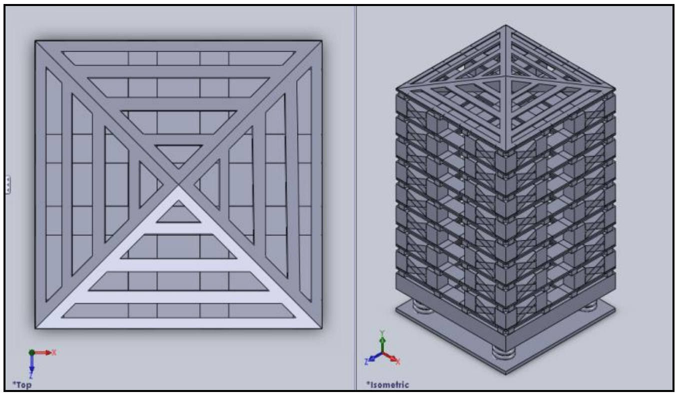

Figure 6: Top and isometric view of design 2

iii. Design 3 using Hollow Beam

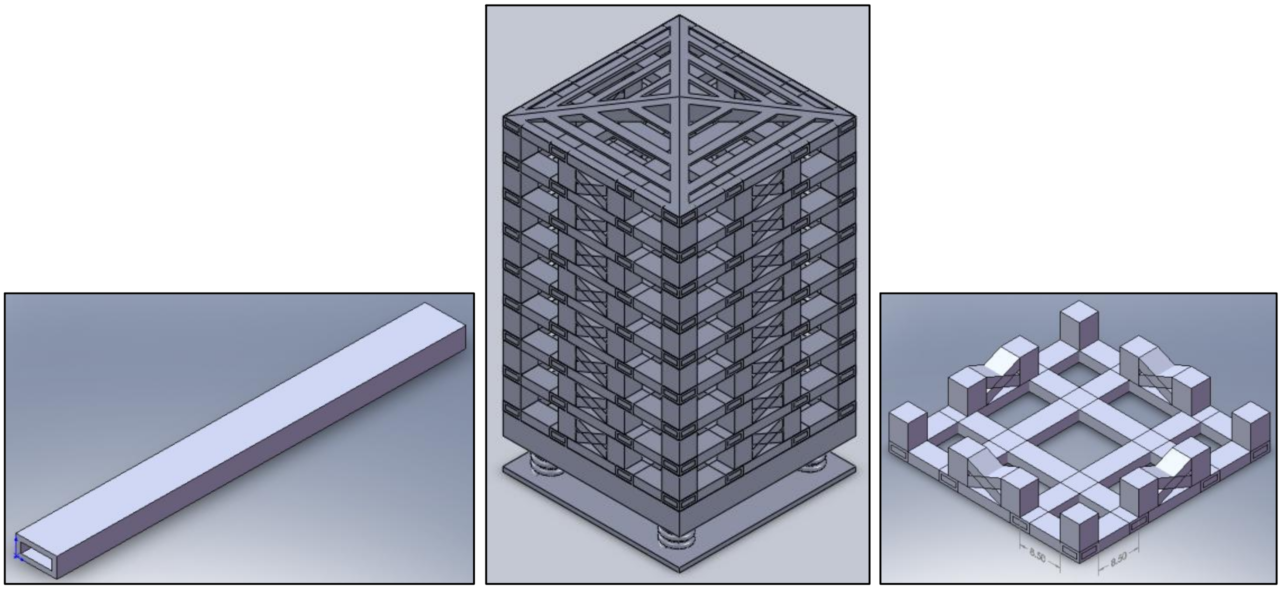

Figure 7: Design 3 of Hollow beam for high rise building

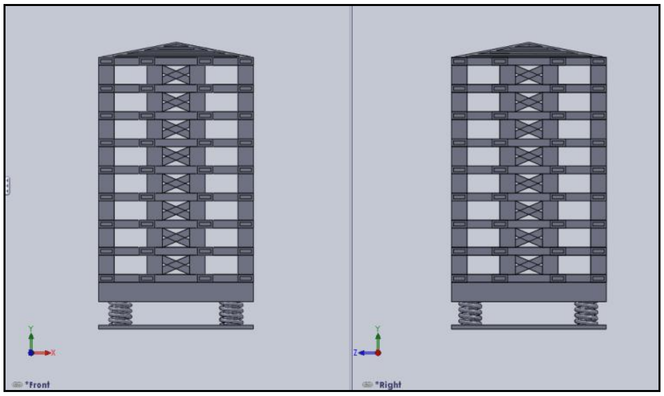

Figure 8: Front and right view of design 3

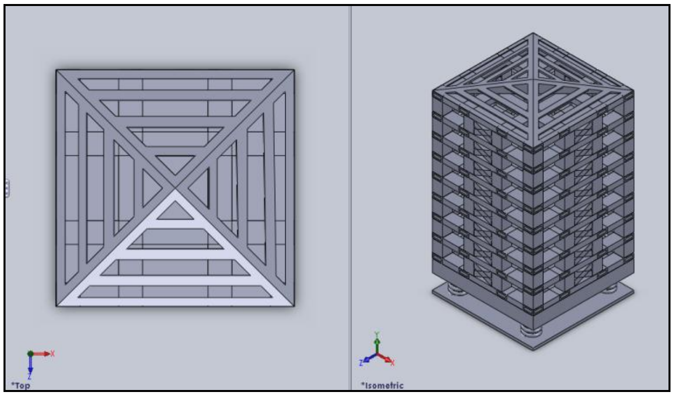

Figure 9: Top and isometric view of design 3

4. Structural Modeling

i. Normal stress second of moment inertia for beam in bending phenomena with T-Beam cross section area.

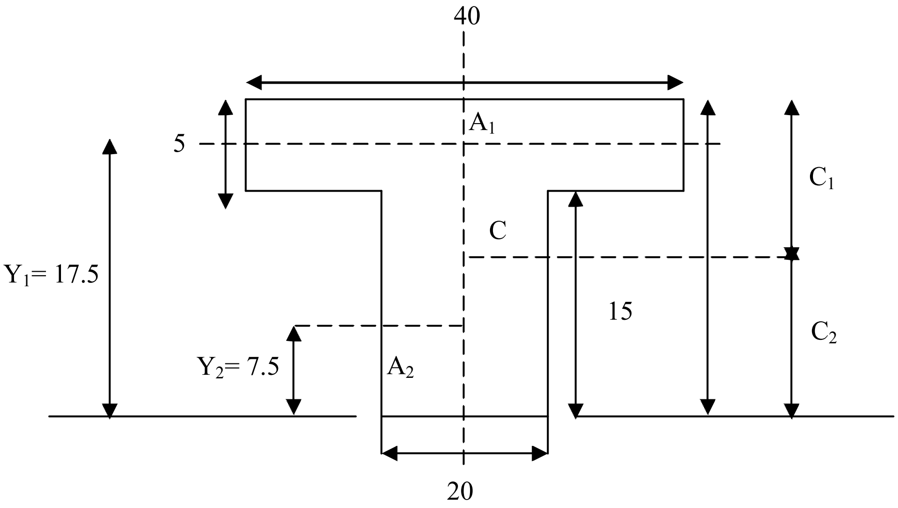

Figure 10: Cross section area of T-Beam

Bending moment, $M = 1600N$

Area, $A = 500mm^2$

Then, T-Beam is divided into 2 rectangles, A_1 and A_2 to find centroid value.

Table 1: Calculation of area

	Area, mm^2	**Y_i ,mm**	**$A_i Y_i$,mm^3**
A_1	(5) (40) = 200	17.5	3500
A_2	(15) (20) = 300	7.5	2250
Total	Σ =500		Σ = 5750

$$C_2 = \frac{5750}{500} = 11.5mm$$

$$C_1 = 20 - 11.5 = 8.5mm$$

Then, second moment of inertia, I was calculated,

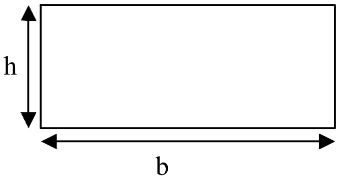

Figure 11: Rectangle cross section area, I for T-Beam

$$I_{A1} = \frac{bh^3}{12} = \frac{(40)(5)^3}{12} = 416.67mm^4$$

$$I_{A2} = \frac{bh^3}{12} = \frac{(20)(15)^3}{12} = 5625mm^4$$

$$A = bh \quad , \quad I_x = \frac{bh^3}{12} \quad , \quad I_y = \frac{bh^3}{12}$$

By applying parallel-axis theorem,

$$I_2 = I_{cg} + Ad^2$$

Then, distance for A_1 and A_2 is

$$d_1 = 8.5 - 2.5 = 6mm \quad , \quad d_2 = 11.5 - 7.5 = 4mm$$

The second moment of inertia, I can be determined using parallel-axis theorem for both rectangles, A_1 and A_2, respectively.

$$I = [416.67 + 5(40)6^2] + [5625 + 20(15)4^2] = 18.042 \times 10^3 mm^4$$

Maximum tensile stress is

$$\sigma = \frac{Mc_1}{I} = \frac{[1600(8.5)10^{-3}]}{[18.042(10^{-6})]} = 753.8MPa$$

Maximum compressive stress is

$$\sigma = \frac{Mc_2}{I} = \frac{[1600(11.5)10^{-3}]}{[18.042(10^{-6})]} = 1019.84MPa$$

ii. Normal stress and second moment of inertia for beam in bending phenomena with Hollow-Beam cross section area.

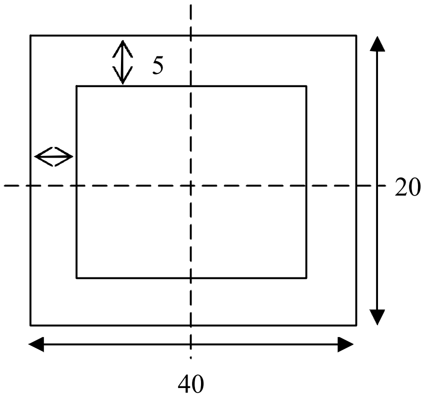

Figure 12: Square cross section area, I for Hollow-Beam

The Hollow-Beam in Figure 12 can be divided into two sections and it shows in Figure 13.

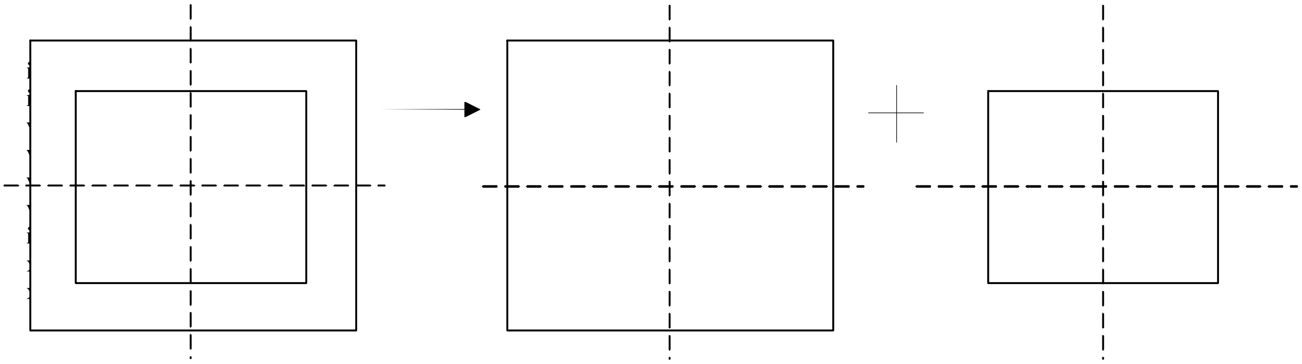

Figure 13: Section of Hollow-Beam

Material property of these Hollow-Beam shows in Table 2.

Table 2: Material properties

No	Description	Value
1	Material	Aluminum alloy
2	Yield stress, σ_y	275 MPa
3	Ultimate stress, σ_u	415 MPa
4	Young Modulus, E	73 GPa
5	Safety factor, F.S	3.00

The second moment of inertia is

$$I = \frac{bh^3}{12} = \frac{(40)(20)^3}{12} - \frac{(30)(10)^3}{12} = 24.17 \times 10^3 mm^4$$

Allowable stress is

$$\sigma_{all} = \frac{\sigma_u}{F.S} = \frac{415}{3} = 138MPa$$

Hence, $\sigma_{all} < \sigma_y$

Then, the bending moment is

$$C = \frac{1}{2} \times 20 = 10mm$$

$$\sigma_{all} = \frac{MC}{I} \quad , \quad M = \left(\frac{I}{C}\right)(\sigma_{all}) = \left(\frac{24.17 \times 10^{-6} mm^4}{0.01m}\right)(138MPa) = 2.417kN.m$$

iii. Finite Element Analysis (FEA)

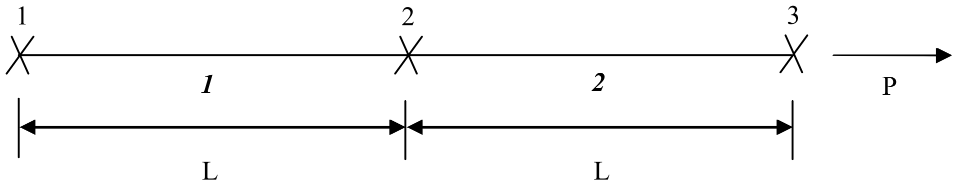

Figure 14: Boundary condition, nodal point of the high rise structural

The dimensional of this structure are: length, $L = 400mm$, Force, $P = 10kN$, Young Modulus, $E = 70GPa$, cross section area, $A_1 = 400mm^2, A_2 = 200mm^2$. The boundary condition, B, C for nodal 1 are U_1, F_2 and F_3, respectively. For element ***1*** and ***2***, there were shown in equation 1.5 and 1.6 below.

$$\frac{AE}{L}\begin{array}{c} U_1 \quad U_2 \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ U_1 \\ U_2 \end{array} \quad or \quad \frac{E}{L}\begin{array}{c} U_1 \quad U_2 \\ \begin{bmatrix} A & -A \\ -A & A \end{bmatrix} \end{array}\begin{array}{c} \\ U_1 \\ U_2 \end{array} \qquad where \qquad A = A_1 \tag{1.5}$$

$$\frac{AE}{L}\begin{array}{c} U_2 \quad U_3 \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ U_2 \\ U_3 \end{array} \quad or \quad \frac{E}{L}\begin{array}{c} U_2 \quad U3 \\ \begin{bmatrix} A & -A \\ -A & A \end{bmatrix} \end{array}\begin{array}{c} \\ U_2 \\ U_3 \end{array} \qquad where \qquad A = A_2 \tag{1.6}$$

Then,

$$\frac{E}{L}\begin{array}{c} U_1 \qquad U_2 \qquad U_3 \\ \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ U_1 \\ U_2 \\ U_3 \end{array}, \qquad where \quad u_1 = 0 \tag{1.7}$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{E}{L}\begin{bmatrix} A_1 + A_2 & -A_2 \\ -A_2 & A_2 \end{bmatrix}\begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} \tag{1.8}$$

So,

$$F_2 = 0; \quad 0 = (A_1 + A_2)U_2 - A_2U_3; \quad 0 = A_1U_2 - \frac{PL}{E} \tag{1.9}$$

Next,

$$\frac{PL}{E} = A_2U_2 + A_2U_3 \tag{1.10}$$

Substitute equation (1.10) into (1.9). The new equation is:

$$U_2 = \frac{PL}{EA_1} \tag{1.11}$$

From equation (1.9),

$$U_3 = \frac{[(A_1 + A_2)U_2]}{A_2} \tag{1.12}$$

Then, substitute equation (1.11) into (1.12).

$$U_3 = \frac{\left[\frac{A_1 + A_2}{A_2}\right]}{\left[\frac{PL}{EA_1}\right]}; \qquad U_3 = \left[\frac{400 + 200}{200}\right] \times \left[\frac{P(400)}{(70GPa)(400)}\right] = 3 \times (0.143mm) = 0.429mm \tag{1.13}$$

On the other hand,

$$U_2 = \frac{PL}{EA_1} = \frac{10kN(400mm)}{(70GPa)(400)} = 0.143mm$$

5. Conclusion

The newly improved designed structural building have been made in order to solve the problem occurred in the high-rise building during earthquake. The structural designs are being analyzed and the research on the structural building design of high-rise building is being made to know the specific problem that related on this structural design. The study about the effect of earthquake on high-rise building, building characteristics and also earthquake engineering are done. The conceptual designs are being made to follow the specification and application that involve on the high-rise building. From the conceptual design, three selected design have been chosen which the most suitable designs are being illustrated by using the Solid work software. The calculation is done in each design to measure the most suitable design that will be used in further analysis. The analysis on the structural design also being made to determine the most strengthen and deformed shape in the analysis. On the analysis the value of force, shear strain, allowable stress, maximum stress, moments and stress (maximum shear) will be determined. Experiments was conducted on the building structure. The graph of mobility against frequency obtained from the experiment shows that when the mobility is high, the frequency also proportional to the mobility. Mobility equals to Vibration divided by Force (M=V/F). The vibration part of the equation can be in either acceleration or velocity and it is frequency related. It is shows that when the building structure start to mobilize, there are deformation occurred which give the phase data against the frequency. This is happen because when the spring received force from the outside. The spring will react to make sure the building will be in stability. The first reaction from the spring to the building will give high amplitude and then when the building becomes stable, the amplitude is starting to decrease until to its final value. So, the objectives to design the suitable high-rise building using numerical approach and to develop an experimental rig on high-rise building have been achieved.

6. Acknowledgement

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