SPHERE FITTING INTO VOLUMETRIC REPRESENTATIONS OF BASKETBALL THROWS

Volkan D. Yaylıoğlu[1], Serdar Arıtan[1]

[1]Biomechanics Research Group, Department of Exercise and Sport Sciences, Faculty of Sport Sciences, Hacettepe University, Ankara, Turkey

This study aims to provide an evaluation, through some simplifications, for the most forgiving throw positions in the game of basketball throughout the playing field. For each position, throws are modelled as differential equations and then solved numerically. Given a position in the playing field, velocity vectors of successful throws constitute a solid and we fit a maximal sphere into these solids to evaluate the error tolerance for each position. A contour graph for the volumes of maximal spheres was plotted for this evaluation.

KEYWORDS: Basketball, modelling, field throw, bank shot, direct shot

INTRODUCTION: Knowing the positions which are more forgiving to throwing errors in a game of basketball may give a player (or the team) the edge necessary to win the match. So, analysis of throws without taking other players into account remains important. Several approaches have been researched to accomplished this. Tran and Silverberg (2008) and Okubo and Hubbard (2015) focuses on throwing kinematics in their papers. Among the previous studies, another point of focus has been free throws: Seppala-Holtzman (2012), Tran and Silverberg (2008), Hamilton and Reinschmidt (1997), Okubo and Hubbard (2006), and Maymin et al. (2012). Silverberg (2013) studies bank shots and compares them to direct shots. Another point of interest has been jump shots: Okazaki et al. (2015).
Silverberg et al. (2003) formulates the equations governing the motion of a basketball, and models the shooter as a probabilistic input to the system, resulting in shooter's probability of making a given shot. Although their work sheds light on positional evaluation of shots, it does not give a total view on all field shots. This study aims to accomplish that, though analytically instead of statistically. This approach further inspects maximal sphere fittings à la maximal rectangles of Seppala-Holtzman (2012).

METHODS: A computer model for basketball throws have been programmed in C++ and evaluated by solving the differential equations numerically via an ode solver function of Boost (v. 1.63.0, Ahnert and Mulansky (2011)) odeint library (namely Dormand-Prince algorithm; 'runge_kutta_dopri5' based on Dormand and Prince (1980)). The model detects through the stepping solver when major events (e.g. backboard or rim collision, successful throw, ground hit) occur and act accordingly.
The program simulates throws for various velocity and position values. Considering the field as a grid of 0.1m x 0.1m squares, the simulation has been run for all the grid intersections in lateral half of the field, since the rim standing in the center of the field laterally allows us to make use of the symmetry of the field. All throws are assumed to be from a height of 2m.

The field used in our model follows FIBA (2014), short of backboard-rim bridge, which is not accounted for in the simulations. Also positions with distance to rim center less than 1m, as well as any position at the back of the rim line are not taken into account.
The method employed is inspired by Freitas (2014), and improves upon that by modelling in 3D, hence generalizing the idea of surface area to a volume. At a given position in the field, we consider the velocities those which result in a scoring throw. Letting these velocity vectors define a 3D solid, the volume of this solid would be an indicator of error tolerance for that position to velocity variations, i.e. player error. Since the exact volume is irrelevant to our purposes, we simply count the number of scoring throws. An example of such a solid constructed from velocities can be seen at Figure 1.

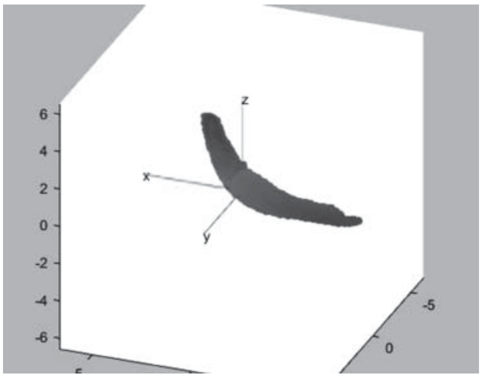
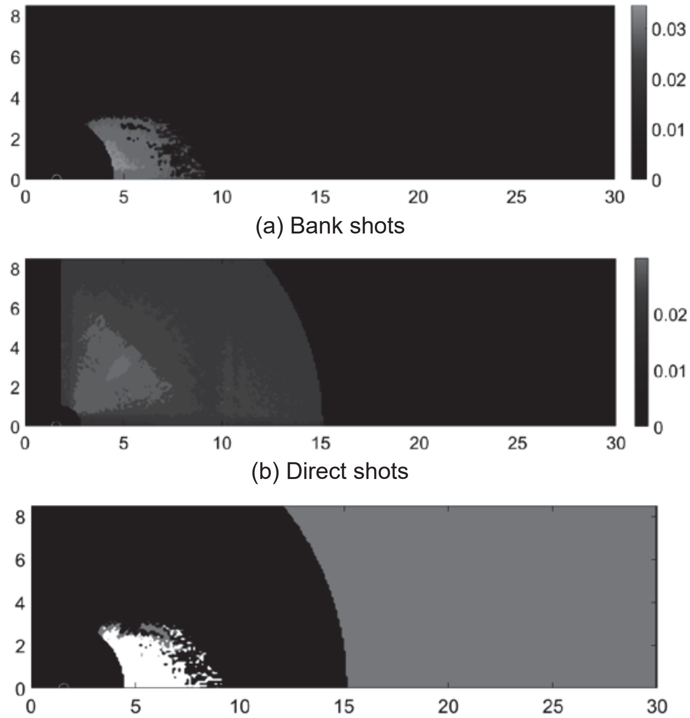


Figure 1. An example of a solid constructed from velocities of successful shots from a single point. Each voxel corresponds to a velocity of a scoring throw. Together, those voxels constitute a solid as shown which we consider its volume as a measure of error tolerance for that position. z axis corresponds to the vertical component of the velocity, whereas y and x axes correspond to components along field width and length, respectively.

Simulations consisted of two sets; one for bank shots, and the other for direct shots. For bank shots, the field was scanned for successful throws colliding with backboard first. For direct shots, the field was scanned for successful throws either passing through the center of the rim, or colliding with the rim first. We then expand the solid at each position by changing these velocities (and by iteration any resulting scoring velocity) of initial successful shots by 0.1m/s in each direction and simulating them.
Our model takes air friction and both rim and backboard bounces into account, but does not account for the ball slipping at the rim nor backspin. When the ball collides with rim or backboard, it immediately bounces back losing its speed but without any compression. After running the simulation until there are no more successful throws with less than 15m/s speed we pass onto analysing them.
Method used so far is the same as in Yaylıoğlu and Arıtan (2016). Improving that, after finding these solids we fit maximal spheres into solids. The outer shell consisting of failed shots was found, and spheres were fit into those shells. We then consider radii of those maximal spheres as a measure of error tolerance, letting a more objective evaluation than simply measuring volumes.

RESULTS: Figure 2 shows the result of simulations. Note in the top two contour graphs that solids formed at further positions were found to have 0 volume. Although they all had at least one successful shot, they did not have enough successful shots (at least 27 –another simplification) to contain a solid in them.



(a) Bank shots

(b) Direct shots

(c) Preference map

Figure 2. Top two images summarize the radii of maximal spheres for successful bank shots (a) and direct shots (b) throughout the field. In these graphs, legend values stand for maximal sphere radius, white half-circle centered at (1.58,0,3.05) denotes the rim and tick marks stand for distance from lateral center and base line. Because of the symmetry of throws, left side (w.r.t. player facing the backboard) is omitted. (c) summarize optimal preference of shot type: White region is where bank shots are preferable, black region is where direct shots are, and grey region is where they are equivalent.

DISCUSSION:

Simulations had some simplifications as mentioned, yet these physics-based simulations are still believed to be fairly close to the reality. Even so, a validation study would be beneficial.

The algorithm assumes if two shots are successful, then any shot between them in a linear fashion is also successful. Further, another important assumption is that any bank or direct shot is a neighbour of another one and they all reside in a single solid.

Silverberg (2013) argues that bank shot can be extremely advantageous over the direct shot, and this study opens a parenthesis to pinpoint the locations for that advantage. Direct shots are regularly applicable overall, yet a bank shot is more preferable than a direct shot at some specific locations, namely close to the rim –but not too close- as shown in Figure 2(a).

Our naïve maximal sphere fitting approach, inspired by the study of Seppala-Holtzman (2012), obviously does not take the nature of player errors into account. That is, a basketball player need not have the same error window in all three axes and this might be improved with fitting player, position and velocity specific ellipsoids instead of spheres. In that sense, this study did not take into account arguably the most important factor, the player which is left for future studies.

CONCLUSION: This study evaluated the basketball field by fitting maximal spheres into volumes obtained from velocities of successful shots that can be taken from each point, both for bank shots and direct shots. By comparing said shots, we have found that both bank shots and direct shots have their preferable positions when it comes to scoring. A player who is aware of the error tolerance of their throws throughout the filed may alter their throw mechanics accordingly, and such a coach may position the players better by player-specific simulations, and even change the game strategy according to this knowledge, especially when further information on both their and opposing team players' capabilities are available.

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