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A Novel Method to Factor Cubic Polynomials

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A Novel Method to Factor Cubic Polynomials: The *ad*-Method

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My colleague and new faculty member, Daniel Rowe, observed me as I taught an intermediate algebra class about the *ac*-method of factoring quadratics. Published in 1979, by Autrey and Austin, the *ac*-method has gained popularity in textbooks and mathematics instruction. Having never seen this method, Rowe was fascinated, which lead to his own investigation of the *ac*-method. He shared his proof and the idea of extending this process to higher-degree polynomials. Our explorations have led to a new method for factoring cubic polynomials, which we will call the *ad*-method. There are several widespread methods for factoring polynomials. However, in practice, these methods do not generalize to factoring higher-degree polynomials. Often, the rational roots theorem is employed to determine whether a polynomial has any rational roots, and then polynomial division is used. The division algorithm however, has very little connection to a student's prior learning, and is not structurally analogous to the *ac*-method of factoring quadratics.

Our method of factoring cubic polynomials, the *ad*-method, uses the ideas from the *ac*-method as scaffolding and, as a result, may lead to a deeper understanding of factoring polynomials (see Sidney & Alibali, 2015). This allows for premier teaching opportunities to introduce new concepts and knowledge to students with firmly established or growing algebraic foundations. Using the *ad*-method to teach factoring of cubic polynomials has turned out to be some of the most satisfying and exciting days in my teaching career.

To demonstrate this satisfaction and excitement, an example is now provided that introduces the *ad*-method. Afterwards, a proof and a series of more intricate examples are presented to illuminate the subtleties of the *ad*-method.

Introductory example

Factor $p(x) = 2x^3 + 9x^2 + 10x + 3.$

Like the *ac*-method, we can decompose the middle terms, except we will decompose each middle term into three summands instead of two:

$$\begin{aligned} 9x^2 &= x^2 + 2x^2 + 6x^2 \\ 10x &= 6x + 3x + x. \end{aligned}$$

Later, we will discuss the origin of these summands. Next, we rearrange the terms and factor by grouping. For clarity, the groups are color coded:

$$\begin{aligned} p(x) &= 2x^3 + x^2 + 2x^2 + 6x^2 + 6x + 3x + x + 3 \\ &= 2x^3 + x^2 + 2x^2 + x + 6x^2 + 3x + 6x + 3 \\ &= x^2(2x+1) + x(2x+1) + 3x(2x+1) + 3(2x+1) \\ &= (2x+1)(x^2 + x + 3x + 3). \end{aligned}$$

Notice that once we factor out the common linear term, the remaining quadratic polynomial is ready to be factored by grouping:

$$\begin{aligned} p(x) &= (2x+1)\left(x^2 + x + 3x + 3\right) \\ &= (2x+1)\left(x(x+1) + 3(x+1)\right) \\ &= (2x+1)(x+1)(x+3). \end{aligned}$$

In the previous example, we merely demonstrated how similar it is to the *ac*-method for quadratics; however, to fully comprehend the choices for the summands, we proceed to the proof.

Proof of the *ad*-method

Let $ax^3 + bx^2 + cx + d$ be a cubic polynomial with integer coefficients, where $\gcd(a,b,c,d) = 1$. Suppose that it has a factorization into three linear terms $(Ax+B)(Cx+D)(Ex+F)$ where $A, B, C, D, E,$ and F are integers. We have the following expression of the polynomial:

$$\begin{aligned} &ACEx^3 + (ACF + ADE + BCE)x^2 \\ &\quad (ADF + BCF + BDE)x + BDF. \end{aligned}$$

Consider the two triples of integers:

$$\begin{aligned} \left(X_1, X_2, X_3\right) &:= (ACF, ADE, BCE) \\ \left(Y_1, Y_2, Y_3\right) &:= (BDE, BCF, ADF). \end{aligned}$$

It is important to note that the pairs $\left(X_1, Y_1\right)$, $\left(X_3, Y_3\right)$ are *dual divisors* of each other relative to the number *ad*. In other words, $X_1 Y_1 = ad$, $X_2 Y_2 = ad$, and $X_3 Y_3 = ad$.

Additionally, notice the following equations:

$$X_1 + X_2 + X_3 = b$$
$$X_1 X_2 X_3 = a^2 d$$

$$Y_1 + Y_2 + Y_3 = c$$
$$Y_1 Y_2 Y_3 = ad^2.$$

The last equation, $Y_1Y_2Y_3 = ad^2$, in fact follows from the second, $X_1X_2X_3 = a^2d$. This is because

$$Y_1Y_2Y_3 = \frac{ad}{X_1}\cdot\frac{ad}{X_2}\cdot\frac{ad}{X_3} = \frac{(ad)^3}{a^2d} = ad^2.$$

Conversely, the collections $\{X_1, X_2, X_3\}$ and $\{Y_1, Y_2, Y_3\}$ are uniquely determined by the previous conditions.

Example 1

Factor $p(x) = 2x^3 + 9x^2 + 10x + 3.$

Recall that our goal is to factor by grouping, analogously to the *ac*-method. Our *ad*-method requires us to find a solution to the following criteria: $b = 9$ must decompose into three summands (that are factors of $ad = 6$), whose product is $a^2d = 12$, and simultaneously, the dual divisors of those terms (relative to $ad = 6$) must sum to $c = 10$ and multiply to $ad^2 = 18$.

Note that it is the factors of *ad* that will drive the work to be done, analogously to the *ac*-method. For clarity, we write out the pertinent items:

$$ad = 6 = 2\cdot 3$$
$$a^2d = 12 = 2^2\cdot 3$$
$$ad^2 = 18 = 2\cdot 3^2.$$

We seek three factors (X_1, X_2, X_3) of $ad = 6$ such that $X_1 + X_2 + X_3 = b = 9$ and $X_1X_2X_3 = a^2d = 12$. We also seek three factors (Y_1, Y_2, Y_3) of $ad = 6$ such that $Y_1 + Y_2 + Y_3 = c = 10$ and $Y_1Y_2Y_3 = a^2d = 18$.

While it is true that we seek solutions to both pairs of previous equations, we search for a solution to only one of the pairs, and then verify that the other pair of equations is satisfied. It is better to work with the first equation if a^2d has fewer prime factors (counting multiples) than

ad^2. It is better to work with the second equation if ad^2 has fewer prime factors (counting multiples) than a^2d. In this case, they each have three prime factors (counting multiples), and so we will work with the first set of criteria, since 12 is smaller than 18.

To find the numbers X_1, X_2, and X_3 systematically, we start by listing all factors of $ad = 6 : \{6, 3, 2, 1\}$. These are the only numbers (positive or negative) we can use in determining the values of X_1, X_2, and X_3.

We check the factors in descending order, as the larger factors are often ruled out quickly. Creating a table helps organize our systematic approach (see Table 1).

The first column lists the factor of *ad* we are considering. The second column writes the factor in terms of its primes to help with visualizing the process. The third column lists the remaining primes of a^2d, since we must use all of them when creating our triple (X_1, X_2, X_3). The fourth column lists the possible triples that multiply to a^2d. It is important to remember that the elements of the triple must be factors of *ad*. After a possible triple has been listed once, it is not necessary to list it again in lower rows of the table. In the last column, we consider whether or not the three numbers (possibly using negatives) can sum to *b*.

In this example, we happen to obtain a viable triple in the first row, but we give the complete table to demonstrate the process.

[Insert Table 1 here, "Table 1" in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 1

Example 1

Factor of $ad = 6$	**Uses from** a^2d	**Leaves from** a^2d	**Possible triples** (*each a divisor of* $ad = 6$)	**Obtains** $b = 9$?
6	$2 \cdot 3$	2	$(6, 2, 1)$	$6 + 2 + 1 = 9$✓
3	3	2^2	$(3, 2, 2)$	no
2	2	$2 \cdot 3$	$(2, 3, 2)$, $(2, 6, 1)$	considered earlier
1	1	$2^2 \cdot 3$	$(1, 6, 2)$	considered earlier

Here is what we have so far: We have a triple $(6, 2, 1) = (X_1, X_2, X_3)$ with the property that $X_1 + X_2 + X_3 = 9$ and simultaneously $X_1 X_2 X_3 = a^2d = 12$.

We need to verify that the dual divisors relative to $ad = 6$ sum to $c = 10$. The dual divisors are

$$\left(Y_1, Y_2, Y_3\right) = \left(\frac{ad}{X_1}, \frac{ad}{X_2}, \frac{ad}{X_3}\right) = (1, 3, 6).$$

Indeed, they satisfy the equation $Y_1 + Y_2 + Y_3 = c = 10$. In the proof, we saw that the equation $Y_1 Y_2 Y_3 = ad^2 = 18$ follows automatically from $X_1 X_2 X_3 = a^2 d$.

We proceed by decomposing bx^2 and cx in order to factor by grouping. Let $9x^2 = 6x^2 + 2x^2 + 1x^2$ and $10x = 1x + 3x + 6x$; it does not matter in what order we choose to write the summands:

$$p(x) = 2x^3 + 6x^2 + 2x^2 + x^2 + x + 3x + 6x + 3.$$

Now pair the first two terms and factor the greatest common factor:

$$p(x) = 2x^2(x+3) + 2x^2 + x^2 + x + 3x + 6x + 3.$$

Now take the next term $\left(2x^2\right)$ and strategically pair it with another term $(6x)$ to ensure that the greatest common factor is $(x+3)$. Afterwards, continue this process of strategically pairing terms to ensure an $(x+3)$ greatest common factor:

$$\begin{aligned} p(x) &= 2x^2(x+3) + 2x(x+3) + x^2 + x + 3x + 3 \\ &= 2x^2(x+3) + 2x(x+3) + x(x+3) + x + 3 \\ &= 2x^2(x+3) + 2x(x+3) + x(x+3) + 1(x+3). \end{aligned}$$

Factor out the binomial $(x+3)$ and what remains is a quadratic polynomial that is ready to be factored by grouping again:

$$\begin{aligned} p(x) &= (x+3)\left(2x^2 + 2x + x + 1\right) \\ &= (x+3)\left(2x(x+1) + x + 1\right) \\ &= (x+3)\left(2x(x+1) + 1(x+1)\right) \\ &= (x+3)(x+1)(2x+1). \end{aligned}$$

This is the final factored form of $2x^3 + 9x^2 + 10x + 3$. This method is structurally similar to the *ac*-method for factoring quadratics. It removes all of the guess-and-check process and connects prior learning of the *ac*-method to a new concept of factoring cubic polynomials.

A second example will show how well this method works for cubic polynomials with larger coefficients.

Example 2
Factor $p(x) = 9x^3 + 39x^2 + 10x - 8.$

As before, we start by identifying the pertinent items:

$$\begin{aligned} ad &= (9)(-8) = (-1)\cdot 2^3 \cdot 3^2 \\ a^2 d &= (9)^2(-8) = (-1)\cdot 2^3 \cdot 3^4 \\ ad^2 &= (9)(-8)^2 = 2^6 \cdot 3^2. \end{aligned}$$

Since $a^2 d$ has fewer prime factors (counting multiples) than ad^2, we will systematically search for three factors of ad that multiply to $a^2 d$ and sum to $b = 39.$

[Insert Table 2 here, "Table 2" in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 2

Example 2

Factor of ad	Uses from $a^2 d$	Leaves from $a^2 d$	Possible triples	Obtains $b = 39$?
72	$2^3 \cdot 3^2$	3^2	$(72,9,1),(72,3,3)$	no
36	$2^2 \cdot 3^2$	$2 \cdot 3^2$	$(36,18,1),(36,9,2),$ $(36,6,3)$	$36+6-3=39$ ✓

We now have a possible triple $(X_1, X_2, X_3) = (36, 6, -3)$ with the property that $X_1 + X_2 + X_3 = b = 39$ and, simultaneously, $X_1 X_2 X_3 = a^2 d.$

We need to verify that the dual divisors relative to $ad = -72$ add up to $c = 10.$ The dual divisors are

$$(Y_1, Y_2, Y_3) = \left(\frac{ad}{X_1}, \frac{ad}{X_2}, \frac{ad}{X_3}\right) = (-2, -12, 24).$$

Indeed, they satisfy the property $Y_1 + Y_2 + Y_3 = c = 10.$ We proceed by decomposing $39x^2$ and $10x$

and then factor by grouping:

$$\begin{aligned}
p(x) &= 9x^3 + 39x^2 + 10x - 8 \\
&= 9x^3 + 36x^2 + 6x^2 - 3x^2 - 2x - 12x + 24x - 8 \\
&= 9x^2(x+4) + 6x^2 + 24x - 3x^2 - 12x - 2x - 8 \\
&= 9x^2(x+4) + 6x(x+4) - 3x(x+4) - 2(x+4) \\
&= (x+4)(9x^2 + 6x - 3x - 2) \\
&= (x+4)(3x(3x+2) - (3x+2)) \\
&= (x+4)(3x+2)(3x-1).
\end{aligned}$$

Example 3

Factor $4x^3 - 21x + 10.$

In this situation, we have a zero coefficient: $4x^3 + 0x^2 - 21x + 10.$ However, the *ad*-method can still be applied by taking $b = 0$:

$$\begin{aligned}
ad &= (4)(10) = 2^3 \cdot 5 \\
a^2d &= (4)^2(10) = 2^5 \cdot 5 \\
ad^2 &= (4)(10)^2 = 2^4 \cdot 5^2.
\end{aligned}$$

We follow our systematic procedure by completing Table 3.

[Insert Table 3 here, "Table 3" in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 3

Example 3

Factor of ad	**Uses from** a^2d	**Leaves from** a^2d	**Possible triples**	**Obtains** $b=0$?
40	$2^3 \cdot 5$	2^2	$(40,4,1),(40,2,2)$	no
20	$2^2 \cdot 5$	2^3	$(20,8,1),(20,4,2)$	no
10	$2 \cdot 5$	2^4	$(10,8,2),(10,4,4)$	$10-8-2=0$ ✓

We have found a viable triple $(X_1, X_2, X_3) = (10, -8, -2)$. As before, the dual equation:

$$\frac{ad}{X_1}+\frac{ad}{X_2}+\frac{ad}{X_3}=c$$

also needs to be verified. In this case it works out nicely:

$$4-5-20=-21.$$

To finish applying our method, we decompose the bx^2 and the cx terms and then factor by grouping:

$$\begin{aligned}
p(x) &= 4x^3+0\cdot x^2-21x+10 \\
&= 4x^3+10x^2-8x^2-2x^2+4x-5x-20x+10 \\
&= 2x^2(2x+5)-x(2x+5)-4x(2x+5)+2(2x+5) \\
&= (2x+5)\left(2x^2-x-4x+2\right) \\
&= (2x+5)\left(x(2x-1)-2(2x-1)\right) \\
&= (2x+5)(2x-1)(x-2).
\end{aligned}$$

In all of the previous examples, the initial factors we found satisfied the dual divisor criteria. However, in the next example, we demonstrate a case where this does not hold.

Example 4

Factor $24x^3-22x^2-5x+6.$

Here, we will explore the situation where we find a triple that satisfies the criteria, but the dual divisors of the triple do not. In this case, the triple must be discarded, and the process simply continues.

$$\begin{aligned}
ad &= (24)(6)=2^4\cdot 3^2 \\
a^2d &= (24)^2(6)=2^7\cdot 3^3 \\
ad^2 &= (24)(6)^2=2^5\cdot 3^3
\end{aligned}$$

Since ad^2 has fewer prime factors (counting multiples) than a^2d, we will search for three factors of $ad=144$ that multiply to ad^2 and sum to $c=-5.$

[Insert Table 4 here, “Table 4” in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 4

Example 4

Factor of ad	**Uses from** ad^2	**Leaves from** ad^2	**Possible triples**	**Obtains** $c=-5$?
144	$2^4\cdot 3^2$	$2\cdot 3$	$(144,6,1),(144,3,2)$	no
72	$2^3\cdot 3^2$	$2^2\cdot 3$	$(72,12,1),(72,6,2)$ $(72,4,3)$	no
48	$2^4\cdot 3$	$2\cdot 3^2$	$(48,18,1),(48,9,2)$ $(48,6,3)$	no
36	$2^2\cdot 3^2$	$2^3\cdot 3$	$(36,24,1),(36,12,2)$ $(36,8,3),(36,6,4)$	no
24	$2^3\cdot 3$	$2^2\cdot 3^2$	$(24,18,2),(24,12,3)$ $(24,9,4),(24,6,6)$	no
18	$2\cdot 3^2$	$2^4\cdot 3$	$(18,16,3),(18,12,4)$ $(18,8,6)$	$-18+16-3=-5$
9	3^2	$2^5\cdot 3$	$(9,16,6),(9,12,8)$	$-9+12-8=-5$ ✓

Notice that although $(-18,16,-3)$ sums to $c=-5,$ the dual divisors of $(-18,16,-3),$ relative to $ad=144,$ are $(-8,9,-48),$ and they do not sum to $b=-22.$ In this situation, we simply discard the triple and continue the process.

We find a viable triple, $(-9,12,-8),$ that sums to $c=-5,$ and then we verify that the dual divisors of $(-9,12,-8),$ relative to $ad=144,$ are $(-16,12,-18),$ and they satisfy $-16+12-18=-22=b,$ as required.

Now we decompose $-22x^2$ and $-5x$ and factor by grouping:

$$\begin{aligned}
p(x) &= 24x^3-22x^2-5x+6\\
&= 24x^3-16x^2+12x^2-18x^2-9x+12x-8x+6\\
&= 8x^2\left(3x-2\right)+12x^2-8x-18x^2+12x-9x+6\\
&= 8x^2\left(3x-2\right)+4x\left(3x-2\right)-6x\left(3x-2\right)-3\left(3x-2\right)\\
&= \left(3x-2\right)\left(8x^2+4x-6x-3\right)\\
&= \left(3x-2\right)\left(4x(2x+1)-3(2x+1)\right)\\
&= \left(3x-2\right)\left(2x+1\right)\left(4x-3\right).
\end{aligned}$$

For our last example, we explore how the method terminates without a solution in two cases where the cubic polynomial does not factor completely into three, linear factors.

Example 5

Factor $p(x)=3x^3+14x^2+12x+6$ and $q(x)=3x^3+8x^2+10x+4$.

In this example, $p(x)$ and $q(x)$ do not factor completely. For both of these polynomials, the *ad*-method terminates without satisfying the required criteria, but for different reasons. We first note that the polynomial $p(x)$ does not factor over the integers; that is, it is prime, while the polynomial $q(x)$ splits into a linear factor and a prime quadratic.

First, we apply the *ad*-method to $p(x)=3x^3+14x^2+12x+6$:

$$\begin{aligned} ad &= (3)(6) = 2\cdot 3^2 \\ a^2d &= (3)^2(6) = 2\cdot 3^3 \\ ad^2 &= (3)(6)^2 = 2^2\cdot 3^3. \end{aligned}$$

We fill in Table 5 using our systematic procedure. Notice that when a row contains a number in column 2 that matches a previous number in column 3, all subsequent rows will contain repeats of previous triples.

[Insert Table 5 here, "Table 5" in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 5

Example 5

Factor of *ad*	**Uses from** a^2d	**Leaves from** a^2d	**Possible triples**	**Obtains** $b=14$?
18	$2\cdot 3^2$	3	$(18,3,1)$	$18-3-1=14$
9	3^2	$2\cdot 3$	$(9,6,1),(9,3,2)$	$9+3+2=14$
6	$2\cdot 3$	3^2	$(6,3,3)$	no

The triples $(18,-3,-1)$ and $(9,3,2)$ are unacceptable. Although they sum to $b=14,$ the dual divisors relative to $ad=18$ are the triples $(1,-6,-18)$ and $(2,6,9),$ and neither of these sum to $c=12.$

At this point, we know that we have exhausted all possible triples, and therefore conclude that the polynomial $p(x)=3x^3+14x^2+12x+6$ does not factor into three linear terms. If we wanted to verify whether $p(x)$ was prime, or if it had any linear factors, the rational roots theorem could be used.

Finally, we apply the *ad*-method to $q(x)=3x^3+8x^2+10x+4$ with

$$\begin{aligned} ad &= (3)(4) = 2^2 \cdot 3 \\ a^2d &= (3)^2(4) = 2^2 \cdot 3^2 \\ ad^2 &= (3)(4)^2 = 2^4 \cdot 3. \end{aligned}$$

We begin our systematic procedure.

[Insert Table 6 here, "Table 6" in bold TNR, centered with no period, next line with title in italics TNR, centered with no period.]

Table 6

Example 6

Factor of ad	**Uses from** a^2d	**Leaves from** a^2d	**Possible triples**	**Obtains** $b=8$?
12	$2^2 \cdot 3$	3	$(12,3,1)$	$12-3-1=8$
6	$2 \cdot 3$	$2 \cdot 3$	$(6,6,1),(6,3,2)$	no
4	2^2	3^2	$(4,3,3)$	no

The dual divisors of the triple $(12,-3,-1)$ relative to $ad=12$ are $(1,-4,-12)$, and they do not sum to $c=10$. This exhausts all of the possible triples, and so we can conclude that $q(x)$ does not factor into three linear factors. If we wanted to go further, to verify whether $q(x)$ was prime, we could use the rational roots theorem to find a root at $x=-2/3$, which leads to a factor of $3x+2$. Polynomial division then gives the expected factorization

$$q(x)=(3x+2)\left(x^2+2x+2\right).$$

Teaching the *ad*-Method

After our collegial exploration of the *ad*-method, we decided to present it to intermediate algebra students. This extension is natural and ties together the concepts of intermediate algebra with concepts traditionally taught in the next course. The lesson was prepared and presented with the help of undergraduate teaching assistants, who are preservice teachers. Previous extension projects used the Rip Van Winkle character, so we continued to use him. (See Appendix A for the handout that we created at http://c.ymcdn.com/sites/www.amatyc.org/resource/resmgr/educator_sept_2017/Barnsleyappen.pdf.) At this point, most of the students were competent with the *ac*-method. Students worked in small groups of three to five, while the teaching assistant and instructor circulated. Students were not given the background or rational for this method; they were guided through the process much like our introductory example. The students were overwhelmingly successful.

Teaching concepts beyond the minimal competency has many benefits. Challenging problems, such as factoring cubic polynomials, engage all students. The advanced students are ripe for the challenge and understand some of the deeper subtleties of the process, while less proficient students gain confidence with the success of a more difficult concept. Student-opinion surveys at the end of the semester mentioned this as one of the most fun lessons of the semester. The teaching assistants were so enthused about this project that they created a set of sample problems with full solutions (see Appendices B and C).

Rewards of Teaching of the *ad*-Method

As with the *ac*-method, the order of the decomposition of middle terms does not matter. When factoring by grouping, we can immediately pick the first two terms, factor out the greatest common factor, and then scan the remaining terms for the one to partner with the third term so that the remaining binomial matches the first binomial, and so forth. This is the same process we use for the *ac*-method. More advanced students pick up on this subtlety. The *ad*-method is an advanced topic that provides exercise on skills needed for competency on the *ac*-method. When students are exploring the possible triples that sum to either b or c, they practice seeing the terms as either positive or negative.

Initially, students benefit from seeing every possible signed combination written out, but as they mature mathematically, they are able to see triples without specific signs and determine if the numbers sum to the required integer. Teaching factoring of cubic polynomials by the *ad*-method provides a concrete connection to prior knowledge, provides practice with skills that are required for intermediate algebra, and builds confidence by having success with really difficult problems.

References

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Daniel Rowe (darowe@nmu.edu) is an assistant professor of mathematics at Northern Michigan University. He grew up on a fishing resort in Northwestern Ontario, Canada. He earned a BMath at the University of Waterloo, and he went on to earn an MS and PhD in mathematics at the University of Toronto. He currently lives in Marquette Michigan.

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