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Coding Theory and Democracy - Part II

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LDPC Codes of Arbitrary Girth

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Abstract

Low density parity check codes of high girth have been known to exhibit better error correcting capabilities than codes of low girth. Moreover, under the sum-product algorithm, high girth also implies faster decoding. In this poster we present a method called *Successive Level Growth* (SLG) for the construction of LDPC codes of arbitrary girth. Our simulations results show that the SLG codes exhibit significant coding gain over randomly constructed LDPC codes of the same length and dimension.

An Observation

Suppose we are given a graph of girth g. If an edge connecting two vertices is removed, then the breadth-first searches starting from the two vertices would not have any nodes in common after $g/2 - 1$ levels of each search have been performed. Thus we can compute a lower bound on the length of a (d_s, d_c)-regular LDPC code of girth g.

$$
\begin{array}{c}
\begin{array}{cccccccccccccccc}
S_0^0 & S_1^0 & S_1^1 & S_2^1 & S_0^2 & S_1^2 & S_2^2 & S_0^3 & S_1^3 & S_2^3 & S_3^3 & S_4^3 & S_5^3 & S_6^3 & S_7^3 & S_8^3
\end{array} \\
\left[\begin{array}{cccccccccccccccc}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}\right]
\begin{array}{c}
C_0^0 \\ C_0^1 \\ C_0^2 \\ C_1^2 \\ C_2^2 \\ C_0^3 \\ C_1^3 \\ C_2^3
\end{array}
\end{array}
$$

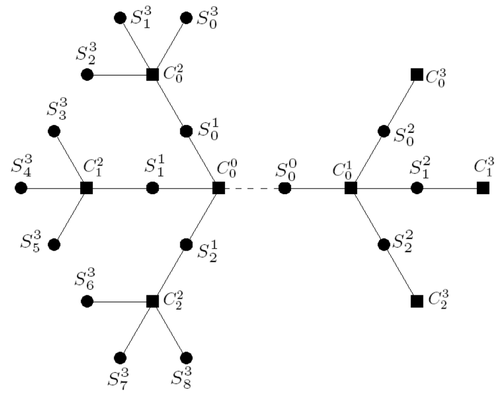


Fig. 1. If $H[0,0] = 0$, $BFS(C_0^0)$ and $BFS(S_0^0)$ would not share any common nodes after 3 levels of each search have been performed.

Lower Bound on the Code Length

Algorithm 1 Compute the minimum possible length for a (d_s, d_c)-regular LDPC code of girth g

$min\_length = 1$; $add = 1$;
for $t = 1$ to $\frac{g}{2}$ **do**
 if $(t \bmod 2) = 0$ **then**
  $add = add \cdot (d_c - 1)$
 else
  $add = add \cdot (d_v - 1)$
 end if
 $min\_length = min\_length + add$
end for
return min_length

Successive Level Growth

Parameters: Girth g, Symbol node degree d_s, check node degree d_c, maximum length $max\_length > min\_length$

Start with a single check node, single symbol node and an edge connecting them.

Expand the graph in a level-by-level fashion

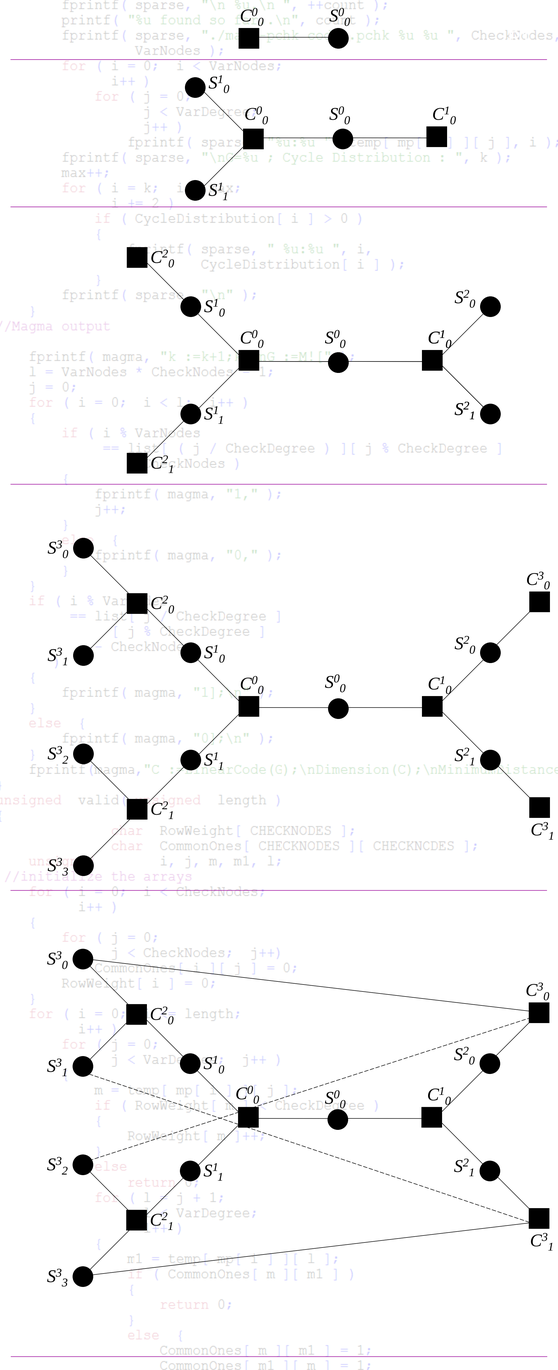
When a suitable level is reached, add edges so that no cycles of length less than g are formed.

The process ends when it is impossible to add a new level.

General SLG Algorithm

Algorithm 1
if$(max\_length \geq min\_length)$ **then**
 $V_s = \{s_0^0\}$; $V_c = \{c_0^0\}$; $E = \{(s_0^0, c_0^0)\}$;
 $level = 0$;
 do
  Add New Level
  $flag = true$;
  if (level selection criterion is satisfied) **then**
   Add Appropriate Edges
   Output Intermediate Graph
   $flag = false$;
  end if
 while (a new level can be added)
 if $(flag)$ **then**
  Add Appropriate Edges
  Output Final Graph
 end if
end if

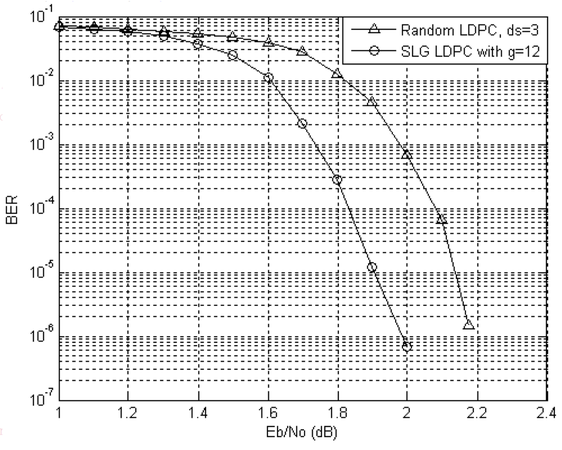
SLG Illustration: $g=6$, $d_s=2$, $d_c=3$



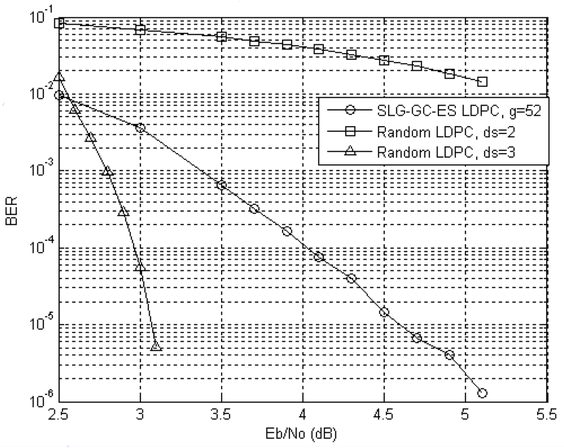
Simulation Results

Results of AWGN channel simulations are shown below. We adopt the rate-adjusted $SNR_1 = 10Log_{10}[E_b / (2R\sigma^2)]$, where $SNR_2 = 10Log_{10}[E_b / (2\sigma^2)]$ is the usual signal-to-noise ratio.

Input			Output					
g	d_s	d_c	g	Avg. d_s	Avg. d_c	V_s	V_c	Rate
12	3	8	12	2.83	7.54	11112	4167	0.625



Input			Output					
g	d_s	d_c	g	Avg. d_s	Avg. d_c	V_s	V_c	Rate
16	3	4	16	2.94	3.92	9312	6984	0.25



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