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Classical Transitions for Tunable Potentials

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Classical Transitions for Tunable Potentials

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Abstract

The conservation of energy is a simple but versatile law that governs almost all observations and occurrences. This interplay between a system’s kinetic and potential (and other) energies can describe something as simple as a ball rolling down a hill, but it can also explain something as staggering as the expansion of the Universe. In the most fundamental sense, the expansion of the Universe is driven by its energy density.

During a first-order phase transition, the state with the lowest potential can be reached by transitioning from a system’s less ideal metastable states. In order to achieve this change, bubbles of lower potential states nucleate within a metastable state and then collide with each other. By colliding, the bubbles produce enough kinetic energy to overcome formerly impenetrable potential barriers. This process allows transitions to a more stable state in a non-classical manner. Collisions like these might have occurred at very early times in the history of the Universe helping us to understand cosmic inflation. We study this model and show under what circumstances these transitions take place.

Forming the Bubble

A bubble is a region of space at a lower potential that has formed in a bulk space of higher potential during a first-order phase transition. Because a potential barrier sits between two stable energy states, a bubble has to be formed through quantum fluctuations. Once the bubble is created, its favorable potential pulls the space around it to the potential of the bubble. In effect, the bubble begins expanding. The expansion of the bubble is governed by the Euclidean Equation of Motion[1]:

$$\ddot{\phi}(r,\tau) + \nabla^2 = -\frac{dV}{d[\phi(r,\tau)]}$$

We can make this equation invariant in time and space using the parameter, $\rho = \sqrt{\tau^2 + r^2}$:

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = \frac{dV}{d\phi}$$

Using the Euclidean Equation of Motion, the bubble can be calculated at all times. Like the behavior of the bubble during its expansion, the initial profile of the bubble provides information and important characteristics that should also be considered. The bubble profile is given by[2]:

$$x = \int_0^{\phi_0} \frac{d\phi}{\sqrt{2V(\phi)}}$$

With the equations to describe the motion and profile of the bubble, we can see its characteristics and observe how it will evolve.

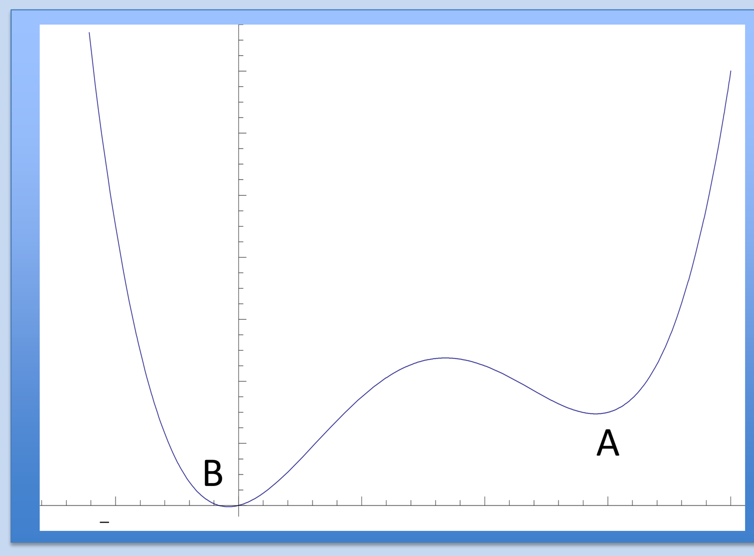

Fig 1: Two-minima potential. (A is metastable)

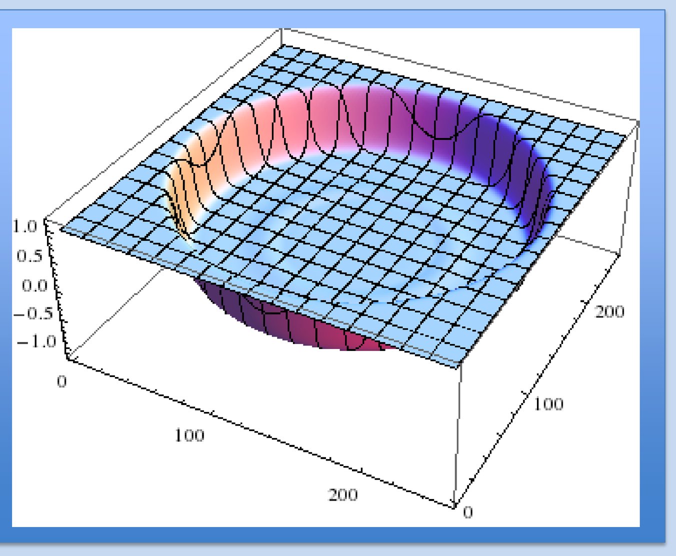

Fig 2: Two dimensional bubble

Three-Minima Model

Normally, a phase transition is limited to the formation of bubbles, a process reliant on quantum fluctuations. In a multiple minima potential, it is possible that two expanding bubbles collide with enough kinetic energy to classically transition over a potential barrier(see Fig 3)[3]. The most fundamental of the potentials allowing this type of classical transition is a potential with three minima: with two metastable states and one stable state. By changing the parameters of a model it is possible to study the limits of the classical transition, such as the minimum distance between nucleated bubbles. The specific three minima model studied was given by:

$$V(\phi) = \frac{\lambda}{4}(\phi - \phi_0)^2(\phi + \eta\phi_0)^2 + \epsilon\phi_0^5(\phi - \phi_0)$$

(Fig. 4)

From this model, the parameter η ,which defined the distance between the two lowest minima of the potential, was the most intently studied. In addition to η, the starting radius of the bubble at nucleation proved to be important as it regulates how much kinetic energy a bubble collision will have. By varying the values of these two parameters and testing their effect on the classical transition, the boundary between transitioning and non-transitioning potentials could be found.

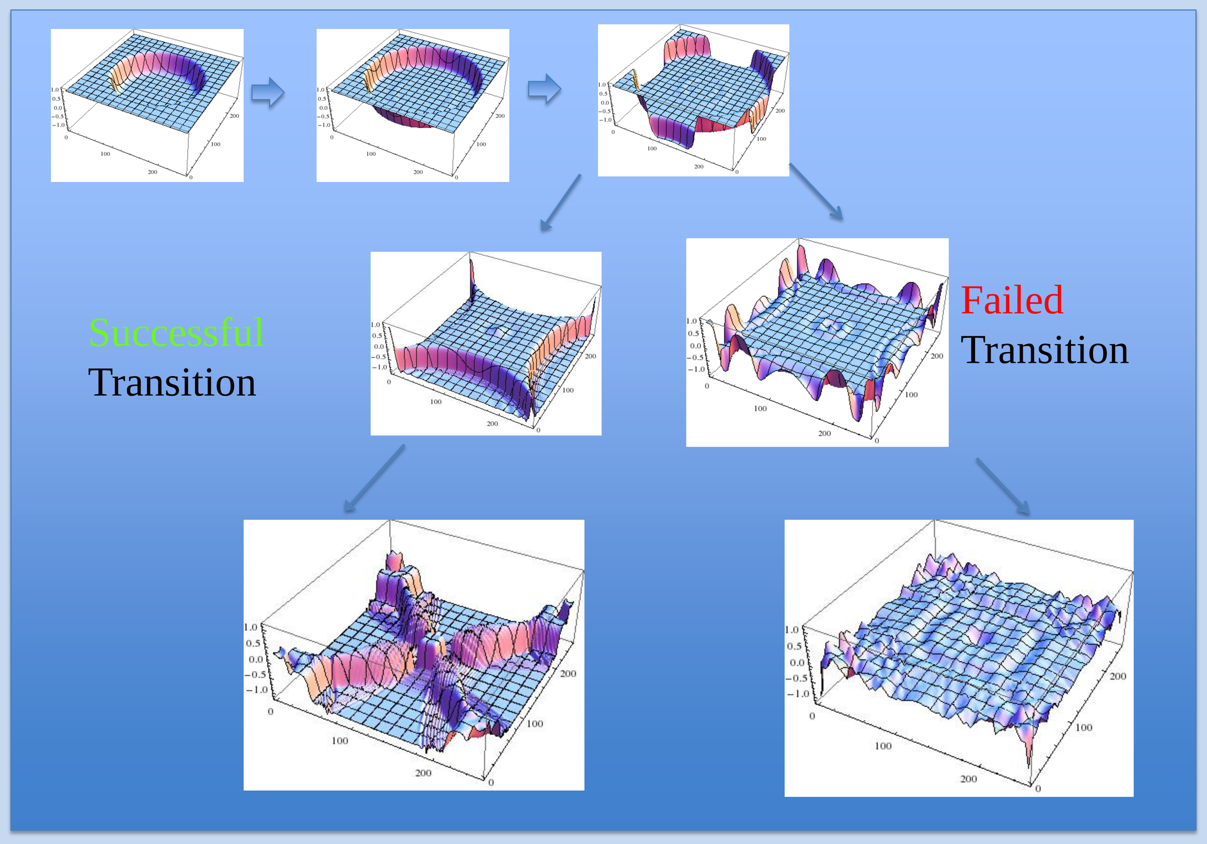

Fig 3: Evolution of Bubble showing both a successful and failed transition

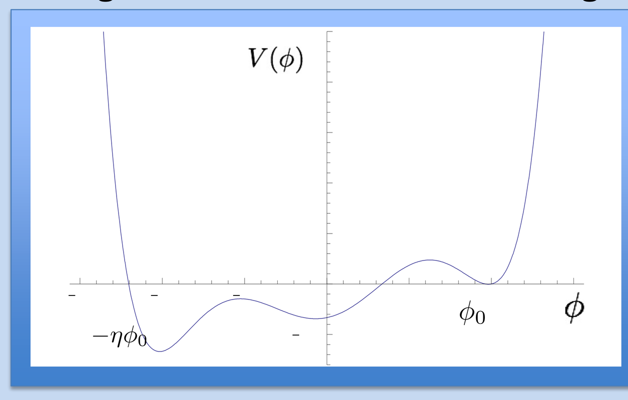

Fig.4: This is the plot of the 3-minima potential studied.

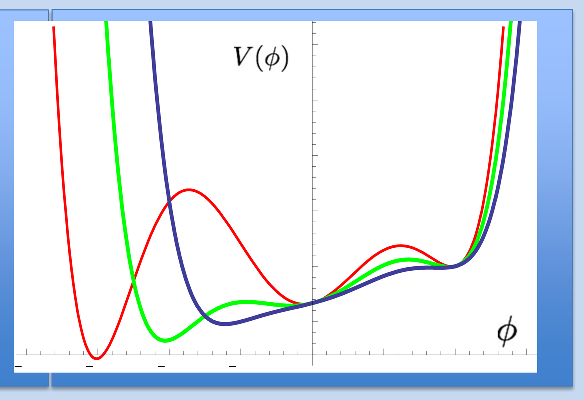

Fig 5: This is a plot of varying eta values. Notice that the potential barrier increases greatly in size as eta increases. (Red is highest eta, blue is least)

References

[1]S.Coleman,Phys.Rev.D 15 (1977).
[2] S. Coleman and F. De Luccia, Phys. Rev. D 21, 3305 (1980).
[3] R. Easther, J. Giblin, L. Hui, and E. A. Lim, Phys. Rev. D 80, 123519 (2009) arXiv:0907.3234 [hep-th].

GABE

The process of evolving scalar fields and potentials required great computational power. To study the evolution and collisions of bubbles, a program called Grid and Bubble Evolver (GABE) was used. GABE uses a second order Runge-Kutta method to calculate field values at all points in a lattice. For this model, a 256^3 point lattice was used. With GABE, the fields are evolved according to the Klein-Gordon equation, given below:

$$\ddot{\phi} + 3H\dot{\phi} - \nabla^2\phi = \frac{-\partial V}{\partial\phi}$$

Initial Bubble distance(in terms of starting radius)

η-values	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0.5	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
0.6	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
0.7	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
0.8	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
0.9	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
1.0	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
1.1	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
1.2	✖	✖	✔	✔	✔	✔	✔	✔	✔	✔
1.3	✖	✖	✖	✖	✖	✖	✔	✔	✔	✔
1.4	✖	✖	✖	✖	✖	✖	✖	✖	✖	✖
1.5	✖	✖	✖	✖	✖	✖	✖	✖	✖	✖
1.6	✖	✖	✖	✖	✖	✖	✖	✖	✖	✖
1.7	✖	✖	✖	✖	✖	✖	✖	✖	✖	✖

Fig.6: This is a plot of the results from the simulations. The varied parameters were η and initial bubble distance. Successful transitions are noted by green checkmarks.

Results

After running many variations on η and the initial bubble radii, the data were collected into the table above (Fig. 6). A clear divide can be seen between transitioning and non-transitioning potentials. Using this divide, a threshold for classical transitions can be found. The data suggests that once a certain η -value is reached, it may not be possible to classically transition at all, no matter how energetic the bubble collision.

Future Work

Progressing from what was gathered this summer, future study could focus on the region between transitions and non-transitions, with smaller and smaller intervals for parameter change. By doing this, it may be possible to find an analytical solution to describe this region. In addition to delving more into this model, it would be interesting to study more complicated potentials, especially with multiple, coupled fields.

Acknowledgements

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