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THE POSSIBLE SHAPES OF NUMERICAL RANGES

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Abstract. Which convex subsets of $\mathbb{C}$ are the numerical range $W(A)$ of some matrix A? This paper gives a precise characterization of these sets. In addition to this we show that for any A there exists a symmetric B of the same size such that $W(A) = W(B)$ thereby settling an open question from [2].

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