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**Supersymmetric leptophilic Higgs model**

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In the leptophilic model, one Higgs doublet couples to quarks and another couples to leptons. We study the supersymmetric version of this model, concentrating on the tightly constrained Higgs sector, which has four doublets. Constraints from perturbativity, unitarity, and LEP bounds are considered. It is found that the lightest Higgs,  $h$ , can have a mass well below 114 GeV, and for masses below 100 GeV will have a substantially enhanced branching ratio into  $\tau$  pairs. For this region of parameter space, traditional production mechanisms (Higgs-strahlung, W fusion, and gluon fusion) are suppressed, but it may be produced in the decay of heavier particles. The second lightest Higgs has a mass of approximately 110 GeV for virtually all of parameter space, with standard model couplings, and thus an increase of a few GeV in the current lower bound on the standard model Higgs mass would rule out the model. The two heavier Higgs are both gauge phobic, one decays almost entirely into  $b\bar{b}$  and can be produced via gluon fusion while the other decays almost entirely into  $\tau^+\tau^-$  but cannot be easily produced.

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**I. INTRODUCTION**

The main purpose of the Large Hadron Collider (LHC) is the study of the mechanism of electroweak symmetry breaking. One of the simplest and most studied extensions of the standard model is the two Higgs doublet model (2HDM), in which two scalar doublets are jointly responsible for electroweak symmetry breaking and fermion mass acquisition [1,2]. This model has a very rich phenomenology, including charged scalars and pseudoscalars. Among the earliest motivations for the 2HDM is its additional  $CP$  violation relative to the standard model [3–9], which can provide an additional source of baryogenesis and the relative abundance of matter to antimatter in the Universe [10,11]. It was also motivated by the fact that supersymmetric models and models with a Peccei-Quinn symmetry [12] will always require a minimum of two Higgs doublets.

In order to avoid unobserved tree-level flavor changing neutral currents (FCNCs), all fermions with the same quantum numbers (and which are thus capable of mixing) must couple to the same Higgs multiplet. The Glashow-Weinberg theorem [13] states that a necessary and sufficient condition for the absence of FCNCs at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of  $SU(2)$ , correspond to the same eigenvalue of  $T_3$ , and that a basis exists in which they receive their contributions in the mass matrix from a single source. In the 2HDM, this is due to the introduction of discrete or continuous symmetries. Generally one may either take both up- and down-type quarks to couple to the same doublet or have each couple to its own doublet. It is usually assumed that the leptons

couple to the same doublet as the down-type quarks, in which case the former scenario describes the type I 2HDM while the latter describes the type II 2HDM. Such couplings can be enforced by imposing a suitable  $\mathbb{Z}_2$  symmetry, which may simply be imposed *ad hoc* or which may arise as a subgroup of a continuous symmetry (as in Peccei-Quinn or supersymmetric models).

Despite the traditional convention that leptons couple to the same doublet as the down-type quarks, there is no *a priori* reason why this must be the case. An alternative possibility is that both the up- and down-type quarks couple to one doublet while the leptons couple to the remaining doublet. While the traditional 2HDMs have received a great deal of attention, relatively little work has been done in investigating this alternative possibility. Those who have focused on this model [14–18] have referred to it by several names, our selection of which is the leptophilic two Higgs doublet model (L2HDM). As noted by Su and Thomas [14], the consequences of a L2HDM could drastically alter the possible detection channels for a light Higgs at the LHC, so it is important that it be considered as incoming data begin to arrive. Furthermore, the possibility of substantially enhanced leptonic couplings (which can only occur in leptophilic models) may shed some insight into explaining recent experimental results from PAMELA, Fermi LAT, and H.E.S.S. [16].

There also remain alternative possibilities. One can couple the up-type quarks and leptons to one Higgs doublet and the down-type quarks to the other (referred to as the “flipped” model [19]) or one can couple all of the charged fermions to one doublet and the right-handed neutrino to another (referred to as the “neutrino-specific” model) [20]. While interesting in their own right, these models do not offer the possibility of substantially enhanced leptonic couplings, and we will not focus on them.

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The most popular extension of the standard model is supersymmetry, which can solve the hierarchy problem and which has a very tightly constrained Higgs sector. Thus, one is led to consider the supersymmetric versions of these alternative 2HDM models. Recently, with McCaskey, we considered [21] the supersymmetric version of the neutrino-specific model, and found some remarkable signatures, including pentalepton and hexalepton events with very high rates at the Tevatron and the LHC. In this work, we extend the L2HDM to incorporate supersymmetry. The resulting supersymmetric leptophilic Higgs model (SLHM) leads to exciting phenomenological prospects. In the scalar sector, the strong constraints on the Higgs potential will substantially alter the phenomenology of the lightest Higgs boson, since decays to leptons can be substantially enhanced, and the decrease in the coupling to the gauge bosons means that the current LEP bounds will not apply, and much lighter Higgs bosons can be tolerated. In addition, the supersymmetric partners to the leptons and the leptonic Higgs doublet are influenced by the unusual Yukawa structure. In the case of  $R$ -parity violation, the lightest supersymmetric particle (LSP) could decay into leptons. Without  $R$ -parity violation the LSP might annihilate into leptons [16]. In this paper, we will focus on the scalar sector, since the results may be testable in the very near future at the Tevatron.

The layout of this paper is as follows. In Sec. II we review the setup of the L2HDM. In Sec. III we introduce the SLHM and calculate the scalar mass matrices. In Sec. IV we consider various constraints on the model's parameter space by focusing on the neutral scalar sector. By combining results from Yukawa coupling perturbativity considerations, unitarity requirements, and direct searches for Higgs bosons at LEP, we obtain severe restrictions on the model's parameter space. In Sec. V we discuss the phenomenology of the lightest and next-to-lightest Higgs bosons at the Tevatron and the LHC, and then in Sec. VI, we conclude.

## II. THE LEPTOPHILIC TWO HIGGS DOUBLET MODEL

The L2HDM contains two scalar  $SU(2)_L$  doublets  $\Phi_q$  and  $\Phi_\ell$ . A discrete  $\mathbb{Z}_2$  symmetry is imposed under which  $\Phi_\ell \rightarrow -\Phi_\ell$  and  $e_{R_i} \rightarrow -e_{R_i}$ , but all other fields are invariant. The resulting Yukawa Lagrangian is given by

$$\begin{aligned} \mathcal{L}_Y = & -\{Y_{ij}^u \bar{u}_{R_i} \tilde{\Phi}_q^\dagger \cdot Q_{L_j} + Y_{ij}^d \bar{d}_{R_i} \Phi_q^\dagger \cdot Q_{L_j} \\ & + Y_{ij}^\ell \bar{e}_{R_i} \Phi_\ell^\dagger \cdot E_{L_j} + \text{H.c.}\}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} Q_{L_i} &= \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix}, & E_{L_i} &= \begin{pmatrix} \nu_{L_i} \\ e_{L_i} \end{pmatrix}, & \text{and} \\ \Phi_X &= \begin{pmatrix} \phi_X^+ \\ \frac{1}{\sqrt{2}}(v_X + \phi_{Xr}^0 + i\phi_{Xi}^0) \end{pmatrix} \end{aligned}$$

for  $X = q, \ell$ , and  $\tilde{\Phi}_q = i\sigma_2 \Phi_q$ . The Higgs sector potential is given by [14,22]

$$\begin{aligned} V = & m_q^2 |\Phi_q|^2 + m_\ell^2 |\Phi_\ell|^2 + (m_{q\ell}^2 \Phi_q^\dagger \Phi_\ell + \text{H.c.}) \\ & + \frac{\lambda_1}{2} |\Phi_q|^4 + \frac{\lambda_2}{2} |\Phi_\ell|^4 + \lambda_3 |\Phi_q|^2 |\Phi_\ell|^2 \\ & + \lambda_4 |\Phi_q^\dagger \Phi_\ell|^2 + \frac{\lambda_5}{2} [(\Phi_q^\dagger \Phi_\ell)^2 + \text{H.c.}]. \end{aligned} \quad (2)$$

The physical scalars consist of two neutral scalars  $h$  and  $H$ , a pseudoscalar  $\chi^0$ , and a charged pair  $H^\pm$ . The other 3 degrees of freedom are the Goldstone bosons  $G^\pm$  and  $G^0$ , which are eaten by the  $W^\pm$  and  $Z^0$ , respectively. If one defines the mixing angle  $\tan\beta = v_q/v_\ell$ , the physical charged scalars can be expressed as

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi_q^+ \\ \Phi_\ell^+ \end{pmatrix}. \quad (3)$$

The physical neutral scalar states are expressed in terms of the mixing angle  $\tan\alpha$ , which can be solved for in terms of the entries of the neutral scalar mass-squared matrix  $\tan 2\alpha = 2M_{12}^2/(M_{11}^2 - M_{22}^2)$ . One then finds the following relation:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_{\ell r}^0 - v_\ell \\ \phi_{qr}^0 - v_q \end{pmatrix}. \quad (4)$$

The vertex factors for the couplings between the charged scalar and fermions are given by [15]

$$\begin{aligned} H^+ u_i d_j &\rightarrow \left( \frac{ig \cot\beta}{2\sqrt{2}M_W} \right) V_{ij} [(m_{u_i} - m_{d_j}) - (m_{u_i} + m_{d_j})\gamma_5], \\ H^+ \nu_i e_i &\rightarrow \left( \frac{ig \tan\beta}{2\sqrt{2}M_W} \right) m_{e_i} (1 - \gamma_5). \end{aligned} \quad (5)$$

For large  $\tan\beta$  the neutrino-lepton coupling to  $H^+$  is magnified while the quarks' coupling to  $H^+$  is diminished. The neutral scalar couplings to the charged leptons will similarly be magnified. An interesting feature of the model is that  $\tan\beta$  can be much larger than in the conventional 2HDMs without causing problems with perturbativity and unitarity, since the standard model leptonic couplings are smaller than the quark couplings.

### III. THE SUPERSYMMETRIC LEPTOPHILIC HIGGS MODEL

In this section we introduce the minimal leptophilic model required to incorporate supersymmetry. A SLHM will require a minimum of four Higgs doublets in order to achieve anomaly cancellation. Therefore, we add to the minimal supersymmetric standard model (MSSM) two Higgs doublets  $H_0$  and  $H_\ell$  with weak hypercharge assignments  $+1/2$  and  $-1/2$ , respectively. The four Higgs doublets along with their weak hypercharges are listed in the table as follows:

$\Phi$	$H_u$	$H_d$	$H_0$	$H_\ell$
$U_Y(1)$	$+1/2$	$-1/2$	$+1/2$	$-1/2$

The scalar doublets  $H_u$  and  $H_d$  are responsible for giving mass to the up and down quarks, respectively. We refer to these doublets as the quark friendly doublets. Of the new doublets, the lepton friendly doublet  $H_\ell$  gives mass to the leptons, while the remaining inert doublet  $H_0$  does not couple to quarks or leptons. This Yukawa structure is enforced by a discrete  $\mathbb{Z}_2$  symmetry, under which the superfields  $E$ ,  $H_0$ , and  $H_\ell$  transform as  $X \rightarrow -X$ , while all other fields remain unchanged. The most general superpotential respecting  $R$  parity, gauge symmetry, and the  $\mathbb{Z}_2$  symmetry is

$$W = y_u U Q H_u - y_d D Q H_d - y_\ell E L H_\ell + \tilde{\mu}_1 H_u H_d + \tilde{\mu}_2 H_0 H_\ell. \quad (6)$$

The  $\mathbb{Z}_2$  symmetry is softly broken by the terms  $(\mu_3^2 H_u H_\ell + \mu_4^2 H_0 H_d + \text{H.c.})$  contained in the Higgs sector soft supersymmetric (SUSY) breaking potential  $V_{\text{Soft}}$  given by

$$V_{\text{Soft}} = \mu_u^2 |H_u|^2 + \mu_d^2 |H_d|^2 + \mu_0^2 |H_0|^2 + \mu_\ell^2 |H_\ell|^2 + (\mu_1^2 H_u H_d + \mu_2^2 H_0 H_\ell + \mu_3^2 H_u H_\ell + \mu_4^2 H_0 H_d + \text{H.c.}).$$

The Higgs sector potential is given by the sum of the  $F$ -terms,  $D$ -terms, and  $V_{\text{Soft}}$ , respectively,

$$V = \sum_{i=1}^k \left| \frac{\partial W}{\partial H_i} \right|^2 + \frac{1}{2} \sum_a \left| \sum_{i=1}^k g^a H_i^\dagger T^a H_i \right|^2 + V_{\text{Soft}}.$$

Expanding the above expression results in

$$V = m_u^2 |H_u|^2 + m_d^2 |H_d|^2 + m_0^2 |H_0|^2 + m_\ell^2 |H_\ell|^2 + (\mu_1^2 H_u H_d + \mu_2^2 H_0 H_\ell + \mu_3^2 H_u H_\ell + \mu_4^2 H_0 H_d + \text{H.c.}) + \frac{g_1^2}{8} \sum_a |H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d + H_0^\dagger \sigma^a H_0 + H_\ell^\dagger \sigma^a H_\ell|^2 + \frac{g_2^2}{8} (|H_u|^2 - |H_d|^2 + |H_0|^2 - |H_\ell|^2)^2,$$

where  $m_u^2 = (|\tilde{\mu}_1|^2 + \mu_u^2)$ ,  $m_d^2 = (|\tilde{\mu}_1|^2 + \mu_d^2)$ ,  $m_0^2 = (|\tilde{\mu}_2|^2 + \mu_0^2)$ ,  $m_\ell^2 = (|\tilde{\mu}_2|^2 + \mu_\ell^2)$ , and  $\sigma^a$  ( $a = 1, 2, 3$ ) are the Pauli matrices. To achieve spontaneous symmetry breaking, the Higgs doublets acquire the following vacuum expectation values (VEVs):

$$\begin{aligned} \langle H_u \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, & \langle H_d \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \\ \langle H_0 \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, & \langle H_\ell \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_\ell \\ 0 \end{pmatrix}. \end{aligned} \quad (7)$$

We define  $v^2 = v_u^2 + v_d^2 + v_0^2 + v_\ell^2$  so that we have  $v^2 = 4M_Z^2/(g_1^2 + g_2^2) \approx (246 \text{ GeV})^2$ . Between the quark friendly doublets we define the mixing angle  $\tan\beta = v_u/v_d$ , while between the lepton friendly and inert doublets we define the mixing angle  $\tan\beta_\ell = v_0/v_\ell$ . We also define  $\tan\alpha = v_q/v_L$ , where  $v_q^2 = v_u^2 + v_d^2$  and  $v_L^2 = v_0^2 + v_\ell^2$ . These definitions allow us to express the individual VEVs in terms of the standard model VEV and the three mixing angles  $\alpha$ ,  $\beta$ , and  $\beta_\ell$ :

$$\begin{aligned} v_u &= v \sin\alpha \sin\beta, & v_d &= v \sin\alpha \cos\beta, \\ v_0 &= v \cos\alpha \sin\beta_\ell, & v_\ell &= v \cos\alpha \cos\beta_\ell. \end{aligned} \quad (8)$$

Each of the four complex Higgs doublets contains 4 real degrees of freedom, so there are a total of 16 degrees of freedom. Three of these are eaten to give mass to the  $W^\pm$  and  $Z^0$ , while those remaining result in a scalar mass spectrum that includes four neutral scalars, three pseudo-scalars, and three charged pairs. From the scalar potential above, the mass matrices can be calculated. We parametrize them in terms of the gauge boson masses and the three mixing angles appearing in Eq. (8).

The neutral scalar mass matrix is

$$M_N^2 = \begin{pmatrix} M_1^2 & -\frac{1}{2} M_Z^2 s_\alpha^2 s_{2\beta} - \mu_1^2 & \frac{1}{2} M_Z^2 s_{2\alpha} s_\beta s_{\beta_\ell} & -\frac{1}{2} M_Z^2 s_{2\alpha} s_\beta c_{\beta_\ell} - \mu_3^2 \\ -\frac{1}{2} M_Z^2 s_\alpha^2 s_{2\beta} - \mu_1^2 & M_2^2 & -\frac{1}{2} M_Z^2 s_{2\alpha} c_\beta s_{\beta_\ell} - \mu_4^2 & \frac{1}{2} M_Z^2 s_{2\alpha} c_\beta c_{\beta_\ell} \\ \frac{1}{2} M_Z^2 s_{2\alpha} s_\beta s_{\beta_\ell} & -\frac{1}{2} M_Z^2 s_{2\alpha} c_\beta s_{\beta_\ell} - \mu_4^2 & M_3^2 & -\frac{1}{2} M_Z^2 c_\alpha^2 s_{2\beta_\ell} - \mu_2^2 \\ -\frac{1}{2} M_Z^2 s_{2\alpha} s_\beta c_{\beta_\ell} - \mu_3^2 & \frac{1}{2} M_Z^2 s_{2\alpha} c_\beta c_{\beta_\ell} & -\frac{1}{2} M_Z^2 c_\alpha^2 s_{2\beta_\ell} - \mu_2^2 & M_4^2 \end{pmatrix},$$

where  $s_x$  and  $c_x$  are shorthand for  $\sin x$  and  $\cos x$ , respectively, and the diagonal terms are given by

$$\begin{aligned}
M_1^2 &= M_Z^2 \sin^2 \alpha \sin^2 \beta + \lambda_1, & \lambda_1 &= \mu_1^2 \cot \beta + \mu_3^2 \cot \alpha \left( \frac{\cos \beta_\ell}{\sin \beta} \right), \\
M_2^2 &= M_Z^2 \sin^2 \alpha \cos^2 \beta + \lambda_2, & \lambda_2 &= \mu_1^2 \tan \beta + \mu_4^2 \cot \alpha \left( \frac{\sin \beta_\ell}{\cos \beta} \right), \\
M_3^2 &= M_Z^2 \cos^2 \alpha \sin^2 \beta_\ell + \lambda_3, & \lambda_3 &= \mu_2^2 \cot \beta_\ell + \mu_4^2 \tan \alpha \left( \frac{\cos \beta}{\sin \beta_\ell} \right), \\
M_4^2 &= M_Z^2 \cos^2 \alpha \cos^2 \beta_\ell + \lambda_4, & \lambda_4 &= \mu_2^2 \tan \beta_\ell + \mu_3^2 \tan \alpha \left( \frac{\sin \beta}{\cos \beta_\ell} \right).
\end{aligned}$$

The pseudoscalar mass matrix is

$$M_A^2 = \begin{pmatrix} \lambda_1 & \mu_1^2 & 0 & \mu_3^2 \\ \mu_1^2 & \lambda_2 & \mu_4^2 & 0 \\ 0 & \mu_4^2 & \lambda_3 & \mu_2^2 \\ \mu_3^2 & 0 & \mu_2^2 & \lambda_4 \end{pmatrix}. \quad (9)$$

The charged scalar mass matrix is

$$M_{H^\pm}^2 = M_A^2 + \Delta M^2, \quad (10)$$

where

$$\Delta M^2 = M_W^2 \begin{pmatrix} s_\alpha^2 c_\beta^2 + c_\alpha^2 c_{2\beta_\ell} & \frac{1}{2} s_\alpha^2 s_{2\beta} & \frac{1}{2} s_{2\alpha} s_\beta s_{\beta_\ell} & \frac{1}{2} s_{2\alpha} s_\beta c_{\beta_\ell} \\ \frac{1}{2} s_\alpha^2 s_{2\beta} & s_\alpha^2 s_\beta^2 - c_\alpha^2 c_{2\beta_\ell} & \frac{1}{2} s_{2\alpha} c_\beta s_{\beta_\ell} & \frac{1}{2} s_{2\alpha} c_\beta c_{\beta_\ell} \\ \frac{1}{2} s_{2\alpha} s_\beta s_{\beta_\ell} & \frac{1}{2} s_{2\alpha} c_\beta s_{\beta_\ell} & c_\alpha^2 c_{\beta_\ell}^2 + s_\alpha^2 c_{2\beta} & \frac{1}{2} c_\alpha^2 s_{2\beta_\ell} \\ \frac{1}{2} s_{2\alpha} s_\beta c_{\beta_\ell} & \frac{1}{2} s_{2\alpha} c_\beta c_{\beta_\ell} & \frac{1}{2} c_\alpha^2 s_{2\beta_\ell} & c_\alpha^2 s_{\beta_\ell}^2 - s_\alpha^2 c_{2\beta} \end{pmatrix}.$$

In Sec. 3.3 of [23] Gupta and Wells outline a procedure for obtaining an upper bound on the tree-level mass of the lightest neutral scalar,  $h$ , in the limit of large SUSY breaking masses (as compared to the  $Z$  mass). The procedure consists of transforming the mass matrices into the so-called ‘‘Runge basis,’’ in which one doublet obtains all of the VEVs, while the others are orthogonal to one another. Details on the Runge basis can be found in [24]. In this basis all but one diagonal entry of the neutral scalar mass matrix grow large in the limit of large SUSY breaking masses. This entry acts as an upper bound on  $M_h^2$  since, for a positive definite matrix, the smallest eigenvalue is bounded above by the smallest diagonal entry. Their result holds in our case as well and results in the inequality

$$M_h \leq M_Z |\sin^2 \alpha \cos 2\beta + \cos^2 \alpha \cos 2\beta_\ell|. \quad (11)$$

Leading order radiative corrections to the Higgs masses will be important in constraining parameter space. As usual, the dominant contributions come from top quark loops, governed by the top quark Yukawa coupling. In this section we have written the neutral scalar mass matrix,  $M_N^2$ , in the  $\{u, d, 0, \ell\}$  basis. Hence the 1-1 entry receives a correction from top quark loop diagrams given by

$$\Delta M_{11}^2 = \frac{3\alpha}{\pi} \left( \frac{m_t^4}{M_Z^2} \right) \frac{\ln(m_t^2/m_i^2)}{\sin^2 2\theta_W \sin^2 \alpha \sin^2 \beta}, \quad (12)$$

where  $m_i$  is the stop squark mass, which we take to be  $\sim 1$  TeV. In addition to top quark loop corrections, other corrections are potentially significant because of the possibility of very large values for  $\tan \beta$  and  $\tan \beta_\ell$ . We therefore also consider the leading correction to the 2-2 and 4-4 entries of  $M_N^2$ , which come from bottom quark loop diagrams and a tau loop diagram, respectively. The 3-3 entry receives no correction since the inert doublet,  $H_0$ , does not couple to quarks or leptons. There are other sub-leading-log corrections to the masses, and these can contribute 5–10 GeV to the masses (see Ref. [25] for a detailed discussion).

#### IV. CONSTRAINTS ON THE SUPERSYMMETRIC LEPTOPHILIC HIGGS MODEL

In this section we outline the main constraints that limit the viable parameter space of the SLHM. The free parameters arising from the scalar sector consist of the four couplings  $\mu_1^2$ ,  $\mu_2^2$ ,  $\mu_3^2$ , and  $\mu_4^2$ , which mix pairs of Higgs doublets in the scalar potential, as well as the three mixing angles  $\tan \alpha$ ,  $\tan \beta$ , and  $\tan \beta_\ell$ , which appear in Eq. (8). The constraints arising from the charged scalar sector are similar to those of the L2HDM, which is studied in [15]. Our interest therefore lies in the neutral sector. We find that LEP data and other constraints severely restrict the size of the allowable parameter space, but leave enough room to comfortably fit the model for a lightest neutral scalar mass substantially less than 110 GeV.



### A. Yukawa coupling perturbativity

The first constraints come from requiring that the Yukawa couplings remain perturbative. By demanding that each Yukawa coupling remains smaller than  $4\pi$  we obtain the following three inequalities:

$$\begin{aligned} \left(1 + \frac{1}{\tan^2 \alpha}\right) \left(1 + \frac{1}{\tan^2 \beta}\right) &< \frac{8\pi^2 v^2}{m_t^2} \approx 13^2, \\ \left(1 + \frac{1}{\tan^2 \alpha}\right) (1 + \tan^2 \beta) &< \frac{8\pi^2 v^2}{m_b^2} \approx 520^2, \\ (1 + \tan^2 \alpha) (1 + \tan^2 \beta_\ell) &< \frac{8\pi^2 v^2}{m_\tau^2} \approx 1235^2. \end{aligned} \quad (13)$$

One can see that the top quark Yukawa coupling becomes nonperturbative for small values of  $\tan \alpha$  or  $\tan \beta$ , while the bottom quark Yukawa coupling does so for small values of  $\tan \alpha$  or large values of  $\tan \beta$ . In addition, the tau Yukawa coupling becomes nonperturbative for large values of  $\tan \alpha$  or  $\tan \beta_\ell$ .

### B. Tree level unitarity

Requiring perturbative unitarity of fermion–antifermion scattering places upper bounds on the fermion masses. The unitarity condition that must be satisfied is  $|\text{Re}(a_J)| \leq 1/2$ , where  $a_J$  is the  $J$ th partial wave amplitude in the partial wave expansion of the fermion–antifermion scattering amplitude. The scattering we consider occurs by the exchange of a Higgs boson. We obtain bounds from imposing the unitarity condition on the  $J = 0$  partial wave amplitude, which is calculated from a sum over  $s$ - and  $t$ -channel helicity amplitudes in the high energy limit. The procedure is described in detail in [26], where contributions to the partial wave amplitudes are provided for a general model. These contributions depend on combinations of the vector and axial vector Yukawa couplings. For the SLHM the resultant bounds are found to be (see [26] for a clear discussion)

$$\begin{aligned} \frac{G_F m_t^2}{4\pi\sqrt{2}} &< \sin^2 \alpha \sin^2 \beta, \\ \frac{G_F m_b^2}{4\pi\sqrt{2}} &< \sin^2 \alpha \cos^2 \beta, \\ \frac{G_F m_\tau^2}{4\pi\sqrt{2}} &< \cos^2 \alpha \cos^2 \beta_\ell. \end{aligned} \quad (14)$$

Here we have used the bounds obtained for third generation fermions as their larger masses yield the most stringent results. The unitarity constraint prevents very large values for  $\tan \beta$ , capping it at around 300. Several combinations of  $\tan \alpha$  and  $\tan \beta$  values on the order of several tenths are also eliminated.

### C. The anomalous muon magnetic moment

As in the standard model, the magnetic moment of the muon receives a contribution from the one-loop diagram formed by connecting the muon lines on a muon-muon-photon vertex with a neutral Higgs boson. Only the lightest neutral Higgs is relevant since the contribution goes as the square of the ratio between the muon and Higgs masses. For the SLHM the contribution is

$$\Delta a_\mu = K^2 \frac{m_\mu^2}{8\pi^2 v^2} \int_0^1 \frac{z^2(2-z)}{z^2 + x^2(1-z)} dz, \quad (15)$$

where  $x = M_h/m_\mu$  and

$$K^2 = \frac{|U_{41}|^2}{\cos^2 \alpha \cos^2 \beta_\ell}.$$

If the Higgs mass,  $M_h$ , is assumed to be the same in the SLHM and the standard model, then the contribution to the muon's magnetic moment from a light scalar in the SLHM is simply its standard model value multiplied by  $K^2$ . The value of  $K^2$ , however, remains  $\leq 1$  across the entire spectrum of parameter space, even for very large values of  $\tan \alpha$  and  $\tan \beta_\ell$ . A review on the anomalous muon magnetic moment is given by [27], while current results and uncertainties can be found in [28,29]. In our case the contribution is much too small to produce any bounds.

In addition, however, there is a two-loop Barr-Zee effect [30], which is generally more significant than the one-loop contribution discussed above. The Barr-Zee effect occurs by connecting an internal Higgs to an internal photon through a massive fermion loop and is given by [31,32]. We consider such effects with third generation fermions in the SLHM and find that the contribution to the muon magnetic moment is

$$\begin{aligned} \Delta a_\mu = & -\frac{\alpha m_\mu^2 U_{41}}{4\pi^3 v^2 \cos \beta_\ell} \left\{ \frac{8U_{11}f(x_t)}{3 \sin 2\alpha \sin \beta} + \frac{2U_{21}f(x_b)}{3 \sin 2\alpha \cos \beta} \right. \\ & \left. + \frac{U_{41}f(x_\tau)}{\cos^2 \alpha \cos \beta_\ell} \right\}, \end{aligned} \quad (16)$$

where  $x_f = m_f^2/M_h^2$  and the function  $f(x)$  is given by

$$f(x) = \frac{x}{2} \int_0^1 \frac{1-2z(1-z)}{z(1-z)-x} \ln \left[ \frac{z(1-z)}{x} \right] dz.$$

Though the contribution from the tau loop diagram is suppressed by  $m_\tau^2/M_h^2$ , it is enhanced for very large  $\tan \beta_\ell$ . In following [33] we measure how well these contributions compare to experiment with the quantity

$$\chi_{a_\mu}^2 = \left( \frac{\Delta a_\mu^{\text{SLHM}}}{6.8 \times 10^{-10}} \right)^2,$$

where  $6.8 \times 10^{-10}$  is the theoretical uncertainty for  $a_\mu$  in the standard model (used because it is larger than the experimental uncertainty). The result is that, though larger than the one-loop contributions, the two-loop Barr-Zee

effect contributions are still too small to provide significant constraints on the parameter space.

#### D. LEP Higgs search data

The largest source of constraints for the neutral sector of the SLHM consists of LEP's failure to discover a neutral Higgs boson. If the lightest neutral scalar's mass is too small, one would expect LEP to have seen it, whereas for a mass  $M_h > 114.4$  GeV, LEP data become irrelevant and no bounds can be obtained [34]. The production mechanism at LEP is the Higgs-strahlung process  $e^+e^- \rightarrow hZ$ , and thus if the coupling,  $g_{ZZh}$ , between the lightest neutral scalar and  $Z$  pairs is sufficiently small, the scalar's non-discovery at LEP can be explained [35–38].

In addition, there is an effect which suppresses the sensitivity with which the experimental results may be applied to constrain models beyond the standard model [33,39]. Bounds from LEP were produced under the assumption that the Higgs boson decays exclusively into  $b\bar{b}$  pairs or exclusively into  $\tau^+\tau^-$  pairs. LEP has provided a bound on the quantity  $\text{BR}(h \rightarrow X\bar{X})\xi^2$  for  $X = b$  and  $X = \tau$ , where  $\xi$  is the ratio of the  $ZZh$  coupling in a model to that of the standard model i.e.  $\xi = g_{ZZh}/g_{ZZh}^{\text{SM}}$ . We find the value of  $\xi^2$  in the SLHM to be

$$\xi^2 = |U_{11} \sin\alpha \sin\beta + U_{21} \sin\alpha \cos\beta + U_{31} \cos\alpha \sin\beta_\ell + U_{41} \cos\alpha \cos\beta_\ell|^2. \quad (17)$$

We will employ both of these bounds to exclude regions of parameter space in the SLHM. Naively, one expects  $\text{BR}(h \rightarrow b\bar{b})$  to approach unity when  $\tan\beta$  is large and  $\tan\alpha$ ,  $\tan\beta_\ell$  are small, since in that case the down-type quark Yukawa couplings are doubly enhanced while the lepton Yukawa couplings remain small. On the other hand, when  $\tan\alpha$  and  $\tan\beta_\ell$  are large while  $\tan\beta$  is small, the lepton Yukawa couplings are enhanced and the down-type quark Yukawa couplings remain small, resulting in an increase in the branching ratio  $\text{BR}(h \rightarrow \tau^+\tau^-)$ .

Since in the interesting region of parameter space, the  $ZZh$  and  $WW_h$  couplings are small, we can approximate the total decay width as simply  $\Gamma(h \rightarrow b\bar{b}) + \Gamma(h \rightarrow \tau^+\tau^-)$ . The two branching ratios for the SLHM can therefore be conveniently expressed as  $\text{BR}(h \rightarrow b\bar{b}) = 1/(1 + \kappa)$  and  $\text{BR}(h \rightarrow \tau^+\tau^-) = \kappa/(1 + \kappa)$ , where  $\kappa = \Gamma(h \rightarrow \tau^+\tau^-)/\Gamma(h \rightarrow b\bar{b})$ . The variable  $\kappa$  is straightforward to calculate and is given by

$$\kappa = \left(\frac{m_\tau^2}{3m_b^2}\right) \tan^2\alpha \frac{\cos^2\beta}{\cos^2\beta_\ell} \left|\frac{U_{41}}{U_{21}}\right|^2 \left(\frac{M_h^2 - 4m_\tau^2}{M_h^2 - 4m_b^2}\right)^{3/2}, \quad (18)$$

where the  $U_{ij}$  are entries of the  $4 \times 4$  diagonalizing matrix defined by  $U^\dagger M_N^2 U = M_{\text{diag}}^2$ .

We have numerically scanned through parameter space, calculating the values of  $\text{BR}(h \rightarrow b\bar{b})\xi^2$ ,  $\text{BR}(h \rightarrow \tau^+\tau^-)\xi^2$ , and  $M_h$  in the SLHM. Those points in parameter space for which either  $\text{BR}(h \rightarrow b\bar{b})\xi^2$  or

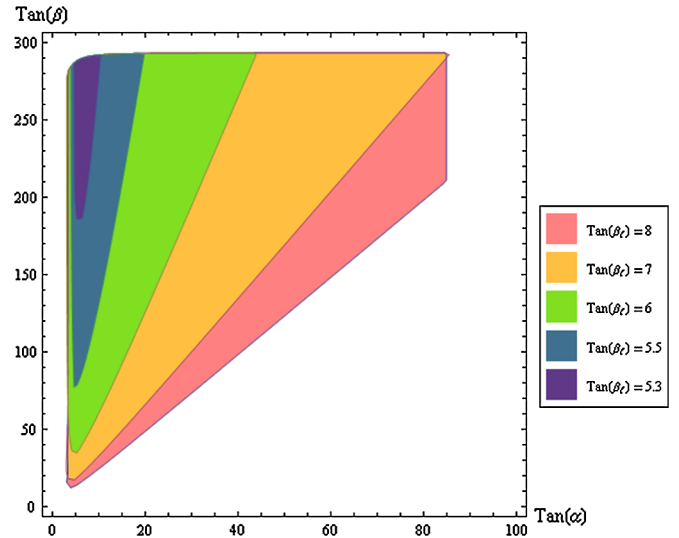


FIG. 1 (color online). The colored regions illustrate the allowed points in the  $\tan\alpha$ ,  $\tan\beta$ , and  $\tan\beta_\ell$  parameter space. Each region is a slice of constant  $\tan\beta_\ell$  in the  $\tan\alpha \times \tan\beta$  plane. The values of  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$  are fixed at 200, 250, 300, and 100 GeV, respectively, but changing  $\mu_1$  and/or  $\mu_3$  has relatively little effect. Increasing  $\mu_2$  and/or  $\mu_4$  shrinks the above space. Increasing  $\tan\beta_\ell$  enlarges the size of the allowed space quite rapidly until around  $\tan\beta_\ell \approx 8$ , when the space stops enlarging and begins to slowly shrink—this can be seen in Fig. 2.

$\text{BR}(h \rightarrow \tau^+\tau^-)\xi^2$  is greater than its LEP bound at the corresponding value of  $M_h$  are excluded. By imposing these two LEP bounds as well as the perturbativity requirements of Sec. IV A and the unitarity requirements of Sec. IV B, we are able to exclude substantial regions of the model's parameter space. In Figs. 1 and 2 the allowed region of the three-dimensional parameter space for the

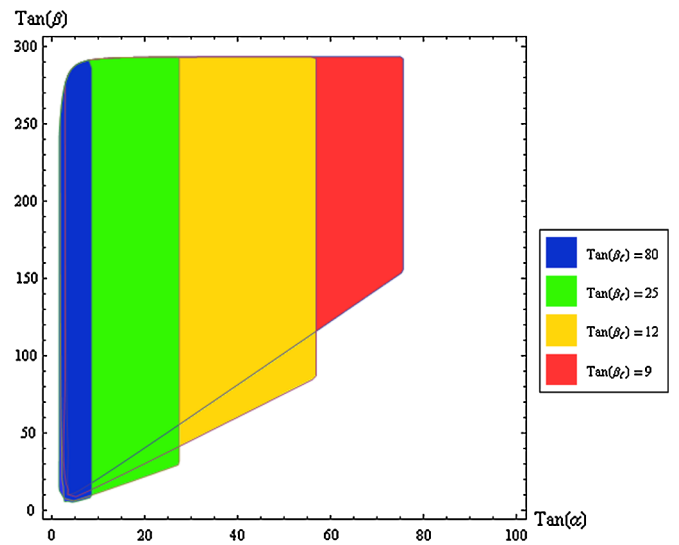


FIG. 2 (color online). A continuation of Fig. 1 for larger values of  $\tan\beta_\ell$ . As  $\tan\beta_\ell$  increases beyond 80, the space very slowly shrinks into an extremely thin sliver of possible  $\tan\alpha$  values centered near 2; it finally disappears completely at  $\tan\beta_\ell \approx 350$ .

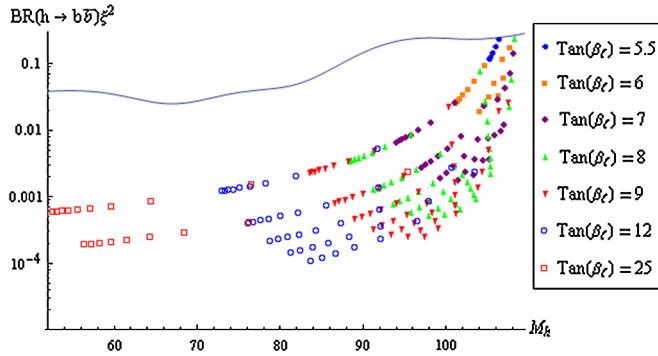


FIG. 3 (color online). Various values of the quantity  $\text{BR}(h \rightarrow b\bar{b})\xi^2$  plotted as a function of the lightest neutral scalar mass  $M_h$ . The plotted values correspond to a uniform sampling of points within the allowed regions of the  $\tan\alpha \times \tan\beta$  plane for the different values of  $\tan\beta_\ell$  that are plotted in Figs. 1 and 2. The LEP bound of Ref. [36] is shown in blue.

variables  $\tan\alpha$ ,  $\tan\beta$ , and  $\tan\beta_\ell$  is shown. For these plots the values of  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$  have been fixed at 200, 250, 300, and 100 GeV, respectively. The plots depict several sections of viable parameter space in the  $\tan\alpha \times \tan\beta$  plane, each being a slice of constant  $\tan\beta_\ell$ . As  $\tan\beta_\ell$  varies over its allowed range, one can see how the sections grow in area, change shape, and eventually shrink back away.

Though the values of  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , and  $\mu_4$  are fixed, the size and shape of the allowed parameter space remains largely unchanged when  $\mu_1$  and  $\mu_3$  are allowed to vary between 50 and 1000 GeV. Their values are consequentially relatively unconstrained. Increasing the value of  $\mu_4$ , however, has the effect of sharply cutting down on the size of the allowed region of parameter space. So too does increasing  $\mu_2$ , though to a slightly lesser degree. Merely increasing  $\mu_4$  to 200 GeV results in a drastically smaller allowed region than that shown in Fig. 1 and completely eliminates the regions corresponding to  $\tan\beta_\ell$  values of 5.3 and 5.5. The other regions are compressed so that  $3 \lesssim \tan\alpha \lesssim 20$  and  $50 \lesssim \tan\beta \lesssim 290$ , while their overall

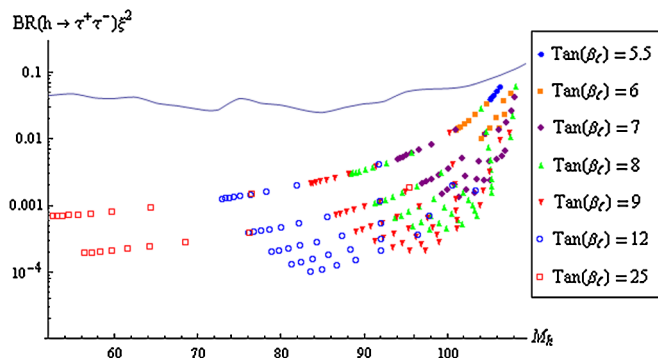


FIG. 4 (color online). Various values of the quantity  $\text{BR}(h \rightarrow \tau^+\tau^-)\xi^2$  plotted as a function of the lightest neutral scalar mass  $M_h$ .

shape remains the same. Enlarging either  $\mu_2$  or  $\mu_4$  further rapidly shrinks the allowed space away until it vanishes completely.

Figure 3 plots an assortment of possible  $\text{BR}(h \rightarrow b\bar{b})\xi^2$  values as a function of the lightest neutral scalar mass  $M_h$ . Each value plotted corresponds to some point in the allowed region of parameter space. The LEP curve is shown in blue (the upper solid line). For very large values of  $\tan\beta_\ell$ , the curves continue down to approximately 25 GeV, with the value of  $\text{BR}(h \rightarrow b\bar{b})\xi^2$  becoming extremely small. We see that Higgs bosons below 114.4 GeV are certainly allowed, but below approximately 90 GeV their couplings to vector bosons become negligible, making detection through vector boson fusion or Higgs-strahlung off a vector boson impossible. The analogous result for  $\text{BR}(h \rightarrow \tau^+\tau^-)$  is plotted in Fig. 4, with similar conclusions.

## V. PHENOMENOLOGY

In this section we discuss the possibility of detecting a supersymmetric leptophilic Higgs. We have focused on the neutral sector, as the charged sector strongly resembles the non-SUSY leptophilic scenario covered in [15]. The quantity of importance to the decay of the lightest neutral scalar is the ratio  $\kappa = \text{BR}(h \rightarrow \tau^+\tau^-)/\text{BR}(h \rightarrow b\bar{b})$ , which is given by Eq. (18) in Sec. IV D.

For the region of parameter space discussed in the previous section, we have shown various values of  $\kappa$  in Fig. 5. For Higgs bosons near 114.4 GeV, the allowed value of  $\kappa$  approaches its standard model value of approximately 0.1. However, for lighter Higgs bosons,  $\kappa$  is much bigger, approaching unity for Higgs masses below 100 GeV.

We see that in this model, the Higgs can be relatively light, and will have a much larger branching ratio to  $\tau^+\tau^-$  than in the standard model. In order to detect the Higgs at the Tevatron or the LHC, however, one also must consider the production rate. As we have seen, for Higgs bosons below 90 GeV, the  $ZZh$  and  $WW_h$  couplings are quite small, and thus Higgs-strahlung is negligible. What about gluon fusion, which is the primary production mechanism for a light Higgs? Here, one must include both top and

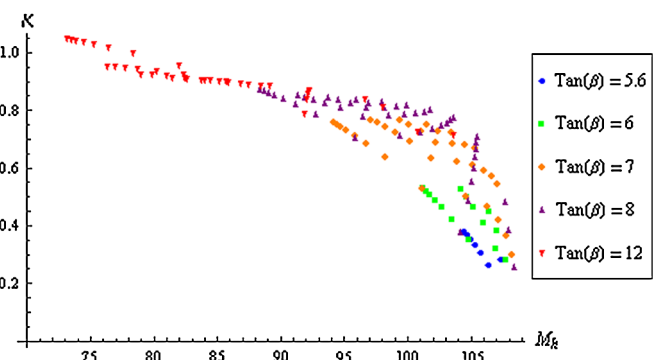


FIG. 5 (color online). Various values of  $\kappa$  plotted as a function of the lightest neutral scalar mass  $M_h$ .



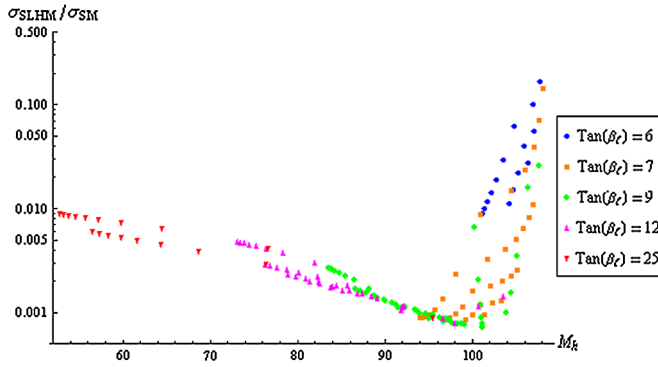


FIG. 6 (color online). Log plot of the ratio of the production cross section of the lightest neutral scalar by gluon fusion in the SLHM to the standard model.

bottom loops, and the coupling to the Higgs will be different. We find that the ratio of the gluon fusion cross section to that of the standard model is

$$\frac{\sigma_{\text{SLHM}}}{\sigma_{\text{SM}}} = \left| \frac{U_{11}}{\sin\alpha \sin\beta} + \frac{A(m_b)}{A(m_t)} \frac{U_{21}}{\sin\alpha \cos\beta} \right|^2, \quad (19)$$

and this is plotted in Fig. 6 for various parameters. The function  $A(m_f)$  is given by  $A(m_f) = 2[x_f + (x_f - 1)f(x_f)]x_f^{-2}$ , where  $x_f = M_h^2/4m_f^2$  and  $f(x_f)$  is given by Eq. 2.47 in [40]. For much of the parameter space, the gluon fusion rate is also very small, making Higgs detection extremely difficult. In the standard model, the only other production mechanism that does not involve gluon fusion or the  $WW_h$  or  $ZZh$  vertex is Higgs-strahlung off a top quark. That is difficult in the standard model, and in this model is even weaker since the top quark Yukawa coupling is smaller. One can think about Higgs-strahlung off a tau, but this is likely to be swamped by backgrounds.

In any event, this is just a specific model. One might have other possibilities for Higgs production, such as production in the decay of one of the charged Higgs bosons in the model, or production through supersymmetric particles. In both of these scenarios, the production rate would depend on many additional parameters. Thus, experimenters should look for Higgs bosons in the 75–110 GeV range with a substantially enhanced coupling to  $\tau$  pairs (below 75 GeV, a very small sliver of parameter space does remain). A study of  $\tau$  pair detection in leptophilic Higgs decays at the LHC was carried out in Ref. [41]. Since they did not consider the supersymmetric version, they concentrated on Higgs in the 100–160 GeV mass range, and gluon fusion production was not particularly suppressed, as it is here. They also focused on models with dark matter candidates (usually involving an additional singlet or an additional inert doublet). Nonetheless, their techniques show that detection of a Higgs decay into  $\tau$  pairs is feasible in the early stages at the LHC. At the Tevatron, CDF and D0 did explicitly search for Higgs decays to  $\tau$  pairs [42], but did not consider Higgs masses below 90 GeV

Throughout this analysis, we have ignored the effects of the heavier neutral Higgs scalars. Consider the second lightest neutral scalar,  $\eta$ . As we scan the entire allowed parameter space, we find that the  $\eta$  always appears to be very close to 110 GeV. This may not be too surprising. Imagine that there was no mixing at all between the quarkophilic and leptophilic Higgs sectors. Then each sector would have a similar mass matrix to that of the MSSM (although with smaller overall VEVs), and thus one would find two relatively light Higgs. Mixing cannot be eliminated, of course, due to  $D$ -terms, but it is not surprising that there are two relatively light scalars in the model. In the region of parameter space in which the couplings of the  $h$  to the gauge bosons are severely suppressed, however, the couplings of the  $\eta$  will not be, and thus the  $\eta$  will be similar to the standard model Higgs. Given the uncertainty in our calculations, including the effects of non-leading-log and higher order corrections to the masses, it is premature to conclude that the current LEP bounds would rule out this 110 GeV Higgs, but an increase of just a few GeV in the current lower bound on the standard model Higgs would rule out this model.

In the region of parameter space of interest, the  $h$  and  $\eta$  are primarily linear combinations of  $H_0$  and  $H_u$ , with small admixtures of  $H_d$  and  $H_\ell$ . Nonetheless, the ratios of vacuum expectation values are large enough that the dominant decay of the  $h$ , for example, is primarily into  $\tau$ 's and  $b$ 's through these small admixtures. The two heaviest Higgs bosons are each almost entirely  $H_d$  and  $H_\ell$ , respectively, with little mixing.

Consider these two heavier Higgs bosons,  $H_1$  and  $H_2$ . Since the coupling of the  $\eta$ , in the region of interest, to  $Z$  pairs is very close to that of the standard model, then the fact that the sum of the squares of the Higgs couplings to  $Z$  pairs must equal the square of the standard model coupling implies that the coupling of  $H_1$  and  $H_2$  with  $W$ ,  $Z$  pairs is negligible. We have confirmed this numerically. Another way to say this is that the narrow window of parameter space forces the direction of the vacuum expectation value to be almost entirely in the  $\eta$  direction, leaving little room for VEV-dependent couplings of the other neutral Higgs. This will also cause a suppression in the  $H_1hh$  and  $H_2hh$  couplings. The  $H_1$  and  $H_2$  will thus be both Higgs phobic and gauge phobic and will only decay into fermion pairs. One of the two,  $H_1$ , will decay almost entirely into  $b\bar{b}$ , and the other,  $H_2$ , will decay almost entirely into  $\tau^+\tau^-$ . This leads to interesting phenomenological consequences. The  $H_1$  can be copiously produced through gluon fusion (through its coupling to the  $b$  quark), and its dominant decay into  $b\bar{b}$  will be quite dramatic. The  $H_2$  would be a heavy Higgs boson that decays entirely into  $\tau$  pairs. However, gluon fusion occurs at a small rate, and thus production through heavier particles or supersymmetric partners would be necessary. This possibility is currently under investigation.

## VI. CONCLUSION

In this work, we have studied the Higgs sector of the supersymmetric version of leptophilic models. The model contains four Higgs doublets, which couple to the up quarks, down quarks, charged leptons, and no fermions, respectively. The Higgs sector, as in all supersymmetric models, is tightly constrained. We consider constraints from perturbativity, unitarity, the muon anomalous magnetic moment, and we also impose constraints from experimental searches at LEP.

We find that in most of the parameter space, the lightest Higgs,  $h$ , has a mass between 75 and 110 GeV (with a very small sliver of parameter space giving smaller masses). For lighter values of the mass, the decay branching ratio into  $\tau$  pairs is substantial, and can even be the dominant decay mode. This would lead to some spectacular signatures at the Tevatron and the LHC. However, the conventional production mechanisms, such as  $W$  fusion, Higgsstrahlung, and gluon fusion, are suppressed in this region of parameter space.

The second lightest Higgs,  $\eta$ , has a mass throughout the allowed parameter space of approximately 110 GeV.

Its production cross section is not as strongly suppressed, and would appear similar to a standard model Higgs. The remaining two neutral scalars are typically heavier, are gauge phobic and Higgs phobic, and would decay into fermions. One decays almost entirely into  $b\bar{b}$  and would be copiously produced through gluon fusion. The other decays almost entirely into  $\tau^+\tau^-$ , but conventional production mechanisms are suppressed.

There are also three charged scalars and three pseudo-scalars in the model. We do not expect the phenomenology to differ substantially from the detailed analysis of Logan and MacLennan [15], who used MSSM parameters to constrain their parameter space (even though the model was not supersymmetric), and thus there would only be  $\mathcal{O}(1)$  changes in their results due to mixing angles. Exploration of the supersymmetric particles in the model are currently under investigation.

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