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Interactions between Fiscal and Monetary Policy:
A New Keynesian Model with Regime Switching Process

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science in Interdisciplinary Studies from
The College of William & Mary

by

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Interactions between Fiscal and Monetary Policy: A New Keynesian Model with Regime Switching Process

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May 12, 2017

This paper examines the interactions between traditional fiscal and monetary policy tools: government spending and the interest rate. Two models are used: a baseline linear model, and a Markov switching model with active/passive fiscal and monetary policy combinations. The linear model is estimated and the posterior mean parameterization is used to calibrate the regime-switching model. Sims (2002) algorithm and policy function iteration are used to solve the models, and a particle filter is used to evaluate the likelihood functions. The results show that government spending alone cannot raise inflation despite the positive effect on output. The duration of the stimulus effect in output increases significantly under active fiscal regime. The strongest effect occurs when both monetary and fiscal policy are active.

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1 INTRODUCTION

Policy responses during the Great Recession were unprecedentedly aggressive. In the past when the government tried to combat adverse economic situations, take the Great Moderation as an example, the main policy tool was the short-term interest rate, the federal funds rate. However, besides the effective federal funds rate staying at the zero lower bound (ZLB) for more than seven years, which has rendered conventional monetary policy meaningless in this period, the Federal Reserve has implemented unconventional policy tools such as quantitative easing and has paid an extensive amount of attention to forward guidance on the future federal funds rate. On the other side, congress passed the Economic Stimulus Act worth approximately \$152 billion and the Trouble Asset Relief Program (TARP) valued at \$455 billion, which were comprised of tax rebates and liquidity injections.¹ In 2009, congress also passed the American Recovery and Reinvestment Act in 2009, which is a long-term stimulus package nominally worth \$831 billion in total. It includes federal tax relief, improved social security and unemployment benefits, investments in infrastructure projects, and increased expenses in healthcare, education, and scientific research.²

According to projections by the Congressional Budget Office (CBO) in 2009, federal debt as a share of Gross Domestic Product (GDP) was expected to rise from 44% in 2008 to over 100% in 2023, which is high compared to the post World War II share between 20% to 40%. In hindsight, March 2017 CBO projections point out that the short-term growth rate of federal debt was more significant while the long-run expected growth of debt has stabilized as the current situation improves.³ In most times, tax revenues are frequently adjusted to stabilize government debt. However, during recessions, in order to have active fiscal policy response, the government pays more attention on injecting liquidities into the market as opposed to debt stabilization.

¹The financial amount is stated in the following reports: "H.R. 5140, Economic Stimulus Act of 2008," Congressional Budget Office, <https://www.cbo.gov/publication/19471>; "Report on the Troubled Asset Relief Program?October 2012," Congressional Budget Office, <https://www.cbo.gov/publication/43662>.

²The financial amount is stated in "Estimated Impact of the American Recovery and Reinvestment Act on Employment and Economic Output from October 2011 Through December 2011," Congressional Budget Office, <https://www.cbo.gov/publication/43013>.

³Long term budget projections are available at <https://www.cbo.gov/about/products/budget-economic-data>

This paper, following the direction of Davig and Leeper (2011), will shed light on the interactions between monetary and fiscal policies over the business cycle of the U.S. macroeconomy using a data-driven method. It aims to answer the following two questions:

1) Despite the history of monetary policy being more active than fiscal policy, the Great Recession has implied a possible new pattern of economic regulation, with both active fiscal and monetary responses depending on the circumstance. How do they interact with each other?

2) Traditional one-regime dynamic stochastic general equilibrium (DSGE) modeling technique may not be able to produce the rich dynamics demanded by the current economic situation. With regime-switching, a natural question to ask is which policy response combination produces the best outcome under each fiscal/monetary regime.

For this paper, I estimate a log-linear New Keynesian model with exogenous processes for income tax rate, total factor of production (TFP), nominal interest rate, and risk premium, using data from 1986Q1 to 2016Q4. The New Keynesian model is the workhorse model for monetary policy analysis around the world. Thus, it is a natural framework to consider the interactions between monetary and fiscal policy. In my model, the effective return in the economy is defined as the product of nominal interest rate and risk premium. The government in my model spends a fixed proportion of GDP, and is funded by one-period nominal bonds and the labor tax. The bond holdings are completely endogenous and unobservable in the model while tax rate responds to the debt-to-GDP ratio and is susceptible to exogenous shocks. The representative infinitely-lived household in the model is utility maximizing and allocation consumption dynamically. Their utility function values consumption and leisure and they are constant relative risk averse (CRRA). The New Keynesian aspect is fulfilled by monopolistically competitive firms that have some pricing power which takes the form of Rotemberg (1982) costly price adjustments.

In my model, the productivity of labor is only affected by the TFP, which has an exogenous shocks and a permanent component. My model also assumes constant capital stock and thus investment. Thus, the channel of crowding out effect of increase in government spending and that

of crowding in of increase in interest rate do not exist. As common specification of market clearing, consumption moves in the same direction as output. In my model, this provides an answer for an important question – an increase in tax rate, thus government spending, stimulates private consumption, as the increase in output exceeds the increase in the tax paid by households, i.e., the substitution effect outweighs the wealth effect. Sticky price, under this situation, dictates the price to adjust gradually, smooths out inflation, and allows the real variables to evolve.

My baseline model is a linear model without regime switching process. In my alternative model, I introduce a Markov process, where the economy moves between two states (expansion and recession) following a given statistical distribution. This regime switching is plausible and realistic as I will show in Section 4 that the estimated regimes generally follows the empirical evidence. This specification is also in line with how people form their expectations about different economic regimes – that policy state changes over time probabilistically. Although for some almost-fully informed agents, future economic states might be deterministic, most people expect the the economy to change based on current available information. In a dynamic model, this is simplified into a distribution that is contingent on the current state of policy.

The linear model is fully estimated using quarterly data on output, inflation, and interest rate from 1986Q1 to 2016Q4. The posterior mean parameterization is then used to calibrate the regime switching model. The models are solved using Sims (2002) algorithm and policy function iteration. A particle filter is used to evaluate the likelihood function and to produce the filtered likelihood for each policy states in the regime-switching model. My results show that an increase in government spending alone cannot effectively raise inflation, and its positive effect on aggregate output is very small compared to that of an interest cut. However, active fiscal policy does substantially increase the length of the stimulus effect, though not its half-life. A one standard deviation increase in government spending and decrease in interest rate produces the largest effect under the active monetary and active fiscal policy state.

The paper develops as follows: Section 2 reviews recent literature related to this paper; Section

3 presents the baseline model with details in specifications, solution method and estimation procedures, quantitative results and interpretations; following the structure of Section 3, the regime-switching non-linear model is discussed in Section 4, with a brief comment on another regime-switching mechanism; Section 5 talks about future extensions of this paper; Section 6 concludes.

2 LITERATURE REVIEW

There has been an enormous amount of different dynamic stochastic general equilibrium (DSGE) models used for policy analysis. The most prominent structure is the New Keynesian model, in which monetary policy responds aggressively to inflation based on a certain policy rule, in most cases, the Taylor principle. Output gap in New Keynesian models is viewed as inflation pressure. This structure is extensively used among central banks for its ability to accommodate both real and nominal quantities. Christiano et al. (2010) outlined the basic structures of the NK model with two variations that are generally viewed as the second generation of the New Keynesian model. Following their chapter in the Handbook of Macroeconomics, there has been various literatures with richer financial sectors and labor market, as a response to the rise in demand of richer dynamics due to the Great Recession. That distinct event has triggered very interesting policy reactions, which may not be produced with traditional policy specifications, as will later be proven in this paper. Thus, some researchers have also shifted their focus when studying monetary policy, from the Taylor rule to unconventional policies used by the Fed in the past decade, such as forward guidance (McKay 2016). Due to the size of literatures falling under this category, in this section, I only review the ones directly related to my model. Some more literatures are mentioned in Section 5.

Davig and Leeper authored a paper in 2011 in similar topics using sample from 1949Q1 to 2008Q4 with Calvo pricing. Their government demand (spending) is specified as a composite of differentiated goods, g_{jt} , in the same portion as households, using Dixit and Stiglitz (1977) aggregator $G_t = \left[\int_0^1 g_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$. In their paper, they used lump-sum tax rather than proportionate

tax, and exogenous shock is turned on for the lump-sum tax and the interest rate. The government budget and fiscal and monetary rule are:

$$G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} - \frac{(1 + r_{t-1})B_{t-1}}{P_t} \quad (1)$$

$$r_t = \alpha_0(S_t^M) + \alpha_\pi(S_t^M)\pi_t + \alpha_y(S_t^M)y_t + \sigma_r(S_t^M)\varepsilon_t^r \quad (2)$$

$$\tau_t = \gamma_0(S_t^F) + \gamma_b(S_t^F)b_{t-1} + \gamma_y(S_t^F)y_t + \gamma_g(S_t^F)g_t + \sigma_\tau(S_t^F)\varepsilon_t^\tau \quad (3)$$

They specified two states for fiscal and monetary policy respectively: active and passive, thus four combinations in total. Their results showed that active monetary/passive fiscal (AM/PF) combination resembles the dynamics of a typical real business cycle model and Ricardian Equivalence rather than traditional Keynesian framework, where an increase in government spending decreases the life-time wealth of the representative households by decreasing consumption and leisure and increasing working time by a small amount. Under the passive monetary/active fiscal (PM/AF) situation, an increase in government spending, which is expected to persist, increases inflation expectations, and thus decreases interest rates, which then crowds in private consumption by lowering the expected return on saving, creating a larger output multiplier than the AM/PF combination. PM/PF option shows similar dynamics as AM/PF, with higher tax changes; AM/AF is explosive and no equilibrium exists.

In Davig and Leeper (2011), the fiscal multipliers after 5 quarters is around 0.8 for AM/PF regime, and around 1.7 for PM/AF and PM/PF regimes. The permanent change of output regarding to a one unit change in government spending is around 0.86 for AM/PF and 1.36 for PM/AF and PM/PF. Simulation of the government spending paths as implied by the ARRA implies a multiplier of 0.68 under a fixed regime of AM/PF, and 3.00 under PM/AF.

While studying the topic of fiscal-monetary interactions, Davig and Leeper (2011) also touched on the disagreement of the effect of government spending shocks on private consumptions. Barro and King (1984) and Baxter and King (1993) argued that government uses tax revenues to finance

an increase in its spending, thus decreasing the households' wealth and moving private consumption in the same direction. However, several empirical studies have revealed a positive correlation between government spending and private consumption, including Blanchard and Perotti (2002). Gali et al. (2007) and Monacelli and Perotti (2008) both suggested a positive government spending multiplier for private consumption though smaller than unitary. Most researchers in this side of the debate argue that representative households' preference and habit formation ensure a larger rise in real wage and a smaller increase in labor supply when tax increases, making the substitution effect more dominant than the wealth effect.

Besides DSGE models, another method to study such subject is to use vector autoregression (VAR), which approaches the question from empirical perspective, largely based on data instead of structural models. The emphasis of such research often lies in identification and recursive ordering. Recent papers such as Dungey and Fry (2009) usually combine traditional exclusion restrictions with cointegration and sign restrictions to disentangle the shocks under the proposed structural VAR framework. Among the VAR literature, Auerbach and Gorodnichenko (2012) developed a smooth transitioning VAR (STVAR) that allows regime switching while maintaining linearity in the model. It uses the seven-quarter moving average of output growth to produce the likelihood of the economy being in expansion or recession. Different from smooth transition autoregressive models (STAR), differential contemporaneous responses to structural shocks exist in STVAR. In my paper, I will adopt the smooth regime switching mechanism described in Auerbach and Gorodnichenko (2012), calibrate the tuning parameter to match the regimes with NBER published recession periods. However, there are fundamental issues of implementing the STVAR into DSGE models. Details are discussed in section 4.4.

On the topic of regime switching, Markov switching is no doubt one of the most famous and popular mechanisms, in which regime switches according to a known transition matrix and the current state is only dependent on the previous state, i.e. an unobservable state variable following a first-order Markov chain exists in the model. Below is the form of the transition matrix of a two state Markov-switching model ($s_t = 1, 2$). This notation of the transition matrix will be consistent

throughout the paper: the row number (the first index) indicates the state that we are conditioning upon, and the column number (the second index) indicates the state that we are transitioning towards.

$$\mathbf{P} = \begin{bmatrix} \mathbb{P}(s_t = 1 | s_{t-1} = 1) & \mathbb{P}(s_t = 2 | s_{t-1} = 1) \\ \mathbb{P}(s_t = 1 | s_{t-1} = 2) & \mathbb{P}(s_t = 2 | s_{t-1} = 2) \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

Filtering is the core in Bayesian and quasi- maximum likelihood methods as required to evaluate the likelihood of a certain parameter set. When models are linear and the innovations are normally distributed, the likelihood can be calculated analytically using the Kalman filter as in Dynare. However, with non-linear features, the time-series behavior of the model must be produced numerically. There has been multiple extensions of the Kalman filter due to its efficiency, robustness, simplicity, and optimality, for example, the Unscented Kalman Filter (UKF), the Extended Kalman Filter (EKF), and the Sigma-point Kalman Filter (SPKF). In this paper, I use a more intuitive and straightforward sequential Monte Carlo method developed by Gordon, Salmond, and Smith (1993). The basic idea of this simulation-based filter is to use discrete points (particles) to collectively approximate the unknown density. Details of implementation are provided in the following sections.

3 LINEAR MODEL

This section provides the specification of the baseline linear model. The extended model is based largely on this linear model with only a few changes to add in regime switching. Four actors co-exist in this model: a representative forward-looking household which owns all the firms and tries to maximize its lifetime utility; a representative firm that sell a final good as a bundle of intermediate goods; a government which issues a one-year bond and collects proportional taxes; and finally a central bank which acts upon fluctuations in inflation and output. A deterministic trend is assumed to exist in the model that drives economic development besides the growth of

total factor of production (TFP). The trend can be related to the population growth in reality and is the endogenous growth of all factors that drives output growth, in our case TFP and labor supply.

3.1 Model Specification

3.1.1 Households

The household agent chooses $\{c_t, n_t, b_t\}_{t=1}^{\infty}$ to maximize its lifetime utility with constant relative risk aversion (CRRA):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t/z_t)^{1-\gamma} - 1}{1-\gamma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right) \quad (4)$$

Both consumption and labor is assumed to be an aggregate of all consumptions and labor supplies, i.e. $n_t = \int_0^1 n_{it} di$ for labor and $c_t = \int_0^1 c_{it} di$ for consumption. β is the discount factor; z_t is the trend defined with respect to the steady state growth rate, $z_t = z_{t-1} \bar{g}$; $1/\gamma$ is the intertemporal elasticity of substitution, or γ is the risk aversion parameter; $1/\eta$ is the Frisch elasticity of labor supply; and χ is a scaling parameter such that in steady state, the household allocates 1/3 of its time for labor.

In this model, the representative bond has a nominal face value of 1 and a premium discounted by the effective interest rate, defined as the product of the nominal rate i and the risk premium s , of the period when the bond is purchased. At any given time t , the household spend money purchasing goods, bonds, and paying taxes on its labor income, and receives money on real bond payments, labor income, and firm dividends. Thus, its choices are constrained by its budget constraint:

$$c_t + b_t/(i_t s_t) = (1 - \tau_t) w_t n_t + b_{t-1}/\pi_t + d_t \quad (5)$$

Bond serves as a saving instrument for households. Excess income not spent on consumption is used to purchased interest-bearing bonds. The optimality condition of household implies the

following equations:

$$w_t/z_t = \chi n_t^\eta (c_t/z_t)^\gamma (1 - \tau_t)^{-1} \quad (6)$$

$$1 = \beta E_t \left[\left(\frac{c_t/z_t}{c_{t+1}/z_{t+1}} \right)^\gamma \frac{z_t}{z_{t+1}} \frac{s_t \dot{t}_t}{\pi_{t+1}} \right] \quad (7)$$

3.1.2 Firms

In the economy, there is a continuum of identical monopolistically competitive intermediate good producers, indexed by i , producing differentiated intermediate goods, as well as one final good retailer or bundler. Assuming the same TFP and level of development and constant capital across the entire economy, each firm produces y_{it} amount of intermediate goods priced at p_{it} , following $y_{it} = z_t a_t n_{it}$. The final good producer bundles y_{it} together with an aggregator displaying constant elasticity of substitution (CES), in our case, the Dixit-Stiglitz aggregator, $y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$. As all producers are identical and have the same labor demand, the aggregate production function is given by

$$y_t = z_t a_t n_t \quad (8)$$

Our final good bundler takes p_t as given by the market. Thus, its profit is defined as $p_t y_t - \int y_{it} p_{it} di$. The optimal condition means that

$$0 = p_t \frac{dy_t}{dy_{it}} - p_{it} di$$

Based on the definition of y_t , it can be shown that

$$\hat{y}_{it} = \hat{y}_t - \theta(\hat{p}_{it} - \hat{p}_t)$$

From the above, it is clear that θ in the Dixit-Stiglitz aggregator serves as the elasticity of sub-

stitution between intermediate goods. When θ is close to zero, the production function becomes Leontief, in which all inputs are perfect complements; and when θ approaches infinity, all inputs become perfect substitutes. To better serve the reality, in this model, θ is restricted to a value larger than 1.

Since intermediate good producers are monopolistically competitive, each has the market power to set prices of its own goods, rather than only being able to accept the market price. This is the source of nominal rigidity in New Keynesian models. There are two commonly used pricing mechanisms, both of which yield the same dynamics. Calvo (1983) dictates that at each time period, only a portion of the firms can adjust their prices; while Rotemberg (1982) poses adjustment cost to firms, which is quadratic to the percentage of the change and proportional to the aggregate nominal output. Though both will give us the same log-linearized form in terms of aggregate production, Calvo (1983) is more computationally-demanding due to the existence of price dispersion among producers. Here, I proceed with Rotemberg pricing.

Since the firms are maximizing their lifetime profits, the adjustment cost produces the trade-off of paying the costs and keeping the price. Using the functional form in Ireland (1997), the real adjustment cost is specified as $\alpha_{it} = y_t \varphi (p_{i,t} / (\bar{\pi} p_{i,t-1}) - 1)^2 / 2$. The φ parameter controls the scale of adjustment costs. Profits are reflected by dividends, which is $d_{it} = (p_{it}/p_t)y_{it} - w_t n_{it} - \alpha_{it}$. The goal is to maximize the present value of all future dividends, expressed by $E_0 \sum_{t=0}^{\infty} q_{0,t} d_{it}$, where $q_{t,t+1} = \beta \left(\frac{c_t/z_t}{c_{t+1}/z_{t+1}} \right)^\gamma \frac{z_t}{z_{t+1}}$ is the pricing kernel from the bond Euler equation, and $q_{0,0} \equiv 1$, $q_{0,t} \equiv \prod_{j=1}^t q_{j-1,j}$. Assuming symmetry, the optimality condition yields

$$\varphi(\hat{\pi}_t - 1)\hat{\pi}_t = (1 - \theta) + \theta \frac{w_t}{a_t z_t} + \beta \varphi E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\gamma \left(\frac{z_t}{z_{t+1}} \right)^{1-\gamma} (\hat{\pi}_{t+1} - 1)\hat{\pi}_{t+1} \frac{y_{t+1}}{y_t} \right] \quad (9)$$

where $\hat{\pi}$ denotes deviation from steady state, i.e. $\pi_t = \bar{\pi} \hat{\pi}_t$. (For detailed derivation of firm optimization please see Appendix B.)

3.1.3 The Government

In this model, the government spending strictly follows the state of the economy, as a fixed proportion of the output after adjusted for resource loss during the price adjustment process. In other models, it is also common that government spending is specified as a function of bonds, tax revenues, and output, or even as a fixed level. However, I am more interested in studying the dynamics produced by tax shocks, and this specification allows me to simplify the problem as most government spending rises are accompanied by a rise in taxes. Admittedly though, such specification produces a wealth effect feeding back to the output and restricted the calculation of government spending multipliers.

Since this is a closed economy model with constant capital, consumption takes up the rest of the GDP. The government makes revenue through selling bonds and levying taxes, and has expenses in repaying the bonds due at the current period and general government spending. As implied by the government budget constraint, the tax rate should respond to the lagged debt level: when debt level at $t - 1$ is high, the government needs to raise taxes and/or issue more debt to finance the repayment. Empirically, the government measures the debt level by debt-to-GDP ratio, and reacts to the deviation of debt-to-GDP ratio to its preset debt target. So is the tax rule specified here. As an instrument of fiscal policy, tax rate is also susceptible to exogenous shocks. The above implies the following equations:

$$y_t^{gdp} = (1 - \frac{\varphi}{2}(\hat{\pi}_t - 1)^2)y_t \quad (10)$$

$$c_t = (1 - \kappa)y_t^{gdp} \quad (11)$$

$$g_t = \kappa y_t^{gdp} \quad (12)$$

$$b_t/(i_t s_t) + \tau_t w_t n_t \geq b_{t-1}/\bar{\pi} \hat{\pi}_t + g_t \quad (13)$$

$$\tau_t = \bar{\tau} + \delta(b_{t-1}/y_{t-1} - \bar{b}y) + \bar{\sigma}_t \varepsilon_{\tau,t} \quad (14)$$

3.1.4 The Central Bank

The central bank controls the nominal interest rate based on inflation and output:

$$i_t^n = (i_{t-1}^n)^{\rho_i} (\bar{\pi}_t^{\phi_\pi} (y_t/y_{t-1})^{\phi_y})^{1-\rho_i} \exp(\bar{\sigma}_i \varepsilon_{i,t}) \quad (15)$$

ρ_i denotes the persistence of interest rate and is between 0 and 1; ϕ_π, ϕ_y are the responses to inflation and output fluctuations respectively. This Taylor rule follows the dual mandate of the Federal Reserve - stabilize inflation and control unemployment. For simplicity reasons, the zero lower bound (ZLB) is not binding.

3.1.5 Miscellaneous

To allow an exogenous output shock and a different source of an 'interest' shock, the laws of motion of TFP and risk premium are defined as

$$a_t = (1 - \rho_a) + \rho_a a_{t-1} + \bar{\sigma}_a \varepsilon_{a,t} \quad (16)$$

$$s_t = (1 - \rho_s) + \rho_s s_{t-1} + \bar{\sigma}_s \varepsilon_{s,t} \quad (17)$$

where ρ_a, ρ_s denotes the persistence of TFP and risk premium respectively. In this model, all shocks are denoted by ε with a subscript indicating the source of the shock. All shocks are assumed to follow standard normal distribution with no serial correlation or correlation with others.

3.2 Equilibrium and Steady State

Due to the existence of unit root in the original system sketched above, no steady state exists unless the system is detrended. Below is a summary of the detrended system with the tilde indicating

transformed variables.

$$\frac{y_t}{y_{t-1}} = z_t \frac{\tilde{y}_t}{\tilde{y}_{t-1}} \quad (18)$$

$$\tilde{w}_t = \chi n_t^\eta \tilde{c}_t^\gamma (1 - \tau_t)^{-1} \quad (19)$$

$$1 = \left(\frac{\beta}{\bar{g}}\right) E_t \left[\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^\gamma \frac{s_t i_t}{(\bar{\pi} \hat{\pi}_{t+1})} \right] \quad (20)$$

$$\tilde{y}_t = a_t n_t \quad (21)$$

$$\varphi(\hat{\pi}_t - 1)\hat{\pi}_t = (1 - \theta) + \theta(w_t/a_t) + \beta\varphi E_t \left[\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^\gamma (\hat{\pi}_{t+1} - 1)\hat{\pi}_{t+1} \left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right) \right] \quad (22)$$

$$\tilde{y}_t^{gdp} = \left(1 - \frac{\varphi}{2}(\hat{\pi}_t - 1)^2\right)\tilde{y}_t \quad (23)$$

$$\tilde{c}_t = (1 - \kappa)\tilde{y}_t^{gdp} \quad (24)$$

$$\tilde{b}_t/(i_t s_t) + \tau_t \tilde{w}_t n_t = \tilde{b}_{t-1}/(\bar{\pi} \hat{\pi}_t) + \kappa \tilde{y}_t^{gdp} \quad (25)$$

$$\tau_t = \bar{\tau} + \delta(\tilde{b}_{t-1}/\tilde{y}_{t-1} - \bar{b}y) + \bar{\sigma}_\tau \varepsilon_{\tau,t} \quad (26)$$

$$i_t^n = (i_{t-1}^n)^{\rho_i} (\bar{i} \hat{\pi}_t^{\phi_\pi} (\tilde{y}_t/\tilde{y}_{t-1})^{\phi_y})^{1-\rho_i} \exp(\bar{\sigma}_i \varepsilon_{i,t}) \quad (27)$$

$$a_t = (1 - \rho_a) + \rho_a a_{t-1} + \bar{\sigma}_a \varepsilon_{a,t} \quad (28)$$

$$s_t = (1 - \rho_s) + \rho_s s_{t-1} + \bar{\sigma}_s \varepsilon_{s,t} \quad (29)$$

The steady state value of TFP and risk premium is set to equal to 1, while steady state wage, which is also the steady state marginal cost of production, equals to the marginal revenue of the firm as shown in the appendix. In steady state, there is no incentive for the firms to adjust prices, therefore there is no adjustment cost to the economy, meaning $\tilde{y}_t^{gdp} = \tilde{y}_t$. To summarize, steady state implies:

$$\bar{a} = 1 \quad (30)$$

$$\bar{s} = 1 \quad (31)$$

$$\bar{i} = \bar{g}\bar{\pi}/\beta \quad (32)$$

$$\bar{w} = (\theta - 1)/\theta \quad (33)$$

$$\bar{\tau} = (1/\bar{\pi} - 1/\bar{i})\bar{b}\bar{y}/\bar{w} + \kappa/\bar{w} \quad (34)$$

$$\bar{y} = [\bar{w}(1 - \bar{\tau})(1 - \kappa)^\gamma]^{1/(\eta - \gamma)} \quad (35)$$

$$\bar{n} = \bar{y} \quad (36)$$

$$\bar{c} = (1 - \kappa)\bar{y} \quad (37)$$

$$\bar{b} = \bar{b}\bar{y}\bar{y} \quad (38)$$

To transform the above system into a linear system, equations are log-linearized around its steady state. Variables with hats in the log linearized system denotes percentage deviation from steady state.

$$\hat{w}_t = \eta\hat{n}_t + \gamma\hat{c}_t + \frac{\bar{\tau}}{1 - \bar{\tau}}\hat{\tau}_t \quad (39)$$

$$\hat{s}_t + \hat{i}_t + \gamma\hat{c}_t = \gamma E_t\hat{c}_{t+1} + E_t\hat{\pi}_{t+1} \quad (40)$$

$$\hat{y}_t = \hat{a}_t + \hat{n}_t \quad (41)$$

$$\varphi\hat{\pi}_t = \theta\bar{w}(\hat{w}_t - \hat{a}_t) + \beta\varphi E_t\hat{\pi}_{t+1} \quad (42)$$

$$\hat{c}_t = \hat{y}_t \quad (43)$$

$$\hat{y}_t^{gdp} = \hat{y}_t \quad (44)$$

$$\frac{\bar{b}}{\bar{i}\bar{s}}(\hat{b}_t - \hat{i}_t - \hat{s}_t) + \bar{\tau}\bar{w}\bar{n}(\hat{\tau}_t + \hat{w}_t + \hat{n}_t) = \frac{\bar{b}}{\bar{\pi}}\hat{b}_{t-1} + (\kappa\bar{y}\bar{\pi} - \frac{\bar{b}}{\bar{\pi}})\hat{\pi}_t + \kappa\bar{y}\hat{y}_t \quad (45)$$

$$\hat{\tau}_t = \frac{\delta\bar{b}\bar{y}}{\bar{\tau}}\hat{b}_t - \frac{\delta\bar{b}\bar{y}}{\bar{\tau}}\hat{y}_t + \bar{\sigma}_\tau\varepsilon_{\tau,t} \quad (46)$$

$$\hat{i}_t^n = \rho_i\hat{i}_{t-1}^n + (1 - \rho_i)\phi_\pi\hat{\pi}_t + (1 - \rho_i)\phi_y(\hat{y}_t - \hat{y}_t^p) + \bar{\sigma}_i\varepsilon_{i,t} \quad (47)$$

$$\hat{a}_t = \rho_a a_{t-1} + \bar{\sigma}_a\varepsilon_{a,t} \quad (48)$$

$$\hat{s}_t = \rho_s s_{t-1} + \bar{\sigma}_s\varepsilon_{s,t} \quad (49)$$

3.3 Solution, Calibration, and Estimation

3.3.1 Solution Method

The solution method follows Sims (2002) Gensys algorithm, which is designed to solve stochastic linear rational expectations models. The model is mapped into the following representation

$$G_0 X_{t+1} = G_1 X_t + \Psi \varepsilon_{t+1} + \Pi \eta_{t+1}$$

where ε is a vector of exogenous shocks, in my case, the shocks for TFP, risk premium, interest rate, and tax rate, η is a vector of forecast errors, defined as $\eta_{t+1}^x = x_{t+1} - E_t x_{t+1}$, and G_0, G_1, Ψ, Π are functions of μ , our structural parameters. The solution is expressed in a VAR(1) form

$$X_t = T X_{t-1} + M \varepsilon_t$$

where T and M dictates the evolution of the model. Based on the results from Gensys, with parameters in the parameter space shown in Figure 4, my model has a unique and stable solution.

3.3.2 Calibration

I calibrated five poorly estimated parameters based on empirical evidence and previous papers. The calibrated values are summarized below. The discount factor can be calculated as $\beta = \bar{g}\bar{\pi}/\bar{i}$ as shown in the model. However, GDP growth turned out to be a poor proxy for the growth of economy in this case. As implied by the model, the growth of the economy is mainly driven by the growth of total factor of product. Therefore, I chose the quarterly growth of TFP introduced in Fernald (2014) as the proxy. The series takes real-time inputs from the U.S. economy and implements an adjustment for factor utilization. The " β 's" are calculated for each quarter for my

sample period and the mean was taken as the calibrated value for β .

$$\beta = \frac{1}{T} \sum_{t=1}^T \frac{(1 + \frac{G_t}{100})^{0.25} (1 + \frac{\Pi_t}{100})^{0.25}}{(1 + \frac{I_t}{100})^{0.25}} \quad 4$$

Below is the table of calibrated parameters and the base of my calibration.

Parameter	Description	Value	Target
β	Discount factor	0.9984	Fernald (2014)
θ	Elasticity of substitution	6	Average markup over marginal cost equal to 0.2
$1/\eta$	Frisch elasticity of labor supply	3	Peterman (2016)
κ	% of govt spending in GDP	0.2	Empirical evidence from U.S. data
$\bar{b}y$	Debt-to-GDP target	0.75	President's office 2012 budget

Table 1: Baseline Calibration

3.3.3 Estimation

The rest of the parameters are estimated using a Metropolis-Hastings algorithm that utilizes a particle filter to evaluate the likelihood of the posterior distribution. Three observables from 1986Q1–2016Q4 are used for estimation: output, interest rate, and inflation. Output is the calculated real GDP per capita value with GDP series obtained from the National Income and Product Accounts (NIPA) tables and civilian noninstitutional population data from the Bureau of Labor Statistics. For the interest rate, I use the effective federal funds rate published by the Board of Governors of the Federal Reserve System; and inflation is based on the implicit price deflator also available from the NIPA tables. ⁵

⁴Capital letters in this paper generally denotes empirical data used in calibration or estimation, unless previously mentioned in the model specification section

⁵Data are available on the website of St. Louis Fed (<https://fred.stlouisfed.org/>) with series names: GDP, GDPDEF, CNP16OV, FEDFUNDS.

The raw data are then transferred into model implied units:

$$\begin{aligned}\hat{y}_t^{data} &= D \left[\log \left(\frac{GDP_t}{DEF_t/100 \cdot (POP_t \cdot 10^6)} \right) \right] \\ \hat{\pi}_t^{data} &= D[\log(DEF_t)] \\ \hat{i}_t^{data} &= D [\log(1 + FFR_t/100)^{0.25}]\end{aligned}$$

where $D(\cdot)$ indicates first difference. Thus all data are percentage deviations from the last period.

Figure 1 shows the comparisons between forecasts made from the model and the original data after transformation. We can see that both inflation and interest rate are centered around zero. At the end of the interest rate, we can see clearly a period that signifies the ZLB, where interest rate has not been changed following two significant decrease in interest rate. Output is not centered around zero because steady state growth rate is embedded in the series. It is exactly the fact that it does not center around zero that allows a precise estimation of the steady state growth rate. From the graph, we can see that the model predicts the data very precisely.

The estimation procedure is outlined following Figure 1. It generally follows the estimation algorithm in Plante et al. (2016) and uses the adapted particle filter in Herbst and Schorfheide (2016), which is based off on Stewart and McCarty (1992) and Gordon et al. (1993) but is modified such that it better accounts for unexpected fluctuations in the data. My estimation algorithm involves four phases: initial mode search, preliminary Metropolis-Hastings, Metropolis-Hastings (1), and Metropolis-Hastings (2). Let $\Theta \equiv \{\delta, \gamma, \varphi, \phi_\pi, \phi_y, \rho_a, \rho_s, \rho_i, g, \pi, \sigma_i, \sigma_a, \sigma_t, \sigma_s\}$ be a vector of estimated parameters.

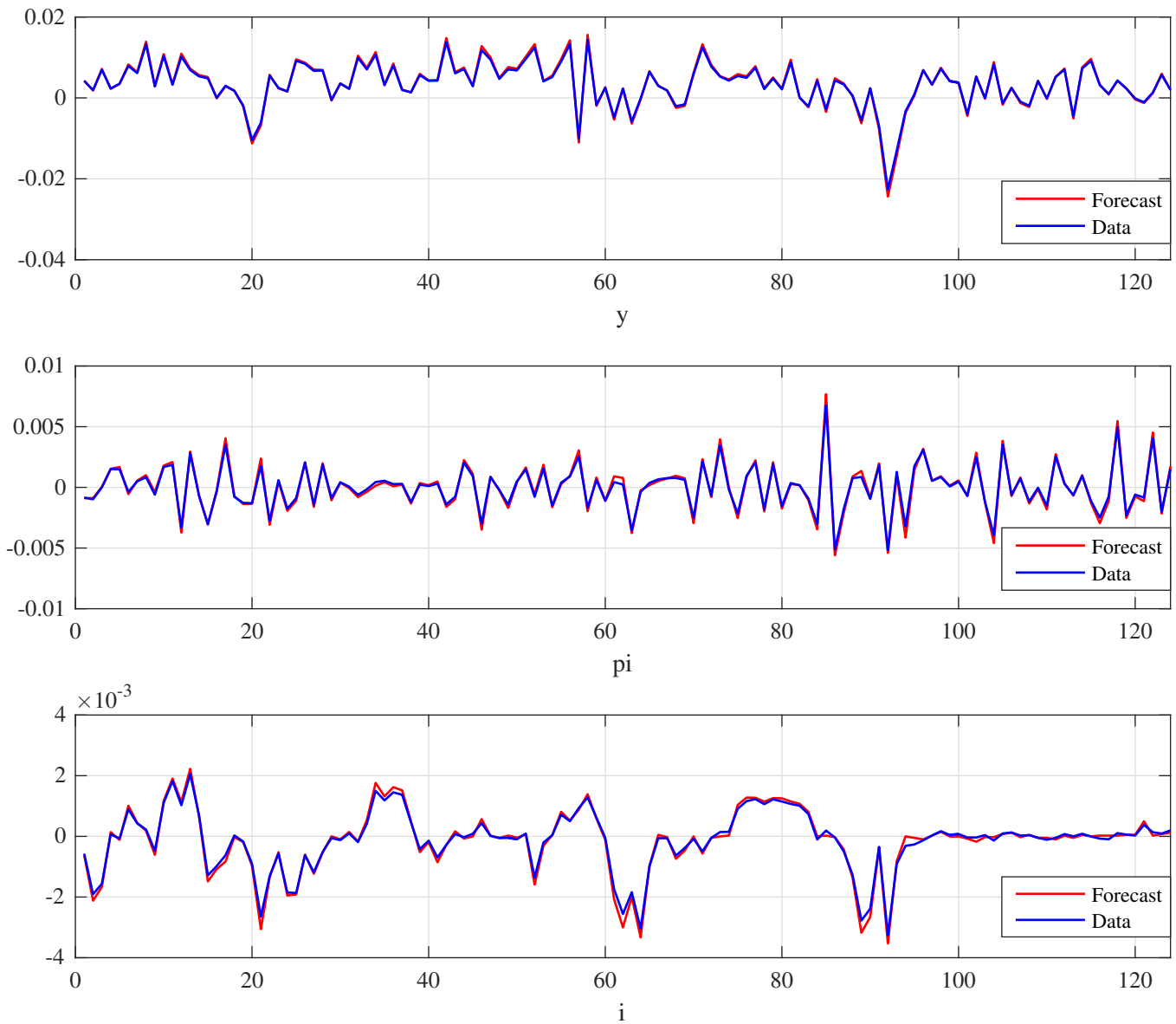


Figure 1: Comparison between forecasts from the model and empirical data

Estimation algorithm: The first step is to randomly draw 10000 set of parameters based on their prior distributions. All parameters are assumed to be independent during the mode search phase. Then the model is solved by gensys using the given parameter values. If a general equilibrium exists, then the algorithm proceeds to using the particle filter to calculate the likelihood for the

model. The posterior log-likelihood is defined as

$$\begin{aligned} \log l_i^{post} &= \log l_i^{prior} + \log l_i^{model} = \log p(\Theta_i | \boldsymbol{\mu}, \boldsymbol{\sigma}) + \log l_i^{model} \\ i &= 1, \dots, 10000 \end{aligned} \quad (50)$$

Then the draws with posterior likelihood above the 90-th percentile, Θ_{sub} , are used to calculate the variance-covariance matrix, Σ , using the following formula, and the draw with the highest posterior log likelihood, Θ_0 is stored for the preliminary MH algorithm.

$$\begin{aligned} \tilde{\Theta} &= \Theta_{sub} - \frac{1}{1000} \sum_{i=1}^{1000} \Theta_i \\ \Sigma &= \tilde{\Theta}'\tilde{\Theta}/1000 \end{aligned} \quad (51)$$

The second step starts with Θ_0 and calculates the posterior log likelihood of this initial draw similar to Equation (50). Θ_0 serves as the first accepted draw, $\hat{\Theta}_1$, with the hat denoting accepted draws. Then the following draws are drawn from $\mathbb{N}(\hat{\Theta}_{i-1}, c\Sigma)$, $i = 1, \dots, 25000$. c is a tuning parameter calibrated such that the acceptance rate is around 21% to 30%, which around the optimal acceptance rate argued in Roberts and Rosenthal (2001). Posterior log likelihoods are then evaluated, and the acceptance rule is as following:

$$\text{accept } \Theta_i^{cand} \text{ if } \exp(\log l_i^{cand} - \log l_{i-1}) > u, \quad u \sim \mathbb{U}(0, 1)$$

If accepted, then $\hat{\Theta}_i = \Theta_i^{cand}$, $\log l_i = \log l_i^{cand}$. If rejected, then $\hat{\Theta}_i = \hat{\Theta}_{i-1}$, $\log l_i = \log l_{i-1}$. After finishing 25,000 draws, the first 5000 draws are discarded because they usually display high volatility and serve as the burn-in period. The rest are used to calculate a $\bar{\Theta}$ and a refined variance-covariance matrix, Σ_1 , as the starting point for the first main MH stage.

The first and second stage of the main MH algorithm follows the same idea of the above preliminary stage. The two main stages are set to make 50000 draws, and the covariance matrix is

recalculated after each stage. Normally after MH(1) is finished, the posterior distributions of the parameters should have already formed their shapes and converged to its true distribution given our sample and our prior specification. MH(2) serves as an extra stage to ensure convergence. Parameter priors and posteriors are reported in the next subsection.

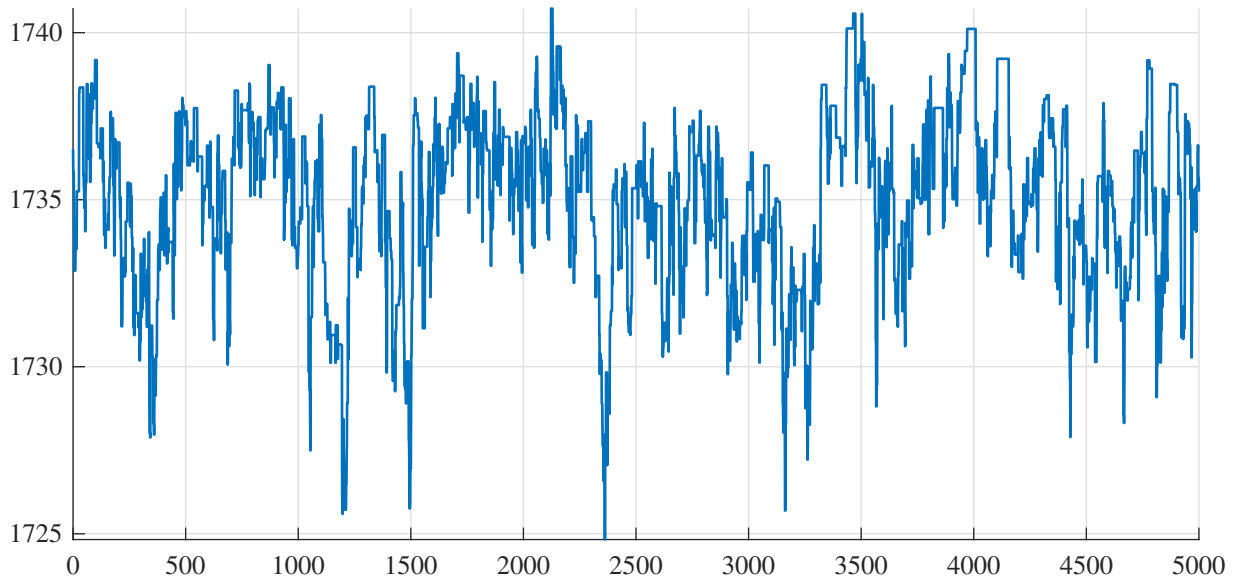


Figure 2: Log Likelihood of the last 5000 accepted draws – MH(2)

Figure 2 shows estimation diagnostics. We can see the clear advantage of Metropolis Hastings algorithm, that it accepts draws with lower log-likelihood probabilistically, thus while exploring the area around a local maximum in our parameter space, and allows the possibility of moving to the next local maximum. However, the acceptance rate ensures that the algorithm would not move too far away from the maxima, as shown by the range of our log-likelihood value which is between 1725 to 1740. The third figure shows the values of each draw after thinning the after-burn draws at 1:100 ratio. The sketch pattern shown in the figures indicates that the accepted values do converge to the posterior modes, as opposed to the case when they do not converge, there would be clear shifts in the graphs.

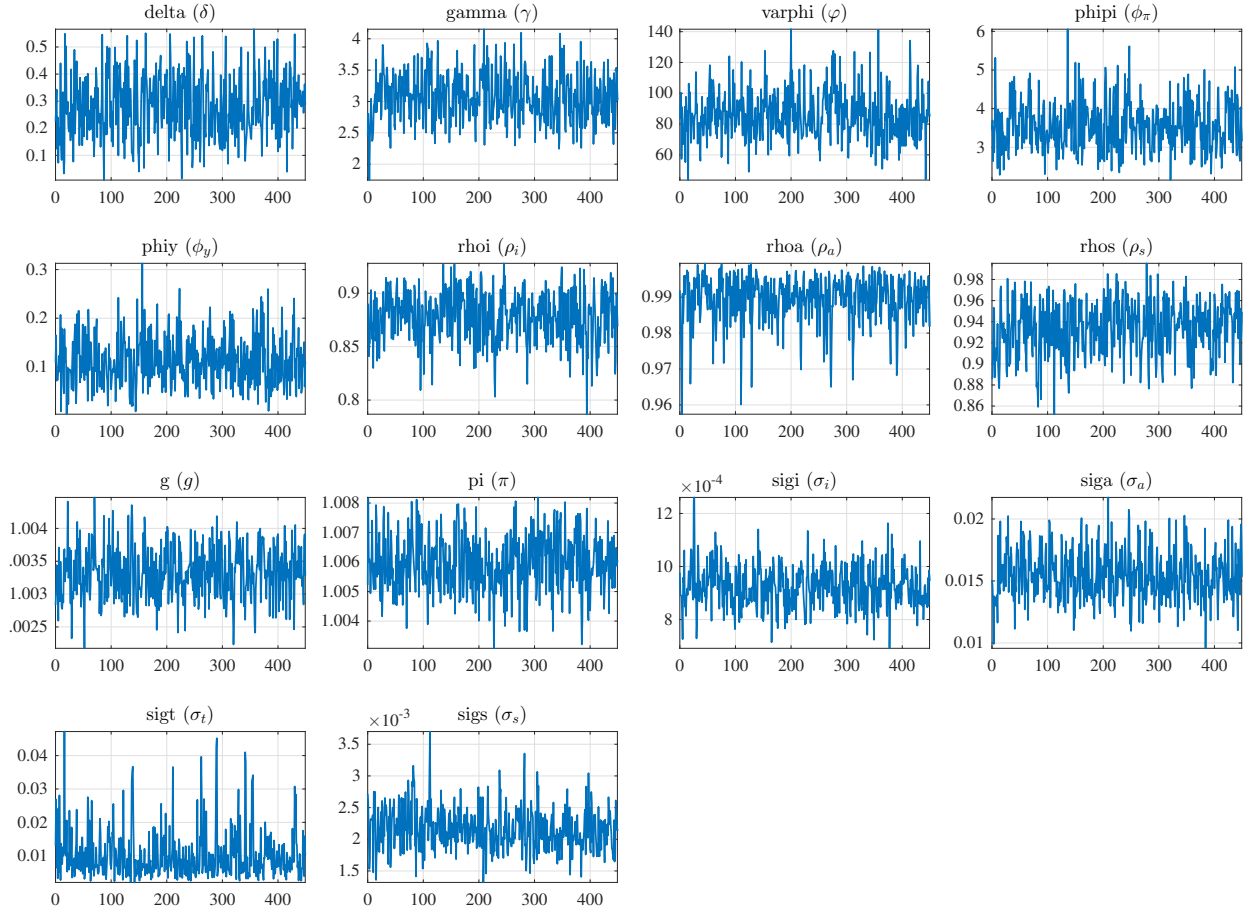


Figure 3: Trace plots of accepted draws after thinning – MH(2)

Adapted particle filter: The particle filter is first initialized by simulating the model 25 times with random shocks $\mathbf{e}_{t,p} = \{\varepsilon_{a,t,p}, \varepsilon_{s,t,p}, \varepsilon_{i,t,p}, \varepsilon_{t,t,p}\}_{t=-24}^0$, where $p \in \{0, \dots, 40000\}$ indicates the index of the particle. The result of the simulation is a draw of the state vector from the ergodic distribution.

The adaption phase gives the mean of the adapted distribution $\bar{\mathbf{e}}_t$, which is solved to maximize $p(\xi_t | \mathbf{z}_t)p(\mathbf{z}_t | \mathbf{z}_{t-1})$, and \mathbf{z}_{t-1} is the state vector. This phase is solved using the `fminsearch` routine of Matlab, which chooses different values of $\bar{\mathbf{e}}_t$ to minimize the objective function defined as

$$-1 \cdot \exp(-\xi_t' H^{-1} \xi_t / 2) \cdot \exp(-\bar{\mathbf{e}}_t' \bar{\mathbf{e}}_t / 2) \quad (52)$$

which is derived from the density of ξ_t , the difference between our model prediction and our actual

data, assumed to be multivariate normally distributed, and from the density of current state given the previous state

$$\begin{aligned}
p(\xi_t | \mathbf{z}_t) &= (2\pi)^{-3/2} |H|^{-1/2} \exp(-\xi_t' H^{-1} \xi_t / 2) \\
p(\mathbf{z}_t | \mathbf{z}_{t-1}) &= (2\pi)^{-3/2} \exp(-\bar{\mathbf{e}}_t' \bar{\mathbf{e}}_t / 2) \\
\Rightarrow p(\xi_t | \mathbf{z}_t) p(\mathbf{z}_t | \mathbf{z}_{t-1}) &\propto \exp(-\xi_t' H^{-1} \xi_t / 2) \cdot \exp(-\bar{\mathbf{e}}_t' \bar{\mathbf{e}}_t / 2) \\
H &\equiv \text{diag}(\sigma_{me, \hat{y}^{data}}^2, \sigma_{me, \hat{\pi}^{data}}^2, \sigma_{me, \hat{i}^{data}}^2)
\end{aligned}$$

The H matrix is defined as a diagonal matrix of measurement error covariance matrix. Without measurement errors, the particle filter suffers from degeneracy. Following Plante et al. (2016), the measurement error variances are set to 10% of the variances of corresponding observable.

After the adaptation, the vector of shocks is drawn, $\mathbf{e}_{t,p} \sim \mathbb{N}(\bar{\mathbf{e}}_t, I)$. Then we can obtain the updated state vector $\mathbf{z}_{t,p}$, and vector of endogenous variables. The unnormalized weight of each particle p is

$$\omega_{t,p} = \frac{p(\xi_{t,p} | \mathbf{z}_{t,p}) p(\mathbf{z}_{t,p} | \mathbf{z}_{t-1,p})}{g(\mathbf{z}_{t,p} | \mathbf{z}_{t-1,p}, \hat{\mathbf{x}}_t^{data})} \propto \frac{\exp(-\xi_{t,p}' H^{-1} \xi_{t,p} / 2) \cdot \exp(-\mathbf{e}_{t,p}' \mathbf{e}_{t,p} / 2)}{\exp(-(\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t)' (\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t) / 2)} \quad (53)$$

Then the model log likelihood of period t can be calculated using

$$\log l_t^{model} = \sum_{p=1}^{N_p} \omega_{t,p} / N_p \quad (54)$$

The weights are then normalized such that the sum equals to 1, and the set of particles is systematically resampled using the algorithm described in Kitagawa (1996). The model log likelihood of the next data point is then calculated following the steps outlined above until we have the log likelihood values for all 124 data points. The model log likelihood of the parameters is defined as the sum of that of individual data points.

3.4 Estimation Results and Analyses

The prior and posterior distributions for the parameters are summarized in Table 2 and a comparison between prior distributions and kernel smoothed density for posterior draws is presented in Figure 3. For the prior values, I generally followed Plante et al. (2016) and Gust et al. (2013), and have set the variances for the prior distributions to larger values, such that they are informative but less restrictive about the posterior modes.

	Prior Distribution					Posterior Distribution			
	Dist.	Mean	Std	Lower	Upper	Mean	0.05	0.95	Std
δ	norm	0.3	0.1	0	10	0.2819	0.096219	0.46842	0.11523
γ	norm	2.31	0.5	0	10	3.0791	2.4347	3.7576	0.4041
φ	gam	99.62	20	10	250	85.66762	61.5012	114.7344	16.3767
ϕ_π	norm	2.44	1	1.01	10	3.5474	2.6127	4.6707	0.62871
ϕ_y	norm	0.5	0.4	0	5	0.11035	0.032672	0.20449	0.052472
ρ_i	beta	0.6	0.2	0	1	0.87784	0.83678	0.91266	0.022854
ρ_a	beta	0.6	0.2	0	1	0.98927	0.97573	0.99738	0.006172
ρ_s	beta	0.6	0.2	0	1	0.9329	0.88741	0.97094	0.024781
\bar{g}	norm	1.004	1E-3	1	2	1.0033	1.0027	1.0039	3.7087E-4
$\bar{\pi}$	norm	1.006	1E-3	1	1.5	1.0059	1.0043	1.0075	9.7449E-4
$\bar{\sigma}_i$	invgam	0.01	0.01	1E-5	1	9.3108E-4	8.0063E-4	1.0825E-3	8.6638E-5
$\bar{\sigma}_a$	invgam	0.01	0.01	1E-5	1	1.5435E-2	1.2283E-2	1.9107E-2	0.00209
$\bar{\sigma}_t$	invgam	0.01	0.01	1E-5	1	1.0525E-2	3.3946E-3	2.6752E-2	7.7213E-3
$\bar{\sigma}_s$	invgam	0.01	0.01	1E-5	1	2.165E-3	1.6734E-3	2.7977E-3	3.4254E-4

Table 2: Prior and posterior distributions of the estimated parameters

From Table 2 and Figure 4, it is shown that most of the parameters are well-informed by the data, especially the interest response to inflation and output, as well as the persistence of TFP, risk premium, and interest rate. The estimated mean and standard error for Rotemberg pricing adjustment cost coefficient (φ) is in line with the estimates from recent literatures, approximately corresponding to a Calvo pricing duration of five quarters. The persistence of interest rate is larger than that in Plante et al. (2016), which can be attributed to the larger post-Great Recession sample included in my model. The steady state quarterly growth rate and inflation rate are 0.33% and 0.59% respectively, very well representing the empirical evidences from my sample period, which are 0.34% and 0.54% respectively. The monetary response to output gap is a bit low comparing

to other estimates, for example, 1.49 in Plante et al. (2016). However, this estimate has fluctuated between 0.1 to 2 in the past. For example, it was around 0.5 in the so-called Taylor 1993 rule, but in later revision, it was changed to 1.0 in Taylor 1999.⁶

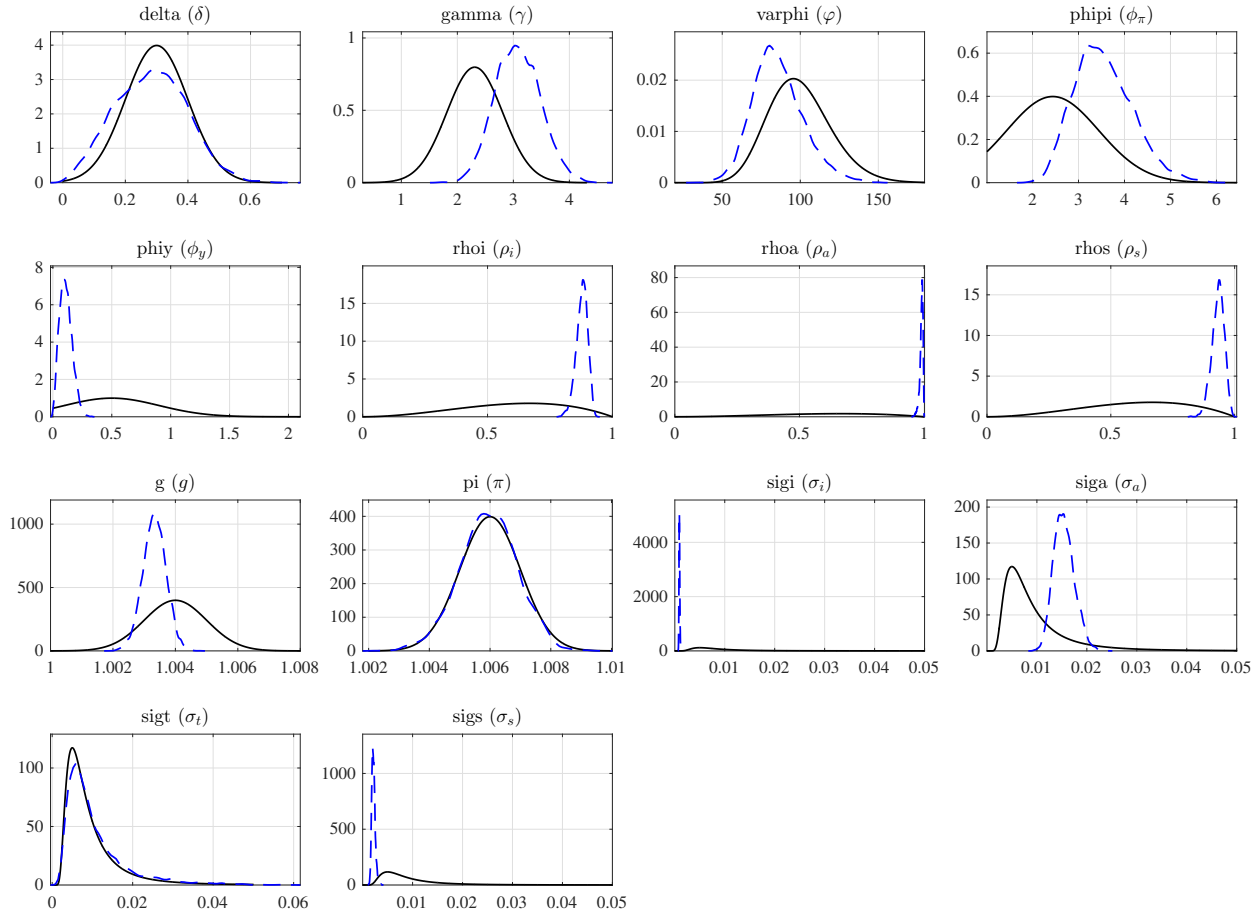


Figure 4: Prior distribution vs. kernel smoothed density for posterior draws

As we can see from Figure 4, that although most parameters are well estimated, the steady state inflation rate and tax response to debt are not informed by the data based on the graph. However, despite of this fact, the two parameters make sense as the posterior mean of inflation moves to the mean of the data, and the tax response parameter has its mean around 0.3, which implies that a switch in the debt target takes the economy about 10 years to adjust, consistent with Congress's budget window, according to Richter and Throckmorton (2015).

⁶See <https://www.kansascityfed.org/publicat/econrev/pdf/12q4Kahn.pdf> for details.

The lack of tax and debt data is speculated to be the cause of the problem. There have been failed attempts to include both series as observables. I have produced the net tax rate according to Blanchard and Perrotti (2002), however, particle filter cannot accommodate the dynamics in the tax rate. For the bond data, the Federal Reserve Bank of San Francisco produces the monthly market value for government bond, which can serve as a good proxy to the one term bond in the model. But because of my specification, government spending is specified as a fixed proportion of GDP and bond serves as an entirely endogenous variable responding to tax deviations and government spending. Thus my model cannot produce the dynamics as we see in the Great Recession, where tax revenue and output fell, but the value of government debt increased extensively in order to fund all the resuscitate measures described in the Introduction section. Davig and Leeper (2011) has included government debt as an observable because of their different way of specifying the policy function for tax rate. In their specification, tax rate responds to debt and output respectively instead of a ratio of the two. Despite the difference, my specification is also valid in the sense that with many reductions and simplifications, my model is less computationally demanding and still produces the basic dynamics as shown in the following impulse response functions.

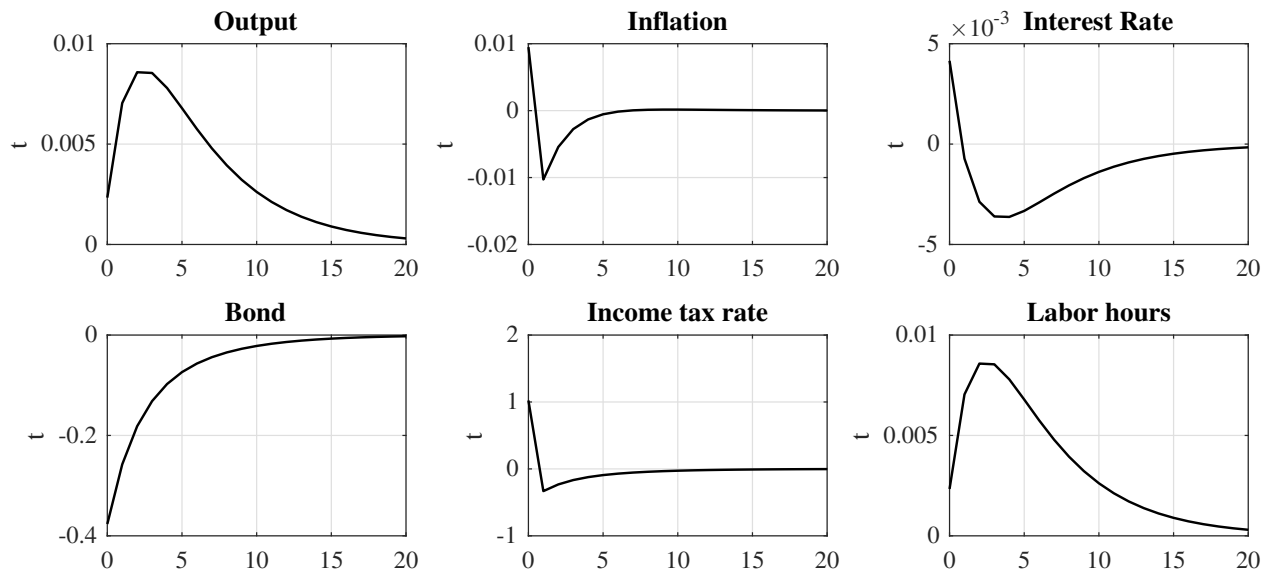


Figure 5: Impulse response function of a 1 standard deviation increase in tax rate

From Figure 5, we can see that an increase in tax rate decreases bond level in the same time

period, forces household to spend. Since tax rate responds to bond-to-GDP ratio, the well-informed household would expect the tax rate to decrease in the future, thus also has the incentive to increase its consumption and thus supplies more labor hours, raising output and lowering interest rate. This is counterintuitive in a sense that an increase in tax is supposed to crowd out private consumption. However, in my specification, as I mentioned before, tax rate is an instrument of fiscal policy, more specifically, it is the way to fund government spending. Thus an increase in tax rate in this model translates to an increase in government spending. The wealth effects take place after the initial time period, as government spending feeds back to output. From the impulse response, we can see that the crowding out of private consumption is outweighed by the increase in the overall output by government spending. The increase in labor hours also signifies a large increase in tax revenue, and therefore less bond is required to finance government spending. However, for inflation, we can see an increase in government spending alone is not enough to increase the inflation rate. This will be discussed more in details when the IRF with interacting fiscal and monetary policy is shown.

Figure 6 on the next page shows the effects of an decrease in interest rate, which is the traditional monetary tool for the Federal Reserve during recessions. A decrease in interest rate disincentivizes households to purchase bonds, as shown by a decrease in the bond holdings, increasing the money demand in the economy as well as the consumption and thus output and inflation. Lower return on bond also increases the labor hours supplied by the household, since people are compensated in another channel rather than mainly labor.

Figure 7 shows the interaction between an increase in government spending and an interest cut. It is clear that output and inflation both rises with these two inflationary policy reactions. When only government spending changes, inflation is not ready to increase, which has been the situation for the period after the Great Recession. During that period, the federal reserve has already hit the zero lower bound (ZLB), thus interest cut is no longer an option (or an impulse) to the economy, and inflation has stayed low until around late 2014 and early 2015. Back to the model, as a result of both interest cut and increased tax revenue, the bond holding decreased significantly, which also helps explain why the model cannot accommodate the data where bond is used to fund most of

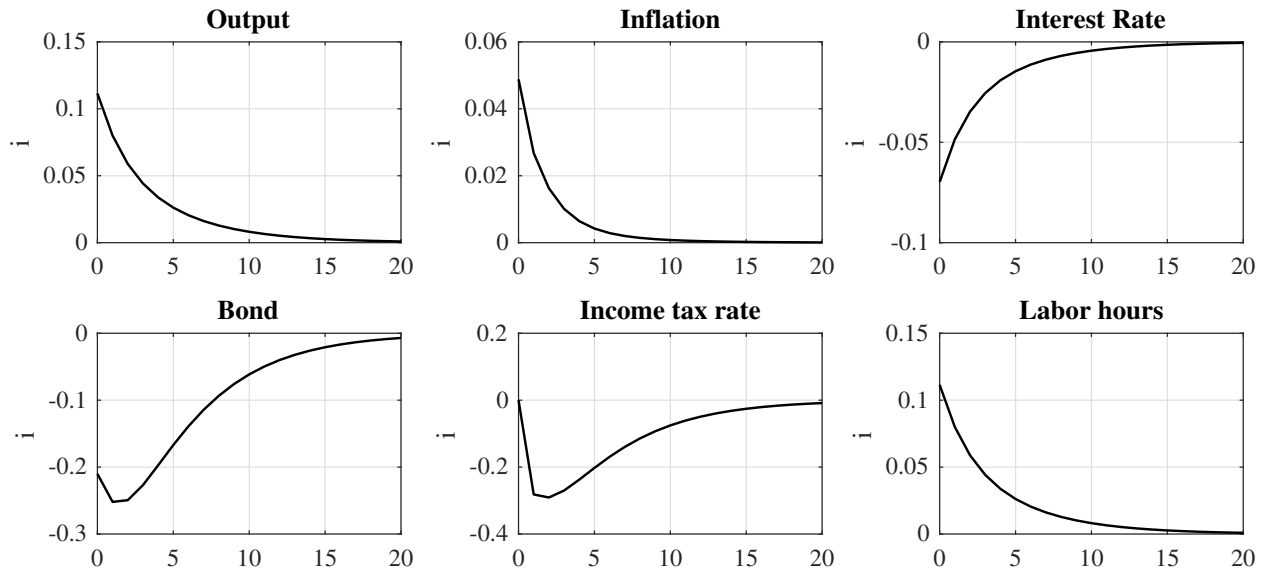


Figure 6: Impulse response function of a 1 standard deviation decrease in interest rate

the government spending during the Great Recession. As for labor hours, an increase in output motivates people to supply more labor, as well as the decreased return on bond holdings. It is clear that the effect on output and inflation is significant with the combination of these two policies, under the condition that interest rate can be lowered, which was not the situation during the Great Recession.

The graphs illustrate the biggest disadvantage of using a linear model without regime switching to analyze policy interactions. After 2007Q4, the Federal Reserve quickly hit the ZLB, leaving only passive policy tools available on the monetary side of the response mechanism. Intuitively and theoretically, such tools do not include manipulating the interest rate in the same way as before, because it has already been 'bound.' Ideally, to study the dynamics, the model should have different regimes determined by passive or active policy states. For better modeling of post-Great Recession periods, the model should use another set of variables driving the decision making process when ZLB is binding, such as uncertainty measures. My next section discusses a specification which aims more towards the general situation.

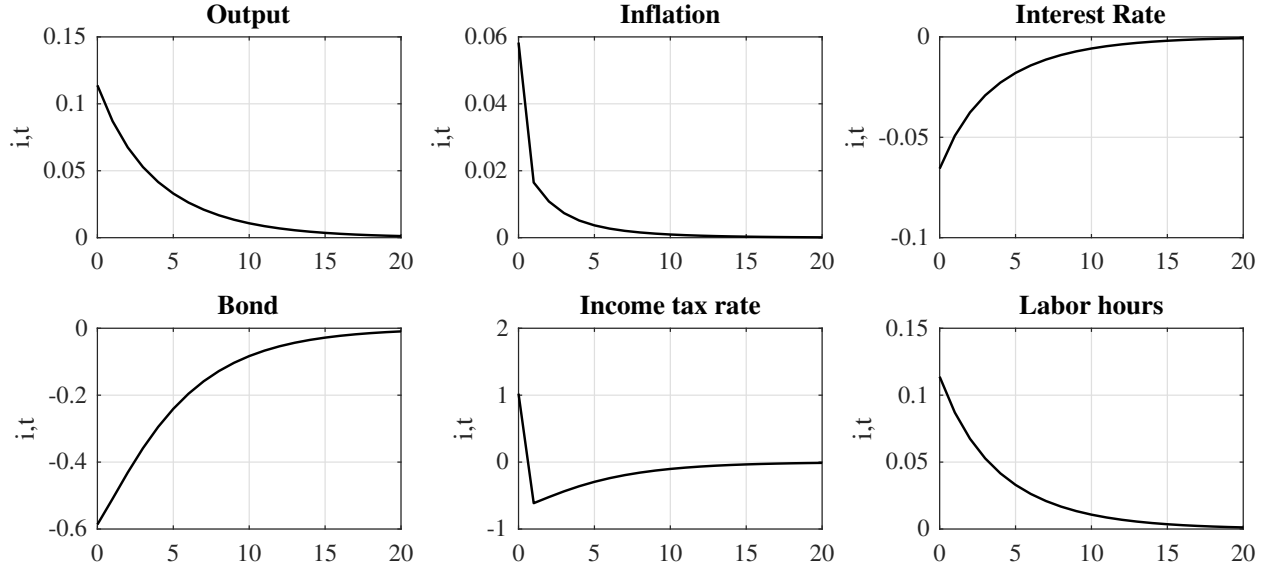


Figure 7: Impulse response function of a 1 standard deviation increase in tax rate and a 1 standard deviation decrease in interest rate

4 REGIME-SWITCHING MODEL

4.1 Model Specification

The regime-switching model discussed here is an extension of the baseline model in the last section. Thus, most of the model specification is adopted except for the policy rules. In order to accommodate for possible policy states, the policy rules are modified into the following form

$$\hat{i}_t^n = (\hat{i}_{t-1}^n)^{\rho_i} (\bar{\pi}_t^{\phi_\pi(S_t^M)}) (y_t/y_{t-1})^{\phi_y(S_t^M)} (1 - \rho_i) \exp(\bar{\sigma}_i \varepsilon_{i,t}) \quad (55)$$

$$\tau_t = \bar{\tau} + \delta(S_t^F)(b_{t-1}/y_{t-1} - \bar{b}y) + \bar{\sigma}_\tau \varepsilon_{\tau,t} \quad (56)$$

where $S_t^X \in \{1, 2\}$, $X \in \{F, M\}$ is the unobserved state variable. The value 1 and 2 correspond to active and passive respectively. With two regimes for each policy type, there are a total of four states in the model. The aggregated state variable S_t takes the following values and follows a

four-state Markov chain.

$$S_t = \begin{cases} 1 \equiv \text{active monetary, active fiscal (AMAF)} \\ 2 \equiv \text{active monetary, passive fiscal (AMPF)} \\ 3 \equiv \text{passive monetary, active fiscal (PMAF)} \\ 4 \equiv \text{passive monetary, passive fiscal (AMAF)} \end{cases}$$

In Davig and Leeper (2011), they also assumed the variance of monetary shocks to take on two different values independent from the coefficients, resulting in a total of four monetary regimes. It is plausible because policy rate can be expected to be more volatile in a certain period of time than the other under the same policy regime. However, the states of the variance would be highly endogenous and unobservable, and even in Davig and Leeper’s paper, they assumed the same policy coefficients for the two ‘variance state’, which implies that the difference in variance does not interfere with policy decisions. Also, as they point out in their paper, such specification do not yield different solution; the only effect lies in the evolution of different shocks. Thus, without compromising the main research question, my model does not take such variations into account.

4.2 Solution and Calibration

4.2.1 Policy Function Iteration

The regime switching feature, specifically the unobserved state, needs me to solve the full non-linear model. I chose the numerical algorithm described in Richter et al. (2014), which iterates on the policy functions until the solution converges.

The model can be written as

$$E[f(\mathbf{v}_t + \mathbf{1}, \mathbf{w}_t + \mathbf{1}, \mathbf{v}_t, \mathbf{w}_t) | \Omega_t] = 0$$

where $\mathbf{v} = (a, s)$ are exogenous variables, $\mathbf{w} = (\tilde{y}, \tilde{y}^{gdp}, \tilde{w}, \tilde{c}, n, i, \pi, \tau, \tilde{b})$ are the endogenous variables, and $f(\cdot)$ is a vector-valued function. The expectation is conditioned upon our information set $\Omega = \{S, P, \mathbf{z}\}$, where S is the steady state value from the linear model, P is the parameter space, and $\mathbf{z} = (\tilde{b}, i, \tilde{y}^{gdp}, a, s, \varepsilon_i, \varepsilon_\tau, S_t)$ is the state vector. Due to the computational limit, the state space is discretized into $(5, 5, 5, 3, 3, 3, 3, 4) = 40500$ points. For $i, \tilde{b}, \tilde{y}^{gdp}$ respectively, the bounds of the state space are set to $\pm 2.5\%$, $\pm 10\%$, $\pm 4\%$ of the steady state value, and the points are equally distributed between the bounds using the `linspace` routine in Matlab. The state space for $a, s, \varepsilon_i, \varepsilon_\tau$ is discretized using Rouwenhorst's method of approximation based on Rouwenhorst (1995) and Kopecky and Suen (2010). Although S_t is a state variable, due to its known property, it is set to have four values: 1, 2, 3, 4. Its transition matrix, which will be specified in the next section, is incorporated into the Rowenhorst weight matrices to form the integration weights ($\Phi(\cdot)$). After initializing the grids, I obtain the initial conjectures for the policy functions, $c_t = f(\mathbf{v}_t, \mathbf{w}_t), \pi_t = g(\mathbf{v}_t, \mathbf{w}_t)$. The initial conjectures are obtained from the solution of the linear system.

The iteration starts by updating the variables at time t except for those containing expectation operators using the calibrated parameters for that specific state. The next step is to use linear interpolation to obtain the updated value for policy functions at each node, which will then be used to calculate the value of the expression inside of the expectation operators in Equation (20) and (22). The value of the expectation operator at time t is obtained by integration with the aforementioned integration weights ($\Phi(\cdot)$)

$$E[f(\mathbf{v}_{t+1}, \mathbf{w}_{t+1}, \mathbf{v}_t, \mathbf{w}_t) | \Omega_t] = \sum \Phi(a_t, s_t, \varepsilon_{i,t}, \varepsilon_{\tau,t}) f(\mathbf{v}_{t+1}, \mathbf{w}_{t+1}, \mathbf{v}_t, \mathbf{w}_t)$$

The above steps are repeated until Chris Sims' `csolve` function finds the optimal value of c_{t+1}, π_{t+1} that satisfy Eqn (20) and (22). The policy function is then updated and the next iteration starts. The algorithm terminates until the value of the policy function converges, meaning the maximum distance between the updated policy function values and the previous ones is less than

the threshold (in my case, $1E-7$).

4.2.2 Simulation and Modified Particle Filter

The simulation algorithm is similar to the above iterations. The aim is to simulate the model enough times such that it converges to the stochastic steady state. At $t = 0$, it is also initialized by the steady state values from the linear model. For $t = 1, \dots, 10000$, it first updates the time t value for the exogenous state variables, and then uses the policy function values from the solution to update consumption and inflation at time t using linear interpolation and Hermite extrapolation, followed by updating all other variables. The values of variables converge after around 100 simulations. Even with stochastic states, the variables still converge to a steady state with no significant decrease in the rate of convergence.

Due to the existence of the unobserved state variable, it is very hard to use the `fminsearch` routine to implement the adapted particle filter. Thus, for this model, I only used the regular particle filter, which draws the four random shocks from normal distribution and the states based on its transition matrix. Unlike the linear model, where the updated state variables can be directly calculated using the T and M matrix, the nonlinear model requires simulation at each single particle due to the difference in state. This significantly decreased the speed of the particle filter, from only couple seconds per time to around 10 minutes per time.

Recall that in Section 3.3.3, following Equation (53), the weights of each particle are then normalized and input into the Kitagawa (1996) algorithm to resample. Here the normalized weights are also used to calculate the probability of that data point being in each state. Therefore, we can obtain the filtered probability of the state variable from the particle filter.

4.2.3 Calibration

Since this model is even more computationally demanding than the linear model, I only calibrated the model based on previous literatures and my linear estimates. Another alternative of estimating

the model is to use VAR estimates to approximate the values of the policy states.

$$\mathbf{P}^F = \begin{bmatrix} 0.94 & 0.06 \\ 0.05 & 0.95 \end{bmatrix} \quad \mathbf{P}^M = \begin{bmatrix} 0.97 & 0.03 \\ 0.01 & 0.99 \end{bmatrix}$$

I obtained the above transition matrices for fiscal and monetary regimes from Davig and Leeper (2011). The entire transition matrix is then calculated by

$$\mathbf{P}^M \otimes \mathbf{P}^F = \begin{bmatrix} 0.9118 & 0.0582 & 0.0282 & 0.0018 \\ 0.0485 & 0.9215 & 0.0015 & 0.0594 \\ 0.0094 & 0.0006 & 0.9306 & 0.0594 \\ 0.0005 & 0.0095 & 0.0495 & 0.9405 \end{bmatrix}$$

Policy parameters are calibrated around the posterior mean from the estimates of the linear model as shown in Table 3. To exacerbate the effect of regime switching, the difference between parameters in different states are set as far as permitted and as not distorting the true dynamics. Different parameter calibration affects the solution time of the model. With the shown calibration, the solution time is around six hours on a four-core machine.

Parameter	Posterior Mean	Active Regime	Passive Regime
ϕ_π	3.5474	4.5	1.01
ϕ_y	0.11035	0.5	0.05
δ	0.2819	0.1	0.5

Table 3: Policy coefficients calibration

4.3 Results and Analyses

Active fiscal policy in this model is specified as the government injecting liquidity into the market by issuing bonds. Thus the tax response to debt in active regime is lower than that in the passive fiscal regime. Active monetary regime is defined as the Federal Reserve actively adjusting the policy rate according to inflation and output. Figure 8 shows the probability of in the active states

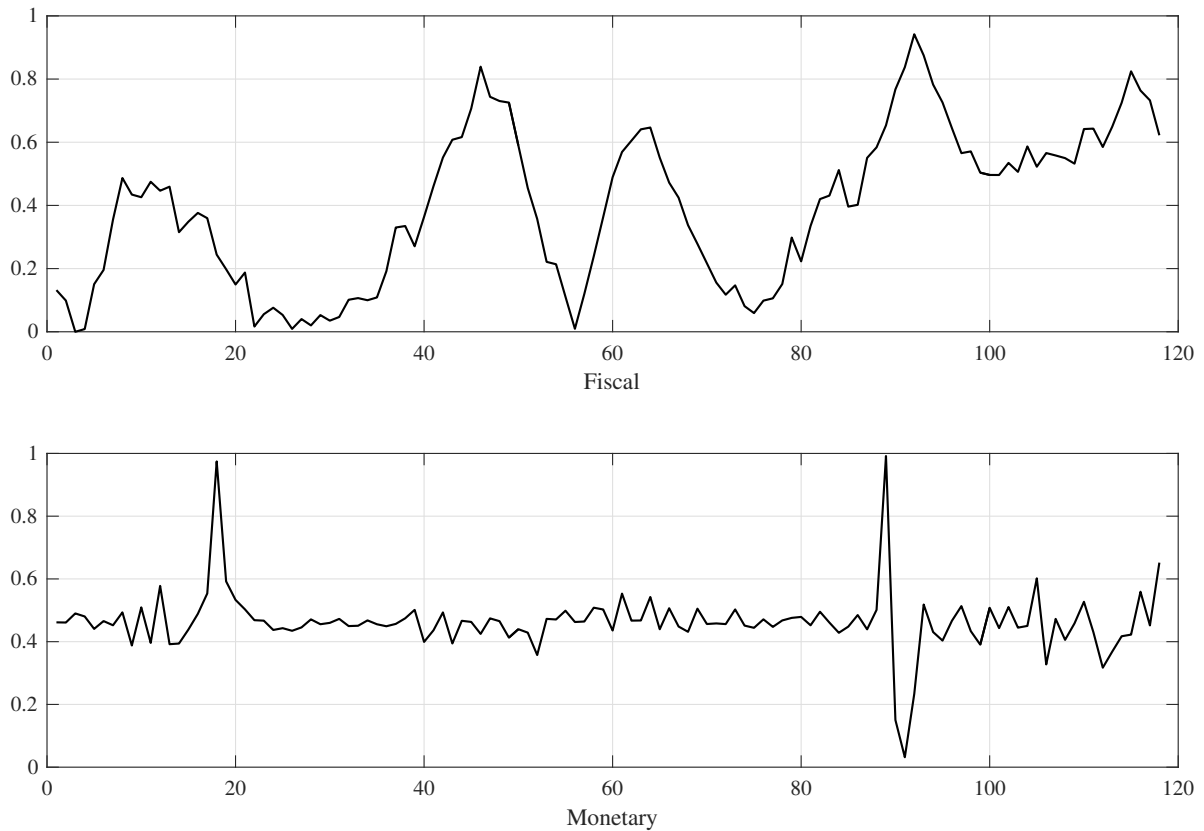


Figure 8: Filtered probability of in the active states

obtained from the particle filter. Overall, this model produces richer dynamics and is more suitable for analysis of policy interactions.

As shown in the graph, the fiscal states are well filtered but it is not the case for the monetary states. For the fiscal states, we can see that the government became active in 1990s when the economy enjoyed fast development, again in early 2000s after the dot-com crisis, and right after the Great Recession. The graph generally resembles the one shown in Davig and Leeper (2009). On the monetary side, the filter does not yield satisfactory results. But at around 2008, it does capture the shift in monetary policy when Federal Reserve quickly lowered the policy rate and hit the ZLB.

It is worth noting that unlike what is produced by the linear model, although an increase in tax

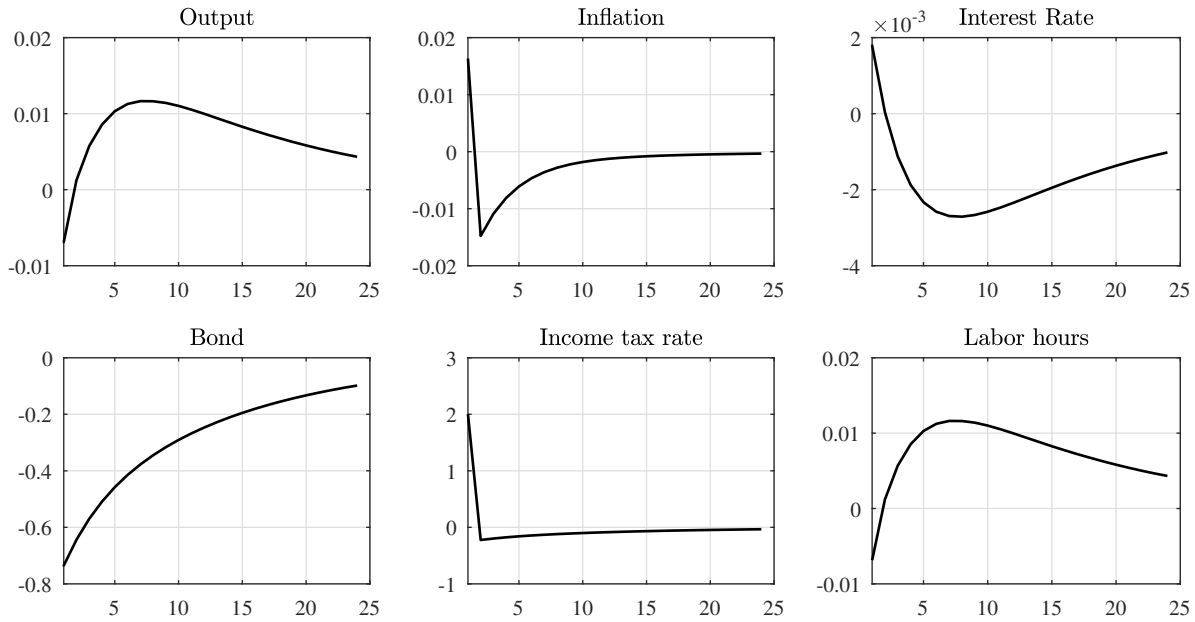


Figure 9: Impulse response function of a 1 standard deviation increase in tax rate under PMAF regime

rate still represents an increase in government spending, such impulse results in an initial crowding out of private consumption in Figure 9. However, the household expects the tax rate to decrease in the future given a lower bond level, thus labor supply increases and consumption also increases soon after the initial impulse as a result of utility smoothing. Similar to Davig and Leeper (2011), under the PMAF regime, an increase in government spending persists, as shown in the above graph by the response of output (because government spending is specified as a proportion of the output). This is the state that is the closest to the the situation in the Great Recession. From the graph, same conclusion as the linear model can be reached: increase in government spending alone is not enough to raise the inflation rate. In this nonlinear model, the crowding out effect is significant enough to decrease the inflation rate during the first couple quarters. Under this regime, tax rate does not respond aggressively towards changes in the debt level. Thus we can see that the change in bond level persists even after 20 quarters, and the tax increase dies out quickly after the initial impulse. The pattern for the interest rate can be attributed to it responding to the inflation change. The passivity of monetary policy is reflected in the scale.

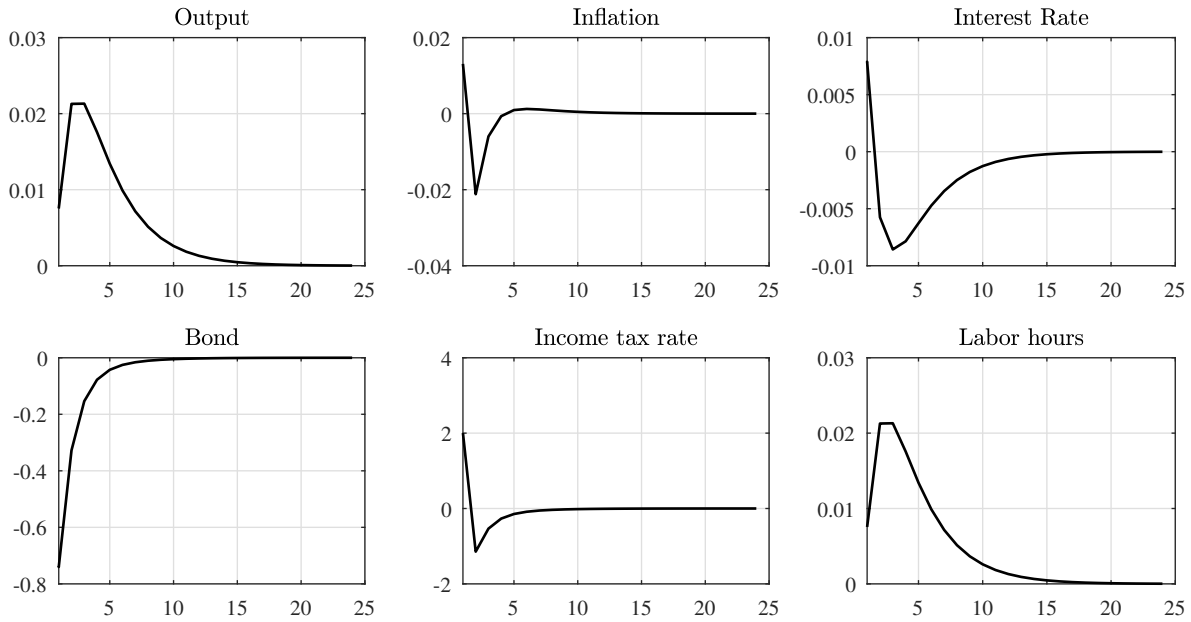


Figure 10: Impulse response function of a 1 standard deviation increase in tax rate under AMPF regime

With Active Monetary/Passive Fiscal combination, Davig and Leeper (2011) argued that it resembles the dynamics of a RBC model and Ricardian Equivalence better than the New Keynesian Model. Figure 10 agrees with their conclusion. However, due to the fact that my model has distortionary tax instead of lump sum tax, there are some variations. Although the tax rate has increased, the crowding out on private consumption is not obvious. An increase in government spending does decrease the household's lifetime wealth, lowers its inflation expectation, and increases its labor supply by a small amount. But the effect of lowered inflation expectation is very quickly followed by a sharp decrease in interest rate as the result of the active monetary policy specification. Generally speaking, this regime displays similar dynamics as the AMAF regime.

For Figure 7 in the last section, I argued that although an increase in government spending and a lowered interest rate can have strong inflationary results, but the situation is not likely to happen during the Great Recession. The impulse response of the PMAF regime supports my argument by giving the interest rate a very small impulse. (See Appendix A.) Even if the Federal Reserve were able to further lower the interest rate, the impulse responses would still be dominated by the effect

of an increased tax rate, and the inflation rate would not rise either.

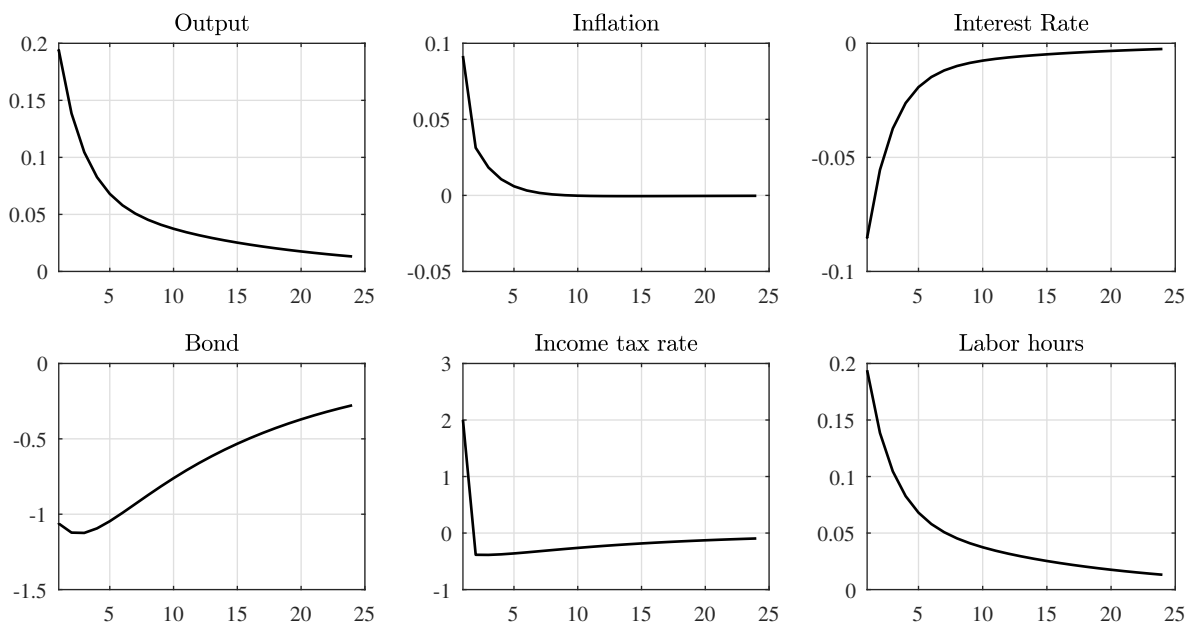


Figure 11: Impulse response function of a 1 standard deviation increase in tax rate and a 1 standard deviation decrease in interest rate under AMAF regime

The combination of the policy responses is the most effective under AMAF regime, where inflation response stays positive and output response quickly becomes positive after the interest rate adjusts and the positive effect still exists 20 periods after the initial shock. With a decrease in bond returns and a decrease in the need of bonds to fund government spending, bond level decreases significantly and the change persists even after 40 periods.

Table 4 summarizes the impulse responses under different regimes with a 1 standard deviation shock. From the table, it is clear that the duration of the effects differ significantly among different fiscal regimes, and tax shocks contribute more to inflation responses comparing to output responses. The effects from 1 standard deviation tax shock are smaller than that of an interest shock with the same magnitude. However, one thing to note is that during the Great Recession, the Federal Reserve was unable to adjust the interest rate while tax shocks were once as high as 6 standard deviations. The standard deviation implied from empirical evidence of a linear regression is also three times as large as the one in our model parameterization.

Impulse		Output Response				Inflation Response			
Var.	State	max	max qt.	half	end	max	max qt.	half	end
i, τ	AMAF	0.1908	1	3	92	0.0911	1	1	9
	AMPF	0.2133	1	3	24	0.0743	1	1	19
	PMAF	0.3600	1	4	96	0.1811	1	2	12
	PMPF	0.3683	1	4	37	0.1560	1	1	32
i	AMAF	0.1922	1	2	84	0.0686	1	2	11
	AMPF	0.2048	1	2	23	0.0609	1	1	19
	PMAF	0.3645	1	3	92	0.1644	1	2	13
	PMPF	0.3709	1	4	37	0.1477	1	2	32
τ	AMAF	0.0185	6	17	80	0.0224	1	1	27
	AMPF	0.0218	3	5	20	0.0133	1	1	15
	PMAF	0.0119	8	19	75	0.0166	1	1	40
	PMPF	0.0107	3	7	25	0.0083	1	1	20

Table 4: Summary of impulse responses under different regimes with a 1 standard deviation shock

4.4 Comments on the Smooth Transitioning Method

The biggest advantage of the smooth transitioning model described in Auerbach and Gorodnichenko (2012) is that it preserves the linear feature of the model, by including the regime switching as an observable rather than an unobservable. The state variable is treated as a continuous variable bounded by 0 and 1, and is expressed as the probability of being in one state. The state is calculated using the following formula

$$F(z_t) = \frac{\exp(-\lambda(z_t - \gamma))}{1 + \exp(-\lambda(z_t - \gamma))} \quad (57)$$

where z_t is the seven quarter moving-average of the GDP growth, and γ is a tuning parameter, such that $t : F(z_t) > 0.8$, which signifies a recession, is roughly 21% of the entire sample, corresponding to the business cycles published by the NBER.⁷ In my calculation, λ and γ are calibrated to 3 and 0.8 respectively. The graph below shows my replication of the probability of being in a recession, which generally resembles the graph in their paper.

The implementation of this method in a structural VAR problem is relatively easy compared

⁷<http://www.nber.org/cycles.html>

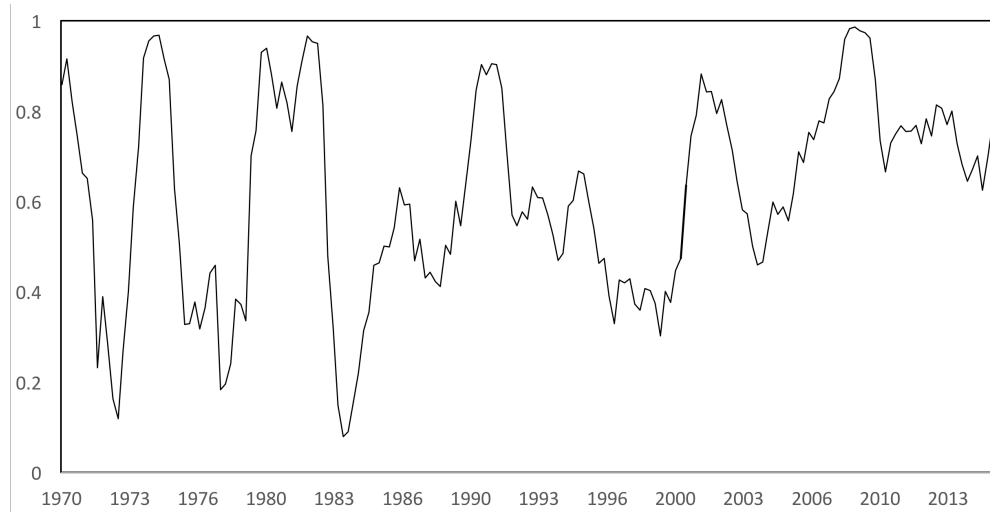


Figure 12: Probability of being in a recession using the smooth transitioning method

with implementing it to estimate a DSGE model. Two sets of coefficients are estimated in the VAR, and for each time period, we can view the usual VAR coefficients as a weighted average of the two sets of coefficients. However, to estimate a DSGE model, the solution depends on the values of the parameters. To implement this method, the model needs to be solved every time we move to the next data point in the filtering process, as opposed to the current linear model, where one solution is obtained for 124 data points. Due to the difference in parameters, when the algorithm moves to the next data point, the weights for the resampling process may no longer be valid, thus decreasing the efficiency of the particle filter.

However, the true difficulty of implementing this method lies in its foundation. This empirical based method is suitable for VAR or other empirical methods without any doubt, but it is very hard to be extended to be included in structural modeling. For example, without good proxies, it is almost impossible and implausible to construct the regimes for fiscal policy and monetary policy individually following this method.

5 FUTURE EXTENSIONS

Due to the constraint on time, computing power, and my own knowledge, this project only scratched the surface of fiscal monetary interactions. Given the above sections, a natural extension should be a better specified government spending and fiscal policy rule. VAR estimation should also be included to confront the dynamics and estimates from the structural model. More thoughts can also be given to the regime switching part. For example, how can the regimes better represent periods before and after the Great Recession, the period when the ZLB was binding, and even the current period, where the ZLB is no longer binding and the economy is recovery but it is not safe to say that the current period is in the same state as any period before the Great Recession happened.

Besides the traditional Markovian switching mechanism, other mechanisms can also be used, for example an extension of the smooth transitioning regime switching. Chang et al. (2017) has developed a similar approach where an autoregressive latent factor determines regimes by examining the value against a threshold level. The difference between it and the standard Markov switching is that it is endogenous or weakly exogenous, and is allowed to be correlated with the innovations. Different from Auerbach and Gorodnichenko's approach, the state variable is still discrete rather than continuous. This method is also developed under a single-equation framework, but current work is being done to extend it into a DSGE model.

Uncertainty measures and stochastic volatility can also be included in the model to produce richer dynamics. Uncertainty measures signify market confidence, which is very important in the post-Great Recession period. Multiple literature has emerged discussing the relationship between market confidence and economic growth, or between forward guidance and economic growth, such as Plante et al. (2016), the paper that is heavily cited in this project, as well as McKay (2016).

However, after all, a direct upgrade would be to conduct a full-scale estimation for the non-linear model. Even according to Dr. Leeper himself, this would be a very significant extension for Davig and Leeper (2011) as well.

6 CONCLUSION

This paper studies the dynamics of interactions between fiscal and monetary policy. The results are obtained from a baseline linear model and a nonlinear model with Markov switching. In both models, the government spending and consumption are specified as a fixed proportion of the aggregate income. Proportional tax on labor income is implemented as a method to fund government spending, meaning that an exogenous rise in the tax rate signifies an increase in government spending. The models are solved using data-driven methods and impulse responses are obtained.

Both models show that only an increase in government spending is able to increase output but is not enough to raise inflation expectations. Under certain regimes, where the crowding out effect of government spending on private consumption dominates the dynamics of the first several quarters, such change can have significant disinflationary impact. Agreeing with Davig and Leeper (2011), I conclude that the effect of increased government spending is likely to persist under passive monetary and active fiscal regime; while active monetary and passive fiscal regime resembles Ricardian Equivalence. The policy combination of lowering interest rate and increasing government spending is most effective under active monetary and active fiscal combination, which can be the policy state that we are currently in or are about to step in. This model is not built to accommodate unconventional monetary policies that have been implemented in the past few years, but there is sufficient empirical evidence suggesting that the Federal Reserve is ready to become active again.

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A Impulse Responses for the Regime Switching Model

A.1 1 standard deviation negative shock on interest rate

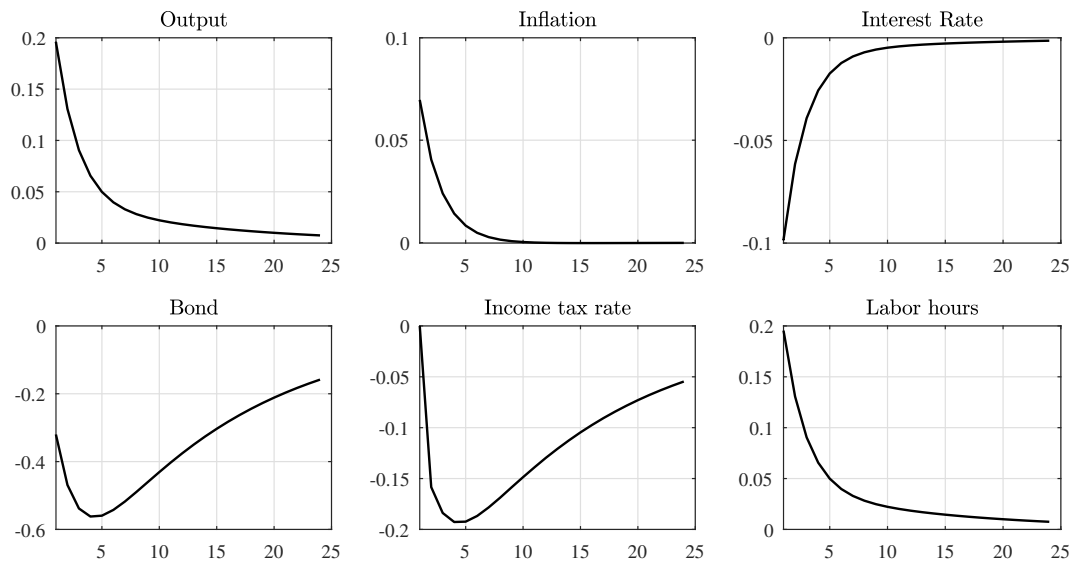


Figure 13: A decrease in interest rate in AMAF regime

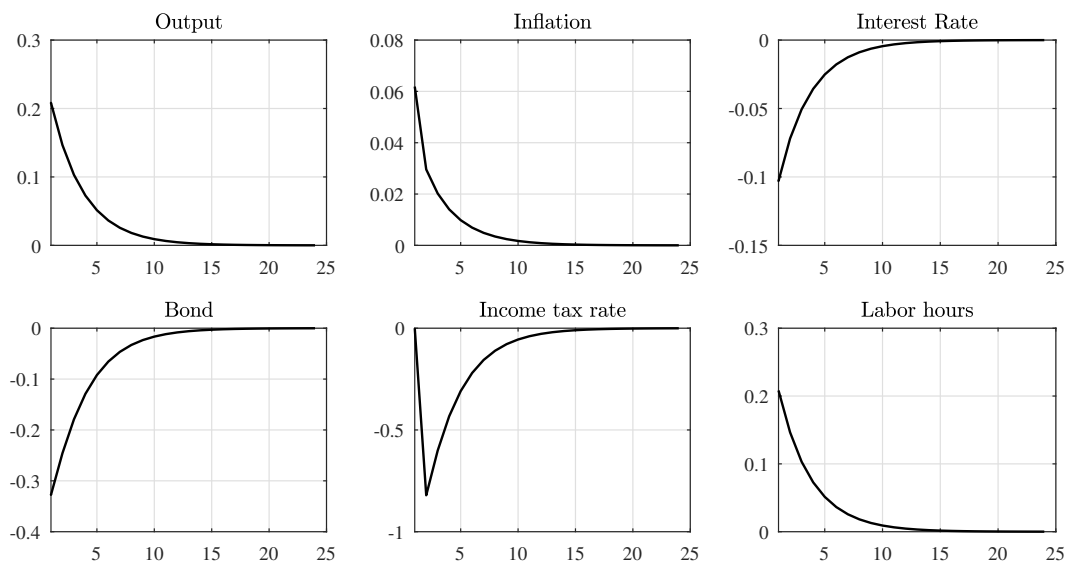


Figure 14: A decrease in interest rate in AMPF regime

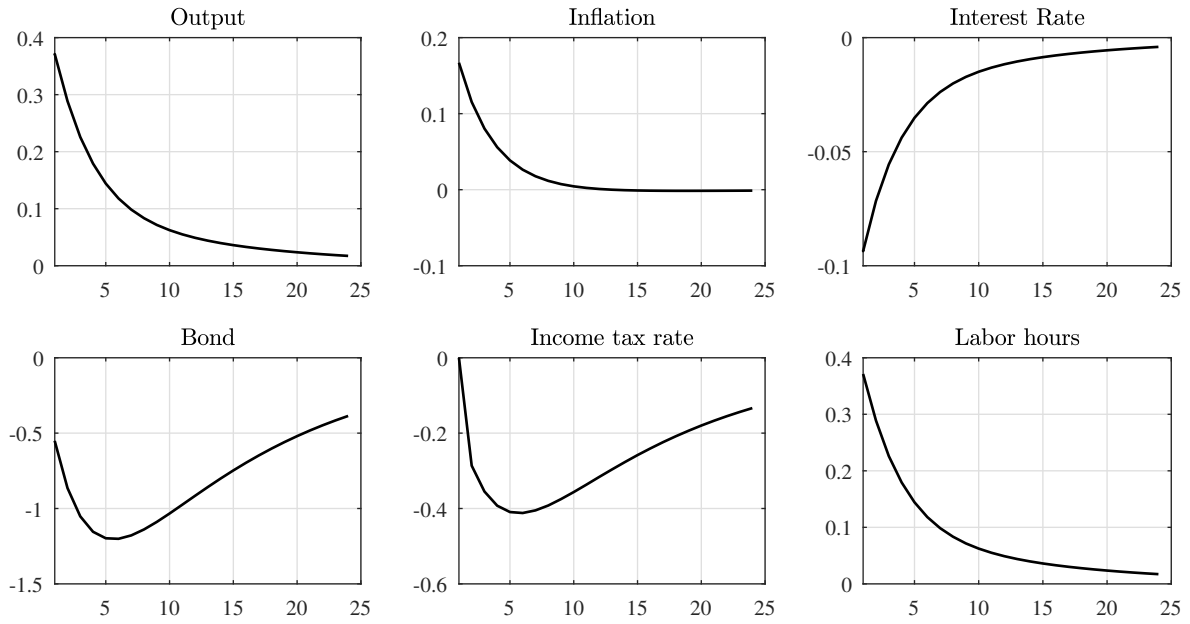


Figure 15: A decrease in interest rate in PMAF regime

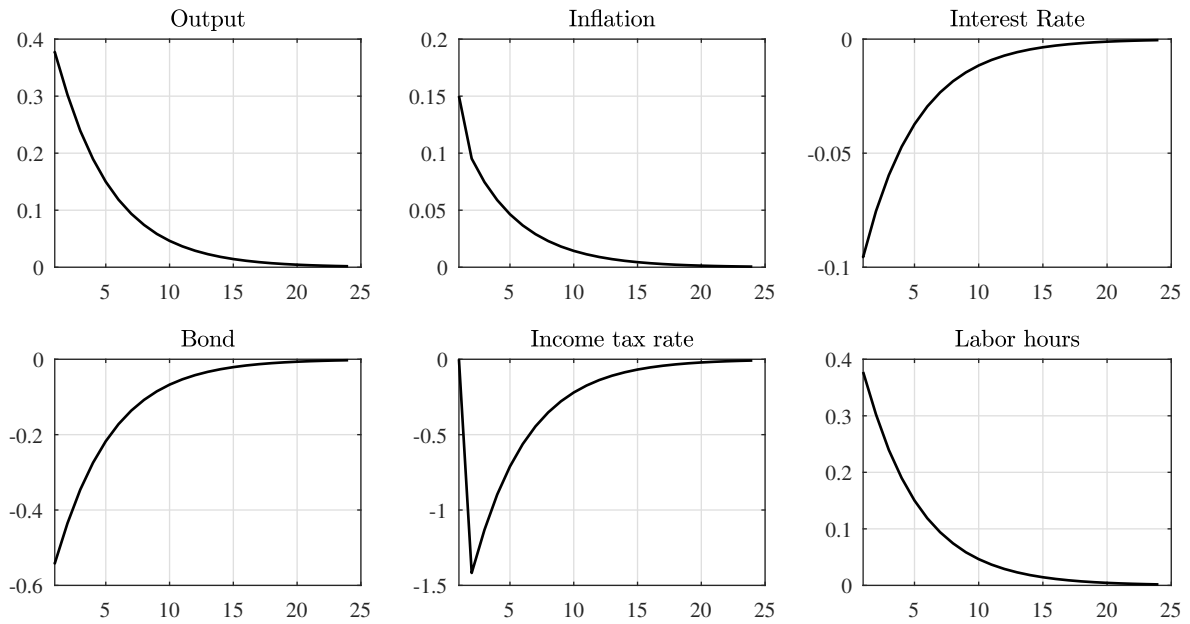


Figure 16: A decrease in interest rate in PMPF regime

A.2 1 standard deviation positive shock on tax rate

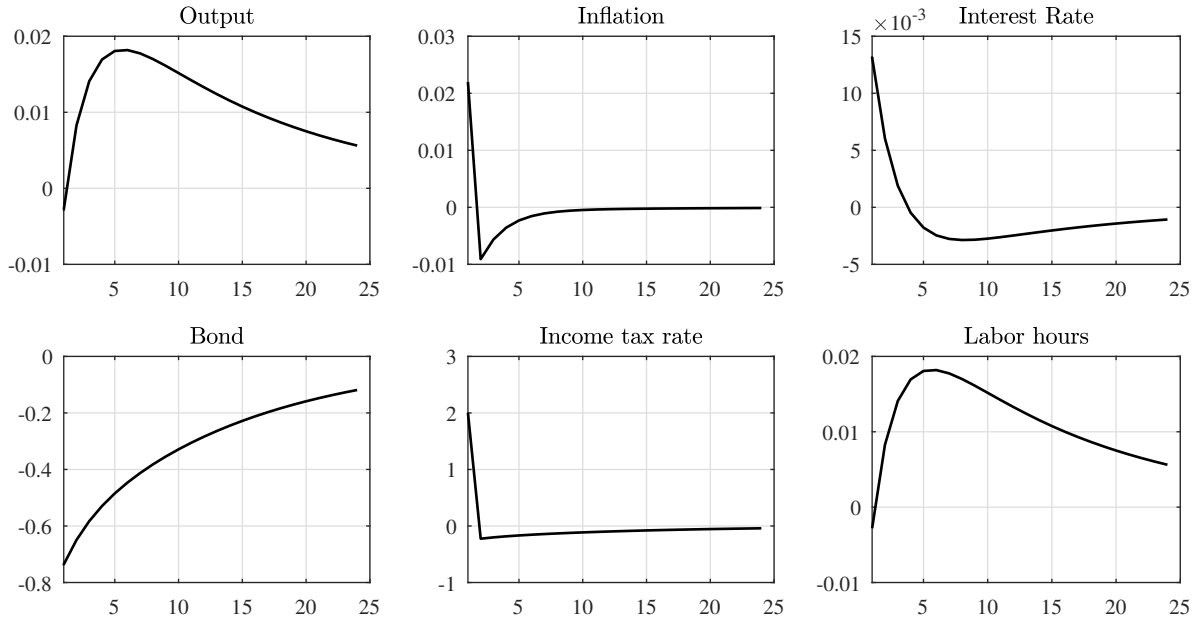


Figure 17: An increase in tax rate in AMAF regime

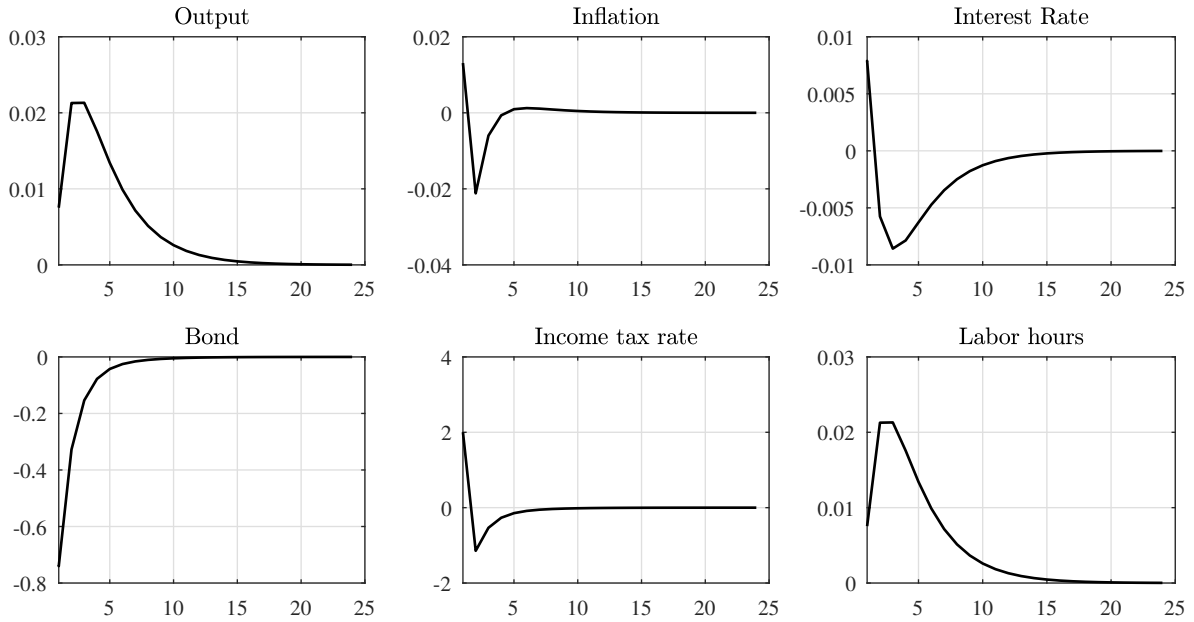


Figure 18: An increase in tax rate in AMPF regime

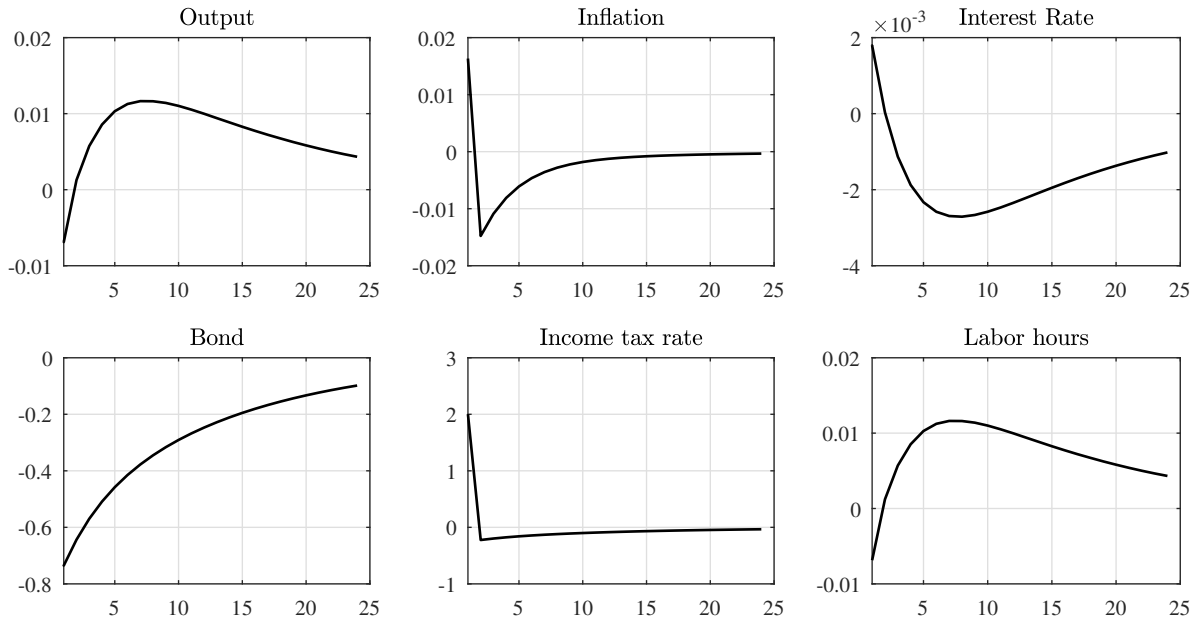


Figure 19: An increase in tax rate in PMAF regime

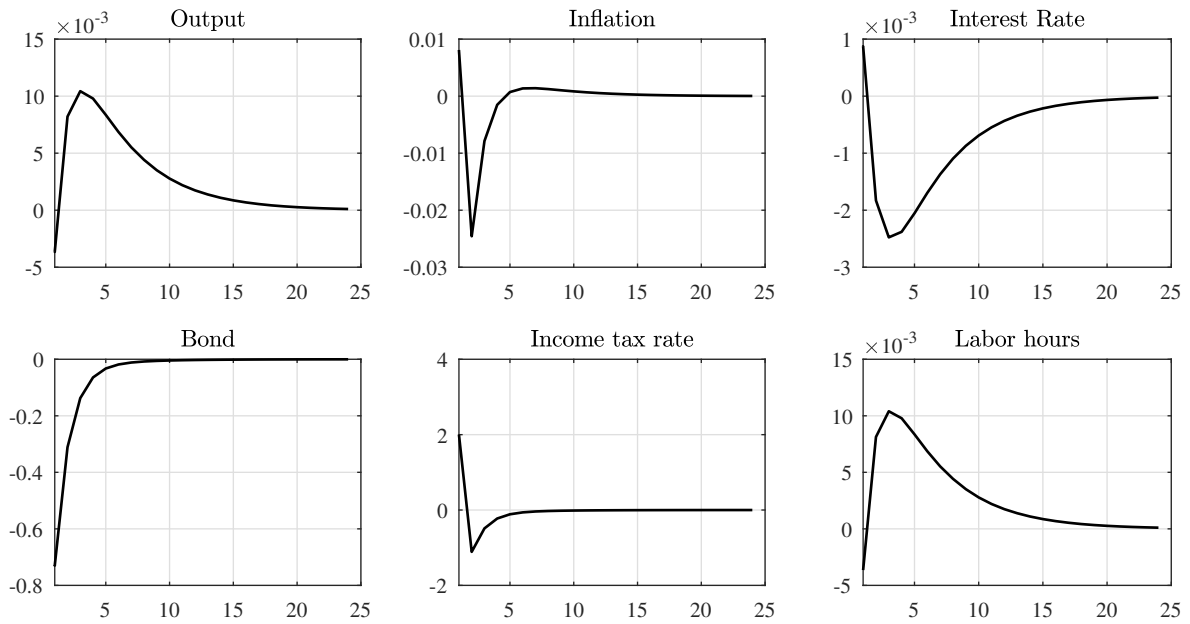


Figure 20: An increase in tax rate in PMPF regime

A.3 1 st.dev. positive shock on tax rate and negative shock on interest rate

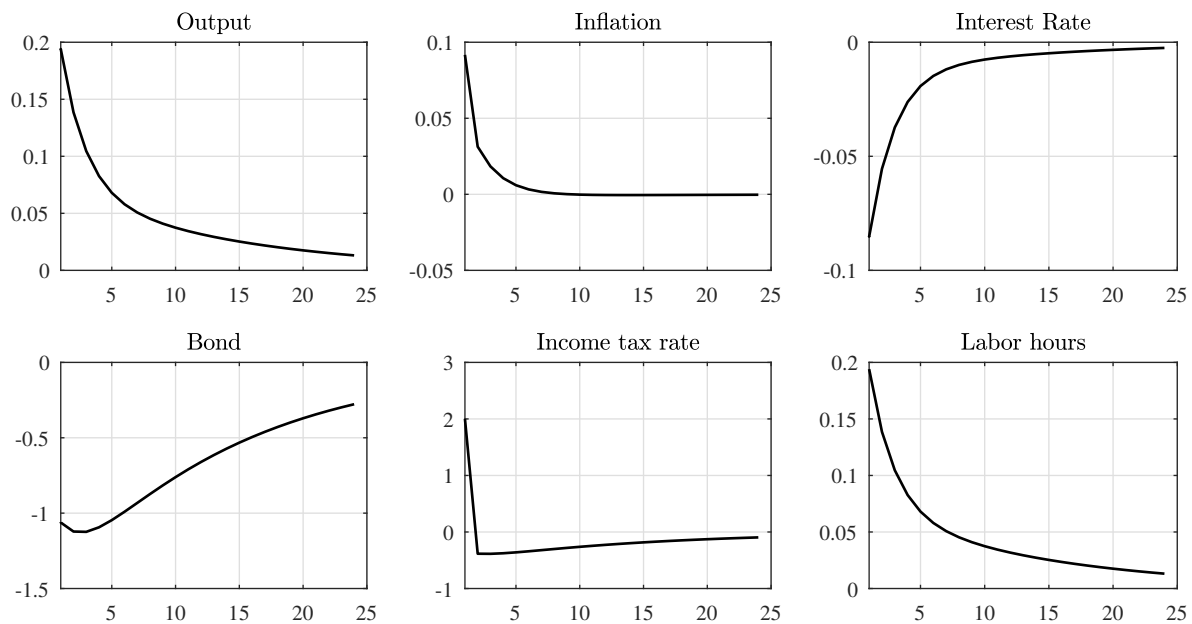


Figure 21: An increase in tax rate and a decrease in interest rate in AMAF regime

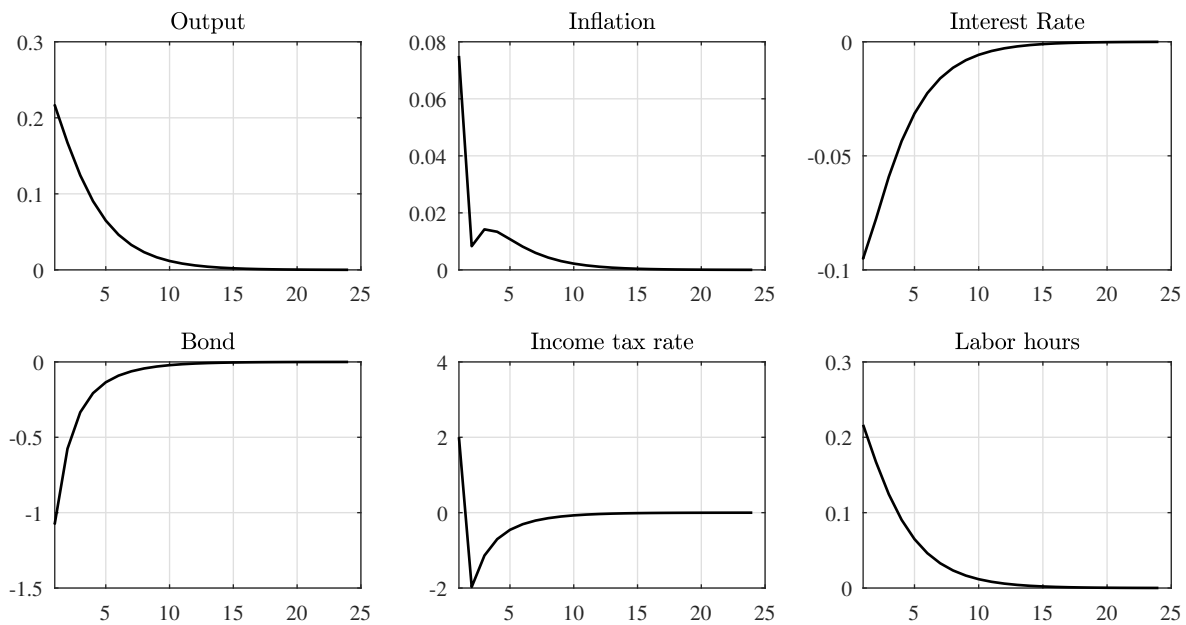


Figure 22: An increase in tax rate and a decrease in interest rate in AMPF regime

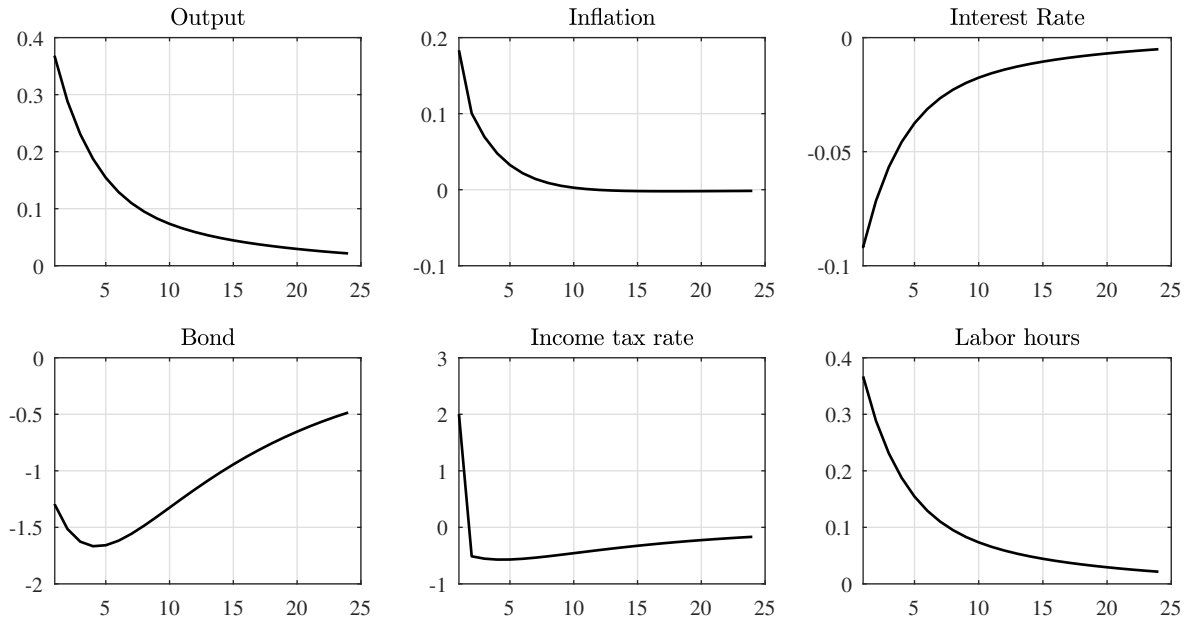


Figure 23: An increase in tax rate and a decrease in interest rate in PMAF regime

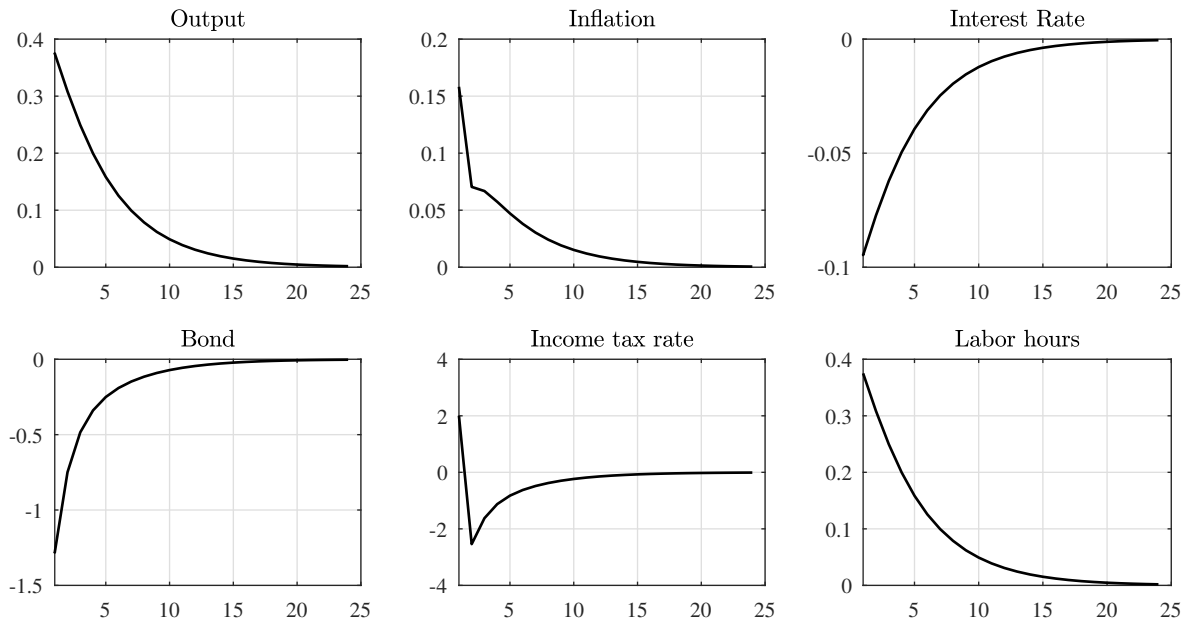


Figure 24: An increase in tax rate and a decrease in interest rate in PMPF regime

B Firms' Problems under Rotemberg (1982) Pricing

As specified in section 3.1.2, there are two types of firms in the economy: a continuum of identical monopolistically competitive intermediate good producers indexed by i (*producers*), and a final good bundler bundling intermediate goods together (*bundler*). Denote the price determined by the producers as p_{it} and the price taken by the bundler as p_t . Denote the output by the producers as y_{it} , and the aggregate output as y_t , which is the output of the bundler assuming no resource loss during the 'bundling' process. Using Dixit-Stiglitz (1977) aggregator, our bundler bundles things together using the following production function that displays constant elasticity of substitution:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (1)$$

As a price taker without monopolistic power, the bundler's goal is to maximize its profit by choosing its demand from all producers

$$\max_{y_{it}} p_t y_t - \int_0^1 y_{it} p_{it} di \quad (2)$$

The optimality condition implies the following

$$0 = p_t \frac{dy_t}{dy_{it}} - p_{it} di \quad (3)$$

$$\begin{aligned} \frac{dy_t}{dy_{it}} &= \frac{\theta}{\theta-1} \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{1}{\theta-1}} \frac{\theta-1}{\theta} y_{it}^{-\frac{1}{\theta}} di \\ &= \left(\left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \right)^{\frac{1}{\theta}} y_{it}^{-\frac{1}{\theta}} di \\ &= y_t^{\frac{1}{\theta}} y_{it}^{-\frac{1}{\theta}} di \\ \Rightarrow y_{it} &= \left(\frac{p_{it}}{p_t} \right)^{-\theta} y_t \end{aligned} \quad (4)$$

Equation (4) shows the optimal output for each producers. The log-linearization form of (4) shows why θ is the elasticity of substitution:

$$\hat{y}_{it} = \hat{y}_t - \theta(\hat{p}_{it} - \hat{p}_t) \quad (5)$$

To find p_t , substitute equation (4) into equation (1):

$$\begin{aligned} y_t &= \left(\int_0^1 \left(\left(\frac{p_{it}}{p_t} \right)^{-\theta} y_t \right)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ \Rightarrow p_t &= \left(\int_0^1 p_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}} \end{aligned} \quad (6)$$

Assuming TFP, real wage, and the exogenous trend are identical for all firms, and each firm follows the same production function (Equation (7)), then we can set up the cost minimization problem for our producers

$$y_{it} = z_t a_t n_{it} \quad (7)$$

$$\min_{n_{it}} w_t n_{it} + \mu_{it} (y_{it} - z_t a_t n_{it}) \quad (8)$$

where the lagrange multiplier μ_{it} is the marginal cost of producing one more unit of output. The optimality condition implies that in equilibrium, every firm faces the same marginal cost.

$$\mu_{it} = \frac{w_t}{z_t a_t} = \mu_t \forall i \quad (9)$$

With marginal cost for producers, we can now maximize the present value for dividends/profits. From the bond's Euler equation (Equation (7) in section 3.1.1), the pricing kernel (the real interest

rate) is defined as

$$q_{t,t+1} = E_t \left[\frac{\pi_{t+1}}{s_t \dot{l}_t} \right] = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\gamma \left(\frac{z_t}{z_{t+1}} \right)^{1-\gamma} \right] \quad (10)$$

Following Rotemberg (1982) and Ireland (1997), producers face adjustment costs when trying to change their price, $\alpha_{it} = y_t \frac{\varphi}{2} \left(\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right)^2$. In this form, firms face higher adjustment costs when the output or price level is high. Since in equilibrium, every producer faces the same marginal cost, $p_{it} = p_t \forall i$. Thus under Rotemberg pricing, there is no price dispersion between producers, unlike Calvo (1983). Producers try to maximize their lifetime dividends, defined in real terms as the profit subtracted by labor costs and adjustment cost. The representation of the problem is shown in (11) and the following equations are the first order condition and its transformations.

$$\max_{\{p_{it}\}_{i=0}^{\infty}} E_0 q_{0,t} \left[\left(\frac{p_{it}}{p_t} \right) y_{it} - w_t n_{it} - y_t \frac{\varphi}{2} \left(\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right)^2 \right] \quad (11)$$

$$\max_{\{p_{it}\}_{i=0}^{\infty}} E_0 q_{0,t} \left[\left(\frac{p_{it}}{p_t} \right)^{1-\theta} y_t - \frac{w_t}{z_t a_t} \left(\frac{p_{it}}{p_t} \right)^{-\theta} y_t - y_t \frac{\varphi}{2} \left(\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right)^2 \right] \quad (12)$$

$$0 = (1 - \theta) y_t \frac{p_{it}^{-\theta}}{p_t^{1-\theta}} + \theta y_t \frac{w_t}{z_t a_t} \frac{p_{it}^{-\theta-1}}{p_t^{-\theta}} - y_t \varphi \left(\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right) \frac{1}{\bar{\pi} p_{it-1}} - E_t q_{t,t+1} y_{t+1} \varphi \left(\frac{p_{it+1}}{\bar{\pi} p_{it}} - 1 \right) \frac{p_{it+1}}{\bar{\pi} p_{it}^2} \quad (13)$$

$$0 = (1 - \theta) y_t \frac{p_t^{-\theta}}{p_t^{1-\theta}} + \theta y_t \frac{w_t}{z_t a_t} \frac{p_t^{-\theta-1}}{p_t^{-\theta}} - y_t \varphi \left(\frac{p_t}{\bar{\pi} p_{t-1}} - 1 \right) \frac{1}{\bar{\pi} p_{t-1}} - E_t q_{t,t+1} y_{t+1} \varphi \left(\frac{p_{t+1}}{\bar{\pi} p_t} - 1 \right) \frac{p_{t+1}}{\bar{\pi} p_t^2} \quad (14)$$

$$0 = (1 - \theta) y_t + \theta y_t \frac{w_t}{z_t a_t} - y_t \varphi (\hat{\pi}_t - 1) \hat{\pi}_t - E_t \beta \left(\frac{c_t}{c_{t+1}} \right)^\gamma \left(\frac{z_t}{z_{t+1}} \right)^{1-\gamma} y_{t+1} \varphi (\hat{\pi}_{t+1} - 1) \hat{\pi}_{t+1} \quad (15)$$

$$\varphi (\hat{\pi}_t - 1) \hat{\pi}_t = (1 - \theta) + \theta \frac{w_t}{a_t z_t} + \beta \varphi E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\gamma \left(\frac{z_t}{z_{t+1}} \right)^{1-\gamma} (\hat{\pi}_{t+1} - 1) \hat{\pi}_{t+1} \frac{y_{t+1}}{y_t} \right] \quad (16)$$

Thus, the optimal condition of profit maximization implies Equation (16), which is shown as Equation (9) in section 3.1.2. In steady state, $\hat{\pi} = 1$, we have the equilibrium wage as $\bar{w}/\bar{z} = \frac{\theta-1}{\theta}$.