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A Numerical Study of Non-Perturbative Unification

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A Numerical Study of Non-Perturbative Unification

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science with Honors in
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Abstract

Non-perturbative unification provides an attractive framework for exploring physics beyond the Standard Model. It assumes nothing about the form of the unified physics, yet provides low-energy predictions of Standard Model couplings. In this investigation, we consider models that add multiplets of $SU(5)$ to the Standard Model in order to unify in this way. We present a search for those that correctly reproduce experimental results and find that some models unify with added matter at the scale of the potential future 100 TeV collider. We conclude with an illustrative example of how these models can be built off of for further model building beyond the Standard Model with extractable phenomenological results.

I Introduction

The gravitational, electromagnetic, weak and strong forces are the four fundamental forces that form the basis of the laws that govern the universe. Much work has been done in the last 75 years to formulate these forces in our modern understanding of physics, the theoretical framework of quantum field theory. In the framework of quantum field theory, every force in the Standard Model has an associated symmetry group and coupling constant. The associated symmetry group captures the fundamental structure of the force, while the coupling constant tells us roughly how strongly the forces acts. In the Standard Model, then, we have three fundamental symmetry groups, $SU(3) \times SU(2) \times U(1)$, and three couplings, g_3, g_2, and g_1 for roughly the strong, weak, and electromagnetic forces respectively.

In such efforts, the electromagnetic and weak forces were unified in the late 1960's into the electroweak theory for which Glashow, Weinberg, and Salam were awarded the Nobel Prize. By unification, we mean that the weak and electromagnetic forces

are the same force above some energy scale, the associated electroweak scale. Below that scale however, this unified force breaks in two. The strong force was described in the same formalism of quantum field theory in the quark model, which was confirmed experimentally in the 1970s when quarks were discovered at particle colliders. The Standard Model, the electroweak and strong force written together, was created. The strong force, however, was never unified with the electroweak force in the same way that the electroweak force is a unification of forces. Unification is too attractive to leave alone, however, and so is a point that we'll come back to.

In the process of making Standard Model theory consistent, by which we mean preventing it from giving us nonsensical answers, we must "renormalize" the coupling constants. As a result, the coupling constants of the fundamental forces are no longer constant! Despite the name, they now depend on the amount of energy involved in an interaction. This means that naively measuring the fine-structure constant at 13 TeV at the LHC will give a different value than on the 10 GeV beam down the road at Jefferson Lab. Just as the maximum resolution of a telescope is dependent on the energy and of the incoming light, higher energy scales are equivalent to probing interactions on shorter and shorter distances.

Though couplings are now energy dependent, this dependence is given by the method of renormalization. The method with which we renormalize the theory depends ultimately on the structure, thus the symmetry groups, of the Standard Model [1]. The method of renormalization leads us to a perturbative description of how the coupling constants change with energy scale. These differential equations are called the renormalization group equations (RGEs), of the coupling constants, which are

$$\frac{dg_i}{dt} = b_i \frac{g_i^3}{16\pi^2} \tag{1}$$

to the lowest order in perturbative expansion [2]. Here the index i runs from 1 to 3, shorthand for three separate equations, with b_i a different constant for each equation and t the energy scale. We can integrate, or "run", these equations up from their measured values at our current energy scale to those yet unexplored to theorize what is happening at higher energy scales. When we do this, we see that the couplings pass each other at some higher energy scale, see left graph in FIG. 1. This means that at energies higher than this crossing, the strong force is weaker than the weak force! This is an interesting occurrence that we will be coming to later.

Hypothesized particles and extra symmetry groups are often theorized to solve problems within the Standard Model, such as those concerning consistency issues and the hierarchy problem [3]. The hierarchy problem asks why gravity is so weak compared to the other four forces, and is a long standing problem in physics. Supersymmetry is one proposed solution to this disparity. Supersymmetry adds an extra symmetry to the Standard Model which pairs each known particle with a hypothesized partner particle that has a different spin. One effect of the addition of supersymmetry is that the weakness of gravity is rendered natural. The separation between the weak and gravitational energy scales can be maintained after quantum corrections are taken into account without a fine tuning of parameters [3]. Another result is that a whole slew of new particles are theorized to exist. As these particles have not yet been found, they are theorized to be massive enough to have hidden from the current generation of particle colliders. As $E = mc^2$, more energetic particle colliders can search for heavier particles, but all colliders have a limit. For the LHC, this is around 13 TeV. For these reasons, researchers searching for new physics often push to higher energy scales for new, yet undiscovered heavier particles that could be evidence for supersymmetry or some another theory of physics beyond the Standard Model.

Important for this discussion is that this new symmetry and matter changes the constants in Eq. (1), thus changing the running of the couplings to high energies. The new running of the coupling constant seems to intersect at a higher energy scale, see right graph in FIG. 1. If all three coupling constants become equal at some scale, then it suggests that our three coupling constants become one. Since a coupling constant is related to each symmetry group, one coupling means that our three symmetry groups become one group and thus one force. This unification is broken at lower scales in much the same way as electromagnetism and the weak force unify into the electroweak. The low energy, "broken" symmetry groups of the Standard Model are pieces of a larger group, in which we say they are embedded. This idea is attractive as in some ways it seems more "natural" for the whole Standard Model to be part of one single, larger group. We call theories in which the forces of the Standard Model unify Grand Unified Theories (GUTs) and the larger group they consider the unified group. This implied unification under supersymmetry is now taken by some to be a motivating factor in exploring supersymmetric models. One of the most appealing and simplest GUTs is $SU(5)$ which was originally proposed by Georgi and Glashow [4]. Though it still faces theoretical and experimental challenges, it remains a well-studied model of supersymmetric unification.

At present, however, supersymmetric particles have failed to be detected by experimentalists at the scales at which they were originally predicted [5]. The idea of unification of the forces in the Standard Model, however, is still attractive and other approaches have been explored. One such approach is the possibility of non-perturbative unification. Normally, coupling constants are small enough that they can be calculated using perturbation theory, giving us progressively more precise equations by adding small corrections. In this non-perturbative unification scheme, the coupling constants all diverge and become strong, or "blow-up", at a finite energy
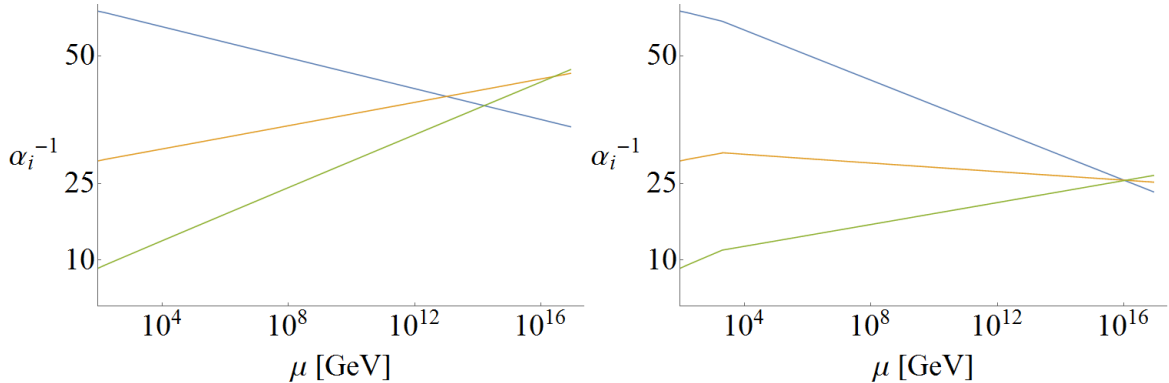
FIG. 1: In all graphs of the running of couplings α_1 will be blue, α_2 orange, and α_3 green. Here, μ is the renormalization scale. On the left we have the running assuming just the Standard Model with no supersymmetry. On the right is the running of Standard Model with supersymmetry and we can see unification at the scale about 10^{16} GeV.

scale, generally taken below the Planck scale. As they blow-up, they are no longer calculable using perturbation analysis as the corrections are no longer small. Since the beta functions of the coupling constants are coupled together, they all blow-up at the same scale, where they are said to unify. What happens beyond the non-perturbative scale is beyond the scope of our current theories.

Non-perturbative unification has a long history in searches for physics beyond the Standard Model, see Refs. [6, 7] for instance. Strong couplings at high energy is desirable for some string theories and theories of compositeness, see Ref. [8] and references therein. As non-perturbative unification does not assume the physics at this high scale, it avoids the theoretical problems such assumptions can introduce. Non-perturbative approaches have the added benefit that the low-energy values of the coupling constants are obtained as fixed points of the differential equations [8]. This type of unification still requires new particles to achieve the desired behaviour of the gauge couplings. Some of these new particles might live within reach of a future 100 TeV collider, such as those being considered in China [9] and at CERN [10].

II   Mathematical Background

These RGEs contain a lot of information, and in this section we attempt to outline how they depend on the structure of the symmetry group. We do not intend to give a full, thorough or technical account of how these RGEs come about. The main idea is that in the process of formulating a quantum field theory, one runs into infinities. The solution to these infinities is to repackage terms in the expression that diverge and to introduce a renormalization scale. The dependence of the couplings on the renormalization scale is reliant on the group structure of the forces and the matter fields involved in the interaction. These RGEs are perturbative equations, however, so are approximations in increasing order of precision, each order called a loop. In this investigation we work to two-loops as that is the lowest order in which the equations are coupled. To help elucidate the coming form of the RGEs, we make a short detour into the group theory and representation theory that underlies particle physics.

Group Theory Primer

When we say that a theory is invariant under the action of a symmetry group, what we mean is that under a set of transformations the theory remains unchanged. If we have some variable Ψ and some function $f(\Psi)$ thereof, then we then say that the class of transformations U which act on Ψ that leave $f(\Psi)$ invariant is the symmetry group of the equations. Symbolically,

$$\Psi \to U\Psi \quad \Rightarrow \quad f(\Psi) \to f(U\Psi) = f(\Psi). \tag{2}$$

In particle physics, these symmetries are a special kind of symmetry groups called Lie groups. To understand why these are special, we briefly consult a geometric example.

Both a square and a circle can be transformed such that the transformed shape looks the same as the original, namely by rotations. But the two shapes are fundamentally different. A square only exhibits symmetry for certain specific rotations, namely multiples of 90°. On the other hand, the circle can be rotated by any angle and remain invariant. This freedom of choice in rotation of the circle is fundamentally related to the fact that its symmetry is also an infinitesimal symmetry. The circle can be rotated by an infinitesimal angle and look the same. Groups of this sort are called Lie groups and are those of interest in particle physics. Specifically, we will be interested in the special unitary group, $SU(N)$ which is the group of $N \times N$ unitary matrices with determinant 1.

In fact, it can be shown that Lie groups are largely characterized by their behavior under infinitesimal transformations. We can form a basis of such infinitesimal transformations such that any finite transformation can be decomposed into repeated applications of these infinitesimal ones. These infinitesimal transformations, which are not unique, are called the generators of the Lie group. These generators are related to each other by the commutation relation

$$[R_i, R_j] = R_i R_j - R_j R_i = i f_{ij}^k R_k \quad , \tag{3}$$

where each R_i is a generator, and the f_{ij}^k are called the structure constants. The structure constants are fundamental and define the local structure of the Lie group. This formulation so far is abstract and general, which is good for classification of symmetries but bad for computation. In physics, our objects of interest are almost always vectors of some form and so we look for *representations* of our Lie group as matrices which can then act on our vectors. As the generators of the Lie group are no longer taken to be abstract objects, we have freedom in choosing their form. We can

construct distinct representations that correspond to using matrices with different properties, such as matrices of larger or smaller dimension. We will henceforth only consider the generator R_i in some specific representation ϕ, some matrix denoted R_i^ϕ.

Convenient representations for us will be the trivial, the adjoint and the fundamental representation. The trivial representation is one in which we map all group elements to the identity matrix, and thus lives up to its name. The adjoint representation, denoted by $\phi = A$, defines the generators in terms of the structure constants themselves. We take each matrix element of a given generator R_i^A as given by $(R_i^A)_j^k = i f_{ij}^k$. The structure constants define the structure of the group, so the adjoint representation is distinguished. It is self-referential as it is a representation in terms of the infinitesimal transformations that characterize the Lie group. It is therefore of the same dimension of the group, which is $N^2 - 1$ for $SU(N)$. The fundamental representation is the smallest dimension representation of the Lie group which is non-trivial. For $SU(N)$, the fundamental representation is given by matrices of dimension N. We now give an illustrative example of the fundamental representation.

For a hopefully familiar example, we look at $SU(2)$. The Lie group $SU(2)$ is the group of 2×2 unitary matrices with determinant 1. The group $SU(2)$ has three generators, which we can choose to write in as 2-dimensional matrices. This is an obvious way to write $SU(2)$ and so is called the fundamental representation. We can write these generators as

$$R_1^F = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad R_2^F = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad R_3^F = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (4)$$

where $\phi = F$ denotes the fundamental representation. One can check that these matrices satisfy the relation

$$[R_i^F, R_j^F] = i\epsilon_{ijk}R_k^F \quad , \tag{5}$$

Where ϵ_{ijk} is the Levi-Civita symbol with $\epsilon_{123} = 1$ and is totally antisymmetric, so exchanging any two indices gives a minus sign. In an introduction to Quantum Mechanics course, students learn about the quantum mechanical spin of an particle in terms of Pauli matrices, which are related to above by $R_i^F = \frac{1}{2}\sigma_i$. With this representation in quantum mechanics, we can act on the vectors which represent the spin state of fermions. In the adjoint representation, denoted by the label A, we take $(R_i^A)_{jk} = i\epsilon_{ijk}$. This gives a adjoint representation of $SU(2)$ in terms of three dimensional matrices,

$$R_1^A = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad R_2^A = i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad R_3^A = i\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad . \tag{6}$$

One can check that these generators likewise satisfy $[R_i^A, R_j^A] = i\epsilon_{ijk}R_k^A$.

Application to Physics

Matter in particle physics is in some representation of every gauge group. The electron, for example, is not charged under color $SU(3)$ so is in a trivial representation of that group. It is, however, charged under $SU(2) \times U(1)$ so is in some non-trivial representation of those groups. Using all of this information in the process of renormalization leads to a general RGE for any group and matter additions. This equation we find in [11] as

$$\frac{d}{dt}g_i = \frac{g_i^3}{(4\pi)^2}[-3C(G_i) + T_i(\phi)] + \frac{g_i^5}{(4\pi)^4}2C(G_i)[-3C(G_i) + T_i(\phi)]$$
$$+ \sum_j \frac{g_i^3 g_j^2}{(4\pi)^4}4T_i(\phi)C_j(\phi), \tag{7}$$

where $C_j(\phi)$ is the Casimir operator of the representation of ϕ in the group G_j, $C(G_i)$ the Casimir operator of the adjoint representation of G_i, and $T_i(\phi)$ is the index of a representation of ϕ in the group G_i. These are all numbers that are invariants of representations of groups. For some representation ϕ of a group G_i, we define them as

$$T_i(\phi)\delta_{xy} = \text{Tr } R_x^\phi R_y^\phi \quad \text{and} \quad C_i(\phi)\mathbb{1} = R_x^\phi R_x^\phi, \tag{8}$$

where x and y label the generators of the representation ϕ. In all terms with ϕ we have suppressed a summation over all ϕ_a, the fields that are charged under the group (such as the quarks under color $SU(3)$). So

$$T_i(\phi) = \sum_a T_i(\phi^a) \quad \text{and} \quad T_i(\phi)C_i(\phi) = \sum_a T_i(\phi^a)C_i(\phi^a). \tag{9}$$

These equations are a result of the renormalization procedure we described earlier, connecting the gauge group and the running of the coupling constant. All we need now is to know what fields we have in our theory, the groups they are charged under, and their charge assignments. From there, we can calculate the supersymmetric beta functions we need. For the previous example of the fundamental representation of $SU(2)$ the Casimir operator is

$$C_{SU(2)}(F)\mathbb{1} = R_x^F R_x^F = R_1^F R_1^F + R_2^F R_2^F + R_3^F R_3^F = \begin{pmatrix} 3/4 & 0 \\ 0 & 3/4 \end{pmatrix} = \frac{3}{4}\mathbb{1}. \qquad (10)$$

In the same way, we find that for the adjoint representation

$$C_{SU(2)}(A)\mathbb{1} = R_x^A R_x^A = R_1^A R_1^A + R_2^A R_2^A + R_3^A R_3^A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 2\mathbb{1}. \qquad (11)$$

By convention, we set $T_i(F) = \frac{1}{2}$ for the fundamental representation. We find that for the adjoint representation

$$T_i(A)\delta_{xy} = \mathrm{Tr}R_x^A R_y^A = 2, \qquad (12)$$

which one can check given the generators above. General results for $SU(N)$ in the adjoint and fundamental representation are given in a table in the Appendix. We will later need such invariants for other representations other than those considered above, and those are found in the same way. We find the generators of the group in the representation that interests us, and we compute invariants as above. Then, we can find the coefficients of the beta functions, given as [11]

$$b_i = -3C(G_i) + T_i(\phi)$$
$$b_{ij} = 2C(G_i)[-3C(G_i) + T_i(\phi)]\delta_{ij} + 4T_i(\phi)C_j(\phi). \qquad (13)$$

To see how this translates into the beta function, we compute the b_3 term in

the supersymmetric beta functions. The group in question is $SU(3)$, which has $C(SU(3)) = 3$. The charged fields in question are the right and left-handed quark fields, which are in fundamental representations of $SU(N)$, so have $T_i = \frac{1}{2}$ by convention. The left handed fields are paired in a doublet, and we have three generations of all the fields so we have

$$
\begin{aligned}
b_3 = -3C(G_3) + T_3(\phi) &= -3 \times 3 + (T_3((u_L, d_L)) + T_3(u_R^C) + T_3(d_R^C)) \times 3 \\
&= -9 + (2 \cdot \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) \times 3 = \boxed{-3 = b_3}.
\end{aligned}
\tag{14}
$$

This agrees with the known results presented in the Appendix.

III Method

We begin by considering the two-loop beta functions which take into account couplings between the fields. These are

$$
\frac{dg_i}{dt} = \frac{g_i^3}{16\pi^2} \left[b_i + \frac{1}{16\pi^2} \sum_{j=1}^{3} b_{ij} g_j^2 \right] \quad ,
\tag{15}
$$

where we have repressed the dynamics of the Yukawa couplings, see full two-loop equations in [2]. The constants b_i and b_{ij}, as we have discussed, depend on the matter content and symmetries present in the theory. As a result, when supersymmetry and matter are added at the scales denoted as m_{susy} and m_f respectively, these constants change. This gives a piecewise defined system of coupled differential equations. We'll define $\alpha_i = g_i^2/4\pi$ for convenience. In a non-perturbative scenario, couplings will be diverging to infinity so we will often consider α_i^{-1} instead, as it is more convenient for visualization.

Non-perturbative unification acts as a boundary condition for these coupled dif-

ferential equations. At some scale Λ all α_i are taken to be 10, which stands in for our unification. Recall that these differential equations are perturbation expansions in α_i, so break down when α_i is order 4π, about 10. The beta functions must all have $b_i > 0$ in the leading order term so that the couplings continue to grow towards higher and shrink towards lower energies. The equations with this boundary condition are then used to numerically integrate down to the scale of the added matter m_f, below which the equations are changed to the supersymmetric Standard Model beta functions as our added matter content only exists above that scale. The equations are then integrated down to m_{susy}, the scale at which supersymmetry emerges, at which point they are changed to the non-supersymmetric Standard Model beta functions. They are finally run down to the electroweak scale, the scale at which the experimental values we match to are measured. The values of the coupling constants at the electroweak scale is completely determined by the scales Λ, m_f and the amount of the supersymmetric matter added. There are no other free parameters to change. Note that if we set the scale of unification and form of the added particles, we only have to determine m_f if we want to try to match low-energy experimental values.

With these free parameters, we can attempt to make the model match current low-energy experimental values for the coupling constants, found in Ref. [5]. For each m_f we can choose a unification scale Λ which correctly reproduces α_{EM}, a linear combination of α_1 and α_2, at the electroweak scale. We match to α_{EM} as it is the experimental value that is most precisely measured experimentally. This condition fixes the scale Λ for a certain m_f. This does not mean, however, that the model necessarily reproduces $\sin\theta_W$, a different linear combination of α_1 and α_2 that is measured experimentally, and α_3 at the electroweak scale. We can still vary m_f, each time fixing a new Λ by the above condition, to try and reproduce $\sin\theta_W$ and α_3. FIG. 2 shows such a relationship between m_f and the electroweak values of $\sin\theta_W$

and α_3. There is a range of values of m_f which accurately reproduces $\sin\theta_W(M_Z)$, and a range of values which accurately reproduce $\alpha_3(M_Z)$. In order for there to be a plausible reproduction of all three coupling constants at the electroweak scale, those two ranges must overlap. If they do not, the model is not a candidate for non-perturbative unification. If they do, then we have a range of values where new matter could lie, and which produces non-perturbative unification which accurately predicts the electroweak scale couplings.
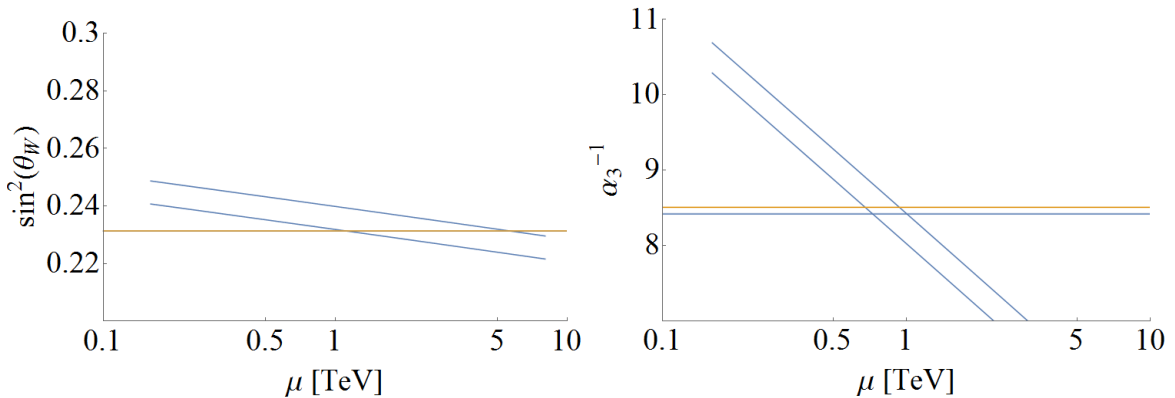

FIG. 2: The electroweak values calculated given a scale for m_f, with experimental and theoretical error bars shown. There is no simultaneous scale for this set of particles which correctly reproduces both to within uncertainty regions, though it comes close.

The approach described above is the same as found in [12], but with updated experimental input. In extending their study, we have separated the scales of new matter content and supersymmetry from each other. We have left m_{susy} at 2 TeV for the time being and have allowed m_f to float to higher energy scales. There is no explicit motivation to connect the scales, and disconnecting them allows for a greater number of particle configurations to work. It is also important to have m_{susy} stay on the scale of 2-100 TeV. The upper bound is set so as to maintain the hierarchy solving properties of supersymmetry and the lower bound is what has been ruled out by experiments [3, 5]. We redo the analysis for values of m_{susy} of 2, 10, and 100 TeV to

sample the range where supersymmetry may arise. We also preform the analysis for the originally proposed scenario that the scale of supersymmetry and the new matter is the same. We increase the precision of the results by including smaller effects such as modifying the beta functions appropriately below the mass of the top quark, at 160 GeV, and quantifying the uncertainty of leaving off the evolution of the Yukawa couplings. We now turn to this task.

IV   Error Analysis

We first aim to show that the RGEs are insensitive to the boundary conditions at the unification scale, as asserted previously. They are insensitive because the differential equations that describe the running of the coupling constants have a stable infrared (i.e. low energy) fixed point (IRFP) structure. Technically, it is the ratio of the couplings that exhibit the IRFP structure. We can see this clearly if we rewrite the RGEs as ratios so that

$$\frac{d}{dt} \ln \frac{\alpha_i}{\alpha_k} = \frac{1}{\alpha_i} \frac{d\alpha_i}{dt} - \frac{1}{\alpha_k} \frac{d\alpha_k}{dt} = 2(b_i \alpha_i - b_k \alpha_k) + \mathcal{O}(\alpha^2). \tag{16}$$

At the one-loop order that we expanded to, we can see that there is a fixed point at

$$\left(\frac{\alpha_i}{\alpha_k} \right)^* = \frac{b_k}{b_i}. \tag{17}$$

To see that this is a fixed point, check with $\alpha_i/\alpha_k = b_k/b_i + \epsilon$ such that ϵ is a small

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positive deviation from the fixed point. This gives

$$\frac{d}{dt}\ln\epsilon = \frac{1}{\epsilon}\frac{d\epsilon}{dt} = 2((b_k+\epsilon)\alpha_k - b_k\alpha_k)$$

(18)

$$\Rightarrow \frac{d\epsilon}{dt} = 2\epsilon^2\alpha_k.$$

Since α_k and ϵ are positive, the deviation gets smaller as we go to smaller values of the energy ($t \to -t$) so the fixed point is stable and attractive when running down from high energies. This holds for all ratios of couplings. Note then, that although we assumed a positive deviation, by taking the inverse ratio we see that the fixed point is stable from both above and below.

If the two-loop corrections are small, then then the fixed point structure remains and the fixed point will shift slightly. Since we fixed the value of α_{EM} to set Λ, we have set part of the ratio so we have an IRFP for $\sin(\theta_W)$ and α_3. These values can be estimated with the above equation. See [8] for a more in-depth analysis of the IRFP structure of the RGEs. The RGEs will flow to this fixed point asymptotically. Since we want to know the value of the couplings at an intermediate scale m_Z and not in the low-energy limit, we instead have a quasi-infrared fixed point (QRFP). We expect there to be some drift from this fixed point in electroweak predictions.

To find the theoretical error in setting the arbitrary value of the non-perturbative boundary condition that $\alpha_i(\Lambda) = 10$, we vary the condition from 1 to 10 to 100 for each coupling. This gives 9 different combinations of boundary conditions as starting points that all give slightly different low-energy predictions. We compute the range of the low-predictions for these different boundary conditions at the specific scale of the new matter for each model. We use this range to set the theoretical error bars. This procedure is thus very important to finding allowable ranges for new particle

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content. As such, it will be important to consider other sources for possible error.

So far we have left off the Yukawa couplings from the RGE equations we consider. We will only consider the effects of the top quark Yukawa coupling, as it is by far the largest at $\mathcal{O}(1)$ at the electroweak scale. To leading order, the top quark Yukawa coupling RGE is

$$\frac{d\lambda_{\text{top}}}{dt} = \frac{\lambda_{\text{top}}}{16\pi^2}\left(-\sum_i c_i g_i^2 + 6\lambda_{\text{top}}^2 + \lambda_{\text{b}}^2\right), \tag{19}$$

where $c_i > 0$ for all i [2]. As such, when the gauge couplings blow up, the Yukawa couplings are driven to zero. Indeed, we set the Yukawa couplings to 0 at the blow-up scale in all simulations. We believe this is reasonable as the Yukawa couplings go to zero as the interaction couplings go to infinity, so their importance becomes small at large energy scales. Their differential equations are proportional to their value so once they are zero, they stay zero. This removes all Yukawa couplings from our simulations.

To estimate the error from leaving off the Yukawa couplings, we varied the top quark Yukawa coupling randomly at the blow-up scale as a stability test. We found that the electroweak values of the coupling constants only scattered to a maximum range of 3%, but with standard deviations all well below 1%. As such, we have kept the Yukawa couplings at 0 for the blow-up scale initial condition and accounted for this error in our theoretical error bars. The effect from varying the boundary conditions of the couplings is larger in general than from Yukawa couplings and the QRFP. We thus estimate the total error by the range obtained from varying the boundary conditions of the couplings. See [13] and [14] for further discussion on quantifying uncertainties from Yukawa and higher-order effects in a non-perturbative unification scenario.

V   Updating Results

Our investigation begins in reproducing the results of an earlier investigation. We began by reproducing and extending this work in order to inform our later analysis. We also aim to evaluate their model with current experimental and numerical error bars. Ref. [12] studied the possibility of non-perturbative unification by adding an extra pair of supersymmetric families to the Standard Model at 1 TeV. They found that this model reproduced low-energy experimental values of the couplings to within the experimental bounds at the time. In their analysis the scale at which supersymmetry emerges is taken as the same energy scale as the matter content they insert into the theory, referred to collectively as m_f in this section.

Although the results of this previous study have been informative in setting the basis for the method of searching, we have expanded off of the analysis in a number of ways. First, as it is a short paper, there is no elaboration of the method used to produce theoretical error bars in the theory calculation for plots such as FIG. 2. As such, we have no way to directly compare methods for error analysis. Second, our plot of α_3 as shown in FIG. 2 is offset from those of the previous study by a non-zero value, the source of which is unknown. Despite these discrepancies, we are confident in the methods in our own analysis in reproducing theoretical error bars and accurate runnings of couplings.

We have found that the original addition of a supersymmetric family pair of particles at 1 TeV no longer reproduces the correct values of the coupling constants. Due to the shrinking of experimental error bars that has occurred over the last 20 years and our difference in experimental error bars, allowable regions have shifted. The allowable range is at 1.2-1.3 TeV. This scale is encroached upon, however, by the absence of supersymmetry at particle accelerators. We can break the scale of

supersymmetry and the added matter to put supersymmetry out of the forbidden range to 2 TeV. When we do this, however, there is a scale inversion and the new matter must be added below supersymmetry. This is not viable either as particles below 2 TeV coupled to the Standard Model would most likely have been detected by now. This may be bad news for a lonely pair of generations, but we note that small changes in theoretical error bars can greatly affect a model whose allowable regions are close. If we extend the search to other matter content, we find candidate models with much more robust allowable regions in scales that are not ruled out by experiment.

VI Adding Matter

We conducted a search for other particle combinations, outside of those considered by Ref. [12]. We have searched through the combinations of additional generations (only adding pairs), Higgs doublets, $5 + \bar{5}$ and $10 + \overline{10}$ pairs. These particle additions transform nicely under the supersymmetric unified group $SU(5)$ so are ideal for simpler model building in the future. Adding complete $SU(5)$ multiplets is also known to preserve perturbative supersymmetric unification and is therefore the most promising additional matter to add in extending to the nonperturbative scenario. Note however, that we have still not assumed the final symmetry of a GUT, just that the new matter fits into a representation of $SU(5)$. The coefficients for the one loop term are [15]

$$
b = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_g + \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix} n_h + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} n_5 + \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} n_{10} + \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix}, \tag{20}
$$

where n_g is the number of generations, n_h of Higgs doublets, n_5 of $5+\bar{5}$ pairs and n_{10} of

$10 + \overline{10}$ pairs. Here we think of the individual b_i as the ith element of the list as written above, but this is purely for bookkeeping purposes. There is a companion equation for the two-loop contribution of the new matter, b_{ij}, presented in the Appendix. With both, we can fully describe how the added content contributes to the running of the couplings. We define the notation $(n_g,\ n_h,\ n_5,\ n_{10})$ to denote the model with that amount added matter. Note that to have all $b_i > 0$ so that a divergence at finite energy scale occurs, we need sufficient added matter to be added. The base supersymmetric case is $(3,\ 2,\ 0,\ 0)$ which is not enough to flip b_3 positive. We need to add matter such that $2(n_g - 3) + n_5 + 3n_{10} \geq 4$. When only adding one type of matter, we find that the base cases are $(5,\ 2,\ 0,\ 0)$, $(3,\ 2,\ 4,\ 0)$, and $(3,\ 2,\ 0,\ 2)$. These and two other minimal models $(3,\ 2,\ 5,\ 0)$ and $(3,\ 2,\ 6,\ 0)$ will form the basis for our study.

In our preliminary analysis, there were many such combinations of particle content that were allowable. For many such models, however, the new matter is introduced at scales at 10^5 to 10^{10} TeV. These are less interesting in practice than those at lower scales, as current particle accelerators probe scales only up to 10 TeV. Motivated by possible phenomenological results, we continue to study those models that push down the mass scale of added matter. More matter content makes the couplings blow up faster, as a new matter field ϕ increases b_i by $T_i(\phi)$ which is always positive. As a result, more matter content pushes up the energy scale at which it is introduced since the boundary condition is fixed. In much the same way, an escalator works on a much longer horizontal distance than an elevator, yet they both reach the same height. As a result, the models with lower mass scales tend to be simpler in the sense that they contain less added matter. The running of the combination of particles with the lowest mass scale is reproduced in FIG. 3. Note, however, that even this predicted masses scale of about 350 TeV is above current accelerator energy levels.

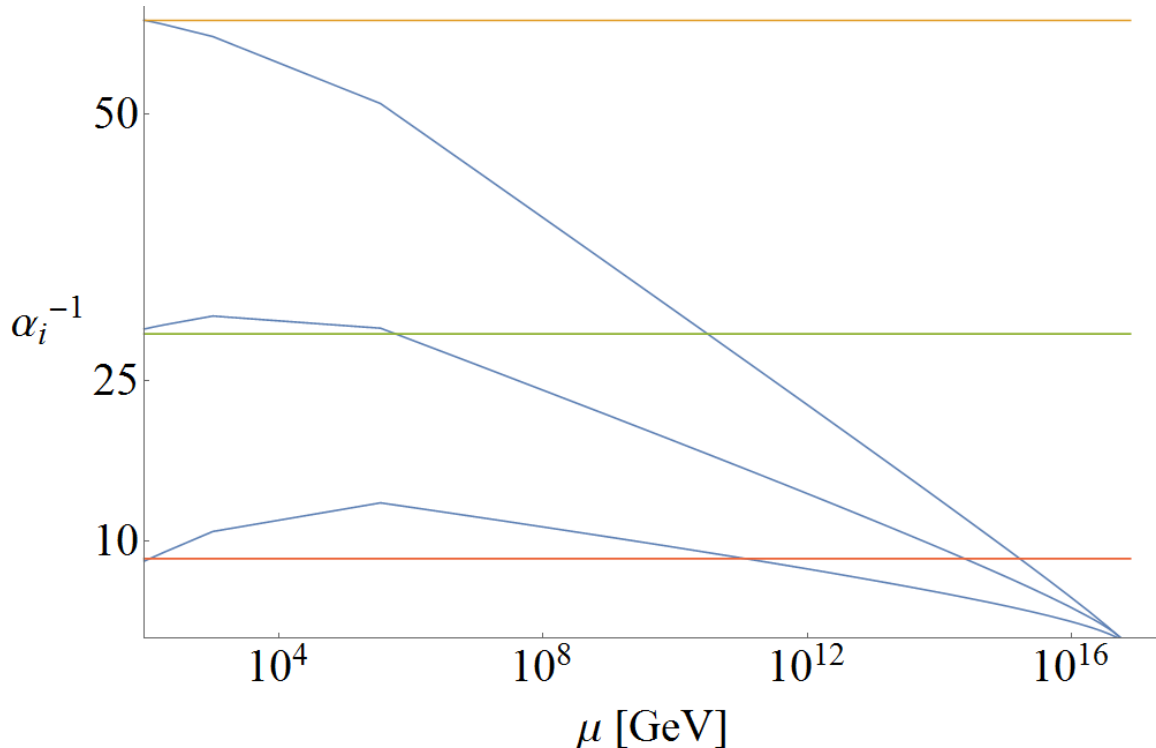See Table 1 for the predicted mass scale of added matter for these minimal models.

FIG. 3: Sample of a successful running of (5, 2, 1, 0) with m_{susy}=1 TeV and m_f= 346.5 TeV. Horizontal lines are the experimental values of the couplings at the electroweak energy scale. Down from the top line is α_1, α_2, then α_3.

VII   Trying to explain all that New Matter

Preliminary Reasoning

In the analysis thus far, we have proceeded in evaluating models of strong unification somewhat blindly. The advantage of this approach is that the tightly constrained structure of strong unification guides all of the phenomenological observables in which we could be interested. We are now left with the question as to the origin of this specific amount of matter content for each combination. We hope to explain the

Model	m_{susy} (TeV)	m_f range (TeV)	Λ (10^{16} GeV)	α_3 % Error	$\sin^2(\theta_W)$ % Error
(3,2,5,0)	2	90 - 190	5.6	+3.3, -2.3	+1.4, -1.2
	10	23 - 36	4.0		
	100	Not Allowed	–		
	m_f	16-18	3.9		
(3,2,6,0)	2	5200-11500	5.3	+3.7, -2.9	+1.4, -1.25
	10	1600-2800	3.7		
	100	Not Allowed	–		
	m_f	Not Allowed	–		
(3,2,0,2)	2	Not Allowed	–	+4.8, -3.6	+1.5, -1.5
	10	3400 - 4500	6.5		
	100	310 - 830	4.0		
	m_f	190 - 340	3.2		

Table 1: Allowable ranges for select models, uncertainty ranges are the same within each model. We have varied m_{susy} in the range 2-100 TeV and set it equal to m_f within each particle combination.

amount of matter content of the most interesting models that we found in the previous wide search. For our study, interesting models will be ones that have low energy scales of new matter and are thus of phenomenological relevance. Remember too that these are also more svelte in their matter content, which will potentially lead to simpler explanations for their origin.

One explanation is the possibility that there exists an extra force in the Standard Model. This extra force manifests as an extra gauge group, G_D where D anticipates "dark", with a coupling that unifies together with the rest of the Standard Model couplings. The new gauge group would be such that the added matter transforms in a d-dimensional representation of the new gauge group. This would thus explain the amount of new matter by giving a fundamental gauge group origin to the number of added $5 + \bar{5}$ pairs.

As an example, take the case that the particle combination (3, 2, 6, 0) unifies

strongly. We could take the extra 6 $5 + \bar{5}$ pairs as transforming in the 6-dimensional representation of $SU(2)$, the 6-dimensional representation of $SU(3)$ or the fundamental representation of $SU(6)$ for example. This would mean that when seen from the viewpoint of the new gauge group, the new particles would fit in perfectly in a representation of the Lie group. This gives a group theoretic origin to the specific amount of new matter. This shifts the question from the amount of matter added to the specific group added. Gauge groups of the Standard Model in some sense "just are"; their origin can possibly only be explained by some GUT which this analysis does not presuppose.

This coupling of this new force, g_D, is described by an extra RGE added to our set of differential equations. The new force also modifies our old RGEs at the two-loop level, where the sum now runs from 1 to 4 (with 4=Dark). Adding a new equation will, of course, change the running of the couplings with the particle content that originally interested us, but we expect that change to be small. This is because the new coupling constant only affects the others at two-loop level, which is suppressed by a factor of $16\pi^2$, and in general the beta coefficients will be such as to drive the new coupling to small values, further suppressing any effects.

Turning Lemons into Lemonade

As we saw with supersymmetry, adding extra symmetries to the Standard Model has wide appeal in solving many of the questions that still plague physics. This includes the popular question as to the origin of dark matter (DM). In particular, a possible DM candidate is a composite particle called a DM 'glueball'. This particle arises as a bound state of DM gluons described by a $SU(N)$ gauge symmetry. Such a composite particle would be similar to the particles in the spectrum of color $SU(3)$ bound states.

These include the proton, neutron, and a corresponding theorized color glueball which has yet to be detected. The mass of the glueball is approximately the scale Λ_D, at which the gauge coupling blows up. References have set an upper bound on Λ_D at about 10^5 GeV for stable glueball DM to exist, but recent papers have estimated a scale of MeV to GeV to fit with astrophysical data [16][17].

Since non-perturbative unification provides a predictive scheme for additions to the Standard Model, we will investigate additions of a dark $SU(N)$ to our previously found models. This condition for a low-energy blow up then exists in addition to the strong unification boundary condition at high-energy. As a result, this extra $SU(N)$ symmetry would blow up at low-energy and high-energy, just as $SU(3)$ does in non-perturbative unification. In this investigation, we will restrict ourselves to models with $(3, 2, D, 0)$. The D 5s will be in some D-dimensional representation, $\mathbf{D}$, of $SU(N)$; and, the D $\bar{5}$s will be in the corresponding D-dimensional conjugate, $\bar{\mathbf{D}}$. The conjugate of some representation, sometimes denoted by "anti-" or by a bar on top, is simply the complex conjugate of the representation. Note that the conjugate representation will have the same invariants as the non-conjugate. We can denote our new matter now by $(5, \mathbf{D}) + (\bar{5}, \bar{\mathbf{D}})$, if given some $SU(N)$ and some such D-dimensional representations thereof. As noted in the last section, we expect the corrections of adding this extra coupling to be small so we will consider the models $(3, 2, 6, 0)$ and $(3, 2, 5, 0)$ which previously unified favorably. We look for groups that have 5 or 6-dimensional representations and consider their viability in non-perturbative unification and model building.

We first consider the simple model where $D = N$, $(3, 2, N, 0)$. Our new matter is $(5, \mathbf{F}) + (\bar{5}, \bar{\mathbf{F}})$ where the $\mathbf{F}$ is the fundamental and $\bar{\mathbf{F}}$ the anti-fundamental representations of $SU(N)$. Recall that for the model to be a candidate for strong unification, the new matter must force all $b_i > 0$ so that the couplings will blow up. For such a

model, however,

$$b_4 = -3C(SU(N)) + 5 \times T_D(\mathbf{N}) + 5 \times T_D(\bar{\mathbf{N}}) = -3 \times N + 5 \times \frac{1}{2} + 5 \times \frac{1}{2} = -3N + 10. \quad (21)$$

This is not positive for $N \geq 4$, which is the minimal number of n_5 for which $b_3 > 0$. As such, no simple model (3, 2, N, 0) with the extra matter in the fundamental representation of $SU(N)$ will be compatible with non-perturbative unification. Instead, we turn to specific 5 and 6-dimensional representations of lower dimensional gauge groups $SU(2)$, $SU(3)$ and $SU(4)$ to explain the matter in the (3, 2, 5, 0) and (3, 2, 6, 0) models. We can immediately rule out $\mathbf{6}$ and $\bar{\mathbf{6}}$, the 6-dimensional representations of $SU(4)$, as well however. For this representation, we find

$$C(SU(4)) = 4 \quad (22)$$

$$C_D(\mathbf{6}) = \frac{5}{2} \quad (23)$$

$$T_D(\mathbf{6}) = 1. \quad (24)$$

For (3, 2, 6, 0) with our new matter as $(5, \mathbf{6}) + (\bar{5}, \bar{\mathbf{6}})$, we have

$$b_4 = -3C(SU(4)) + 5 \times T_D(\mathbf{6}) + 5 \times T_D(\bar{\mathbf{6}}) = -3 \times 4 + 5 \times 1 + 5 \times 1 = -2. \quad (25)$$

We can see that $b_4 < 0$ so this scenario is not compatible with non-perturbative unification either.

Dark $SU(2)$

We consider the general model (3, 2, D, 0) with the new matter charged under $SU(2)$. This group is nice because it has representations in every dimension in a consistent

manner, so we can find the invariants for any $D > 0$ in general. We assume our added matter is of the form $(5, \mathbf{D}) + (\bar{5}, \bar{\mathbf{D}})$, where $\mathbf{D}$ and $\bar{\mathbf{D}}$ are representations of $SU(2)$. For a D-dimensional representation of $SU(2)$, we find that

$$C(SU(2)) = 2 \tag{26}$$

$$C_D(D) = \frac{1}{4}(D^2 - 1) \tag{27}$$

$$T_D(D) = \frac{5}{6}D(D^2 - 1). \tag{28}$$

From these, we find the beta functions above the scale of the added matter are

$$b_i = \left(\frac{33}{5} + D, \ 1 + D, \ -3 + D, \ -6 + \frac{5}{6}D(D^2 - 1) \right) \tag{29}$$

and

$$b_{ij} = \begin{pmatrix} \frac{199}{25} + \frac{7}{15}D & \frac{27}{5} + \frac{9}{5}D & \frac{88}{5} + \frac{32}{15}D & D(D^2 - 1) \\ \frac{9}{5} + \frac{3}{5}D & 25 + 7D & 24 & D(D^2 - 1) \\ \frac{11}{5} + \frac{4}{15}D & 9 & 14 + \frac{34}{3}D & D(D^2 - 1) \\ \frac{1}{3}D(D^2 - 1) & D(D^2 - 1) & \frac{8}{3}D(D^2 - 1) & -24 + \frac{5}{6}D(D^2 - 1)(D^2 + 3) \end{pmatrix}. \tag{30}$$

Below the scale of new matter, these equations are valid but we set D=0. We can see this decouples g_D from the other three couplings. We expect this as below this scale, there are no matter fields that are charged under the Standard Model and the Dark $SU(2)$. Below the scale of supersymmetry, however, the beta functions will switch to the non-supersymmetric beta functions with no matter fields. The general equations for the non-supersymmetric case in terms of group theory invariants can

be found in [18]. From these we find that

$$b_4 = -\frac{22}{3} \quad \text{and} \quad b_{44} = -\frac{136}{3},\tag{31}$$

with $b_{i4} = b_{4i} = 0$ for $i \neq 4$. We can now go ahead and add this equation for g_4 to our set of RGEs and run the new model down to low energies. Our findings for this are summarized in Table 2. We note that these have a problem, however. In FIG. 4 you can see this more clearly. The dark coupling is pushed so low at the electroweak scale and the one loop beta function is not enough for it to blow-up again at any reasonable scale. As such, we find that these models don't blow-up until such ridiculously low scales that they might as well be zero for physical purposes. So, though this scheme does unify, it is not what we are looking for. Notice, however, that allowable ranges for extra matter are close to those without the extra gauge group, so as we expected the new group has only a small effect on the running of the other couplings.
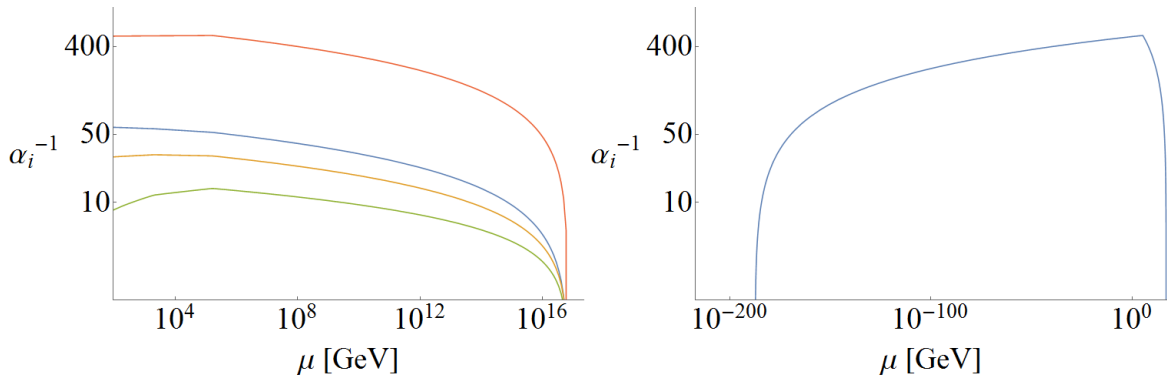

FIG. 4: On the left is the running of (3, 2, 5, 0) with added matter in a 5-dimensional representation of $SU(2)$, α_D shown in red. On the right is the running of just α_D which we can see will only blow-up at energies that are too low to be physical.

Model	m_{susy} (TeV)	m_f range (TeV)	$\alpha_D^{-1}(m_Z)$	Λ_D (eV)
(3,2,5,0)	2	110 - 250	510	10^{-600}
in 5-dim rep	10	28 - 50	540	10^{-650}
of $SU(2)$	100	Not Allowed	–	–
	m_f	17 - 20	560	10^{-680}
(3,2,6,0)	2	6200-14500	770	10^{-1000}
in 6-dim rep	10	1900-3700	800	10^{-1100}
of $SU(2)$	100	Not Allowed	–	
	m_f	Not Allowed	–	

Table 2: Results with $SU(2)_D$ gauge group added, Λ_D is the predicted scale of DM glueballs.

Dark $SU(3)$

We now consider adding a Dark $SU(3)$ to our supersymmetric RGEs. This group has 6-dimensional representations, $\mathbf{6}$ and $\bar{\mathbf{6}}$, which the $SU(5)$ multiplets in the (3, 2, 6, 0) non-perturbative unification model can fit into. Our added matter then transforms as $(5, \mathbf{6}) + (\bar{5}, \bar{\mathbf{6}})$. For a 6-dimensional representation of $SU(3)$ we find

$$C(SU(3)) = 3 \tag{32}$$

$$C_D(\mathbf{6}) = \frac{10}{3} \tag{33}$$

$$T_D(\mathbf{6}) = \frac{5}{2}. \tag{34}$$

From these, we find the beta functions above the scale of the added matter are

$$b_i = \left(\frac{63}{5},\ 7,\ 3,\ 16\right) \quad \text{and} \quad b_{ij} = \begin{pmatrix} \frac{269}{25} & \frac{81}{5} & \frac{152}{5} & 80 \\ \frac{27}{5} & 67 & 24 & 80 \\ \frac{19}{5} & 9 & 82 & 80 \\ 10 & 30 & 80 & \frac{1288}{3} \end{pmatrix}. \tag{35}$$

Model	m_{susy} (TeV)	m_f range (TeV)	$\alpha_D^{-1}(m_Z)$	Λ_D (eV)
(3,2,6,0)	2	10000-25000	49	.5
in 6-dim rep	10	3000-6800	53	.05
of $SU(3)$	100	580-590	59	.001
	m_f	Not Allowed	–	

Table 3: Results with $SU(3)_D$ gauge group added, Λ_D is the predicted scale of the DM glueball spectrum.

As before, below the scale of heavy matter the beta functions are still supersymmetric but g_D decouples from the running of the other couplings. We find that

$$b_4 = -9 \quad \text{and} \quad b_{44} = -54, \tag{36}$$

with $b_{i4} = b_{4i} = 0$ for $i \neq 4$. Below the scale of supersymmetry, we again use the general equations in [18] and find that

$$b_4 = -11 \quad \text{and} \quad b_{44} = -102, \tag{37}$$

with $b_{i4} = b_{4i} = 0$ for $i \neq 4$ as g_D is still decoupled. For such a running we find that Λ_D is on the order of an eV. See FIG. 5 for an example running with m_{susy}=2 TeV and m_f= 1.6×10^5 TeV. For such a model we find that $\Lambda_D = 0.5$ eV. Results for a sampling of m_{susy} are presented in Table 3. Note that as m_{susy} is raised, Λ_D drops lower. Soni and Zhang in Ref. [17] estimate using astrophysical evidence a scale of 20 MeV for self-interacting dark glueballs in $SU(3)$, much higher than we have predicted here.
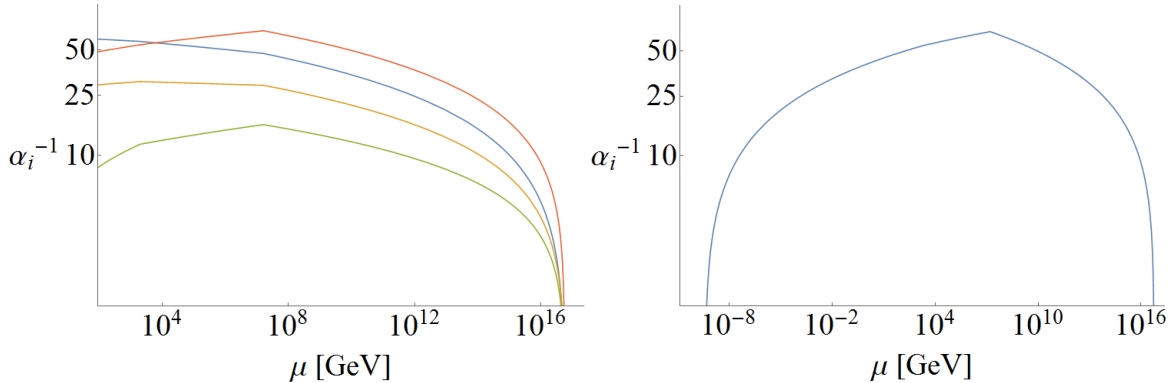
FIG. 5: On the left is the running of (3, 2, 6, 0) with added matter in a 6-dimensional representation of $SU(3)$, α_D shown in red. On the right in the running of just α_D down to its low-energy blow-up at about 0.5 eV.

Glueball DM Outlook

From a model building perspective, this approach to glueball DM is not ideal. Studies that we reference have studied glueball DM as a stand alone model, not one that is supersymmetric and coupled to the Standard Model at higher energy scales. Such additions lead to other stable or nearly stable particles, so that the phenomenological viability of the model is much more complicated to study. There may be a model in which these scenarios are favorable, but more work would need to be done on the theoretical side. Such a study would require a full analysis of options for particle decay and of supersymmetric complications.

This investigation, however, has been illustrative in how the non-perturbative scheme can be used for further model building endeavors. Additional groups and matter can be added easily to the RGEs. Once added, relevant observables can be extracted simply. This makes the non-perturbative unification scheme especially nice for further phenomenology of extensions to the Standard Model. One such addition that can be explored in this way is that of an extra $U(1)$ symmetry group, a popular option to tackle problems in supersymmetry and as a dark matter candidate.

VIII Conclusions

In this study we have explored the framework of non-perturbative unification. This framework is a general, yet simple scheme to study additions to the Standard Model. We have updated a previous study on nonperturbative unification. We find that we can improve on the method of that study and use new experimental error bars for parameters and added corrections. The specific model proposed in that previous study is no longer desirable given new experimental parameter values.

We expanded the search for viable models with other possible additions of matter. We chose to add particles that are especially well studied as an aid for future model building. Many of these additions we found had viable mass scales. Of these, the most promising combinations are the most minimal particle additions. These are also those with the lowest mass scales. Some of these will possibly be detectable at a future 100 TeV collider, being considered in China [9] and at CERN [10].

For these phenomenologically interesting models, we attempted to give a group theoretic origin to the amount of added matter. Motivated by recent theoretical studies of glueball Dark Matter, we described this new matter as a single representation of a dark $SU(N)$ gauge group. We found that our specific models had mass scales that were too low based on previous theoretical investigations of glueball Dark Matter and astrophysical evidence of Dark Matter. These previous studies do not consider a Dark Matter sector that is also supersymmetric, so their bounds do not necessarily apply to our model. More investigation is needed to evaluate whether or not this specific, more complicated, model of Dark Matter and non-perturbative unification is viable.

Non-perturbative unification provides a novel approach to unification that is attractive for exploring physics beyond the Standard Model. It assumes nothing about

the form of a GUT and so can be quite general, yet provides precise phenomenological predictions that can be searched for at current and future colliders. Non-perturbative unification thus provides predictivity without introducing many of the theoretical model building problems that come from assuming the form of high-energy physics. Our specific findings in this study produce the possibility of non-perturbative unification with extra particles at the scale of about 100 TeV, which might be probeable in the near future.

Acknowledgements

A   Appendix

The two-loop RGE without Yukawa couplings is [2]

$$\frac{dg_i}{dt} = \frac{g_i^3}{16\pi^2}\left[b_i + \frac{1}{16\pi^2}\sum_{j=1}^{3} b_{ij}g_j^2\right] \quad .$$

$$(\text{A.1})$$

In the Standard Model, beta functions are given by [2]

$$b_i^{SM} = \left(\frac{41}{10}, \ -\frac{19}{6}, \ -7\right) \quad \text{and} \quad b_{ij}^{SM} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}.$$

$$(\text{A.2})$$

The general beta functions for the minimal supersymmetric model with particle content $(n_g, \ n_h, \ n_5, \ n_{10})$ are [15]

$$b = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_g + \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix} n_h + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} n_5 + \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} n_{10} + \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix},$$

$$(\text{A.3})$$

and

$$b_{ij} = \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_h + \begin{pmatrix} \frac{7}{15} & \frac{9}{5} & \frac{32}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} n_5$$

$$+ \begin{pmatrix} \frac{23}{5} & \frac{3}{5} & \frac{48}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{6}{5} & 6 & 34 \end{pmatrix} n_{10} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix}.$$

$$(\text{A.4})$$

General supersymmetric beta functions for any group G_i and matter fields $T_i(\phi)$ charged under that group are [11]

$$b_i = -3C(G_i) + T_i(\phi), \tag{A.5}$$

and

$$b_{ij} = 2C(G_i)[-3C(G_i) + T_i(\phi)]\delta_{ij} + 4T_i(\phi)C_j(\phi). \tag{A.6}$$

Group theory invariants for the fundamental representation, $\mathbf{F}$, of $SU(N)$ are

$$C_{SU(N)}(\mathbf{F}) = \frac{N^2 - 1}{2N} \tag{A.7}$$

$$T_{SU(N)}(\mathbf{F}) = \frac{1}{2}. \tag{A.8}$$

For the adjoint representation, $\mathbf{A}$, of $SU(N)$ we find

$$C(SU(N)) \equiv C_{SU(N)}(\mathbf{A}) = N \tag{A.9}$$

$$T_{SU(N)}(\mathbf{A}) = N. \tag{A.10}$$

References

[1] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory," Reading, USA: Addison-Wesley (1995).

[2] V. D. Barger, M. S. Berger and P. Ohmann, "Supersymmetric grand unified theories: Two loop evolution of gauge and Yukawa couplings," Phys. Rev. D **47**, 1093 (1993).

[3] H. E. Haber and G. L. Kane, "The Search for Supersymmetry: Probing Physics Beyond the Standard Model," Phys. Rept. **117**, 75 (1985).

[4] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces," Phys. Rev. Lett. **32**, 438 (1974).

[5] C. Patrignani *et al.* [Particle Data Group Collaboration], "Review of Particle Physics," Chin. Phys. C **40**, no. 10, 100001 (2016).

[6] L. Maiani, G. Parisi and R. Petronzio, "Bounds on the Number and Masses of Quarks and Leptons," Nucl. Phys. B **136**, 115 (1978).

[7] N. Cabibbo and G. R. Farrar, "An Alternative To Perturbative Grand Unification: How Asymptotically Nonfree Theories Can Successfully Predict Low-energy Gauge Couplings," Phys. Lett. **110B**, 107 (1982).

[8] D. Ghilencea, M. Lanzagorta and G. G. Ross, "Strong unification," Phys. Lett. B **415**, 253 (1997).

[9] CEPC-SPPC Study Group, "CEPC-SPPC Preliminary Conceptual Design Report. 1. Physics and Detector," IHEP-CEPC-DR-2015-01, IHEP-TH-2015-01, IHEP-EP-2015-01.

[10] M. Bicer *et al.* [TLEP Design Study Working Group], "First Look at the Physics Case of TLEP," JHEP **1401**, 164 (2014).

[11] D. R. T. Jones, "Asymptotic Behavior of Supersymmetric Yang-Mills Theories in the Two Loop Approximation," Nucl. Phys. B **87**, 127 (1975).

[12] T. Moroi, H. Murayama and T. Yanagida, "The Weinberg angle without grand unification," Phys. Rev. D **48**, R2995 (1993).

[13] G. Amelino-Camelia, D. Ghilencea and G. G. Ross, "The Effect of Yukawa couplings on unification predictions and the nonperturbative limit," Nucl. Phys. B **528**, 35 (1998).

[14] A. A. Andrianov, D. Espriu, M. A. Kurkov and F. Lizzi, "Universal Landau Pole," Phys. Rev. Lett. **111**, no. 1, 011601 (2013).

[15] C. D. Carone and H. Murayama, "Phenomenology of minimal SU(5) unification with dynamical supersymmetry breaking," Phys. Rev. D **53**, 1658 (1996).

[16] G. D. Kribs and E. T. Neil, "Review of strongly-coupled composite dark matter models and lattice simulations," Int. J. Mod. Phys. A **31**, no. 22, 1643004 (2016).

[17] A. Soni and Y. Zhang, "Hidden SU(N) Glueball Dark Matter," Phys. Rev. D **93**, no. 11, 115025 (2016).

[18] M. E. Machacek and M. T. Vaughn, "Two Loop Renormalization Group Equations in a General Quantum Field Theory. 1. Wave Function Renormalization," Nucl. Phys. B **222**, 83 (1983).