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Production of B Modes in the CMB from Physics Beyond the Standard Model

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Arts / Science in Department from
The College of William and Mary


by

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Production of B-modes in the CMB from Physics Beyond the Standard Model

Chris Zoghby

May 8th, 2015

Abstract

In this thesis, I investigate several phenomena that affect the polarization of the cosmic microwave background (CMB). First, I examine what effects a hypothetical axion-photon coupling would have on the CMB polarization. I found that axion-photon coupling affects the polarization pattern, but alone is not enough to create B modes from E modes. I also investigate whether Faraday Rotation could rotate an E mode polarization state into a B mode, thereby mimicking a signature of primordial gravity waves.

1 Introduction

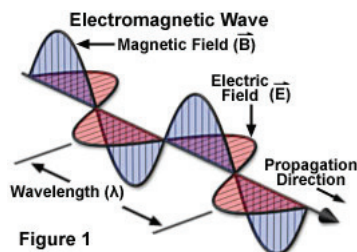


Figure 1: Light as an Electromagnetic Wave[20]

Light, or electromagnetic waves, consists of a pair of oscillating electric and magnetic fields that are perpendicular to both each other and the direction of motion (chosen for

example as the z-axis). There is a freedom in how these fields are oriented, for example the electric field could oscillate through the x-axis, or the y-axis, or at an angle to them.

When unpolarized light reflects off of a surface, it tends to become polarized in a direction parallel to the surface it reflects off of. For example, this is how sunglasses help reduce glare. Sunglasses act as a filter for the polarization of light, only letting light through that is polarized in a particular direction. The polarization of light dictates how the light might appear to an observer on Earth, a key concept in this paper that will be elaborated on later.

The Big Bang theory says that, about 13.8 billion years ago the universe was a very hot, very dense plasma, full of electrons and protons. Spacetime then expanded and the universe began to cool as space expanded, and clouds of dust and gas eventually collapsed under gravity to form stars and galaxies, and eventually us.

But, for the first 380,000 years after the Big Bang, the universe was still this hot, dense plasma, and light could not pass through all of these electrons and protons. If light was emitted, it would scatter off of an electron and then run into another electron and just keep bouncing around in this plasma, unable to leave it. Now as the universe expanded and began to cool, these protons and electrons in this very hot plasma bound together to form neutral Hydrogen. This decoupled the trapped photons, and so emitted light would scatter one last time, and then propagate through the universe generally unimpeded. The last scattering of these photons influenced how they became polarized.

Break down a light rays polarization into two components, which I've chosen oscillate along the z- and y-axes for example, after choosing the outgoing propagation direction to be the z-axis. The component of the electric field parallel to the direction of propagation would not be scattered off of the electron, but the component in the y-direction would be scattered and continue on. Now, I'll introduce another ray of light, this time polarized along the z- and x-axes. This ray of light will, by extension, scatter its x-component of the electric field, which will mix with the outgoing y-component from the other ray of light. Now, if these beams of light had an identical intensity, or amplitude, then there would be an equal

contribution from both beams of light to the outgoing scattered light rays.

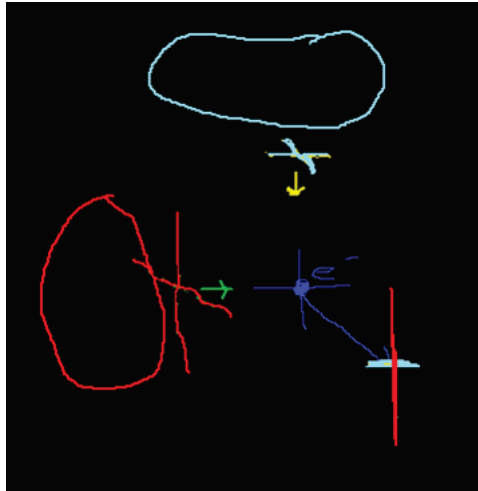


Figure 2: The red circle indicated light coming from a hot region of gas, and the blue circle indicates a relatively colder region where light is coming from.

If the light from one source is coming from a hot region of gas, and the other light beam coming from the second source comes from a gas region cold relative to the first region. The light coming from the hotter gas region will have a larger intensity compared to the light coming from the colder region. So, the outgoing light beam will have a stronger electric field component in the direction that the light from the hot region scattered off of the electron, or, continuing from the last example, the y-axis. The component of the light coming from the cold region that scatters off of the free electron into the outgoing ray will have a smaller amplitude electric field oscillating in the x-direction. So, overall, the scattered light is said to have a net polarization in the y-direction. The temperature anisotropies that influenced this type of scattering were present during the time the universe was a hot, dense plasma. This is the type of polarization the BICEP2 team claims to be measuring.

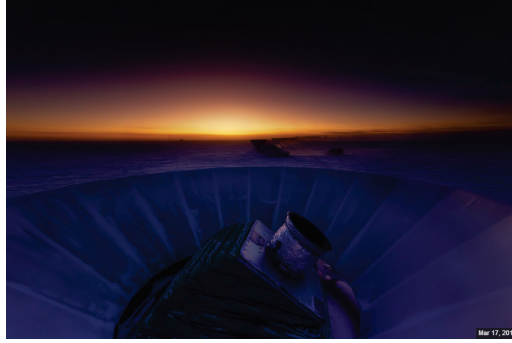


Figure 3: The BICEP Telescope[19]

The BICEP2 experiment is a large telescope set up on the south pole, that studies the polarization of radiation coming from the Cosmic Microwave Background.

The South Pole is an ideal place to study the CMB for these polarization effects because of the naturally dry, cold, air, and high altitudes. From 2010 to 2012, the BICEP2 telescope scanned an empty patch in the sky that the team calls the southern hole. This southern hole is generally free of noisy effects that could be coming from other clusters of galaxies, in addition to our own. The BICEP2 would break down the sky into tiny patches, and measure the net polarization in each patch of the sky and then create a map from this information. The BICEP2 team then mathematically broke down these maps of the net polarization into patterns referred to as E modes, and B modes. The reason for breaking down these maps into these separate polarization patterns is the different physical phenomena that give rise to E and B modes.

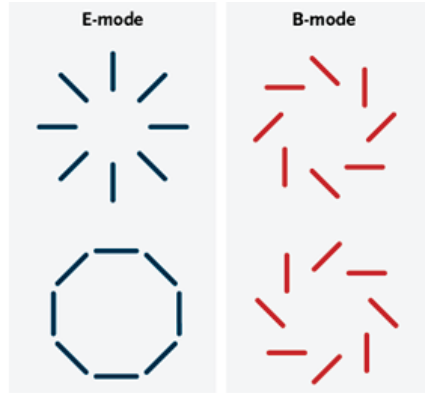


Figure 4: E Mode and B Mode polarizations patterns, and their different orientations when viewed on the BICEP2 temperature maps[13]

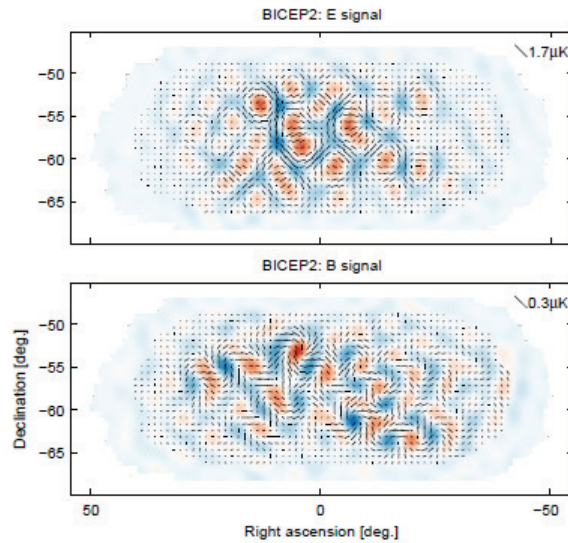


Figure 5: Temperature map breakdown of E and B modes. Note the order of magnitude difference in strength between E and B mode signals in the top right corner.[13]

The early universe, when it was a hot dense plasma, was generally smooth except for tiny density perturbations. These density perturbations in turn created regions with slightly varying temperatures, called temperature anisotropies. So, when the light that was bouncing around in the plasma last scattered, it scattered near these anisotropies, acquiring a net polarization.

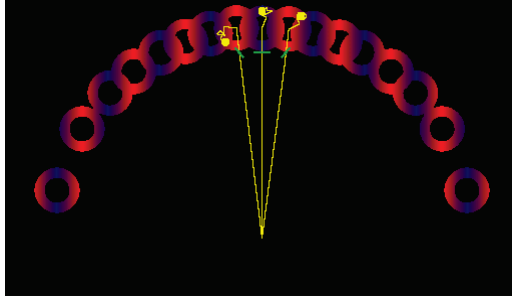


Figure 6: Density wave created temperature anisotropies and light scattering from these anisotropies[21]

These density waves, as it turns out, will only produce E modes in the CMB radiation. They only produce E modes because of the way E modes and B modes behave under parity. Under parity is another of saying if I were to arbitrarily stick a negative sign on one of the positive components of an E modes polarization vector, would its overall sign change? The answer is no. E modes are then known as even under parity. B modes, by contrast, are odd under parity: by reversing one spatial coordinate in the vector describing the polarization of the B mode, the overall sign on B mode polarization would change.

Density waves, similarly, are even under parity. This means that density waves and the temperature fluctuations they cause, also being even under parity, could not give rise to B modes, which are odd under parity. So other types of parity violating physics are needed to produce B modes.

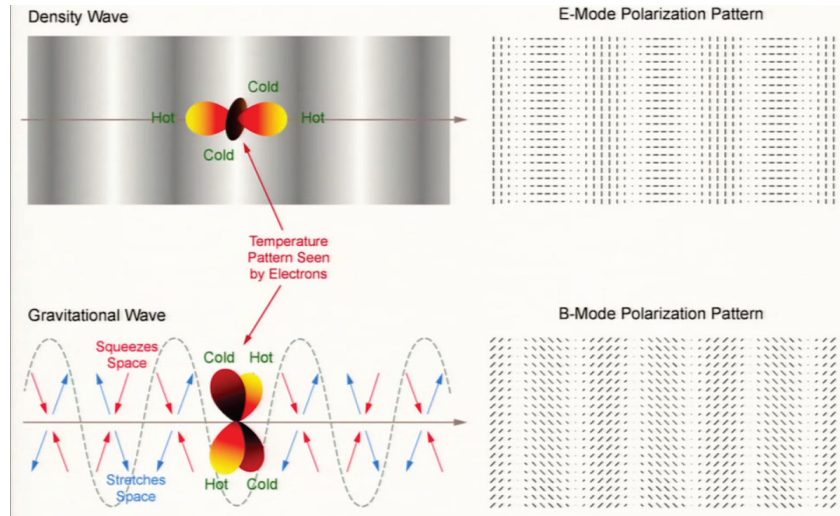


Figure 7: How Gravity and Density Waves Interact with the Space around them[13]

Gravitational waves are ripples in the curvature of spacetime. Gravitational waves propagate at the speed of light in any direction, and don't interact with anything physically, so they could bypass the hot dense plasma of the early universe. As gravity waves pass through the plasma though, they will distort spacetime, stretching and squeezing space in alternate directions. This distortion of spacetime affects how light propagates through it. From the perspective of an electron when these gravitational waves pass by, the stretched regions will appear like the hot and cold temperature anisotropies produced by density waves. Light coming from these regions will therefore be produced with a similar net polarization effect. However, gravitational waves can produce both B modes and E modes. A strong source of gravitational waves would be needed to produce a measureable amount of B modes, stronger than the amount of gravitational waves present during the basic model of the Big Bang. However, if the universe underwent a period of exponential expansion, known as inflation, then there would be a strong enough source of gravitational waves to produce the B modes that BICEP2 detected.

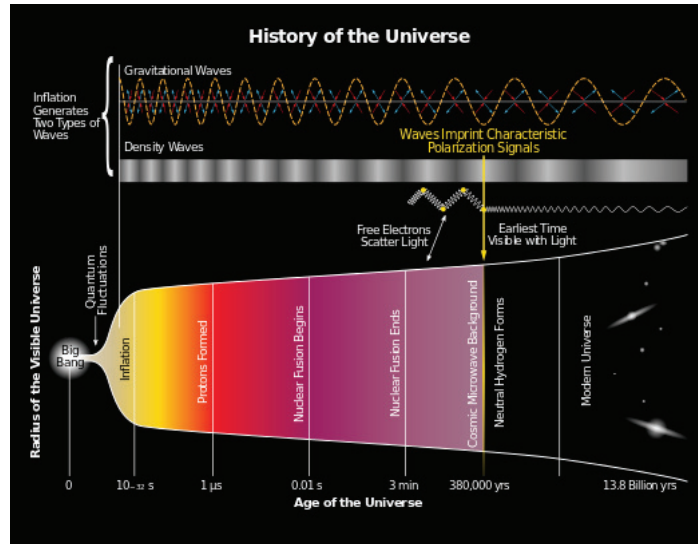


Figure 8: Inflation[13]

A rough approximation of the state of the universe exists to about one has been the best way to learn the state of the universe, but it was trapped in the same hot dense plasma that has been speculated about. Some ideas exist about what must have happened based on how the universe looks now. Inflation is predicted to have lasted from 10^{-36} seconds after the Big Bang to $10^{-33/32}$ seconds. Following the inflationary period, the universe continues to expand but at a less accelerated rate. Inflation is attractive as a theory to physicists because it solves several of problems that appear in the non-inflationary model of the big bang. These problems are the horizon problem, the monopole problem, and the flatness problem.

Think of a closed, isolated container full of gas particles. These gas particles, while maybe initially energetically different, will interact with each other and eventually reach a thermodynamic equilibrium, smoothing out any inhomogeneities in the system. Under the big bang model without inflation, we have no explanation for how the temperature of the CMB appears to be universally the same, despite the fact light traveling from one end of the universe has not had time to reach the other side of the universe, thus meaning those regions have not had a chance to come into causal contact.

The horizon problem declares that, without inflation, the universe should not have had enough time to thermally equilibrate because opposite ends of the universe have not come

into causal contact. Think of causal contact as the gases in a sealed container interacting with each other until they reach thermal equilibrium; the universe is larger in diameter (in light years) than the time since the Big Bang, and as such information sent from one end has not reached the other yet. To continue the container analogy, the gas particles haven't found each other to interact yet, and so the container could not reach equilibrium.

The monopole problem exists in grand unified theories and states that formational events in the very hot, early universe should have produced certain particles, one of which is the magnetic monopole; the problem arises in that we have not yet observed any magnetic monopoles. During an inflationary cool period (cold enough that magnetic monopoles would no longer form), the universe's rapid expansion would have scattered the pre-existing magnetic monopoles, greatly reducing their chance to be observed.

The density of matter and energy in the universe is a critical value when calculating for its space-time curvature. The current observed density shows that our universe is very flat, which means the density of matter and energy in the universe has to be at a very specific value. Because the density falls with time, that means the universe at the time of the Big Bang must have been very close to this critical value. Inflation provides an explanation as to how the universe started so close to this critical value.

BICEP2 measured the sky polarization patterns looking for B modes, because BICEP2 believed the primary source of B modes were gravitational waves. If B modes were found, there must therefore be gravitational waves, and the only way enough gravitational waves could be produced to generate B modes would be if inflation occurred shortly after the time of the Big Bang.

Axions arose initially as an alternative solution to why Supernovae appear less bright to us, instead of the accepted (and now proven method) of an expanding universe, causing the supernovae to shift away from us. Axions would make supernovae appear less bright by interacting with photons in the presence of an extra-galactic magnetic field and encouraging them to oscillate into very light axions.

The axion is a particle that exhibits odd parity behavior, much like gravitational waves.

The initial goal of my project was to investigate whether or not axions, through their interactions with photons, might be able to influence a change on the polarization power spectra when detecting E and B modes, and potentially make an E mode appear to be a B mode. This prospect had potential, because by making photons oscillate into axions, the net polarization of observed light from the CMB might appear to resemble B mode patterning rather than E mode patterning.

2 Axions and B Modes

$$F^{\mu\nu} \equiv \begin{pmatrix} 0 & \frac{-E_x}{c} & \frac{-E_y}{c} & \frac{-E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix} \equiv \frac{\partial A^\nu}{\partial x^\mu} - \frac{\partial A^\mu}{\partial x^\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \quad (1)$$

The electromagnetic field tensor above, where $A^\mu = (\frac{V}{c}, A_x, A_y, A_z)$ is the four-vector potential, was important when studying the way the axion couples to the photon. In particular, the Lagrangian describing the coupling between the axion and the photon was explored. Previous estimations of axion-photon coupling used an ultralight axion [14], such that the term to which the axion coupled was so small, that the effect would disappear in observations and explain why current researchers haven't observed any axions. The Lagrangian describing how the axion couples with electromagnetism, in tensor-component form, is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}\phi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{f}\phi F_{\mu\nu}F_{\alpha\beta}\epsilon^{\mu\nu\alpha\beta} \quad (2)$$

Where $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Cevita symbol, ϕ represents the field of the axion, m is the axion mass, and f is the axion decay constant. In the example of the ultralight axion above, f is very large. From here, we used the Euler-Lagrange equations to begin to calculate the

solution for the equations of motion of the axion:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} \right) = \frac{\partial \mathcal{L}}{\partial A_\nu} \quad (3)$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi} \quad (4)$$

From Eq. (3), we arrived at:

$$-\partial_\mu F^{\mu\nu} + \frac{4}{f} (\partial_\mu \phi) F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} = 0 \quad (5)$$

And Eq. (4) gives:

$$\partial_\mu \partial^\mu \phi = -m^2 \phi + \frac{1}{f} F^{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} \quad (6)$$

Eq. (5) can be expressed in terms of the more familiar \vec{E} and \vec{B} :

$$\frac{1}{c} (\vec{\nabla} \cdot \vec{E}) - \frac{8}{f} \phi (\vec{\nabla} \cdot \vec{B}) = 0 \quad (7)$$

$$-\frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} + (\vec{\nabla} \times \vec{B}) + \frac{8}{f} \phi [\vec{\nabla} \times \vec{E} + \frac{\partial}{\partial t} \vec{B}] = 0 \quad (8)$$

Solutions to these equations are described together by the Klein–Gordon and Maxwell equations. For relativistic axions, defined by having a mass significantly smaller than their energy (writing mass as MeV per c^2 , and using Gaussian natural units so $c = 1$), the above equations reduce to a first-order equation of propagation. Considering a light beam of a single wavelength traveling along the z-axis in the presence of a magnetic field \mathbf{B} , the propagation equation looks like

$$(\omega - i\partial_z + \mathcal{M}) \begin{pmatrix} A_x \\ A_y \\ \phi \end{pmatrix} = 0 \quad (9)$$

where A_x and A_y define the photon's polarization states, ω is the photon or axion energy, and the mixing matrix \mathcal{M} looks like

$$\mathcal{M} = \begin{pmatrix} \Delta_{xx} & \Delta_{xy} & g_{a\gamma\gamma}B_x/2 \\ \Delta_{yx} & \Delta_{yy} & g_{a\gamma\gamma}B_y/2 \\ g_{a\gamma\gamma}B_x/2 & g_{a\gamma\gamma}B_y/2 & \Delta_\phi \end{pmatrix} \quad (10)$$

$\Delta_\phi = -m_\phi^2/2\omega$. Δ_{ij} terms are dependent upon the QED vacuum polarization effect, the Cotton–Mouton (hence abbreviated CM) effect, and the effect of traveling through a plasma which gives the photons an effective mass that scales with the plasma frequency $\omega_{pl}^2 = 4\pi\alpha n_e/m_e$ where n_e is the electron density in the plasma [4]. The QED vacuum polarization effect, relative to the other two contributing factors, is small and safely ignored for the demonstration at hand.

Next, assuming the magnetic field \mathbf{B} is homogenous, I choose the y -axis to be the projection of \mathbf{B} perpendicular to the z -axis. Therefore, $B_x = 0$, $B_y = |\mathbf{B}_T| = B \sin \theta$. Defining $A_x = A_\perp$, and $A_y = A_\parallel$, equation (10) becomes

$$(\omega - i\partial_z + \mathcal{M}) \begin{pmatrix} A_\perp \\ A_\parallel \\ \phi \end{pmatrix} = 0 \quad (11)$$

with the new mixing matrix \mathcal{M}

$$\mathcal{M} = \begin{pmatrix} \Delta_{\perp} & \Delta_R & 0 \\ \Delta_R & \Delta_{\parallel} & \Delta_{\phi\gamma} \\ 0 & \Delta_{\phi\gamma} & \Delta_{\phi} \end{pmatrix} \quad (12)$$

where

$$\Delta_{\phi\gamma} = \frac{g_{\phi\gamma\gamma}}{2} |B_T| \quad (13)$$

$$\Delta_{\perp,\parallel} = \Delta_{pl} + \Delta_{\perp,\parallel}^{CM} \quad (14)$$

Δ_{pl} rises from effects of the plasma frequency ω_{pl}^2 , and can be expressed as

$$\Delta_{pl} = -\frac{\omega_{pl}^2}{2\omega} \quad (15)$$

$\Delta_{\perp,\parallel}^{CM}$ describes the Cotton–Mouton effect, the birefringence of fluids moving through a transverse magnetic field, where $|\Delta_{parallel}^{CM} - \Delta_{bot}^{CM}| \propto B_T^2$ [17]. The Faraday rotation term Δ_R depends on the energy and component of B along the line of sight of the beam path. While initially, safely, ignored for the purposes of axion–photon coupling, the potential for Faraday rotation, without axions, is explored later on in this paper.

$$(\omega - i\partial_z + \mathcal{M}_2) \begin{pmatrix} A_{\parallel} \\ \phi \end{pmatrix} = 0 \quad (16)$$

$$\mathcal{M}_2 = \begin{pmatrix} \Delta_{\perp,\parallel} & \Delta_{\phi\gamma} \\ \Delta_{\phi\gamma} & \Delta_{\phi} \end{pmatrix} \quad (17)$$

These equations are solved by diagonalizing the mixing matrix according to a mixing

angle θ

$$\theta = \frac{1}{2} \arctan \left(\frac{2\Delta_{\phi\gamma}}{\Delta_{pl} - \Delta_a} \right) \quad (18)$$

Similar to neutrino oscillations, the chance for a photon initially in the state A_{\parallel} to oscillate into an axion is given by:

$$|\langle A_{\parallel}(0)|a(s)\rangle|^2 = \sin^2(2\theta) \sin^2(\Delta_{\text{osc}}s/2) = (\Delta_{a\gamma}s)^2 \frac{\sin^2(\Delta_{\text{osc}}s/2)}{(\Delta_{\text{osc}}s/2)^2} \quad (19)$$

$$\Delta_{\text{osc}}^2 = (\Delta_{pl} - \Delta_a)^2 + 4\Delta_{a\gamma}^2 \quad (20)$$

After solving for the probability to convert into an axion, over distances r significantly larger than domain size s [14],

$$P_{\gamma \rightarrow a}(r) = \frac{1}{3} \left[1 - \exp\left(-\frac{3P_0 r}{2s}\right) \right] \quad (21)$$

where P_0 is given by equation (19). When moving over a large enough distance, the intergalactic magnetic field can be safely assumed to be uniform (and small, about 10^{-9} G) over a domain the size of a megaparsec.

The loss in power from photons that have converted to axions was studied, and how that loss in power impacts their polarization power spectrum in E and B mode components. If the axion transition is significant enough, initially E mode polarized radiation might appear primarily B mode polarized after photon \rightarrow axion conversion. However, after calculation, it was concluded that axion photon conversion was not strong enough to distort the power spectrum of initial E mode radiation coming from the CMB.

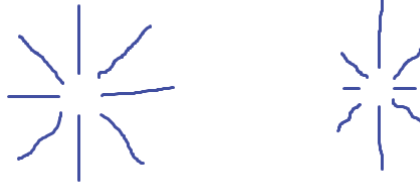


Figure 9: The effect of axion-photon transitions on an incident ray of light from the epoch of last scattering. The overall 1/3 of photons transitioning into axions may change the magnitude of the polarization, but does not distort, so a scattered E mode (left) will still appear an E mode when it reaches Earth (right).

3 Faraday Rotation and B Modes

At the time of decoupling, the CMB should have acquired a measurable amount of polarization. Assuming that a primordial magnetic field was present at the time of last scattering, there would be a faraday rotation of these photons' linear polarization. By comparing two different frequencies of the polarization vector of the CMB, the rotation angle of the polarization can be studied.

Monochromatic radiation of frequency ν passing through a plasma in the presence of a magnetic field B along a given direction $\hat{\mathbf{q}}$ can be roughly estimated to be [1]

$$\frac{d\varphi}{dt} = \frac{e^3 x_e n_e}{2\pi m^2 \nu^2} (\mathbf{B} \cdot \hat{\mathbf{q}}) \quad (22)$$

e , m are the respective charge and mass of an electron, n_e is the total number density of electrons, x_e is the ionization fraction. The rms rotation angle can be estimated with a time integral of the equation above after noticing that $\frac{B}{\nu^2}$ is time-independent and averaging $\varphi^2 \propto (\mathbf{B} \cdot \hat{\mathbf{q}})^2$ over all possible orientations of the magnetic field \mathbf{B} and substituting $\int x_e n_e dt \approx 1/\sigma_T$.

$$\langle \varphi^2 \rangle^{1/2} \approx \frac{e^3 B_0}{2\sqrt{2}\pi m^2 \sigma_T \nu_0^2} = 1.6^\circ \left(\frac{B_0}{10^{-9} \text{ G}} \right) \left(\frac{30 \text{ GHz}}{\nu_1} \right)^2 \quad (23)$$

In the equation above, B_0 is the current amplitude of the cosmological magnetic field, and ν_0 is the observed frequency of the CMB photon. This estimation was further refined to include dependency on comparing two different frequencies of the polarization vector (for derivation see [1]). For a primordial field of strength $B_0 = 10^{-9}$ G, and observed frequency of 30 GHz, the mean Faraday rotation is found to be about 1° or, more explicitly,

$$\langle \varphi_{12}^2 \rangle^{1/2} = \frac{1.6^\circ}{\sqrt{2}} \left(1 - \frac{\nu_1^2}{\nu_2^2} \right) \left(\frac{B_0}{10^{-9} \text{ G}} \right) \left(\frac{30 \text{ GHz}}{\nu_1} \right)^2 \quad (24)$$

The Faraday rotation of an initially polarized E mode photon is not enough to produce a B mode.

4 Conclusion

Axion–photon mixing, while affecting the polarization state, does not seem to generate enough net polarization change that the dominant polarization state of an E mode would be transformed into a B mode.

Additionally, Faraday rotation of a last–scattering photon by means of a primordial magnetic field present at the time of decoupling has been shown to have not had enough of an effect to rotate and initially polarized E mode into a B mode.

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