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"Infallible Proofs": Math, Knowledge, and Religion in the Medieval Islamicate World

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Math, Knowledge, and Religion in the Medieval Islamicate World

A thesis submitted to the Department of History at
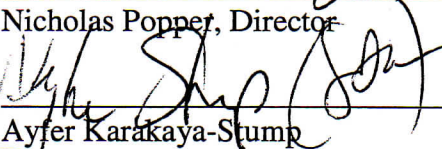The College of William and Mary

by

Deborah Wood

Accepted for ___Highest___ ___Honors___
(Honors, High Honors, Highest Honors)

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Williamsburg, VA
April 30, 2014

I | Introduction

Communities in different times and places in the world have given varying weight to forms of knowledge and the variety of processes by which knowledge is believed to be created. Bound to each society's unique culture, these epistemological hierarchies are constantly in flux: Social and religious factors, for example, shaped such hierarchies by emphasizing some processes of creating knowledge ("ways of knowing") over others, while those ways of knowing could in turn influence the society in which they were performed.

In this study, I examine the changing status of mathematics in the epistemological hierarchy of the medieval Islamicate world, especially in terms of its relationship to other forms of natural and secular knowledge.[1] Part of my research included a quantitative study, detailed in Appendix A, which suggested two trends: (1) Overall, the field of pure mathematics drew more Muslim than non-Muslim scholars, and (2) this trend was not consistent across time; rather, interest spiked in the late ninth to early tenth century and remained high through the rest of the 'Abbasid period. I use these results as guidelines in this study, but the preponderance of my evidence is qualitative in nature as I demonstrate how perceptions of mathematics transformed over the course of three and a half centuries.

From c. 750 to 1100, mathematics underwent a substantial status change in the Islamic world that enabled it to survive a dramatic theological and epistemological shift in the eleventh century that denied legitimacy to other reason-based processes for creating secular knowledge. In those three and half centuries, mathematics was first established as an robust investigative

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[1] The Islamicate world and the Islamicate Empire to which I will be referring are typically just called "Islamic" or "Arabic," but I believe these terms obscure the true religious, ethnic, and linguistic diversity of the region. Because my argument has a religious dimension, I am particularly interested in distinguishing individuals, communities, and other social elements that were "Islamic," thus exclusive of non-Muslims, from those "Islamicate" ones that included actors of varying religious orientations.

field, then gained epistemic clout through its increasing reliance on geometric and algebraic proofs to support its claims. Over the course of the eleventh century, human reason was increasingly denigrated as an invalid way of knowing. Proofs demonstrated with mathematics, however, were increasingly interpreted as transcendent of the foibles of mortal intellect, and subsequently, math remained unassailed by the epistemological atmosphere of the Islamicate world. In fact, all ways of knowing about the natural world, such as the medieval Islamicate versions of what are today called astronomy and physics, were forced subsequently to rely on either sacred revelation or on mathematical foundations, rather than on rational contemplation, until at least the end of the period (c. 750 – 1258) examined here.

The shift in mathematics' epistemological standing between c. 750 and 1100 reflected intertwined political and theological changes occurring in the Islamicate world. At the center of politics in the region was the Islamicate Empire, which had started as the original Muslim community founded by the Prophet Muḥammad in the year 622. The 'Abbasid dynasty came to power in the central Islamicate Empire c. 750, and it was more interested in institutional consolidation than in territorial conquest. Mathematics was initially secured a place in Islamicate society during these early years of the 'Abbasid period through its utility in solving practical problems that faced the ruling Islamic institutions, including the administration of worship rites and the adjudication of Qur'anic inheritance law. As part of early 'Abbasid rulers encouragement of cultural and intellectual growth, they instigated a comprehensive translation movement that introduced Indian and then Greek texts to Islamicate scholars. These texts included books on mathematics, among other fields, and the Islamicate scholastic community incorporated elements from both mathematical traditions into their own, including decimal arithmetic from India and geometric principles from Ancient Greece. Such additions expanded

the field encompassed by mathematics and built on the interest established by Islamic social concerns to engender further scholastic attention to math.

The introduction of foreign mathematics had epistemological consequences, as well as practical ones. Ancient Greek geometry—especially Euclid's *Elements*—introduced the notion of general geometric proofs to Islamicate mathematics. Such proofs claimed to have the advantage of being apodictic and thus transcended the limitations of subjective human reasoning. Islamicate scholars furthered the proof theory they adapted from the Ancient Greeks to apply geometric proofs to algebraic problems and to create algebraic proofs, which were particularly novel when applied to geometric problems. This development of Islamicate proof theory equipped mathematics with an epistemic authority beyond that of other rational, un-mathematized forms of natural knowledge, setting it apart and above human reason. Political and theological changes in the eleventh century worked in tandem to produce a new epistemological *milieu* that rejected the ability of the human intellect to create valid and reliable knowledge. However, due to the unique degree of authority granted to mathematics, it—and mathematized ways of knowing—remained securely in place in Islamic society.

I.a | Knowledge in Context

I.a.i | Knowledge in Arabic

In referring to "fields of knowledge," as I have already done, I am drawing on the Arabic term *'ilm*.[2] Franz Rosenthal points that it can have multiple definitions; there is "*'ilm*, the concrete, specialized discipline of learning," and "*'ilm*, the abstract concept," or it could be

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[2] Translation as "field of knowledge" comes from Jens Høyrup, "The Formation of 'Islamic Mathematics': Sources and Conditions," *Filosofi og Videnskabsteori på Roskilde Universitetscenter*, 3. Række, *Preprints og Reprints* Nr. 1 (1987): 18.

formulated as the "process of knowing."[3] In many cases, this word is translated into English as "science" when used in its first definition. Seyyend Hossein Nasr, for example, translates Abu'l-Ḥasan al-Bastī's ("al-Qalṣādī") *Kashf al-asrār 'an 'ilm al-ghubār* as *The Unveiling of the Mysteries Concerning the Science of the 'Dust-Board'*.[4] David Eugene Smith and Salih Mourad give a similar title, Sinān ibn al-Fatḥ's *'Ilm hisāb al-takht*, as *The Science of Arithmetic of the Takht [Dustboard]*.[5] Even if we take their idea of "science" to be contextually suitable, which is not an insignificant condition, this would be an inappropriately limited way of conceptualizing *'ilm*. After all, Rosenthal claims that no other term, even central religious ones, "equals *'ilm* in depth of meaning and wide incidence of using," and to confine its translation to a single word, "science," is to obscure some of its variation.[6] To avoid, then, both the ambiguity of the Arabic term at my disposal and to highlight distinctions between meanings, I use variations on several terms. "Field of knowledge" is meant to have the same flexibility as "*'ilm*, the concrete, specialized discipline of learning," without the stricter modern connotations of "discipline" or "science." As an abstract concept, *'ilm* can be translated as "knowledge," and the phrase "ways of knowing" is meant to capture the processes by which knowledge is thought to be created.

The term "mathematics" refers to both a subject and a process—a "field" and a "way"—existing in constant interaction. Although I have not found any single medieval Arabic word that carried the same meanings as the English "mathematics," I do believe it accurately represents a unique field as conceptualized in the medieval Islamicate context, distinguished from other forms of *'ilm* by its distinctive manipulation of abstract numerical and spatial concepts that might

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[3] Franz Rosenthal, *Knowledge Triumphant: The Concept of Knowledge in Medieval Islam* (Boston: Brill, 2007), 43, 52-69.
[4] Seyyed Hossein Nasr, *Islamic Science: An Illustrated Study* ([London?]: World of Islam Festival Publishing, 1976), 86.
[5] David Eugene Smith and Salih Mourad, "The Dust Numerals Among the Ancient Arabs," *The American Mathematical Monthly* 34 (1927): 258.
[6] Rosenthal, *Knowledge Triumphant*, 2.

or might not have real life counterparts. Despite perhaps lacking a single, unifying umbrella term in Arabic, a certain language grew to define the processes and fields of knowledge that I capture as "mathematics": For example, the phrases *'ilm al-takht* or *'ilm al- ghubār*, above, referred to arithmetical methods derived from Indian sources; *al-jabr* became "algebra" today, both topically and etymologically; *al-handasa* referred to geometry and *al-ḥisāb* to calculation. Although I try to make the distinctions inherent in these Arabic terms when it is relevant, these subfields became increasingly muddled—especially *al-jabr* and *al-handasa*—with time. However, it is their collective practical utility and epistemological authority in which I am interested, so I predominantly refer to them collectively as "mathematics."[7]

I.a.ii | Knowledge in early Islam

Islam's affinity for *'ilm* stretches back to its inception. Setting a tone that emphasized learning, the angel Gabriel's first injunction when he began to impart the Qur'an to the illiterate Prophet was "Read! In the name of your Lord who created: He created man from a clinging form. Read! Your Lord is the Most Bountiful One who taught by [means of] the pen, who taught man what he did not know."[8] Throughout the whole Qur'an, relatives of the word *'ilm* (knowledge) occur over 750 times and make up approximately 1% of the vocabulary of the entire holy book.[9]

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[7] Or, sometimes, "math." To some experts in mathematics education, "math" suggests in particular the kinds of ideas that are taught in K-12 classrooms; however, I use "mathematics" and "math" interchangeably, with no aim but variety.

[8] Qur'an 96:1-5. Although these verses begin what is now the 96th *sūrah* of the Qur'an, they are most commonly believed to be the first revelation Muhammad received. This is in accordance with *ḥadīth*, the assemblage of recorded fragments of the Prophet's life which have only less authority than the Qur'an in prescribing the rules of Islamic life. The story about the first revelation can be found among *hadith* of al-Bukhari, namely Volume 1, Book 1, Number 3.

[9] Rosenthal, *Knowledge Triumphant*, 19-20. Rosenthal points out that this makes the word group around *'ilm* one of the most popular families in the Qur'an.

Hadith further abound with instructions to learn and to teach. Perhaps the most compelling spiritual case for pursuing knowledge can be found in the *Sunan Abu Dāwūd*:

> If anyone travels on a road in search of knowledge, Allah will cause him to travel on one of the roads of Paradise. The angels will lower their wings in their great pleasure with one who seeks knowledge, the inhabitants of the heavens and the Earth and the fish in the deep waters will ask forgiveness for the learned man. The superiority of the learned man over the devout is like that of the moon, on the night when it is full, over the rest of the stars. The learned are the heirs of the Prophets, and the Prophets leave neither dinar nor dirham, leaving only knowledge, and he who takes it takes an abundant portion.[10]

Under such encouragement, it is little wonder that Muslims make up three-quarters of the scholars identified by religion in the quantitative study. Rather like theist natural philosophers of the European Enlightenment, Muslim scholars in the early ʻAbbasid period saw their investigations into the natural world as religious experiences, plumbing the mysteries of the earth in order to grow closer to the Creator. In the words of Seyyed Hossein Nasr, "Islamic science…seeks ultimately to attain such knowledge as will contribute toward the spiritual perfection and deliverance of anyone capable of studying it."[11] Thus, in large part, early Muslim scholars sought both theological and natural knowledge, and it is inappropriate to draw a hard and fast line between practitioners of theology and other ways of knowing in the until the eleventh century.[12] Even among those early scholars, however, one interest outweighed the rest,

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[10] Ahmad Hasan, trans., "Knowledge" (Kitab al-ʻIlm), *Translation of Sunan Abu-Dawud* (Center for Muslim-Jewish Engagement, University of Southern California, 2011) <http://www.usc.edu/org/cmje/religious-texts/hadith/abudawud/025-sat.php>. This is a partial translation of a ninth century *hadith* collection.

[11] Seyyed Hossein Nasr, *Science and Civilization in Islam* (Cambridge: Harvard University Press, 1968), 39.

[12] It is necessary to specify *early* Muslim scholars here because a separation of "secular" and "holy" knowledge occurred in the eleventh century, under the influence of parallel political and theological shifts. At that time, it is also inappropriate to speak of a monolithic "Islamic" approach to knowledge, as the major sects of Islam, Sunnism

earning them one label or the other. The crisis in the eleventh century was even a manifestation of this: As rational ways of creating secular knowledge were increasingly perceived as a threat to Islam, rather than complements to it, by the theological elite newly empowered by political changes, the elite's rejection of reason as a legitimate way of knowing in favor of revelation was intended to reunite spiritual and natural knowledge.

One social distinction even more basic than that distinguishing scholars by their interests was the divide between professional "seekers after truth," to use a twelfth-century term, and laymen. While the individuals who became professional scholars did not have to come from social elite, they acquired a degree of privilege through patronage. That is, though rarely possessing any social clout of their own, scholars in the Islamicate world were instead elevated above laymen by the favor bestowed by political elites. Extending patronage became an important status symbol in the 'Abbasid era: As Ruth Stellhorn Mackensen described the phenomenon, "Learning, in a sense, may be said to have become fashionable at court."[13] Consequently, significant political figures, including caliphs and viziers, raised favored theological specialists to political posts and other clients to institutions for learning.[14] The individuals in the second category formed an intellectual elite to which I frequently refer as an Islamicate scholastic community. It was characterized by frequent moves—both from rural to urban centers and from one urban center to another—and by consistent communication, via

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and Shi'ism, diverged in their understandings of what constituted legitimate ways of know. See (II.a) for a fuller discussion.

[13] Ruth Stellhorn Mackensen, "Four Great Libraries of Medieval Baghdad," *The Library Quarterly* 2 (July, 1932): 279.

[14] For example the Mu'tazilite theologian 'Abd al-Jabbar lost his position as magistrate when his Buyid patron died. His story can be found in Richard C. Martin and Mark R. Woodward, *Defenders of Reason in Islam: Mu'tazilism from Medieval School to Modern Symbol*, with Dwi S. Atmaja (Oxford: Oneworld, 1997), 35.

letters or commentaries, between its participants, even across temporal, political, and theological

lines.[15]

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[15] For examples of their correspondence, see al-Biruni and ibn Sina's contentious exchange over Aristotle in Rafik Berjak and Muzaffar Iqbal, "Ibn Sīnā–Al-Bīrūnī Correspondence," *Islam and Science* 1 (2003): 91-98, 253-260; 2 (2004): 57-62, 181-188; 3 (2005): 57-62, 166-170; 4 (2006): 165-172; 5 (2007): 53-60. For English extracts of the correspondence between Abū Isḥāq Ibrāhīm ibn Hilāl al-Ṣābī (925-944) and Abū Sahl Wayjan (Wījan) al-Kūhī (al-Qūhī) about geometric principles and their relation to center of mass, see J. Lennart Berggren, "Mathematics in Medieval Islam," in *The Mathematics of Egypt, Mesopotamia, China, India, and Islam*, ed. Victor Katz (Princeton, NJ: Princeton University Press, 2007), 567-573.

II | Background

The transformation of mathematics with which this paper is preoccupied necessarily occurred within a broader historical context, and indeed, this project evolved under the influence of potential and existing research. In order to position this thesis within an appropriate intellectual context, this section outlines first how its foundational research questions developed, then how the present product fits within the historiography of "Islamic science." Finally, the section will provide the political and theological background necessary to set up the next three chapters, which discuss in greater depth mathematics' rise as a subject of interest and as an epistemic authority.

II.a | Modern Intellectual Context

II.a.i | Research Questions

When I began researching for this project over a year ago, my initial research questions differed greatly from the ones posited above. My interest has remained over time fixed on the interaction of math and religion in the Islamicate world, and due to its religious diversity and cultural efflorescence, the 'Abbasid period persisted as the natural temporal focus of the study, even as my questions changed dramatically. Although I eventually elected to investigate the role of mathematics in Islamic epistemology, I at first intended to examine how religious affiliation influenced the mathematical topics individual scholars chose to pursue. For example, would Nestorian Chrisitans in general be driven by their typically Byzantine heritage to study geometry, which was highly influenced by the ancient Greeks? Or, since their religion was descended from Babylonian star-worship, would Sabians such as Thabit ibn Qurra be drawn to trigonometry and mathematical astronomy?

This approach to mathematics and religion in the medieval Middle East was troubled by the problem of insufficient data, however. My proposed prosopographical methodology seemed increasingly unlikely to generate useable results as I encountered difficulties in determining, first, a scholar's sectarian identity and, second, a mathematical subfield for which a scholar had an "affinity." Moreover, I began to worry about the validity of extrapolating relationships between sects and subfields out of a limited number of case studies.

Many of my doubts arose as I read secondary sources for background research, none of which gave any hint that questions like mine could feasibly be answered, and a variety of other half-formed alternative questions presented themselves as I struggled onward. As I perused Boris Rosenfeld and Ekmeleddin Ihsanoğlu's *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th-19th c.)* (MAOS), however, I seemed to note a rising association of mathematics with Muslim scholars, which made sense with primary and secondary sources I had already read. Thus, I pursued the quantitative study based on MAOS detailed in Appendix A. Out of that, my final research questions emerged.

II.a.ii | Historiography

As shown in (I.a) above, this study is not exceptional in highlighting the importance of rational inquiry in the early 'Abbasid period or in demonstrating its decline near the end of the dynasty. Its importance relates instead to the distinction emphasized here between mathematics and other ways of knowing. In particular, the demonstration of mathematics' endurance as a Muslim way of knowing all the way to the end of the 'Abbasid era is, as far as I am aware, unique.

Although the question of why "Islamic science" declined in the Middle Ages is not at present a popular topic in academia, it is at least indirectly addressed in most works on the

subject of Islamic science. Moreover, it is the primary theme of many classic texts in the field, and it appears in popular science at times, so it is certainly a well-known topic.[16] Traditionally, however, academic and non-academic works alike in the history of Islamic or Arabic science use the term "science" to refer monolithically to rational ways and forms of knowledge. When mathematics is distinguished from other sciences, it is usually done in a chapter designated to recount the mathematical accomplishments accomplished by medieval Islamicate scholars, but notice is rarely drawn to the fact that math continued to advance in this tradition well after the "decline" of Islamic science.[17] Therefore, this study is intended to explicate the division of math from the other forms and ways of knowing subsumed under the traditional term "science," and in doing so, to provoke additional consideration about treating mathematics as something that may not always be readily grouped with other rational ways of knowing.

Before reason-based knowledge faced serious epistemological challenge in the eleventh century, Islamic traditions encouraged all ways of knowing as processes by which God could be revealed. But in the midst of a complicated *milieu* of politico-religious changes in the eleventh century involving the dominant sects of Islam, Sunnism and Shi'ism, new theological schools patronized by new government forces elevated revelatory knowledge above all other forms. It is important to note that Islam was never in this period an opponent to learning or knowledge; instead, dominant ideas of *how* knowledge was to be created validly and reliably changed from reason to revelation.

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[16] See for example Dennis Overbye, "How Islam Won, and Lost, the Lead in Science," *New York Times*, October 30, 2001. For an example of a recent academic piece on the decline of science in the medieval Islamic world, something rarely covered so explicitly today, see Hillel Ofek, "Why the Arabic World Turned Away from Science," *The New Atlantis* 30 (Winter 2011): 3-23.

[17] The only exception I noted was Victor J. Katz, *A History of Mathematics: An Introduction*, 3rd ed. (Boston: Addison-Wesley, 2009), 267.

The Prophet Muhammad (570-632) lived for most of his life in Mecca, where in 610 AD he began to receive divine messages through the angel Gabriel. On the authority of this communication with God, he became both a political and religious leader, establishing a Muslim community in the city of Medina in 622, and as shown above, the religious message he related emphasized the pursuit of knowledge—both of God and of the world created by Him. It also stressed the need to spread this new religion, so Muhammad led military campaigns to extend the Islamic politico-religious system. Mecca surrendered in 630, and the Muslim armies swept onward, promulgating Islam. Within a century of the birth of Islam, Muhammad's thirteenth successor ruled over an empire that stretched from Iberia to the Indus Valley.[18]

The question of succession lay at the heart of the sectarian split between Sunni and Shiʿa Islam. When Muhammad had died in 632 without any male heirs, a new leader was chosen for the Muslim community by a consensus of his closest associates. At the core of Shi'a dogma is the (political) conviction that Muhammad had named ʿAlī, his cousin and son-in-law, his heir before he died; hence, only ʿAli was the legitimate leader of the Islamicate Empire. Even the sect's name, "Shiʿa," is derived from "shiʿat ʿAli," meaning partisans of ʿAli. In contrast, the term "Sunni" comes from the phrase "the people of [Muhammad's] example [*sunna*] and community," for these Muslims accepted the caliphs (literally "successors") appointed by the consensus of the *ʿulamā'*, the community of religious elite, which included theologians, as well as judges and jurists.[19] Although religious authority ultimately rested in the Sunni view with the

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[18] Like my use of the term "Islamicate" in referring to the scholarship produced by individuals under the rule of a Muslim caliph regardless of their own religious identity, I call Muhammad's empire the Islamicate Empire because

[19] These figures were responsible for arbitrating Islamic law (*Shar'ia*) for the courtroom and everyday life. Jurists in particular overlapped with theologians by interpreting and writing on the Qur'an. These private scholars were very well-respected in society—and powerful since they essentially defined the "law" that the Muslim populace felt compelled to obey. For more information about their place in Islamic society, see Egger, *A History of the Muslim World to 1405*, 115-122, 255.

ulama, the caliph was expected to exert his political power in accordance with and for the benefit of Islam, so he was assigned a religio-political role.

Shi'ites on the other hand believed that God would not leave the Muslim community without a single divinely-selected leader (Imam) who, like the Prophet, held both political and religious power. This leader possessed esoteric knowledge that the community would never know, and he had to designate his own successor. They posthumously named 'Ali the first Imam and his sons his successors. While Shi'a Imams never gained much by way of political power, they were widely recognized, even by Sunnis, as wise theologians. Indeed, many Muslims who accepted the authority of the *ulama* also sympathized with the plight of 'Ali and his family and even agreed that the true leader of the *Dār al-Islam* ought to belong to the House of the Prophet.

Even the barest of that theology, however, did not arise complete in 632. Rather they were developed over time, and according to Haider Ala Hamoudi, they are still in a degree of flux today.[20] In the seventh century, after four unrelated caliphs ruled in turn and the Prophet's associates began to die, the Umayyad clan established itself as a familial dynasty from 661 to c. 750, after 'Ali (the fourth caliph and Muhammad's alleged heir) was murdered by a member of the Khāriji sect. Although the Umayyads were responsible for expanding the Islamicate Empire to the Pyrenees in the West and the Indus Valley in the East, they did not much endear themselves to the home front. After capitalizing on 'Ali's death in the midst of war with him, the Umayyads were later responsible for killing his son (the Prophet's grandson) Ḥusayn in 680 at Karbala. Considered "by Muslims of all persuasions as perhaps the greatest single calamity that befell the [Islamic] community in its early history," Husayn's martyrdom at Karbala became a

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[20] Haider Ala Hamoudi, "Understanding the Sunni-Shi'a Split: From Theology to Law," lecture, March 14, 2014, The College of William & Mary.

focus of opposition to the Umayyads, in addition to their perceived decadence and treatment of the caliphate as a "secular kingship."[21]

In 750, the ʿAbbasids toppled the Umayyads by harnessing the proto-Shiʿa elements of opposition to the incumbent dynasty. In the mid-eighth century, Shiʿism had yet to consolidate into a defined sect with a coherent doctrine, so the ʿAbbasid effort took advantage of ambiguity in the definition of "the House of the Prophet" in order to secure proto-Shi'a support. The ʿAbbasids took their clan name from the progenitor al-ʿAbbas, one of Muhammad's uncles. Because of that relationship, ʿAbbasid rule appealed to individuals who defined the House of the Prophet the most liberally: as his whole clan. In soliciting support from those who limited the House of the Prophet to ʿAli's line, the ʿAbbasids claimed that one of ʿAli's sons had designated al-Abbas his heir before his death.

The ʿAbbasids themselves were not Shi'a, but in the ambiguity of sects in the eighth century, their strategy of framing themselves as members of the House of the Prophet helped legitimize their claim to the caliphate. But the office of the caliph was itself a proto-Sunni construct, and once proto-Shi'ites and Shi'a-sympathizers elevated the ʿAbbasids to the caliphate, the new dynasty initially continued Umayyad policies of persecuting proto-Shi'ism. Eras of particular scholastic achievement, however, coincided with Shi'a-sympathetic leadership. Caliph al-Ma'mūn (r. 813-833), for example, attempted to heal the divide between proto-Shi'ism and proto-Sunnism. He was in fact so conciliatory to Shi'ism that he adopted its green flag over the ʿAbbasid black for a time, and he even named the Shi'ite Imam Ali al-Ridha his heir.[22] Al-

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[21] Douglas Karim Crow, "The Death of al-Ḥusayn b. ʿAlī and Early Shīʿī Views of the Imamate," *Alserat* 12, Papers from the Imam Husayn Conference, London, 6-9 July 1984 (London: The Muhammadi Trust of Great Britain and Northern Ireland, 1986): 71; Berkey, *The Formation of Islam*, 78-79, 88.

[22] But the Imam al-Ridha died (some say suspiciously) before al-Ma'mun. Jim al-Khalili, *The House of Wisdom: How Arabic Science Saved Ancient Knowledge and Gave Us the Renaissance* (New York: Penguin Press, 2011), 14-15.

Ma'mun was at the same time one of the greatest patrons of the sacred and secular scholarship, continuing a tradition that began with the very first 'Abbasid caliphs.

Although they were unable to maintain the same geographic enormity of their empire, the early 'Abbasids ushered in a comparative golden age of high culture. Within their own boundaries, they inherited the considerable intellectual traditions of Mesopotamia, Persia, and Ancient Egypt as well as the schools of Alexandria, and their neighbors included the shrinking Byzantine Empire and kingdoms of the Indian subcontinent. The practice of gathering texts from these various traditions and translating them into Arabic is said to have originated in 770 in the newly founded capital city, Baghdad. There, Caliph al-Manṣūr received an official delegation from India, which included an astronomer and at least one Sanskrit text.[23]

Translating foreign sources became a concerted movement under Caliph Hārūn al-Rashīd (r. 786-809) and flourished under his son, al-Ma'mun.[24] The latter was something of a scholar himself, and he established in Baghdad an institute of collaborative scholarship, the House of Wisdom (*Bayt al-Ḥikma*), where he installed and patronized scholars from all over the caliphate, regardless of birth or religion. As the center of the translation movement, the House of Wisdom enabled Islamicate scholars to synthesize a variety of intellectual traditions.[25] Ḥunayn ibn 'Isḥāq

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[23] Recorded by al-Qifti, translated by Woepcke, *Mémoire sur la propagation des chiffres indiens* and quoted in Georges Ifrah, *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, trans. by David Bellos, E.F. Harding, Sophie Wood, and Ian Monk (New York: John Wiley & Sons, 2000), 529. Also reported by the eleventh-century historian Ṣāʿid al-Andalusī, *Science in the Medieval World: "Book of the Categories of Nations"*, trans. and ed. Semaʿan I. Salem and Alok Kumar (Austin: University of Texas Press, 1991), 46; Roshdi Rashed, *Encyclopedia of the History of Arabic Science* (London: Routledge, 1996) 1: 80.

[24] Ṣaʿīd al-Andalusī, from the eleventh century, reported that Caliph al-Rashid was the most ardent in wishing to continue the work begun by al-Mansur in translating as many foreign texts as he could. Al-Mansur's original work at the observatory was additionally informed by his own extensive education, especially in logic and law, and he was fond of "philosophy and observation…and of the people who worked in these fields." Al-Andalusi, *Science in the Medieval World*, 44.

[25] Some historians are skeptical about the existence of the House of Wisdom since no physical evidence remains. Even if the House of Wisdom was not the codified institution it is generally believed to be, but rather a loose coalition of scholars patronized by the caliphate and members of the upperclass, Baghdad certainly became a center of learning in the ninth century. For the doubts of its existence and for the projects charged to the House of Wisdom, see respectively al-Khalili, *The House of Wisdom*, 68, 80.

(809-873), an eminent physician better-remembered today as a skilled, prolific translator, traveled himself around Mesopotamia, Syria, Palestine, Egypt, and the Byzantine Empire to collect manuscripts.[26] Caliph al-Ma'mun sought foreign texts diplomatically and demanded them from nations defeated in battle. Indeed, according to legend, the Islamicate Empire received Ptolemy's highly influential *Almagest*, among other Greek manuscripts, as part of a peace treaty al-Ma'mun signed with the Byzantine Emperor, Theophilus.[27]

This was also the time that a (Sunni) theological school called Muʿtazilism became a dominant influence on the intellectual atmosphere of the Islamicate Empire and, to a certain extent, of the whole Islamicate world. Starting c. 800, Mu'tazilite theologians began to enjoy the patronage of 'Abbasid leaders, including al-Ma'mun and his father, and it flourished throughout the ninth century, receiving patronage until the mid-1000s.[28] It was a comprehensive theological school with developed tenets on a wide variety of issues facing Islam, including the nature of the Qur'an, Allah, and free will, but its relevance here is for its epistemology: Mu'tazilism held that the universe must be rational, for "God would not deceive His creatures by creating an irrational universe," and if the universe was rational, then it could be known by human reason (*'aql*).[29] Mu'tazilism even maintained that the sacred texts, the Qur'an and the *hadith*, could be

[26] Ali A. Al-Daffa and John J. Stroyls, *Studies in the Exact Sciences in Medieval Islam* (Chichester, UK: John Wiley & Sons, 1984), 3-4; al-Khalili, *The House of Wisdom*, 74-75; Bayard Dodge, ed. and trans., *The Fihrist of al-Nadim: A Tenth-Century Survey of Muslim Culture* (New York: Columbia University Press, 1970), 2: 584.

[27] A number of authors—Joseph A. Angelo, *Encyclopedia of Space and Astronomy* (New York: Infobase Publishing, 2009), 78; Jean-Claude Pecker, *Understanding the Heavens* (New York: Springer-Verlag, 2001), 143-144; Anton Pannekoek, *A History of Astronomy* (London: George Allen & Unwin, 1961; Mineola, NY: Dover, 1989), 166—repeat the story that al-Ma'mun stipulated the receipt of texts (some say "many", others "annually") as part of at least one peace treaty, whether or not Ptolemy was among them, but I cannot verify this. I am unable to find any evidence that Theophilus and al-Ma'mun (or their immediate successors) ever actually declared peace between them. The eleventh century historian al-Andalusi, in contrast, relayed that al-Ma'mun made friends with the emperors of Byzantium and used gifts to extract copies of Ancient Greek books, such as those by Plato, Aristotle, Euclid, and Ptolemy. Al-Andalusi, *Science in the Medieval World*, 45.

[28] Al-Khalili, *The House of Wisdom*, 14; Martin and Woodward, *Defenders of Reason in Islam*, 1.

[29] Martin and Woodward, *Defenders of Reason in Islam*, 11.

interpreted in context by human intellect.[30] In so far as what reason could understand, it did not distinguish between holy and secular knowledge. Patronized in particular by Shi'a-sympathetic leaders like al-Ma'mun and dominating the intellectual trends of the Islamicate Empire for almost two and a half centuries, Mu'tazilism enabled rational ways of knowing of all kinds to flourish, including mathematics.

While Mu'tazilism was predominantly a Sunni theology, it matched Shi'a encouragement of rational inquiry. A number of Shi'ite Imams delved into natural knowledge, especially the sixth imam, Jaʿfar al-Ṣādiq.[31] As a result of Imamate encouragement, reason became a valid way of knowing in Shi'ite theology. The same source, usage by Imams, legitimized the incorporation of Greek texts into Shi'ite intellectual traditions. The mystical elements of Shi'ism gave additional spiritual weight to the study of nature, which was considered analogous to a "book" whose esoteric meanings could only be determined by intellectual contemplation, not by observation with the senses.[32] Like Mu'tazilism, then, Shi'ism valued human intellect as a valid vehicle by which knowledge—exoteric (obvious, superficial) or esoteric (hidden)—could be discovered. Indeed, despite their Sunni origins, Mu'tazilist texts continued to be studied by Shi'ite theologians after the school fell out of favor in Sunnism.[33]

The advantages of Mu'tazilism and Shi'a-sympathetic leadership joined in 945 when the warlord Buyids took control of the caliphate, producing an atmosphere highly conducive to the output of secular scholarship. Shi'ites or at least sympathizers, they left the 'Abbasid caliph nominally in his Sunni-validated position, even as they assumed all real political power. The tenth century saw several other Shi'ite polities form from peripheral fragments of the Islamicate

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[30] Ibid., 15.
[31] Nasr, *Science and Civilization*, 295. MAOS lists two texts for the Imam: one categorized as mineralogy and geology, the other as astronomy.
[32] Ibid., 295-296.
[33] Martin and Woodward, *Defenders of Reason in Islam*, 1.

Empire: In Bahrain, the Qarmatians began their rule in the year 899; the Hamdanids in northern Iraq and Syria took power in 905; and the Fatimid caliphate first formed in modern-day Tunisia in 909.[34] The Buyids themselves had command of the central Baghdad caliphate from 945 to 1055. Due to the predominance of Shi'ite rule in the Islamicate world during that time, the period from 950 to 1050 can be characterized politically as the "Shi'a Century." In parallel, it is the tenth and eleventh centuries that are praised for the production of truly distinguished Islamicate "science," to use the term in the literature. Ali Abdullah al-Daffa characterizes them, respectively, as "The Muslim Age" and "The Golden Age of Muslim thought."[35] Some of the most famous Muslim producers of natural knowledge flourished between the mid-tenth and mid-eleventh centuries, including ibn al-Haytham, al-Bīrūnī, al-Kūhī, ibn Sīnā, Abū'l-Wafā', and al-Baghdādī. They and their compatriots in the Islamicate scholastic community advanced a variety of rational forms of knowledge, including but not limited to mathematics and metaphysics. Ibn Sina remains to this day one of the most famous Islamicate scholars and a famous philosopher. Ibn al-Haytham, as shown below, made particular contributions to mathematics—particularly in expanding its epistemological role. As a result of ibn al-Haytham's efforts, those of al-Kindī a century previous and of the whole Islamicate scholastic community, as well as the influence of Greek geometry and proof theory on Islamicate mathematics and the interaction of math with the administration of Islamic society, only mathematics out of all the reason-based ways of knowing would have the epistemic strength to survive the political and theological changes of the eleventh century.

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[34] The Fatimid Caliphate, given last, was the empire under which ibn al-Haytham flourished when he moved to Cairo at the invitation of the Ismai'ili (a branch of Shi'ism) caliph there. He had been born, however, under in Iraq under Buyid-'Abbasid rule, so he only ever knew the kind of intellectual atmosphere fostered by Shi'a sympathies and Mu'tazilismm.

[35] Al-Daffa, *The Muslim Contribution to Mathematics*, 12.

These changes began in the late years of the tenth century. Buyid princes and caliphs had quickly taken to the power of patronage, so when in the late 900s, they began to struggle for power among themselves, their scholastic retainers sometimes suffered the consequences.[36] Many of these retainers were Mu'tazilite and Shi'ite intelligentsia, and in the turmoil of lost libraries, posts, or lives, their intellectual supremacy weakened. Over the course of the eleventh century, schools of "traditionalist" thought began to dominate Sunni thought over Mu'tazilism.[37] Essentially, these traditionalist schools rejected the application of human reason to sacred Islamic texts, the Qur'an and the *hadith*, repudiating in particular interpretations of Islamic sacred texts. Ultimately, it resulted in an epistemological shift in the Islamicate world from reason to revelation that denied the validity and reliability of human intellect as a way of knowing. Conveniently for mathematics, it was no longer considered subject to the foibles of human reason: As in the words of ibn Khaldun, "It is hardly possible for errors to enter into geometrical reasoning, because it is well arranged and orderly."

The Shiʿa Century came to an end in 1055 with the fall of the Būyids to the Seljuk Turks. The Seljuks favored strictly Sunnism, which was increasingly dominated by traditionalist schools who repudiated reason for revelation. Elsewhere, the Qarmatians collapsed in 1078, and Hamdānid rule in northern Iraq had ended in 1004. The Egyptian Fāṭimid caliphate would last well into the twelfth century (1171) but would be replaced by the Sunni Ayyūbids. The era of Shi'a-sympathetic rule and its patronage of rationality came to an end. The new dominant Islamic epistemology growing under Sunni rule culminated in a monumental treatise, "Confessions, or Deliverance from Error" by al-Ghazālī, to be discussed in the last section, that dismissed the legitimacy of all ways of knowing but revelation—and mathematics/logic.

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[36] Martin and Woodward, *Defenders of Reason in Islam*, 35.
[37] See Ibid., 14-15, for a discussion of the term "traditionalists."

The progression toward mathematics as an unassailable way of knowing took two and a half centuries to manifest in 1100 into a written condemnation of other forms of knowledge. At the beginning of the 'Abbasid period, mathematics was only just growing into an established field with inherent importance to Islamic society. The next section will describe how, in those early years, mathematics was developed as a tool for Islam, inspiring initial Muslim interest in it as a discipline. The influx of translated Greek texts in the beginning in the ninth century encouraged growing Islamicate interest, established by the interests of Islam, in the field mathematics. Later sections will consider the process by which mathematics separated topically and epistemologically from other natural knowledge and from the vicissitudes of theology as math's position in Islamicate society transformed.

III | Islamic Motivation and Foreign Inspiration

The quantitative study mentioned above revealed a shift in the interests of Muslim scholars toward mathematics in the ninth century. Both internal and external factors contributed to this century-long transition—in particular, the concerns of Islam and an influx of foreign, especially Greek, texts. Between these factors, mathematics became established in Islamic society for its utility, and it engaged the Islamicate scholastic community with an increasing variety of academic challenges.

While Islam was still not the majority religion of the Empire in 800, just a century and a half after Muslim armies began conquering huge swaths of diverse peoples and lands, it was the religion of the Muslim elite; therefore, its strictures and rites dominated Islamicate society. In the peace of the 'Abbasid period, scholars became increasingly aware of the utility of mathematics in administering spiritual rites and inheritance law. While religious leaders sparingly adopted the solutions offered by mathematical scholars, the impetus initially issued by challenges faced in administering Islam encouraged Muslim intellectuals to rely on mathematics.

The widespread introduction of Greek geometry into Arabic in the ninth century further fanned Muslim interest in mathematics as a whole field. Before the translation movement, which flourished in the 800s, Greek texts were decentralized and linguistically inaccessible to most (Arab) Muslims, but as Arabic copies became increasingly available in Islamicate centers of learning, Muslims embraced the mathematics contained in them. They wrote copious commentaries on the ancient books, and they expanded Greek ideas into new texts. By the year 900, the field of mathematics had gained a significant portion of secular Muslim scholastic interest, largely as a result of the internal impetus given by Islamic interests and the external influence of Greek geometry.

III.a | The Practical Impact of Islam

In the early years of Islam, the lines between Islam, Christianity, and Judaism were ambiguous; subsequently, religious identity was fluid.[38] It was only under the Umayyads (661-750), especially Caliph 'Abd al-Malik (r. 685-705), that a truly Islamic identity distinguished itself from the other monotheistic, Abrahamic religions.[39] For the Umayyads, this was an identity privileged to the Arab ethnic elite, but the 'Abbasids fostered the spread of Islam among ethnic minorities as well. The movement of voluntary conversion that flourished under the early 'Abbasids diminished at the beginning of the 1000s, not long after it is believed Muslims finally outnumbered non-Muslims in the Islamicate Empire.[40]

While the reign of the 'Abbasid caliph began without a Muslim majority, his empire was still ruled by the prescriptions of the Qur'an, and the everyday lives of its subjects, especially the Muslim citizenry, were shaped by its ritual obligations. Islamic interests affected Muslim scholars not only as citizens bound to its precepts but as "seekers of truth" too: Several practical concerns of the religion enabled the development and application of mathematics. The performance of some worship practices, such as ritual prayer and fasting, had geographic and calendric dimensions that mathematical astronomy could address more precisely than folk astronomy. Additionally, the complexities of Islamic inheritance law created a desire for better problem-solving techniques that encouraged the development of Islamicate algebra. Broadly speaking, these practical problems faced by administrators of Islamic rites and law fostered a long-lasting relationship between Islam and mathematics.

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[38] Jonathan P. Berkey, *The Formation of Islam: Religion and Society in the Near East 600-1800* (Cambridge: Cambridge University Press, 2003), 116.
[39] Ibid., 80.
[40] Ibid., 162; Egger, *A History of the Muslim World to 1405*, 251.

III.a.i | Math in the service of Islamic worship

The practical concerns that affected Muslim scholarship began with some of Islam's most fundamental charges: its five Pillars. Two of these five Pillars of Islam in particular concerned Muslim scholars. Ritual prayer to be performed five times a day (*ṣalāt*) and fasting during the month of Ramadan (*ṣawm*) depended on mathematical astronomy to determine how (for the former) and when they ought to be executed.[41] Although classified in modern terminology as astronomy, these methods in fact relied on geometry and spherical trigonometry.[42] In the role of solving administrative problems, math demonstrated its practical utility and initiated the growing interest of Muslims in mathematics as its own field or form of knowledge.

The Pillar of ritual prayer required observation not only five times a day but at certain times and while facing a certain direction, called the *qibla*. Verse 144 of the second *surah* in the Qur'an explains the latter prescription on account of the Prophet's own behavior, "Many a time We have seen you [Prophet] turn your face towards Heaven, so We are turning you towards a prayer direction that pleases you. Turn your face in the direction of the Sacred Mosque; wherever you [believers] may be, turn your faces to it."[43] This "Sacred Mosque" is known as the Kaʿba, an ancient shrine in Mecca. Considered in Islam the most sacred site on Earth, it is also the destination of the *hajj*, and Muslims are expected to face it in prayer wherever they might be for the five daily prayers. Moreover, mosques ought to face the Kaʿba, each built with a prayer-niche (*miḥrāh*) to point toward the *qibla*. The dead, too, are oriented along the *qibla*, although in the Middle Ages Muslims were buried on their side with faces turned toward the Kaʿba. In life, people were expected to direct themselves to the *qibla* while engaging in pious exercises—such

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[41] The other three pillars are profession of faith in Allah alone and Muhammad's prophethood (*shahādah*), an annual alms tax (*zakāt*), and a pilgrimage to Mecca at least once in a lifetime if at all possible (*hajj*).

[42] Trigonometry would not be considered separate from astronomy until Nasir al-Din al-Tusi in the thirteenth century.

[43] Qur'an 2:144.

as reciting the Qur'an, calling people to prayer, or slaughtering animals in ritual for prayers—and perpendicular to the sacred direction while performing bodily functions.[44] As a result, the *qibla* was perhaps the most noticeable way in which Islam pervaded daily life, and Muslim scholars addressed the topic with mathematical tools starting in the late eighth and early ninth centuries.[45]

The mathematical methods developed by Muslim scholars for finding the *qibla* were about as precise as they could be, given that contemporary measurements of longitudinal differences were consistently flawed. While it is unclear whether religious leaders instigated the relevant investigations in mathematical astronomy or Islamicate (mostly Muslim) scholars took the initiative to solve a problem that they recognized in their society, some records show Islamicate scholars working on construction projects. Perhaps the most famous example of this kind of cooperation was in the building of Baghdad. This massive project was undertaken at the will of Caliph al-Mansur, and "engineers," "astronomers," and "mathematicians" all had roles to play. These roles were none too separate either: "The measurements were made by the engineers 'Abdallāh ibn Muḥriz, al-Hajjāj ibn Arṭāt, 'Imrān ibn al-Waḍḍāḥ, and Shihāb ibn Kathīr in the presence of the astronomers and the mathematicians Nawbakht and Ibrāhīm ibn Muḥammad al-Fazzārī and al-Ṭabarī."[46] In this instance, engineers, astronomers, and mathematicians were not only working together on the same project, but the mathematicians and the astronomers were given some supervisory power. The engineers measured to their direction, suggesting that mathematics and mathematized astronomy had earned a degree of social importance in recognition of their subjects' utility and intellectual authority.

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[44] Rashed, *Encyclopedia*, 1: 128.

[45] Ibid., 128-129. It should also be noted that the *qibla* was not taken always and by everyone to be the direction of Mecca. Since Medina was 210 miles north of Mecca, the Prophet himself had prayed due south while living in Medina, and many of his followers took due south of anywhere to be the *qibla*. In other places, such as Egypt and Iraq, mosques were built to "face" certain walls of the Kaʿba by direct them to the winter sunrise or sunset (Ibid., 130-132).

[46] Al-Yaʿqūbī, *Buldan,* as quoted in Bernard Lewis, trans. and ed., "Capital and Provinces," *Islam from the Prophet Muhammad to the Capture of Constantinople* (Oxford: Oxford University Press, 1987) 2: 76-77.

Despite a history of engineers and calculators working together in Islamicate society on large, politically-motivated projects like the construction of Baghdad, the orientations of extant medieval mosques shows that these methods were rarely used in the construction of institutes of local import. Instead, some were made to face simple cardinal directions sanctioned by Islamic tradition and many others used folk astronomy to approximate the *qibla*.[47] It is possible that provincial architects found the mathematical prescriptions too complicated to implement or that local religious leaders dismissed them offhand for their complexity. Alternatively, perhaps when some of the earliest ʿAbbasid mosques were built, religious scholars were reluctant to trust important spiritual requirements to mathematical astronomy and continued to rely instead on tradition, subsequently strengthening the authority of precedent in mosque-building.

Although their solutions were rarely applied to daily life, determining the *qibla* with mathematical tools endured as a topic for works in astronomy and mathematics. Information about each title listed in MAOS is sparse, but from what little information there is, at least eight different scholars (six Muslim) can be said to have written on the subject between the ninth and eleventh centuries. The persistence of the problem among the scholastic elite, even in the face of indifference from religious leaders at the local level, suggests that the *qibla* problem was transformed into a purely scholarly enterprise, rather than one of immediate social application. As late as the eleventh century, the astronomer Abū'l-Rayḥān al-Bīrūnī (974-1048) wrote three texts that refer to problems of the *qibla*, including a predominantly mathematical one called "Letter to Abū Saʿīd" ("Kitāb ilā Abī Saʿīd"). He intended in the letter to relay a geometrical method for finding the *qibla* that al-Biruni said belonged to an earlier scholar, Habash al-Hasib (c. 770 – c. 870), but this letter is neither addressed to nor mentions any religious or political leaders, who might be involved in building mosques or directing prayers. Rather, its recipient,

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[47] Ibid., 129.

Abū Saʿīd al-Sijzī (or al-Sijizī or al-Sijī) (c. 950 – c. 1025), and all other identified names mentioned in the text were preoccupied with natural knowledge and mathematicians.[48] Although Kennedy and ʿId consider the method "elegant" and "well suited to the needs, say, of an architect laying out the ground plan of a new mosque," al-Biruni clearly judged that the audience interested in such problems was exclusively intellectual, not political.[49] While the *qibla* problem initially called mathematics to develop methodological tools by which a spiritual rite could best be administered by religious leaders, it had in a few centuries become itself a tool for academic study as subsequent generations of scholars instead disseminated new and old techniques for admiration or absorption on the basis of their mathematical merit.

Religious leaders more thoroughly absorbed the utility of mathematical astronomy used to determine the times of the five daily prayers, as well as their holy days, including the month of Ramadan. The times of prayers were standardized in the eighth century with respect to intervals of the sun's journey, and by the first decades of the next century, al-Khwārizmī had prepared the first known tables laying out the times of the daily prayers for Baghdad. Thereafter, other Muslim astronomers developed new formulae that could be used at all latitudes, and timekeeping methods continued to evolve in the ninth and tenth centuries. In thirteenth-century Egypt, religious institutions took on professional astronomers who were then responsible for performing the calculations to regulate prayer times.[50]

Thus, mathematics first integrated into Islamic society as a device for the administration of worship practices, but, by the eleventh century, it had clearly surpassed this role. While problems facing Islamic administration engendered interest in mathematics as a tool by which

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[48] E.S. Kennedy and Yusuf ʿId, "A Letter of al-Bīrūnī: Ḥabash al-Ḥāsib's Analemma for the Qibla," *Historia Mathematica* 1 (1974): 10. A translation with commentary of al-Biruni's letter.
[49] Ibid., 5.
[50] Rashed, *Encyclopedia*, 1: 170-177.

such problems could be solved, the field perpetuated even when local bodies of that administration ignored or rejected their solutions. By instead absorbing the problem of the *qibla* into a mostly academic vehicle for the exhibition of mathematical technique, Islamicate mathematicians demonstrated how established their field had become in the intellectual community. This same cycle was additionally fed by the arithmetic demands of traditional Islamic inheritance law.

III.a.ii | Math in the service of law

The 'Abbasid Revolution in the mid-eighth century heralded not only a new family of caliphs but a whole new age, characterized at least at first by cultural efflorescence and the rise of bureaucracy. In the words of Vernon Egger, "The old Sasanian cosmopolitan and imperial tradition had triumphed over Arab particularism, and the revolution signaled a shift from the Umayyad focus on conquest to one of institutional consolidation."[51] One such institution to be regularly standardized in the absence of military campaigns was that of inheritance.

In three verses, the Qur'an gives specific commands regarding how a person's property is to be distributed after death. These instructions are found at the beginning and the end of the fourth *surāh*, and although explicitly stated, they can be complicated. For example, the eleventh verse begins,

> Concerning your children, God commands you that a son should have the
> equivalent share of two daughters. If there are only daughters, two or more
> should share two-thirds of the inheritance, if one, she should have half. Parents
> inherit a sixth each if the deceased leaves children; if he leaves no children and

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[51] Egger, *A History of the Muslim World to 1405*, 85.

his parents are his sole heirs, his mother has a third, unless he has brothers in which case she has a sixth.[52]

Thus, although "God [made] clear this clear to you so that you do not make mistakes," adjudicating matters of inheritance could be complicated, especially when involved with matters of wills or disputed testacy, which were discussed in other *surah*.[53] Testators later created legal mechanisms through reinterpretation of verses, loopholes, and precedent, by which they gained more freedom to bequeath their estates inequitably than the Qur'anic strictures necessarily intended.[54]

For the rapidly bureaucratizing empire, the administration of this complicated aspect of Islamic life became a significance concern. Initially the Islamicate world lacked convenient mathematical methods for determining inheritances for different members of the family. Motivated at least in part by this serious, practical problem in their society, Muslim scholars developed a whole new field of mathematics—algebra—under the external influence of Indian mathematics. Not long after the 'Abbasids came to power in 750 and began the process of bureaucratization, mathematics from India and then Greece had started to become available to Muslims via the translation movement. The mathematics the Islamicate Empire inherited from the Hellenistic world was both extremely unsuitable for solving problems of inheritance in accordance with Muslim law and too late to be of much use anyway, since it lagged behind Indian influence for the first crucial decades in its development.[55] Indian arithmetic had started appearing in the Islamicate world as early as the seventh century but its procedures helped

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[52] Qur'an 4:11.
[53] Quote from Qur'an 4:176.
[54] David S. Powers, "The Islamic Inheritance System: A Socio-Historical Approach," *Arab Law Quarterly* 8 (1993): 20-29. David Powers argues that these mechanisms developed with the *surah* to create a "system" of Islamic inheritance, standardized in the ninth century, that lasted until the mid-nineteenth century. Ibid., 13-29.
[55] Ali Abdullah al-Daffa', *The Muslim Contribution to Mathematics* (Atlantic Highlands, NJ: Humanities Press, 1977), 51. It was possibly also too late, depending on the dates one accepts for the earliest translations of Greek texts.

inspire Islamicate algebra in the late eighth and early ninth century after texts began to be translated from Sanskrit more systematically.

The limitations of the Hellenistic system were primarily arithmetical. The Ancient Greeks used an alphabetic numeral system, in which all of the letters, including some fading out of usage as letters, also represented numbers—except one, ς. This limited them to solving problems with only one unknown. As a result, intellectual labor concentrated on the task of eliminating variables before beginning a problem, rather than over the course of solving it. It also frustrated an algebraist's ability to handle indeterminate equations—equations with multiple solutions—since he would be forced to assume given values for all but one unknown variable at a time, if he could not arrange to reduce all variables in terms of just one.[56] Greek calculations further lacked a pure place value system. While they distinguished between ones and tens, etc., they represented the numbers 10, 20, 30… with different letters of the alphabet than were used to write the ones digits 1-9.[57] Yet more debilitating was the failure of Greek notation to connect the concept of x^2 (denoted in Greek as Δ^Y) to the unknown x (ς in Greek) in any obvious manner. According to Sir Thomas Little Heath, this inhibited the development of general solutions that could be applied to multiple unknown quantities.[58]

The advantage of Greek arithmetic was its use of symbolic notation. Although the symbols of Ancient Greece had an intrinsic relationship to the concepts they replaced, unlike mathematical symbols of today, their use stands in stark contrast to the expository mathematics that was used by Islamicate scholars at that point.[59] The Indian algebra on which they drew was

[56] Thomas Little Heath, *Diophantus of Alexandria: A Study in the History of Greek Algebra*, 2nd ed. (1885; New York: Dover, 1964), 51-52.
[57] Ifrah, *The Universal History of Numbers*, 220.
[58] Heath, *Diophantus of Alexandria*, 38-39.
[59] To Nesselmann, the use of symbolic notation put Ancient Greek algebra into the second of three "historical stages of development" in in algebraic equations, whereas Islamicate algebra is confined to the first stage. (Ibid., 49-50.) This evaluation seems to simplistic and teleological to me. While the Islamicate scholastic community did lack a

also expository in nature—that is, explained with words, not symbols. For example, one postulate in the *Āryabhaṭīya* reads,

> The distance between the ends of the two shadows multiplied by the length of the
>
> shadow and divided by the difference in length of the two shadows gives the *koṭī*
>
> [upright leg of a triangle]. The *koti* multiplied by the length of the gnomon and
>
> divided by the length of the shadow it gives of the *bhujā* [a side of the triangle
>
> parallel to the *koti*].[60]

The non-symbolic form of Islamicate algebra perhaps derived from the Indian fashion of presenting equations. Moreover, the Indian use of general terms such as *koti* and *bhuja* without reference to any specific values was very different from Diophantus, who had to follow or even explain many of his claims with demonstrative examples.

Purportedly "encouraged" by Caliph al-Ma'mun's "fondness for science," the famous scholar Muḥammad ibn Mūsā al-Khwārizmī composed a book on the subject of a new, Indian-inspired "algebra."[61] The result, his *Abbreviated Book on the Reckoning of Algebra and Almucabala* (as translated in MAOS from *al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr wa'l-muqābala*), integrated the algebra and arithmetic of India into a new form of problem-solving. Frederic Rosen, editor and translator of the first English edition of this significant text testified to the originality of this Muslim rendition of the subject:

> But under whatever obligation our author may be to the Hindus, as to the subject
>
> matter of his performance, he seems to have been independent of them in the

symbolic notation for textual mathematics, the accomplishments of their algebra should not be neglected simply because they explained with words, not symbols.

[60] Walter Eugene Clark, trans. and ed., *The Āryabhaṭīya of Āryabhaṭa: An Ancient Indian Work on Mathematics and Astronomy* (Chicago: University of Chicago Press, 1930), 32.

[61] Muhammad ibn Musa al-Khwarizmi, *The Algebra of Mohammed ben Musa*, ed. and trans. Frederic Rosen (London, 1831), 3. See note (78) for references for and against this.

manner of digesting and treating it: at least the method which he follows in expounding his rules, as well as in showing their application, differs considerably from that of the Hindu mathematical writers.[62]

Like Rosen, the twelfth- and thirteenth-century bio-bibliographer Abu'l-Ḥasan ibn Yūsuf al-Qiftī saw the Indian influences on al-Khwarizmi's integrative algebra as fairly obvious when he praised it as "the swiftest and most complete method of calculation, the easiest to understand and the simplest to learn; it bears witness to the Indians' piercing intellect, fine creativity and their superior understanding and inventive genius."[63] Al-Khwarizmi himself, however, made no claims for originality, nor did he try to explain any source for his algebra. He only claimed to have been inspired by Caliph al-Ma'mun's example. The work produced survives today as one of the oldest extant to manuscripts to treat the subject.[64]

Al-Khwarizmi's solutions were particularly useful for problems of inheritance law. In his introductory remarks, al-Khwarizmi specified that he had designed the text for public consumption, "confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with another."[65] From the list of possible applications for his algebra, he clearly had in mind an audience of judges and jurists. Indeed, of the 174 pages comprising the English translation of al-Khwarizmi's *Abbreviated Book on the Reckoning of Algebra*, almost 90 are

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[62] Ibid., x..

[63] Translated into English in Ifrah, *The Universal History of Numbers*, 530, quoting from the French translation provided by M. Franz Woepcke, *Mémoire sur la propagation des chiffres indiens* (Paris: Imprimerie Impériale, 1863), 146-147. For the full Arabic text, see 'Alī ibn Yūsuf Qifṭi, *T'arīḫ al-Ḥukamā'*, ed. August Müller and Julius Lippert (Leipzig: Dieterich'sche Verlagsbuchhandlung, 1903).

[64] Joseph calls al-Khwarizmi's *Algebra* "the starting-point for Arab work in algebra." Whether or not it was the first book in Arabic to demonstrate the Islamicate synthesis that became known as algebra, it was the first to be translated into Latin, so it certainly became the starting-point for European work in algebra if nothing else. George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics* (Princeton: Princeton University Press, 2000), 305.

[65] Al-Khwarizmi, *Algebra*, 3.

given over to a chapter "On Legacies." Sixty-six examples, at great expository length, illustrate how algebraic methods can be used to discover an unknown legal share. Although presented in different forms than textbook problems of today, al-Khwarizmi's algebra is certainly familiar to a modern reader. Take the solution to the first inheritance example in which a man has died, leaving behind two sons, one of who owed him ten dirhems yet was the receive ten dirhems of property at the time of his father's death. The patriarch also left one-third of his capital to a stranger (the most an individual could bequeath to a stranger, according to a statement sometimes credited to the Prophet).[66] Al-Khwarizmi begins by defining the variable in which he is interested:

Translation of al-Khwarizmi's original text	**Transcription into modern notation**[67]
You call the sum which is taken out of the debt 'thing.' Add this to the capital which is ten dirhems. The sum is ten and 'thing.' Subtract one-third of this, since he has bequeathed one-third of his property, that is three dirhems and one-third of 'thing.' The remainder is six dirhems and two-thirds of 'thing.' Divide this between two sons….This [result] is equal to the 'thing' which was sought for. Reduce it, by	$x \equiv$ sum removed from the debt; 'thing' Capital $\equiv 10$ dirhems $$10 + x$$ $$(10 + x) - \frac{1}{3}(10 + x)$$ $$= (10 + x) - 3\frac{1}{3} - \frac{1}{3}x$$ $$= 6\frac{2}{3} + \frac{2}{3}x$$ That is, between two sons,

[66] See David S. Powers, "On Bequests in Early Islam," *Journal of Near Eastern Studies* 48 (July, 1989): 185-200.
[67] A transcription into modern notation is provided for the sake of clarity to a modern reader. It does not accurately represent how al-Khwarizmi and his contemporaries conceived this algebra symbolically.

removing one-third from 'thing,' on account of the other third of 'thing.'…It is then only required that you complete the 'thing,' by adding to it as much as one half of the same…This gives five dirhems, which is the thing that is taken out of debts.[68]

$$6\frac{2}{3} + \frac{2}{3}x = 2x$$

$$3\frac{1}{3} + \frac{1}{3}x = x$$

$$3\frac{1}{3} = \frac{2}{3}x$$

$$\frac{1}{2}\left(3\frac{1}{3}\right) + 3\frac{1}{3} = \frac{2}{3}x + \frac{1}{2}\left(\frac{2}{3}x\right)$$

$$5 = x$$

Al-Khwarizmi's algebraic examples, such as the one above, demonstrated a procedure by which many similar problems of finding allotments could be solved. Although al-Khwarizmi did not describe his procedure in general terms or provide proof that it worked, he surpassed the analytical tools of Greek algebra, constrained as it were by an unwieldy arithmetic. In fact, since he operated at just the very beginning of the period in which Greek texts were being translated into Arabic, it is unlikely he had much contact at all with Diophantus' algebra. As Rosen pointed out,

> [Quadratic equations] he [al-Khwarizmi] solves by the same rules which are followed by Diophantus, and which are thought, though less comprehensively, by the Hindu mathematicians. That he should have borrowed from Diophantus is not at all probable; for it does not appear that the Arabs had any knowledge of Diophantus' work before the middle of the fourth century after the Hejira [mid- to late-tenth century BCE], when Abu'l-wafa Buzjani rendered it into Arabic.[69]

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[68] Al-Khwarizmi, *Algebra*, 86-87.
[69] Rosen in al-Khwarizmi, *Algebra*, ix.

Al-Khwarizmi instead integrated the efficiency of Indian arithmetic, as well as its rhetorical style, with purely Muslim problems. The Islamic system of inheritance laws provided impetus for the creation of the field and a vehicle by which its utility could be demonstrated to the general population. In particular, the development of algebra that could simplify the adjudication of Islamic inheritance law would have been useful for the judges and jurists around the Islamicate world who had to do just that in the courts. While the intricacies of al-Khwarizmi's algebra were probably reserved for specialists, there was a relatively substantial demand for the public demonstration his text, the *Algebra*, provided.

Al-Khwarizmi was not the only authority on the division of inheritance. His contemporary Ayyūb al-Basri was another "early algebraist" who wrote on the topic but whose manuscript is lost.[70] Similarly, Abū'l-Ḥamīd al-Qāḍī's (d. 905) *Core of Inheritance* (*Lubāb al-farāiḍ*) is only known by mention.[71] In the ninth century, the Sabian Sinan al-Fath took up many of the same topics as al-Khwarizmi, such as the Indian reckoning board and Indian methods of calculation, as well as inheritance.[72] Lastly, Muwaffaq al-Dīn al-Raḥbī in the twelfth century prepared 180 verses on the topic of dividing inheritances in the poem with the rather ambitious title "His Aim (Wealth) in Investigating All that is Related to Inheritance" (Bughya [Ghunya] al-bāḥith 'an jumal al-mawārīth).[73] Islamic inheritance law inspired works among those four mathematicians, and it crucially demonstrates how Islamic society and scholarship cultivated each other. As J. Lennart Berggren wrote using this example to make a more general point, "medieval Islam created a mathematics whose contents reflected not only its sources but, as in

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[70] Boris Rosenfeld and Ekmeleddin Ihsanoğlu, *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th-19th c.)* (Istanbul: Research Center for Islamic History, Art, and Culture, 2003), 21.
[71] Ibid., 58.
[72] Ibid., 91-92.
[73] Ibid., 187.

al-Khwarizmi's application of algebra to inheritance law, the Muslim society that created and sustained it."[74]

The internal influences of Islamic concerns were central to establishing mathematics as an important form of knowledge with social import initially defined by religious requirements. As foreign traditions continued to flood into the Middle East through translation, they expanded the field of pure mathematics as known to the Islamicate scholastic community. Invigorated with ideas previously barred from non-Greek-speaking Muslim scholars, Islamicate mathematics became even more engaging.

<p style="text-align:center">III.b | The Impact of Foreign Elements on Pure Mathematics</p>

As the absorption of Indian mathematical techniques into inheritance law indicates, Islamicate scholarship was highly receptive to foreign ideas they encountered. Al-Kindi captured the openness of early Islamicate scholarship to outside influence, no doubt the result of Islam's emphasis on learning, when he insisted,

> We ought not to be ashamed of appreciating the truth and of acquiring it wherever it comes from, even if it comes from races distant and nations different from us. For the seeker of truth nothing takes precedence over the truth, and there is no disparagement of the truth, no belittling either of him who speaks it or of him who conveys it.[75]

In practice, the Islamicate scholastic community found truth in particular from India and Ancient Greece. These traditions introduced new theories and new methods to Islamicate intellectuals,

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[74] Berggren, "Mathematics in Medieval Islam," 517.
[75] Yaʿqūb ibn Isḥāq al-Kindī, *Al-Kindi's Metaphysics: A Translation of Yaʿqūb ibn Isḥāq al-Kindī's Treatise "On First Philosophy" (*fī al-Falsafah al-Ūlā*), trans. and ed. by Alfred L. Ivry (Albany, NY: State University of New York Press, 1974), 58.

who were building a new and distinct tradition of their own, one based on ancient Mesopotamian ideas but enriched substantially by inheritances from India and Ancient Greece.[76]

Decimal arithmetic was India's most pervasive impact on Islamicate mathematics, but it had to be adapted to the mathematical tradition already in place. In the ʻAbbasid period, it was Babylonian sexigesimal system that Indian decimal arithmetic struggled to replace in Islamicate astronomical calculations.[77] The pace at which decimal arithmetic was adopted in other areas of calculation varied. Indian traditions also made significant contributions to trigonometry and perhaps algebra, but further influence, however, was stymied by the relatively low social status associated with Indian arithmetic, and in the ninth century onward, translations of Greek work supplanted Indian mathematics in significance. These Greek texts introduced the extensive geometry of their authors and the concept of systematic proof in mathematics.[78] Greek geometry and proofs both provoked considerable interest in mathematics in the Islamicate scholastic community and were subsequently internalized by it, ultimately resulting in a unique melding of geometry and algebra. In short, pieces of both the Indian and Greek traditions, and some others to a much lesser extent, became synthesized and standardized into a distinctly Islamicate intellectual tradition that engaged large parts of the scholastic community.

III.b.i | Indian Influence

Whereas Mesopotamian ideas were indigenous to the Middle East, Indian traditions arrived from outside the region unsystematically between the seventh and ninth centuries. The process of transmitting and absorbing Indian math and astronomy into Islamicate understanding

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[76] I use here the term "Mesopotamian" where it has been traditional in the history of science to say "Babylonian." In this paper, these terms are used infrequently but interchangeably, with the understanding that the word "Babylonian" unjustly privileges one Mesopotamian dynasty over all the others who created scientific and mathematical work, preserved on cuneiform tablets, between 2500 BCE and 300 BCE.

[77] Al-Khalili, *The House of Wisdom*, 99.

[78] The latter idea will be discussed in greater depth in (IV.a.i).

is still fairly mysterious to historians today. As early as 664, the sixth- and seventh-century Christian bishop Severus Sebokht recorded a Syrian monk who understood and appreciated the Indian "nine signs."[79] We know that in the 770s, Caliph al-Mansur received an Indian delegation that gave the Islamicate Empire a Sanskrit astronomical text, and the caliph immediately ordered translated. And we also know that in his lifetime (c. 750 – c. 850), al-Khwarizmi wrote at least one book explaining the process of calculating with the Hindu numerals, which was among the earliest extant works describing the arithmetic that could be performed with decimal numbers. However, the mathematics that arrived from Indian was essentially "anonymous."[80] According to Donald Hill, "[A]part from [in] astronomical works, no references to Indian authors or titles have yet been found in Arabic treatises on mathematics."[81] As a result, the extent of its influence on the development of Islamicate mathematics is controversial. Even its impact on al-Khwarizmi's *Algebra* is still debated by historians, although I have argued for its likelihood above.[82] In contrast, historians more conclusively agree on India's impact on medieval Islamicate astronomy. In that field, Indian sources introduced the *sine* chord, inspiring Islamicate astronomers to

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[79] Quoted in English in John N. Crossley and Alan S. Henry, "Thus Spake al-Khwārizmī: A Translation of the Text of Cambridge University Library Ms. Ii.vi.5," *Historia Mathematica* 17 (May 1990): 104. For the original French translation of the manuscript in Paris, see F. Nau, "Notes d'Astronomie," *Journal Asiatique*, 10e ser., 16 (May 1990): 229.

[80] Jens Høyrup, "The Formation of 'Islamic Mathematics'," 5.

[81] M.J.L. Young, J.D. Latham, and R.B. Serjeant, ed., *Religion, Learning, and Science in the 'Abbasid Period* (Cambridge: Cambridge University Press, 1990), 251.

[82] Compare for example, Donald Hill ("From the frequent attribution of the place-number system to Indian sources we can, however, be sure that Indian influence was strong." Ibid.) to Jens Høyrup ("Islamic algebra was untouched by Indian influence." "The Formation of 'Islamic Mathematics'," 5). For more perspectives on Indian and Greek influences on al-Khwarizmi's *Algebra*, see Soloman Gandz, "The Sources of al-Khwarizmi's Algebra," *Osiris* 1 (1936): 263-277, where he argues that the *Algebra* was in fact a copy of a contemporary Hebrew text; Høyrup, "The Formation of 'Islamic Mathematics'," 13-15, for its relation to syncretic Babylonian algebra. Also, al-Khalili, *The House of Wisdom*, 117-118; Rashed, *Encyclopedia*, 2: 448-449; and Rosen in al-Khwarizmi, *Algebra*, ix-xi.

develop the six basic trigonometric ratios, and planetary theories that predominated until the tenth century.[83]

Until the ninth century, the primary external influence on Islamicate mathematics was Indian, but as the movement to translate Greek texts took off in the early 800s, Ancient Greek traditions supplanted Indian ones in significance to Islamicate scholars. This shift was in large part a matter of social status. Despite its significance for the development of algebra and its contribution of the Hindu numerals, Indian mathematics did not possess the same status as Greek: Whereas Ancient Greek geometry was a field belonging almost exclusively to scholastic elite, Indian arithmetic was associated with commercial transactions. The traditional Indian practice of manipulating their decimal numbers on a dustboard leant itself to the market place because it allowed arithmetic to be performed quickly and easily. This practice thus ultimately became associated in the Islamicate Empire with grocers and street astrologers, although the scholastic elite were likely expected to know it. Whereas the famous polymath Abū ʿAlī ibn Sīnā (980-1037, Latinized Avicenna) studied Aristotle and Ptolemy with a man who considered himself a scholar of "philosophy," he had to learn "Indian calculation" from a vegetable-seller in Bukhārā, according to his autobiography.[84]

In contrast, the books of the ancients (*al-awāʾil*, which Gohlman takes to mean the Greeks) were stored in the library of the Sultan Nūḥ ibn Manṣūr, to which ibn Sina gained access by saving the sultan's life.[85] A similar regard for Greek sources was demonstrated a few decades earlier by Abūʾl-Ḥasan al-Uqlīdisī in Damascus. This tenth-century scholar adapted the

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[83] Ronald Calinger, ed., "Arabic Primacy with Hindu, Chinese, and Maya Contributions," *Classics of Mathematics* (Oak Park, IL: Moore Publishing, 1982)172; Joseph, *The Crest of the Peacock*, 338; Rashed, *Encyclopedia*, 1: 8, 2: 532; Young, Latham, and Serjeant, *Religion, Learning, and Science*, 253, 276; Nasr, *Islamic Science*, 106.
[84] William E. Gohlman, *The Life of ibn Sina: A Critical Edition and Annotated Translation* (Albany, NY: State University of New York, 1974), 21-22. The term "philosophy" may be the same "philosophy"—meaning ways of knowing through human reason—attacked by al-Ghazali in an 1100 essay to be discussed in a later section.
[85] Ibid., 35-37.

numerical methods of Indian arithmetic from the messy dustboard to paper and pen so that those who did not want to be associated with such low people might still use the convenient system. In introducing his system, al-Uqlidisi wrote, "We may hide it [the method of calculation] also by means we shall mention, so that one who sees him that computes with it does not know that it is *Hindī*, but thinks that it is *Rūmī*," where "Rūmī" here means essentially "Greek."[86] One could hide the Indian origin of his calculations by replacing the nine Hindu numerals with the first nine letters of the Greek alphabet.[87] Here al-Uqlidisi clearly suggested that to be caught performing Indian dustboard arithmetic would be embarrassing, whereas to be seen performing a calculation in the Greek fashion was something comparatively advantageous. It is unknown whether this essay was intended to benefit any scholastic elite who might choose to practice mathematics or the judges and jurists who had to arbitrate inheritance law in court, but it was clearly intended to resolve a situation that individuals of higher social status saw as a problem.

The significant disparity in the status of Indian and Greek mathematics was perhaps rooted in religion. The mathematics of India came from a politically- and culturally-fragmented subcontinent that was loosely unified by Hinduism, a polytheistic faith. While Islamicate society extended protection to *mono*theistic non-Muslims, tolerance did not extend to individuals who worshipped more than one god. To have more than one god was to give Allah an equal partner, and in the words of the Qur'an, "God does not forgive the joining of partners with Him: anything less than that He forgives to whoever he will, but anyone who joins partners with God has concocted a tremendous sin."[88] Thus, the traditional polytheism of contemporary Indian states

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[86] As quoted in A. S. Saidan, "The Earliest Extant Arabic Arithmetic: *Kitāb al-Fusūl fī al Ḥisāb al-Hindī* of Abū al-Ḥasan, Aḥmad ibn Ibrāhīm al-Uqlidisi," *Isis* 57 (1966): 487. The term "Rūm" literally meant "Rome," and it referred to the Byzantine Empire (a Roman relic) or Greeks in general.

[87] Ibid., 478. Alternatively, when using paper and ink that could not be rubbed out, a calculator could instead write numbers to be preserved in Arabic *jummal* letters and write numbers to be crossed out in the Hindu numerals.

[88] Qur'an 4:58.

would remain a source of political strife between them and the Islamicate Empire for centuries to come. [89]

At the time of the Greek translation movement, the texts so esteemed by Islamicate scholars were largely transmitted from the Byzantine Empire, which was dominated by the monotheistic Orthodox Church. While the Ancient Greek writers themselves were polytheistic, the medieval culture that propagated their works fit the criterion for religious toleration. Doubting Islamicate scholars' concern for historical detail, it is possible that the monotheism of the society from which Ancient Greek texts were transmitted replaced the polytheism of the long-dead authors and imbued them with an authority denied to those of contemporary Hindus.

Alternatively, since Indian arithmetic was designed to be performed on a dustboard, it was perhaps associated with uncleanliness. This is not necessarily suggested by al-Uqlidisi in his quote above, but earlier in the paragraph he addressed directly individuals who disliked the dust of a dustboard: "If others dislike it because of the dust that makes the hands dirty and injures some figures that rub out [the sand], we say that we may use for that a crooked stylus to write with its point and rub with its back."[90] The association with (potential) dirtiness no doubt clung to the dustboard. Cleanliness is extremely important in Islam, for "God loves those who turn to Him, and He loves those who keep themselves clean."[91] If Indian mathematics *was* associated with dirtiness, then the disdain expressed toward it by medieval Muslim scholars can be understood in religious terms. Ancient Greeks also performed their calculations with "pulvere et radio" ("sand and wand" in the words of Cicero), but considering the form and distance of its

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[89] To this day, Hindus and Muslims sometimes struggle to coexist in India. In 2002, for example, violent riots broke out in the Indian state of Gujarat after 60 Hindu pilgrims were killed in a train fire that was blamed on the local Muslim population. More than a thousand individuals died in the religious riots; most of the casualties were Muslim. For a 2012 article summarizing the Naroda Patiya massacre and the results of subsequent trials, see "India Riots: Court Convicts 32 over Gujarat Killings," *BBC*, August 29, 2012, http://www.bbc.com/news/world-asia-india-19407100.

[90] Saidan, "The Earliest Extant Arabic Arithmetic," 478.

[91] Qur'an 2:222.

migration from Greece to the Middle East, this association was likely lost. If Greek geometry thus emerged into Arabic devoid of its own sandy connotations, its perceived purity could explain why scholastic elite preferred the calculations of Greece over that of India.[92] Whether Indian mathematics was denigrated for its association with polytheism or with dirtiness, Greek geometry certainly impressed the Islamic scholastic community on a more long-term scale than the mathematics of India did.

III.b.ii | Greek ascendency

After al-Mansur instigated the translation movement in 770, many Greek-speaking non-Muslims became integral members of the Islamicate world's academic community, primarily as translators but also as original scholars in their own right. Al-Mansur's translators were initially charged with Sanskrit texts, but in the ninth and tenth centuries, a fervor for a variety of Greek texts emerged. Before this time, knowledge of Greek ideas was largely limited to non-Muslims, many of whom knew Greek. The Islamicate Empire's population of Nestorian Christians, for example, was in large part comprised of immigrants, or their descendants, from the Byzantine Empire, who fled from persecution at the hands of its Orthodox Church to the relative religious toleration of their southern neighbor. For mathematics in particular, the translation of Greek texts into Arabic in the ninth and tenth centuries opened the considerable expanse of Greek

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[92] Cicero referenced the "pulvere et radio" before describing his discovery of Archimedes' tomb in Syracuse. For the original Latin and an English annotation, see Marcus Tullius Cicero, *Cicero's Tusculan Disputations: With English Notes, Critical and Explanatory*, ed. Charles Anthon (New York: Harper & Brothers, 1860), 64, (notes) 371. For an English translation, see Cicero, *Cicero's Tusculan Disputations*, trans. and ed. Andrew P. Peabody (Boston: Little, Brown, and Co., 1886), 289. Cicero further demonstrated the traditional association of ancient Greek and Roman math to their own versions of dustboards by associating ignorance with "having never handled the student's dust" ("numquam eruditum illum pulverem attigistis"). For the Latin, see Cicero, *De Natura Deorum Libri Tres*, trans. [from German] and ed. Austin Stickney (Boston: Ginn & Co., 1881), 93. The English translation comes from Cicero, *De Natura Deorum (On the Nature of the Gods)*, trans. Francis Brooks (London: Methuen, 1896), 99.

geometry to Muslim scholars.[93] The new mathematics drew the attention of many Islamicate scholars and helped inspire the shift toward pure math demonstrated in the quantitative study detailed in Appendix A. The interest it engendered is easily seen in the number of commentaries and original geometric works that followed the introduction of Greek geometry into Arabic.

Allegedly, the very first Arabic commentary on Euclid belonged to the mysterious alchemist Jabir ibn Hayyan, who flourished in the second half of the eighth century. The tenth-century bio-bibliography *Index* (*al-Fihrist*) records the treatise's existence, although the manuscript is not extant today and may not have been even by the time of al-Nadim.[94] If Jabir ibn Hayyan is thus discounted, then the earliest commentary belonged to ibn Rāhiwayh al-Arrajānī from the ninth century.[95] Not much later Yaʿqūb ibn Isḥāq al-Kindī (d. c. 873) wrote three more commentaries on the *Elements* and one on Euclid's *Optics*, as did a slew of others. While Islamicate commentators also addressed Greek physics and metaphysics, Euclid's *Elements* (the central text to Greek geometry) generated by far the most commentaries—no less than 31 from the ninth to mid-thirteenth centuries. The second most popular book was Ptolemy's *Almagest* with at least 23 commentaries.[96] Apollonius and Aristotle came next, respectively, barely hitting the double-digits. In support of the significance of the *Elements* to Islamicate mathematics following its translation into Arabic, see for example the beginning of al-Sijzi's

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[93] Since many non-Muslims knew Greek, Syriac, or Persian—the latter two languages in which some Greek texts were preserved—the translation of those texts into Arabic was less jolting. Muslims, by contrast, were primarily Arab and spoke predominantly Arabic; hence the traditions to which they had access following the translation movement caused a more considerable tremor in their subgroup. As a result, the specifically Muslim view of mathematics (and knowledge more generally) differed from those of non-Muslims in some ways since the translation movement affected each of these Islamicate subgroups differently. Moreover, the theologies held by Muslims and non-Muslims interacted differently with categories of knowledge. Islamic inducements to begin the study of mathematics are discussed above, and the place of math in Islamic epistemology is discussed below.

[94] Casting additional doubt over ibn Hayyan's authorship of such a text, a number of people continued to write under Jabir ibn Hayyan's name long after his death. See al-Khalili, *The House of Wisdom*, 52, and Nasr, *Science and Civilization*, 42-43.

[95] I only looked at the titles of texts belonging to the 165 individuals whose religions I knew.

[96] As suggested by the height of this total, Ptolemy's *Almagest* had a significant impact on Islamicate cosmology. Ptolemy will be discussed at greater length in (IV.a.ii).

Book on Easier Ways to the Derivation of Geometric Propositions (*Kitāb tashīl al-subul li-istikhrāj al-ashkāl al-handasiyya*). After assuring students of geometry that they need not have some innate talent for the subject but must instead work hard, he told them, "It is necessary for someone who wants to learn this art [geometry], to thoroughly master the theorems which Euclid presented in his *Elements*."[97]

Table 1

Greek Writer	Number of Islamicate Commentaries[98]	Notes
Euclid	39	
Elements	*31*	Plus one translation into Sanskrit
Optics	*3*	
Celestial Phenomena	*1*	
Data	*1*	
Division of Canon	*1*	
Gravity and Lightness	*1*	
Ptolemy	24	
Almagest	*23*	Plus one translation into Sanskrit
Harmonics	*1*	
Apollonius	11	
Aristotle	11	
Archimedes	9	
Menelaus	4	

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[97] Berggren, "Mathematics in Medieval Islam," 667. Title taken from MAOS. Berggren translates it as *Making Easy the Ways of Deriving Mathematical Figures*. Note also that this al-Sijzi is the same as Abu Sa'id al-Sijzi above.

[98] From all subjects and excluding commentaries that addressed multiple Greek scholars at once. Titles in MAOS were taken to be commentaries if they addressed a writer or his text by name in the title or in Rosenfeld and Ihsanoğlu's short descriptions of them. Since my data comes from a book entitled *Mathematicians, Astronomers, and Other Scholars*, I was almost inevitably going to find that mathematical and astronomical texts topped this list of Greek works commentated on by the individuals listed in MAOS, rather than philosophical or metaphysical treatises, for example. My point here is not to claim that Ancient Greek mathematicians caused a greater impact on Islamicate intellectual endeavors than Ancient Greek philosophers did, but rather that Euclid was a greater influence than other Ancient Greeks who did math (such as Nicomachus and Diophantus).

Plato	3	
Theodosius	3	
Autolycus	2	
Diophantus	2	
Galen	2	
Aristarchus	1	
Hipparchus	1	Text "unknown" to historians of mathematics
Hypsicles	1	
Nichomachus	1	
Socrates	1	
Theon	1	

Islamicate interest in geometry did not stop at the production of translations and commentaries. Despite nineteenth-century claims to the contrary, Islamicate scholars set quickly to expanding the geometrical knowledge they inherited.[99] The three Banū Mūsa, whom Nasr credits with popularizing Greek geometry in Baghdad, belonged to the House of Wisdom in the midst of the Greek to Arabic translation movement and have five mathematical titles to their collective legacy in MAOS: two are additions to Apollonius' *Conic Sections* and three are original geometric texts.[100] Rosenfeld and Youschkevitch enumerate their geometrical accomplishments at greater length in Rashed's *Encyclopedia*, including several Archimedean proofs.[101] Ya'qub al-Kindi was a contemporary of the Banu Musa—later, a rival—and he was even more prolific. Out of 37 known mathematical texts altogether, twenty were original works in geometry, rather than commentaries on geometry: Five described methods of mensuration or construction for specific physical systems, and fifteen discussed geometrical figures and

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[99] "Having been little accustomed to abstract thought, we need not marvel if, during the ninth century, all their [Arabs'] energy was exhausted merely in appropriating the foreign material. No attempts were made at original work in mathematics until the next century." Florian Cajori, *A History of Mathematics* (New York: Macmillan, 1894), 105.

[100] Nasr, *Islamic Science*, 82; MAOS.

[101] See Rashed, *Encyclopedia*, 2: 450-451.

principles in a general sense.[102] Similarly, the Banu Musa's student Thābit ibn Qurra (836-901) produced 29 mathematical works total (as known to MAOS), out of which eight were commentaries, 16 covered geometry—especially its principles—and one more developed geometric proofs to algebraic problems.[103]

It is in this last area in particular, the merging of algebra and geometry, that historians of mathematics have long acknowledged Islamicate supremacy. In the words of Heinrich Suter, "In the application of arithmetic and algebra to geometry, and conversely in the solutions of algebraic problems by geometric means, the Muslims far surpassed the Greeks and Hindu."[104] Despite the terminology bestowed upon Suter in translation, the advances to which he refers belong to the whole Islamicate scholastic community. Indeed, Thabit ibn Qurra, given as an example above, was Sabian, not Muslim. The advances Islamicate scholars made in geometry and algebraic geometry was largely founded on Greek sources, as they became available through the House of Wisdom and in some ways replaced the influence of Indian traditions. However, the House of Wisdom primarily made those texts available—and accessible—through translation. While many non-Muslims and some Muslims, such as Thabit ibn Qurra and the Banu Musa respectively, knew Greek, the translation of these texts into the *lingua franca* of the Islamicate world, Arabic, made the knowledge contained therein available to a wider, non-Greek-speaking majority. This particularly included many Muslims, such as al-Kindi, who had no religious and few social reasons to know Greek. The translation movement of the ninth century, then, enabled

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[102] Seven were commentaries, eight more related to arithmetic or number theory, and two could not be categorized. Of the seven commentaries, three elaborated on Euclid's *Elements*, two talked about Platonic solids, one revised Nichomachus' *Introduction to Arithmetic*, and one covered Archimedes' reasoning with respect to the ration of the circumference of a circle to its diameter. The above numbers are derived from the list of his titles in MAOS.
[103] For more information about the geometric content of his contributions, see Rosenfeld and Youschkevitch in Rashed, *Encyclopedia*, 2: 451-452.
[104] Quoted in al-Daffa, *The Muslim Contribution to Mathematics*, 89.

many Muslim scholars to learn about Greek geometry and to become fascinated by the new ideas and methods contained within the translated corpus.

III.c | The Integration of Greek and Islamic Traditions

The enthusiasm of the Islamicate scholastic community for Greek geometry built on a pre-existing interest in mathematics for its administrative utility, as demonstrated by its use in solving practical problems of Islamic administration. The *qibla*, prayer times, and the dates of Ramadan could all be found to remarkable precision with mathematical astronomy, and the Middle East's new, syncretic algebra simplified the process of adjudicating inheritance law. Although their methods for finding the *qibla* were not always used by individuals in charge of building mosques, Muslim mathematicians continued to use the *qibla* problem as a vehicle for developing mathematical techniques. Through the determination of prayer times and Ramadan, as well as the calculation of inheritances, math did become an integral, useful part of Muslim society. Thus, the introduction of Indian and then Greek texts in the late eighth through tenth centuries capitalized on a foundation of interaction between Islam and mathematics. In this atmosphere, Islamicate scholars assimilated decimal arithmetic, astronomical notions and trigonometric ideas, and possibly also some algebraic ideas from India. They even more enthusiastically absorbed Ancient Greek geometry and began to expand that cultural inheritance into something their own.

However, it is important to note that neither the concerns of Islam nor the intervention of foreign traditions were straightforwardly causal factors in the growth of Muslim interest in mathematics in the ninth century. Rather, they existed within a matrix of social, religious, and intellectual elements. The factors considered above did foster a particularly Muslim inclination for the field of mathematics, but they were concurrently fostered *by* that inclination. Indeed, the

whole process of acquiring and translating Greek texts began as a deliberate endeavor to cultivate scholarship—not just in mathematics—rooted in a cultural appreciation (among elite) for the authority of the Ancient Greeks. Esteeming the ancients in this way prepared the Islamicate scholastic community to absorb their mathematics and, more importantly, to embrace their methods of arguing by the authority of math—that is, by proof.

IV | Mathematics' Epistemological Singularity

The domestic pressures of Islamic society and an openness to the external influence of foreign ideas contributed to initiating substantial Muslim interest in mathematics in the early centuries of ʿAbbasid rule. In later centuries, changes in the political and theological fabric of the Islamicate world would generate challenges to the epistemic authority of human reason, but mathematics would withstand the broad attack on reason-based ways of knowing. The epistemological elevation demonstrated by this endurance was enabled by trends in the Islamicate scholastic community to internalize Greek notions of geometric proof and to develop its applications further.

IV.a | The Importance of Proof

In the words of Vassilis Karasmanis, Greek thought was dominated by "rationality," a term he uses to mean a tradition of supporting their ideas with arguments and evidence. This manifested in their mathematics through "proofs," as we today call rigorous demonstration of the validity of a mathematical claim.[105] The real value of this approach is not in its ability to show, for example, that the only solution to $x^2 - 4x + 4 = 0$ is $x = 2$ but rather to show that all quadratic equations of the form $x^2 - (2c)x + c^2 = 0$ have just one solution, $x = c$. The method of general proof subsequently reveals *patterns* in mathematical relationships and enables the field to take broader steps with each new development by discussing whole categories of problems at a time.[106]

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[105] Vassilis Karasmanis, "On the First Greek Mathematical Proof," *Hermanthena* 169 (2000): 7, 8.
[106] As G.H. Hardy wrote in his famous apology, "A mathematician, like a painter or a poet, is a maker of patterns." G.H. Hardy, *A Mathematician's Apology*, rev. ed. (1940; Cambridge: Cambridge University Press, 1992), 84.

Before the introduction of Greek mathematics and its reliance on such general proofs, Islamicate mathematics relied predominantly on specific examples.[107] Thus, the Greek tradition's most significant contribution to the development of Islamicate mathematics in the 'Abbasid period was not the geometric problems it introduced, or even methods for solving those problems, but the concept of *proving* those solutions—and proving them for a *general* category. Islamicate scholars internalized this idea and adapted the fundamentals of Greek proofs to their own solutions of geometric problems. Then, they went further: They created geometric proofs to algebraic problems and algebraic proofs to geometric problems. Recognizing its capacity to demonstrate the validity of an initial claim, some Islamicate scholars applied mathematical proofs to other secular, rational ways of knowing, including metaphysics and forms of natural knowledge. By the interconnected nature of the Islamicate scholastic community, the value that these individuals extended to demonstrations of proof then influenced the rest of the community and consequently propagated Islamicate esteem for mathematics.

IV.a.i | Interest in Greek logic and proof

Under the Islamicate Empire's policy of religious tolerance, it became fashionable in the 'Abbasid era for political leaders, including the caliph himself, to arrange for "highly ritualized and highly civilized" theological debates between representatives of different religious sects: sometimes between members of different Muslim schools; sometimes between Muslims, Christians, Jews, and Zoroastrians. In all cases, the opponents attempted to outperform each other in argument, seeking the approval of their audience.[108] Nasr claims that representatives of Islam often lost these debates because they could not frame their arguments in the same

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[107] The change is demonstrated though examples below. Note, however, that Islamicate mathematics before the influence of Greek geometry did not rely *exclusively* on specific proofs.

[108] David Thomas, "Relations with Other Religions," *The Islamic World*, ed. Andrew Rippon (New York: Routledge, 2008), 249.

sophisticated manner as the Christians and Jews, who had access to Greek logic. These failures

risked undermining Muslim law on which all of Islamicate society and caliphate authority rested,

so Caliph al-Ma'mun in the ninth century heavily encouraged his House of Wisdom to translate

as much of the Greek canon as they could in search of better tools for logical argumentation.[109]

David Thomas argues that in fact the ninth century Christians integrated Islamic theological

procedures into their written defenses.[110] It seems likely that Muslims in turn sought to

assimilate some of the Christians' most advantageous forms of argumentation. In an

environment of ritualized debate, equilibrium would naturally develop as each participant

learned from experience which logical strategies, regardless of their origins, were most effective.

Whether he felt his authority was threated or not, al-Ma'mun's reputation for active interest in

non-Muslim religions and in intellectual growth suggests that he would have spearheaded the

initiative to access and incorporate principles of Greek logic. As Ronald Calinger concluded,

"he [al-Ma'mun] may have seen Greek logic with its mathematical proof theory as another path

to truth complementing the path of faith, prophetic traditions (*hadīth*) and the *Quran*."[111] To

Muslims like al-Ma'mun, knowledge and methods of creating it that were imported from Greece

could contribute to the Islamic mission of growing spiritually through learning.

This investment in proof developed just as quickly and securely as the interest in

geometry with which it grew in tandem. The three Banu Musa, who were in large part running

the House of Wisdom in the early ninth century, had proposed an original geometric proof in an

astronomy context even as the translation movement was still at its height. This was Aḥmad's

Book on the Mathematical Proof by Geometry that outside the Sphere of Fixed, there is not a

Ninth Sphere (*Kitāb bayyana fīhī bi ṭarīq ta `līmī wa madhhab handasī annalhū laysa fī khārij*

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[109] Nasr, *Science and Civilization*, 70.
[110] Thomas, "Relations with Other Religions," 252-253.
[111] Calinger, "Arabic Primacy," 168.

kurat al-kawākib al-thābita kura tāsi`a). This is only the second title in the MAOS using the term "proof" but by no means the last. Not long thereafter, their student Thabt ibn Qurra furthered this idea in his *Reasoning on Establishment of Correctness of [Solutions of] Problems of Algebra by Geometric Proofs* (*Qawl fī taṣḥīḥ masāil al-jabr bi'l-barāhīn al-handasiyya*). In proving geometrically the rules of solution of quadratic equations by propositions 5 and 6 in Book 2 of Euclid's *Elements*, ibn Qurra applies the idea of proof to a general category of equations, rather than to a single problem. Closer to the turn of the century, Muḥammad al-Rāzī (865-925) could take geometric proofs for granted and scoff at the person ignorant of them in his *Treatise on [the Fact] that the Man who did not Learn [Mathematical] Demonstration Cannot Imagine that the Earth is a Sphere and People Live on it* (*Risāla fī annahū lā yutaṣawwaru li man yartaḍi bi'l-burhān anna al-arḍ kuriyya wa'l-nās ḥawlahāa*).

In his *[Solutions of] Problems of Algebra by Geometric Proofs*, Thabit ibn Qurra represented the true internalization of Greek methods of proof into Islamicate mathematics. Until the introduction of Greek texts, the "proofs" in Islamicate math were often demonstrations by example. The problem shown above in the discussion of al-Khwarizmi's algebra as applied to inheritance law represents proof-by-example: His intention was to demonstrate his algebraic method, and he did so with a series of scenarios in which he specified how many *dirhems*, family members, and bequests were involved. This kind of proof is seen earlier in his *Algebra* as well. Take, for example, his demonstration that "Roots and Squares are equal to Numbers," where a root is "any quantity which is to be multiplied by itself" (that is, x), a square is correspondingly equivalent to x^2, and a "number" refers to a constant (c, in modern notation).[112] Al-Khwarizmi immediately provided an example in which "one square, and ten roots of the same, amount to thirty-nine dirhems" ($x^2 + 10x = 39$). After solving this case, his treatment for the general case

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[112] Al-Khwarizmi, *Algebra*, 6.

was brief: "The solution is the same when [there are] two squares or three, or more or less be specified; you reduce them to one single square, and in the same proportion you reduce also the roots and simple numbers which are connected therewith."[113] He then launched into two more specific examples. In short, his method of "proof" resembles a textbook attempting to show a student *how* to do a problem, not *why* the procedure works.[114]

In contrast, Euclid's aim was clearly to convince the reader that the theorems he proposed in his *Elements* must be valid. He provided no examples before or after proving each theorem. He laid out his definitions, the axioms on which his theorems depended, and the postulates he intended to prove. When he arrived at the seventh postulate in Book VII, for example, he immediately stated his proposition in the most general of terms: "If a number AB be the same part of a number CD, that a part taken away AE is of a part taken away CF; then shall the residue [remainder] EB be the same part of the residue FD, that the whole AB is of the whole CD."[115] The proof that followed retained the general variables "AB," "CD," etc., without ever giving them constant values.[116] Moreover, his proof called on axioms and postulates previously established, and in doing so, it demonstrated the interlocking nature of the *Elements*' systematic proofs. Especially considering the popularity of the *Elements* among Islamicate scholars (see

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[113] Ibid., 8-9

[114] This is in line with al-Khwarizmi's claim to have written the book for individuals engaged in law and trade, among other things, but it is also worth noting that he never wrote a book on algebra for specialists. While we cannot say what his attitude toward specialists was, he did not consider it necessary to prove to non-specialists *why* his methods worked. His authority and demonstrations seemed sufficient proof. If he was aware of the notion of general proof, then he did not give it, like his successors did, more authority than his status and the use of examples.

[115] Euclid, *Euclide's Elements; the whole fifteen books compendiously demonstrated…*, trans. Isaac Barrow (London, 1705), 148. This postulate essentially states (geometrically) that if you have two geometric objects (lines, squares, circles, etc) AB and CD of proportional magnitude (length, area, etc.) and you take away equivalently proportional segments AE and CF, then the remaining portions EB and FD are also proportional in the same way that AB and CD are. In hindsight, this can be interpreted algebraically as saying that, given $AB = \frac{CD}{n}$ and $AE = \frac{CF}{n}$, $AB - AE = \frac{CD}{n} - \frac{CF}{n} = \frac{CD-CF}{n}$.

[116] Ibid., 148-149.

Table 1), this form of proof would have a substantial impact on the development of Islamicate mathematics.

Under the influence of the *Elements* and other Greek texts in the ninth century, Islamicate scholars began emphasizing generalizability in their proofs. As mentioned above, Thabit ibn Qurra's *[Solutions of] Problems of Algebra by Geometric Proofs* discussed whole categories of equations at a time. In fact, he claimed that most problems of algebra could be reduced to the three he discussed, so he clearly believed that he was making significant statements. The first of these three categories is "*māl* [square] and roots equal a number."[117] In contrast to al-Khwarizmi's treatment of the topic, he gave no examples. He first referenced Proposition II.6 of Euclid's *Elements* for authority, then launches into an extensive proof of a single geometric method (see Figure 1), demonstrating *why* that method can solve all algebraic equations of the form "*māl* and roots equal a number" ($x^2 + bx = c$). Before moving on the second form of equations, he discussed how his geometric method corresponded with the "procedure of the algebraists" in terms more general than those of al-Khwarizmi

Figure 1. Thabit ibn Qurra *[Solutions of] Problems of Algebra by Geometric Proof.*

Namely, their [the algebraists'] taking one half of the number of the roots, is as if we take half of the line BE. That they multiply it in itself, is just as if we take the square of half of the line BE. That they add the number to the result obtained is
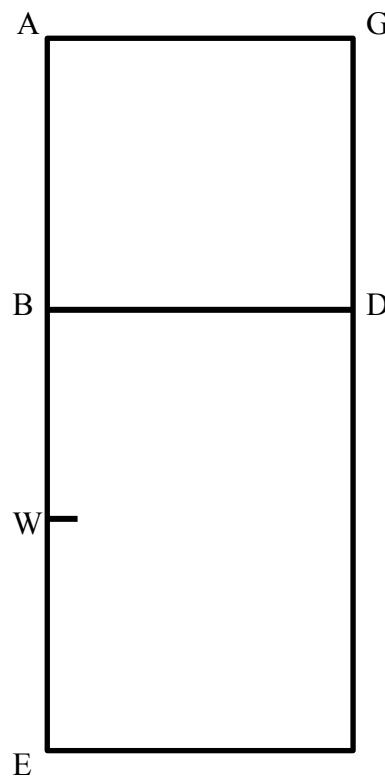
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[117] Thabit ibn Qurra's treatise is translated in part in Berggren, "Mathematics in Medieval Islam," 548-550. He chooses not to translate *māl* throughout the treatise, possibly to distinguish the square of the unknown value from squares of known values referred to elsewhere.

like our adding the product EA in [by] AB. So that, out of all this, the square of the sum of AB and half of the line ($BW = BE/2$) are put together. That they take the root of the result is as if we say: The sum of AB and half the line (BE) is known when its square is known. That they subtract from this <half the number of the roots, so that they obtain the remainder, namely the root, as if we take away half of BE> so that the remainder results, as AB resulted for us. They multiply it in its like, and thus they determine the *māl*, (just) as we determined from AB its square, and that is the *māl*.[118]

Thabit ibn Qurra's interest in providing a method that can be used to solve the greatest number of problems possible clearly suggests a concern for the generalizability inherent in Greek proofs. More than that, he felt compelled to support that method with a geometric proof explaining why his method can be trusted to produce a valid answer to all equations of the same form. His concerns were representative of the wider Islamicate scholastic community's interest in developing and proving solutions in their general forms. Ibn al-Haytham, to be discussed in greater detailed below, criticized Abū Sahl al-Kūhī, called by Abu l-Jūd "master of his age in the art of geometry" (*shaykh ʿaṣrihi fī ṣināʿati l-handasa*), for failing to solve the problem of a paraboloid's volume in all generality.[119] Thus, in the vein of Greek proofs, Islamicate solutions were held in highest esteem when they were in as general form as possible. More to the point, a proof in support of a general solution had to be expounded; therefore, ibn al-Haytham proceeded to construct a solution to the element missing from al-Kuhi's text and to prove its validity.

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[118] Ibid., 549. My annotations are in brackets, and Berggren's additions have been transferred into parentheses. The words in pointed brackets are in Berggren's translation but are based on a restoration offered by an earlier translator to a portion of the manuscript that was destroyed.

[119] For a translation of the relevant text, see Ibid., 588. Al-Jud's reference to al-Kuhi is in J.P. Hogendijk, *Ibn al-Haytham's Completion of the Conics* (New York: Springer-Verlag, 1985), 113.

Islamicate pure mathematicians in short internalized the Greek concept of demonstrable proof as an argument for a method's validity and reliability. Euclid's *Elements* in particular demonstrated these ideas, and Table 1 shows that it was the most popular text for a commentary until the very end of the 'Abbasid period. In the tenth-century, al-Kuhi described mathematicians as a group "whom neither Galen nor anyone else could criticize, neither them or their knowledge, because they depend on proofs in all their sciences and books."[120] Mathematics was hence by this point recognized for its epistemological strength resulting from the ideas of proof theory the Islamicate world inherited from his Hellenistic predecessors. Ibn Khaldun, writing in the fourteenth or fifteenth centuries, stressed the certainty derived from mathematics:

> It should be known that geometry enlightens the intellect and sets one's mind right. All its proofs are very clear and orderly. It is hardly possible for errors to enter into geometrical reasoning, because it is well arranged and orderly. Thus, the mind that constantly applies itself to geometry is not likely to fall into error. In this convenient way, the person who knows geometry acquires intelligence. The following statement was written upon Plato's door: No one who is not a geometrician may enter our house."[121]

Ibn Khaldun's quote clearly shows an immense regard for mathematical, especially geometrical, proof. Even about three hundred years after other reason-based ways of knowing fell out of favor in Islamic theology, Islamicate scholars continued to value them for the dialectic strength they imbued mathematics, in particular geometry. Although the Islamicate community certainly embraced the geometric ideas enclosed in those works as well, it was the notion of proof that

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[120] Quoted in Berggren, "Mathematics in Medieval Islam," 666.
[121] As quoted in al-Daffa, *The Muslim Contribution to Mathematics*, 81-82.

they most importantly absorbed.[122] The next section will even discuss how two particular scholars used math's growing epistemological strength to further advance its power and how their choices contributed to the understand of mathematics as a whole in the Muslim academic community.

IV.a.ii | Impact on the scholastic community

Yaʿqub ibn Isḥāq al-Kindī (c. 801-873) and Abū ʿAli al-Ḥasan ibn al-Haytham (c. 965-1040) belonged to different points of the 'Abbasid period, but they were both Muslim polymaths in the Islamicate scholastic community. More importantly, they shared an appreciation for mathematical proof that exceeded their peers', though the community as a whole had embraced Greek geometry and its proofs. While Heinrich Suter could commend five hundred years of scholars collectively for developing geometric proofs to algebraic problems and vice versa, as quoted earlier, al-Kindi and ibn al-Haytham stand out for championing mathematics as a conclusive argumentative tool. Believing so strongly in its validity, they began to use math and mathematical proofs to justify their arguments in subjects outside of pure math, and in doing so, they imbued it with increasing power. The scholastic community as it existed then allowed al-Kindi and ibn al-Haytham's ideas about the nature of mathematics to propagate among the intellectual elite of the Islamicate world, contributing to the general Islamicate, potentially just Muslim, perception of mathematics' high value.[123] Though the Islamic world became less open to non-revelatory ways of knowing in the eleventh through thirteenth centuries, mathematics

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[122] Katz, *A History of Mathematics*, 271.
[123] In some ways, the specifically Muslim view of knowledge and mathematics in particular differed from those of non-Muslims since the translation movement affected each of these Islamicate subgroups differently and since their theologies interacted differently with different ideas. Two Muslims form the focus of this section, and their conception of mathematics should thus be considered in a Muslim context. However, they were also very influential members of the Islamicate scholastic community more broadly, not just of its Islamic contingent, which is why the wider category will be more explicitly discussed.

remained a legitimate pursuit for Muslim scholars, in part because of the growing value assigned to it by al-Kindi, ibn al-Haytham, and the whole Islamicate scholastic community.

Al-Kindi (historically Latinized as Alkindus) lived in the ninth century, born c. 801 and dead in 873. A contemporary of al-Ma'mun and a colleague of the Banu Musa at the House of Wisdom, al-Kindi flourished in the heart of the translation movement. Although it is generally believed that he did not know Greek himself, he apparently gathered translators around him at the House of Wisdom and may even have developed code-breaking methods to help him translate texts in languages utterly unknown to him.[124] He was very quick to appreciate the Greek philosophies his translators provided for him. Today he is known as the "Philosopher of the Arabs," since his translators were among the first to introduce Aristotle to the Islamicate world and al-Kindi himself is credited with beginning the Islamicization of Greek philosophy.[125]

In his *Epistle on the Number of Books by Aristotle and on What is Required to Study Philosophy*, al-Kindi provides his notions about classifications of forms of knowledge by categorizing the works of Aristotle.[126] Even before enumerating those works in the first part al-Kindi specified that, in the study of "philosophy," a new student ought to begin with mathematics. He also wrote a whole book (no longer extant) entitled *Epistle on the Fact that One Only Comes to Philosophy through Mathematics*.[127] To him, mathematics laid the foundation of knowledge upon which all other knowledge could be built. He wrote in his *Epistle on the Number of Books by Aristotle*,

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[124] Al-Khalili, *The House of Wisdom*, 132-134.

[125] Ibid., 75-76; Calinger, "Arabic Primacy," 169; Nasr, *Science and Civilization*, 293.

[126] Title given as it is translated by Jean Jolivet in his chapter of Roshdi Rashed, *Encyclopedia*.

[127] Calinger, "Arabic Primacy," 169-170; Rashed, *Encyclopedia*, 3: 1009-1010. Al-Kindi's *Epistle…on What is Required to Study Philosophy* is the book with the romanized title *Risāla fī kammiyyat kutub Arisṭūṭālis wa mā yuḥtājū ilayhī fī tahṣīl al-falsafa*, translated in MAOS as *Treatise on the Number of Books of Aristotle and What Is Needed to Learn Philosophy*.

Indeed, if anyone is devoid of knowledge of the introductory disciplines, which are arithmetic, geometry, astronomy, and music, then (even) throughout his whole life, he will never possess perfect knowledge of any other discipline, and his efforts will procure him nothing but the ability to transmit (mechanically) the basics, if he has a good memory; but he will never have intimate knowledge of these disciplines or achievement in any field, if he is devoid of the introductory disciplines.[128]

Channeling Aristotle, al-Kindi felt very strongly that only after the "introductory sciences"—the *quadrivium*, which included mathematics and mathematized subjects, astronomy and music, at its periphery—were well-established in a student's mind could they be followed by the study of anything else. Considering that al-Kindi held that "the human art which is highest in degree and most noble in rank is the art of philosophy," it was to him very important that the study of metaphysics and morality should only follow the study of math; only then would the student be capable of learning them thoroughly and "to attain the truth."[129]

However, al-Kindi was known as the "Philosopher of the Arabs" because he transformed Aristotelian philosophy into an Islamicate context. In doing so, he rejected Aristotle's refusal to ascribe mathematics any power as a source of knowledge. In contrast, he pushed the boundaries of how math—or rather proof by mathematics—could be used to produce knowledge outside of pure mathematics. He demonstrated the value to which he held mathematical proof by using at a form of argumentation in metaphysics. In direct contradiction both to Aristotle's cosmology and his epistemology, al-Kindi argued mathematically against Aristotle's idea of an infinite universe

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[128] My translation from Al-Kindi, *Uno scritto introduttivo allo studio di Aristotele*, trans. by M. Guidi and R. Walzer (Rome: Dott. Giovani Bardi, 1940), 394 [Arabic], 407-408 [Italian].
[129] Al-Kindi, *Al-Kindi's Metaphysis*, 55.

in the treatise *On First Philosophy*.[130] He started with an infinite quantity—say, in modern notation, A. If one were to subtract from that infinite quantity a finite quantity, B, then the remainder $C = A - B$ must be either finite or infinite. If C is finite, then $C + B$ is the addition of two finite quantities and subsequently results in another finite quantity, but $C + B = A$, which is infinite. Therefore, C must be infinite. However, one would then have to conclude that C is less infinite than A by the finite magnitude B. Although today mathematicians recognize infinities of different sizes, al-Kindi believed this to be impossible, and he thus considered it proven—mathematically, no less—that no infinities could exist, certainly not Aristotle's infinite universe.[131] Moreover, by using mathematics to make this argument, al-Kindi rejected Aristotle's inclination to give mathematics no epistemological power in favor of a distinctly Islamic belief that it as a process could create knowledge.

Likewise, al-Kindi utilizes mathematical proof in astronomy beyond even the notions of Ptolemy, whose monumental work, titled in English the *Almagest* from its Arabic title *al-Majisṭī*, formed the cornerstone of planetary theory in Europe and the Islamicate world until well after the death of Copernicus. In the introduction to their translation of al-Kindi's *Epistle to Aḥmad ibn al-Muʿtaṣim: "That the Elements and the Outermost Body are Spherical in Form,"* the first such translation, Haig Khatchadourian and Nicholas Rescher remark on his use of geometry in the letter.[132] While the astronomical theory of the work was in no way original, the "elaborate geometric machinery" in which al-Kindi dressed his argument was far more mathematical than its presentations in Ptolemy's *Almagest* or Aristotle's *Physics*. Khatchadourian and Rescher suspect from his closing remarks that al-Kindi had a greater epistemological argument in mind:

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[130] Also known as *Letter to al-Muʿtaṣim billāh on the First Philosophy* (*Kitāb ilā al-Muʿtaṣim billāh fī'l-falsafa al-ūlā*) according to MAOS.
[131] Al-Khalili, *The House of Wisdom*, 132-133; al-Kindi, *Al-Kindi's Metaphysics*, 68-69.
[132] In MAOS, the same letter is related as *Letter to Aḥmad ibn al-Muʿtaṣim on the Round Form of Elements and the Farthest Body* (*Risāla ilā Aḥmad ibn al-Muʿtaṣim fī anna al-ʿanāṣīr wa'l-jirm al-aqṣā kuriyyat al-shakl*).

"[F]rom the closing remarks," they write, "it would appear that he wishes the epistle to serve as an inducement to the study of geometry by presenting a simple instance of the usefulness of this discipline as an instrument for scientific understanding of the world."[133] Khatchadourian and Rescher's reading of his text is in line with al-Kindi's use of mathematics to construct knowledge about the cosmology of the universe, as demonstrated above. It is evident from how he presented and handled math that al-Kindi believed it to be valid and reliable way of creating knowledge about philosophy and cosmology. He drew a line, however, at using mathematics to learn about material (physical) systems.

While al-Kindi certainly demonstrated his value for mathematical proof and encouraged the study of mathematics by using it to create knowledge about the universe, there was a limit to the extent to which he intended it to reveal understanding about the world of everyday life. According to his *On First Philosophy*, al-Kindi considered mathematics' epistemological role to be limited to "what has no matter." This, he explained, is because

> …matter is a substratum for affection, and it moves, and nature is the primary cause of everything which moves and rests. Therefore every physical thing is material and hence it is not possible for mathematical investigation to be used in the perception of physical things, since it is the property of that which has no matter. Since, then, mathematics is such that its investigation concerns the non-physical, whoever uses it in the investigation of physical objects has left and is devoid of the truth.[134]

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[133] Haig Khatchadourian, Nicholas Rescher, and Yaʿqub ibn Ishaq al-Kindi, "Al-Kindi's Epistle on the Concentric Structure of the Universe," *Isis* 56 (1965): 191.
[134] Al-Kindi, *Metaphysics*, 65.

Since "a number [has] no matter," it could not be used to measure the properties of physical objects and systems governed by the characteristics of matter.[135] Perhaps additionally motivated by his belief that philosophy was the "highest" and "most noble" way of knowing, he disparaged attempts to capture the motion of material things with mathematics. From this perspective, he may have intended to elevate mathematics' status by disassociating it from knowledge about the physical world and instead associating it with the philosophical.

In general, al-Kindi encouraged the study of math in general by placing it at the very foundation of all knowledge, and he crucially promoted math's dialectic role by extending it to subjects beyond algebra and geometry. In doing so, he followed the Aristotelian tradition of favoring mathematics as a fundamental subject on which other knowledge is built; however, he broke with Aristotle by recognizing math as a source of knowledge, not just a form, in its own right. He demonstrated the value he had for mathematics and its proof theory by applying it to his metaphysics. To al-Kindi, he could augment that value by limiting mathematics' applications to abstract, non-physical problems, but to ibn al-Haytham, mathematics in fact gained its merit as a descriptor of the material world.

Ibn al-Haytham (historically Latinized Alhazen or Alhacen) was born in Iraq c. 965, more than a full century after al-Kindi died. Invited to Cairo in 1010, he lived the rest of his life in Egypt, dying there in 1040, and leaving behind a significant corpus and a lasting legacy. Ibn al-Haytham supposedly wrote over 200 texts in his lifetime. MAOS lists over a hundred, more than 50 of which are mathematical and almost 45 in astronomy and physics combined. He is best remembered today for first developing a unified theory of light and optics that forms the foundation of modern understanding in these subjects.

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[135] Ibid., 98.

In formulating this theory, ibn al-Haytham chose to base his ideas about optics on *geometric* accounts, such as those of Euclid and Ptolemy, but he adjusted those theories to accommodate the notion that light enters the eye, rather than is emitted from it, after noting that looking at bright objects causes pain, as if the eye is subjected to a visual attack.[136] Fundamentally, his theory was a conceptual change from the Greek theories inherited by the Islamicate Empire. Rather than relying solely on contemplation and thought experiments, he developed physical experiments and geometric models to justify his ideas on the nature of light and vision.[137] In doing so, he demonstrated his belief that the natural world could be described in mathematical terms, but he also put his geometric model to empirical test in order to prove its empirical accuracy. Describing light in terms of lines allowed ibn al-Haytham to use Euclidean principles to predict how he would perceive light in controlled situations. Given that real-world experiments confirm the validity of his geometric model as a descriptor of vision, he could then take the process of vision as a mathematical system and use geometry to explain why forms are perceived without distortion of their shape and why individuals see more clearly at the center of their vision than at the periphery.[138] He essentially re-founded optics on a geometrical basis, and in doing so, began the process of "mathematizing" physics.[139]

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[136] Jim al-Khalili, "The Empire of Reason," *Science and Islam*, episode 2 (BBC, 2009) on *Films on Demand*, http://digital.films.com.proxy.wm.edu/PortalViewVideo.aspx?xtid=47837; Seyyed Hossein Nasr, *Ismāʿīlī Contributions to Islamic Culture* (Tehran: Imperial Iranian Academy of Philosophy, 1977), 190.

[137] Al-Khalili, *The House of Wisdom*, 158; Rashed, *Encyclopedia*, 2: 703. Other works take ibn al-Haytham's proclivity for experimentation and mathematical justification as suggestions of the beginning of the modern scientific method as early as the eleventh century. Al-Khalili, *The House of Wisdom*, 170-171; al-Khalili, "The Empire of Reason," 42:59-55:00.

[138] See Book 2 in the second volume (English translation) of Abu Ali al-Hasan ibn al-Haytham, *Alhacen's Theory of Visual Perception: A Critical Edition with English Translation and the First Three Books of Alhacen's* De Aspectibus*, the Medieval Latin Version of ibn al-Haytham's* Kitab al-Manazir*, Volumes 1-2* (Philadelphia: American Philosophical Society, 2001).

[139] Naseem Ahmed, *Science in Islam* (New Delhi: Reference Press, 2003), 292; al-Khalili, *The House of Wisdom*, 161; Rashed, *Encyclopedia*, 2: 687-688. Also, Dr. Nader El-Bizri, as quoted in al-Khalili, "The Empire of Wisdom," at time 50:22.

Unlike Islamicate physics, Islamicate cosmology before ibn al-Haytham already fairly mathematical, thanks to Ptolemy's *Almagest*. Scholars in the Middle East had quickly accepted his abstract geocentric, geometric models—see, for example, the previously mentioned *Book on the Mathematical Proof by Geometry that outside the Sphere of Fixed, there is not a Ninth Sphere*, written by Aḥmad, one of the Banu Musa, in the mid-ninth century. However, ibn al-Haytham took issue with Ptolemy's (fundamentally Aristotelian) belief that his mathematical models were tools for conceptualizing and computing the motion of the planets, not for actually capturing them. One of the first scholars to launch a serious critique against the *Almagest*, he not only pointed out errors in Ptolemy's books, but he challenged Ptolemy's epistemology. In censuring a defender of Ptolemy for "believ[ing] in Ptolemy's words in everything he says, without relying on a demonstration or calling on a proof, but by pure imitation," ibn al-Haytham revealed a perception that Ptolemy insufficiently proved the validity of his theory. Indeed, he claimed to have shown by "irrefutable demonstration" that Ptolemy used flawed models in the second chapter of his *Book on Hypotheses*.[140]

In a deliberate attempt to reform Islamicate astronomy, ibn al-Haytham advanced the anti-Ptolemaic idea that mathematics could describe the real world.[141] Accepting still Ptolemy's geocentric universe but believing that mathematical models ought to reflect reality, ibn al-Haytham transformed it into a physical *and* mathematical astronomy.[142] His new model of the universe, as explained in his three-volume book *The Model of Motions of Each of the Seven*

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[140] Roshdi Rashed, "The Celestial Kinematics of ibn al-Haytham," *Arabic Sciences and Philosophy* 17 (2007), 11.
[141] Nasr, *Illustrated Study*, 133. While his corrections of Ptolemy are only mentioned in passing here, they should not be thought to be a passing concern of his. Indeed, one might suggest that ibn al-Haytham hated Ptolemy; he certainly took sufficient enjoyment of denigrating the *Almagest* to devote three books to it: *Book on Doubts about Ptolemy* (*Maqāla fī'l-shukūk 'alā Baṭlamyūs*), *Resolution of Doubts about the Work* Almagest *Which Are Difficult for Some People of Science* (*Ḥall shukūk fī kitāb al-Majisṭī yashukku fīhā ba'ḍ ahl al-'ilm*), and *Corrections to the* Almagest *(Fī Tahdhīb al-Majistī)*. (The last of these three texts is no longer extant and is excluded from the MAOS.) See Rashed, "The Celestial Kinematics of ibn al-Haytham," 9-10.
[142] Ibid.; Al-Khalili, *The House of Wisdom*, 212; Nasr, *Ismāʿīlī Contributions*, 132.

Planets, intended to describe the observed motions of the planets—that is, to describe the reality one could witness in the heavens—in purely geometric terms. Indeed, of the single surviving volume, slightly less than half is a purely mathematical text that first laid out, in the manner of Euclid, fifteen propositions that he subsequently proved and used later in the text.[143] Motivated, in Roshdi Rashed's judgment, by "mathematisation, avoiding Ptolemy's contradictions and accounting for the observations," ibn al-Haytham's *Model of Motions* was entirely uninterested in speculating how or why the planets moved, focusing entirely on capturing with mathematical precision how the movements of the planet were perceived from Earth.[144] Essentially, he intended to replace an astronomy that attempted to fit a mathematical model to a cosmological theory with one that instead emphasized the creation of a mathematical model that reflected the physical reality of the planets. Whereas Ptolemy's astronomy used mathematics as a rhetorical tool for cosmology, ibn al-Haytham elevated mathematics to a position of describing observed planetary motion and predicting such motion in the future. That is, in ibn al-Haytham's astronomy, mathematics became a way of knowing about planetary motion.

As a result, ibn al-Haytham is said to have "mathematized" astronomy, as he had "mathematized" physics. Both processes demonstrated that ibn al-Haytham advanced math as a way of knowing about material as well as immaterial forms. He subordinated other rational forms of natural knowledge to its epistemological power, and he put even physical reality to its tests. His predecessor, al-Kindi, would have been scandalized at his acceptance of geometric descriptors for the physical world, but ibn al-Haytham was following a process instigated by al-Kindi in applying mathematical ideas as proof outside of pure math.

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[143] Rashed, "The Celestial Mechanics of ibn al-Haytham," 17, 21-31.
[144] Ibid., 20; Al-Khalili, *The House of Wisdom*, 168-169; Nasr, *Illustrated Study*, 133.

While al-Kindi and ibn al-Haytham stand as the earliest and most famous (respectively) of the scholars involved in the process of advancing mathematics epistemologically in this fashion, they were not alone. Régis Morelon highlights, for example, the contributions of Thabit ibn Qurra (836-901) and Abu'l-Rayḥan al-Biruni (973-1048) to the mathematization of astronomy, even attributing its origins to ibn Qurra.[145] The organization of the Islamicate scholastic community engendered cooperation and interaction among scholars. Many of them worked together in more or less cohesive, publically-supported institutions of learning in political and academic centers, such as Baghdad and Cairo. They even moved around, corresponded with each other, and commentated on their predecessors' and contemporaries' works, ensuring that intellectuals in the Islamicate world often interacted even over great distances and political divides. The extensive communication within the scholastic community transmitted and preserved individuals' ideas and texts.

Influenced by al-Kindi and ibn al-Haytham's implicit expansionary claims for the power of mathematical proof, the scholastic community as a whole recognized the validity of mathematics as a way of knowing reliably. They not only straightforwardly emphasized its study—as al-Kindi did by placing it first sequentially among disciplines—but they gave it lasting power. As changes in the political and theological atmosphere of the Islamic world, discussed below, made other so-called "philosophical," meaning reason-based, ways of knowing seem less valid, mathematics remained epistemologically sound and even highly valued in Islamic society. Having evolved into an expanding system of immaterial but systematic principles with the potential—but no necessity—for application to other fields, mathematics was increasingly perceived as existing outside of reason. Consequently, it remained an attractive outlet for

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[145] Rashed, *Encyclopedia*, 1: 54.

Muslim secular investigators, even as other rational pursuits fell out of favor in the epistemological atmosphere of the Islamic world in the eleventh century and beyond.

<center>IV.b | Mathematics Alone Remaining</center>

In the 1000s, Sunni theological schools increasingly perceived rational processes as threatening or oppositional to Islam, rather than complementary to it, as individuals relied too much on man and too little on God. Subsequently, the intellectual atmosphere of the Islamicate world became increasingly hostile toward ways of knowing that relied on human reason, rather than on revelation, and rational forms of knowledge tumbled from grace. In 1100, one of Islam's greatest theologians would issue a sweeping condemnation of human reason as a reliable way of knowing about the world; however, he would explicitly *exclude* mathematics from his indictment. In doing so, al-Ghazali recognized mathematics as a legitimate *Muslim* way of knowing. That is, by 1100, mathematics had become a form of knowledge that transcended human reason and its failings. It created abstract and descriptive knowledge where it was employed, and it was perceived to be such a powerful way of knowing that a school of Islamic thought that rejected the power of human reason in favor of divine revelation was obliged to accept and legitimize it.

IV.b.i | Mathematics as a *Muslim* way of knowing

Islam initially encouraged the study of mathematics by providing problems from worship and from law that challenged the field to develop further. Its most crucial effect, however, came in the form of epistemological validation. As the foundation on which all of Islamicate society was built, Islam was its most critical element, and validation in Islamic terms carried significant weight. Therefore, in granting mathematics epistemological validation, Islam provided the subject with its *real* power—but it only did so because all of the factors discussed above: Between its provision of solutions to problems facing administrators of Islamicate society and its

broader appeal to scholars after the introduction of new methods and ideas from foreign sources, mathematics was initially established as a field generating significant interest in the Islamicate world. To that strong foundation, reliance on demonstration by proof added an epistemological argument for the use of mathematics. Individual intellectuals and the Islamicate scholastic community as a whole saw that and increasingly expanded its role as a source of knowledge about metaphysics and the real world, which subsequently set mathematics apart from other rational ways of knowing.

As seen above, the pursuit of "secular knowledge" in any form was not initially considered to be antagonistic to spiritual growth but rather complementary to it. Thus, seeking truth about nature was a legitimate pursuit in the early ʿAbbasid period and one intimately connected to religion. One of the most famous Muslim scholars of the time was ibn Sina (still commonly Latinized as Avicenna), who wrote in his autobiography that when problems of logical syllogism puzzled him, he would go to the mosque in order to pray for clarity.[146] The eleventh-century scholar al-Biruni highlighted the importance of mathematical study in preparing the mind for the understanding of spiritual truths. He thus defended the practice of mathematics on the basis of Islam in the preface of a geometrical text:

> [Y]ou reproached me my preoccupation with these chapters of geometry, not
> knowing the true essence of these subjects, which consists precisely in going in
> each matter beyond what is necessary…Whatever way he [the geometer] may go,
> through exercise will he be lifted from the physical to the divine teachings, which
> are little accessible because of the difficulty to understand their meaning…and

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[146] William E. Gohlman, trans. and ed., *The Life of ibn Sina: A Critical Edition and Annotated Translation* (Albany, NY: State University of New York Press, 1974), 29.

because of the circumstance that not everybody is able to have a conception of them, especially not the one who turns away from the art of demonstration.[147]

As shown above, al-Kindi similarly considered knowledge of mathematics the foundation of upon which knowledge of metaphysics and morality, related to theological questions, could be built. It is clear that the scholars themselves embraced the mutually-sustaining relationship between Islam and their logical or mathematical pursuits.

Al-Biruni represents a continuation into the 1000s of this feeling about mathematics, just as the support of the theological elite for rational knowledge began to cool. During the eleventh century, the Mu'tazilite theological school waned in power, and reason became increasingly associated with subversion of Islam.[148] By 1100, math was no longer thought to lift a scholar "from the physical to the divine teachings," but it was known not just by mathematicians but also by the theological elite to rest on "infallible proofs."[149] Therefore, of all reason-based forms of knowledge, only math survived the major epistemological shift from reason to revelation in the Muslim community.[150]

The significance of the translation movement and of the incorporation of Indian and Greek traditions has been discussed at length above. The introduction of external ideas certainly nurtured mathematics and other forms of knowledge in the Islamicate world, but it also bred the growing association in the eleventh century of secular knowledge with foreign ideas that were potentially subversive to Muslim society.[151] The divergence of secular from holy knowledge

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[147] This is an English translation provided by Victor J. Katz on page 265 in the chapter "The Mathematics of Islam," of his *A History of Mathematics: An Introduction* (Boston: Addison-Wesley, 2009), of a German translation given in Jens Høyrup, "The Formation of 'Islamic Mathematics'," 30.

[148] Katz, *A History of Mathematics*, 267; Martin and Woodward, *Defenders of Reason in Islam*, 1, 18.

[149] Abu Hamid al-Ghazali, "Confessions, or Deliverance from Error," in *The Sacred Books and Early Literature of the East: With historical surveys of the chief writings of each nation*, ed. Charles F. Horne (New York: Parke, Austin, and Lipscomb, 1917), 114. The source of the title, as well.

[150] Cf. "[Math] has no connection with the religious sciences, and proves nothing for or against religion." Ibid., 112.

[151] Katz, *A History of Mathematics*, 267.

paved the way for the famous theologian al-Ghazali (1058-1111) to attack the worth of the "philosophical sciences" in his essay, "Confessions, or Deliverance from Error," published c. 1100. His "philosophers" were those "who profess to rely on logic," and the six "philosophical sciences" he addressed were math, logic, physics, metaphysics, politics, and moral philosophy. He criticized the latter four subjects as simply impossible to reduce to rational, humanly-comprehensible laws. Even physics, by which he meant the process of theorizing about the workings of nature, was dismissed by al-Ghazali, for "all physical [natural] science rests, as we believe, on the following principle: Nature is entirely subject to God; incapable of acting by itself, it is an instrument in the hand of the Creator….Nothing in nature can act spontaneously and apart from God."[152] Without mathematization, physics was simply a contemplative way of knowing that relied on reason to create and defend theories. However, since God according to al-Ghazali was an active force in the world, those rational theories of causal patterns that excluded the possibility for divine intervention were unreliable.

Math and logic received different treatment from al-Ghazali. Logic, he was forced to admit, contained "nothing censurable," so instead of attacking its foundation, he criticized it for being "liable to abuse"—indicative of his participation that holy and secular processes of knowledge had grown antagonistic to Islam. A student of logic, he argued, was vulnerable to fall into heresy by trusting that the religion of a forefather in the subject must surely rely on substantial proof.[153]

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[152] Al-Ghazali, "Confessions, or Deliverance from Error," 115. This attitude, rejecting physical causes, apparently continued well into the modern day. Pakistani physicist and professor, Dr. Prevez Hoodbhoy (b. 1950) remembers a time when educational guidelines administered by the Institute for Policy Studies in Pakistan recommended causes were not attributed to physical effects. That is, "it was not Islamic to say that combining hydrogen and oxygen makes water. 'You were supposed to say,' Dr. Hoodbhoy recounted, 'that when you bring hydrogen and oxygen together then by the will of Allah water was created.'" Overbye, "How Islam Won, and Lost, the Lead in Science," *New York Times*, October 30, 2001.
[153] Ibid., 114-115.

Like logic, mathematics was epistemologically robust: Al-Ghazali admitted that math "rests on a foundation of proofs which, once known and understood, can not [sic] be refuted." Like the case of logic again, he then found a weakness by which mathematics might lead individuals away from the truth of Islam: Namely, hearing of mathematicians' "disregard for the Divine law, which is notorious" their students or laymen might then also reject Islam, for "if there was truth in religion, it would not have escaped those who have displayed so much keenness of intellect in the study of mathematics."[154] However, mathematics—consisting in al-Ghazali's definition of "the knowledge of calculation, geometry, and cosmography"—stands apart from the other "philosophical sciences," for al-Ghazali ended its section by defending it in a sense.[155] He warned "sincere but ignorant Muslims" against rejecting all mathematical and mathematized forms of knowledge in the name of religion.[156] When devotees did so, however well-meaning they were, Islam earned a reputation for ignorance among those who know that

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[154] Ibid., 113. Later in the same paragraph, he returns to the topic of mathematicians' disregard for Divine law: "[T]hose who study mathematics should be checked from going too far in their researches. For though far removed as it may be from the things of religion, this study, serving as it does as an introduction to the philosophic systems, casts over religion its malign influence. It is rarely that a man devotes himself to it without robbing himself of his faith and casting off the restraints of religion." This is perhaps the quote behind L. E. Goodman's passing statement that al-Ghazali "reject[ed] mathematics." See M.J.L. Young, J.D. Latham, and R.B. Serjeant, eds., *Religion, Learning, and Science in the 'Abbasid Period*, 496. However, Goodman is clearly mistaken. Besides the argument in the rest of this section, it is worth noting that—in spite of this attack on mathematicians' faithfulness—al-Ghazali maintains that, "The religious law contains nothing which approves them or condemns them, and in their turn they make no attack on religion." Although he probably would not encourage an individual to study mathematics when they could study religion instead, al-Ghazali does bestow a kind of theological sanction or blessing on the field of study.

[155] The inclusion of "cosmography" in this definition of mathematics is unexpected but not necessarily problematic. If the "cosmography" intended here was the kind interested in describing terrestrial or celestial space by virtue of measurement, then it overlapped with "geometry." If, however, al-Ghazali did mean to convey "cosmology" and its connotations of rationally-determined planetary theory, this exception would undermine the significance of math in this text. However, elsewhere in "Confessions," al-Ghazali denied one of the fundamental assumptions of rational cosmology in writing that the "sun, moon, stars, and elements are subject to God and can produce nothing of themselves. In a word, nothing in nature can act spontaneously and apart from God." Al-Ghazali, "Confessions, or Deliverance from Error," 115.

[156] (See above for a discussion about this "mathematizing" process and, especially, ibn al-Haytham's role therein.) Al-Ghazali explicitly emphasizes that he does not attack the astronomical calculations that define the solar and lunar orbits. In fact, he also wrote three astronomical treatises, according to MAOS, including one called *On Motion and the Nature of Planets* (*Fī ḥarakat wa ṭabī' at al-kawākib*).

mathematics was epistemologically sound, and as a result, they spurned religion in favor of the certainty of the mathematics.[157]

By distinguishing mathematics in this way from the other "philosophical sciences," al-Ghazali showed that scholars of the exact (meaning mathematical) sciences already conceived of mathematics as an epistemological authority—able to create purely mathematical knowledge as well as to validate forms of natural knowledge that drew on it—and he backed it with his own theological clout.[158] While the point of his essay is to advance mysticism and reliance on revelation as more canonically-sound ways of knowing about the physical and spiritual worlds, it also effectively empowered mathematics as a legitimate *Islamic* way of knowing. It simultaneously denied that same *carte blanche* legitimacy to other forms of secular, rational knowledge, which thereafter were only valid when supported mathematically. The quantitative study shows that Muslims were already inclined toward mathematics by 1100, but al-Ghazali's famous tract cemented its place within Islamic epistemology. The graph (Figure 2) does also show a general upward climb in the number of Muslims performing mathematics in the Islamicate world after 1100, which could be in part attributable to the paradigm shift propagated by al-Ghazali as well as to population growth and the proliferation of centers of learning.

Since Islam was the single most significant element of Islamic and the broader Islamicate society, the legitimacy it bestowed on mathematics by the end of the eleventh century was

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[157] Ibid., 113-114. It may also be worth pointing out that while Field's translation uses the term "exact sciences" in the subsection I refer to as al-Ghazali's "defense" of mathematics, I have reinterpreted "exact" to mean mathematical. I do not know any Arabic to make a translation of the original word myself; however, I found a second translation, in which the same term had been rendered as "mathematical sciences," validating my assumption about the intended meaning. Richard J. McCarthy's translation can be found on the website of American University of Beirut: <http://www.aub.edu.lb/fas/cvsp/Documents/reading_selections/CVSP%20202/Al-ghazali.pdf>.

[158] As a brief reminder, private theological scholars like al-Ghazali were the true elite of Islamic society since they arbitrated on the rules that governed everyday society. They were in many cases respected far more than governmental officials, and as it were, al-Ghazali was particularly well-known in his time period. Indeed, Martin and Woodward call him "the greatest Sunni theologian" and point out that his texts "have been dictated, copied, studied (and now printed and reprinted) continually in all parts of the Islamic world." *Defenders of Reason in Islam*, 35.

crucial for preserving math on the epistemological hierarchy of the medieval Islamicate world. As Islamic society underwent significant changes in the eleventh century, including the ebbing of Mu'tazilism from its dominant position in the caliphate, rational ways of knowing were increasingly repudiated; mathematics survived because it acquired recognition from Islamic theology as existing outside the failings of human reason. Other forms of secular knowledge were thereafter subject to proving their theories by virtue of mathematics. Essentially, mathematics in 1100 occupied a position separate but virtually equal to revelation on the epistemological hierarchy of the medieval Islamicate world.

V | Conclusion

"Knowledge in the world is spread,
To it is the wise man sped."[159]

Islam has a long and complicated history with knowledge. The very first words the illiterate Prophet Muhammad received from the angel Gabriel were an injunction to read. The *hadith* further support and encourage the acquisition of knowledge, promising eternal riches to the learned man. In the early centuries of Islam, knowledge was not divided into "holy" and "secular"; rather, the study of any aspect of God's universe could contribute to a Muslim's spiritual health. At the focus of this study are Muslims who chose to seek knowledge outside of the *sura* of the Qur'an and the *hadith*. These scholars instead relied on their own intellect and on written sources outside of the sacred texts to learn about the universe around them and to craft a syncretic but distinctly Islamicate mathematics.

According to a quantitative study involving 120 scholars with known religion and mathematical performance, Muslim interest in mathematics at the beginning of the 'Abbasid period started low; but in the ninth century, the proportion of Muslim scholars performing math rose steeply. Thereafter, mathematics remained a significant rational interest of Muslim scholars, even persisting as the only legitimate Muslim, rational interest in the face of major political and theological shifts in the eleventh century that resulted in the divorce of secular from holy knowledge. By this point, however, mathematics had been established as far more than just a tool to other rational ways of knowing, which collapsed as Muslim fields of study under the new Islam epistemology. Math was instead an epistemic and dialectic authority in its own right,

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[159] A poem fragment quoted by grammarian Abū Saʿīd al-Ḥasan al-Sīrāfī in a debate with the philosopher Abū Bissr Matta. See a translation of Yāḳūt's report in D.S. Margoliouth, "The Discussion between Abu Bishr Matta and Abu Sa'id al-Sirafi on the Merits of Logic and Grammar," *The Journal of the Royal Asiatic Society of Great Britain and Ireland*, 2nd ser., 37 (1905): 114.

independent of the faults of human reason, and acknowledged by the dominant Islamic theology of the time to possess a legitimacy it denied to the other processes of knowing.

Islamic society was, in the 'Abbasid era, a fertile environment for the development of mathematics. Ritual obligations in everyday life provided vehicles (in the form of problems requiring solutions) for the advancement of mathematical methods as Islamic life became standardized in the early decades of the 'Abbasid period. These problems were mostly in the field of mathematical astronomy, but they established math as a useful Islamic subject, which became the platform on which math's growing significance in the subsequent centuries built. Other institutions being consolidated under the 'Abbasids—namely that of inheritance law, which finally standardized into a system of Qur'anic injunctions and man-made loopholes in the ninth century—absorbed and promoted mathematical interest. A distinctly Islamicate version of algebra appeared in this time, becoming a language through which God's injunctions about inheritance could be adjudicated. By developing an efficient algebra with applications to legal affairs, Islamicate mathematicians connected their subject directly to the administration of Islamic society and consequently improved the value of mathematics to the culture under study.

Not long after the concerns of Islam engendered specifically Muslim interest in mathematics, an influx of foreign intellectual traditions introduced new problems and new methods to the Islamicate field. Although Indian mathematics introduced decimal arithmetic with the "nine signs" and the dustboard, which vastly improved the efficiency of calculation, and it partly inspired the new Islamicate algebra, it was also associated with lower-status elements of society. As a result, Greek geometry had a far greater long-term impact on "mainstream" (meaning patronized or institutionalized) Islamicate mathematics because it was unassociated with the market place, unlike Indian arithmetic. The Greek impact included not only geometric

principles but, more significantly, the principle of systematic, general proof. In creating geometric solutions to algebraic problems and algebraic solutions to geometric problems, Islamicate scholars incorporated the new geometry itself. By developing proofs that explained why their methods must provide the correct answer, they further revealed an internalization of Ancient Greek proof theory. Islamicate scholars increasingly valued solutions and proofs that, if performed entirely in variables, could be applied to whole categories of problems.

Al-Kindi so valued proof by mathematics that he employed it in a metaphysical argument, intending to defeat Aristotle's infinite universe theory with a conclusive, mathematical blow. To him, mathematics formed the basis of all forms of knowledge; consequently it had to be thoroughly mastered by a student before he could move on to other subjects. In advocating for math in these ways, al-Kindi encouraged Muslim study of mathematics, and he distinguished its unique strength for creating knowledge about the universe. He limited that strength, however, to the evaluation of "immaterial" systems of the universe; that is, systems that by virtue of lacking matter were unaffected physical processes. Ibn al-Haytham, a century later, utilized geometry as a descriptor of "material" systems, trusting it to capture even movement caused by "nature."[160] By taking geometrical models as accurate renditions of complex astronomical and mechanical phenomena, ibn al-Haytham could use Euclidean axioms and proven postulates to predict or extrapolate reliably the behavior of these natural systems. He and al-Kindi both championed mathematics' authority as a way of knowing even in fields outside of pure math, and the Islamicate scholastic community in which they worked preserved and propagated their ideas to subsequent generations.

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[160] Al-Kindi and ibn al-Haytham both saw geometric regularity in nature, although al-Kindi focused on the immaterial of nature—namely cosmology—and ibn al-Haytham emphasized the geometry of physical systems. The patterns they saw and their mathematical extrapolations, however, paralleled each other, and connect al-Kindi and ibn al-Haytham in an intellectual genealogy.

Al-Kindi's generation flourished under Caliph al-Ma'mun, a political leader characterized by his particular "fondness for science," in the words of al-Khwarizmi. His Mu'tazilite and proto-Shi'a sympathies helped motivate him to establish the House of Wisdom and to patronize the contemporary sciences extensively. Similarly, ibn al-Haytham lived first in Iraq while the 'Abbasid caliph was under the control of the Shi'a-sympathetic Buyids and later under the explicitly Shi'ite Fatimid caliph in Egypt. Mu'tazilism was still popular but on the wane in his lifetime. When the Seljuk Turks replaced the Buyids in 1055, both Mu'tazilism and Shi'ism—characterized by their recognition of human reason as a way of knowing—lost the patronage of political leaders. In their stead, schools of "traditionalists" exerted command over the dominant Islamic epistemology. These figures repudiated rationality in favor of revelation, resulting in the waning status of reason-based knowledge in Islamic scholarship. Unlike other reason-based forms of knowledge, the field of mathematics was unaffected by the traditionalist attack on reason because, by the eleventh century, it was perceived as existing outside of human intellect. Though it was not considered a revelation-based way of knowing any more than it was at that point considered to be reason-based, Al-Ghazali's "Confessions" explicitly confirmed math's authority, granting it Islamic recognition. Therefore Muslim interest in mathematics could persist in spite of the political and theological changes that sank other rational processes.

Accordingly, the Islamicate world continued to produce significant mathematical achievements after 1100. Among the great names to live through or to follow al-Ghazali's legitimization of mathematics were 'Umar Khayyam and Naṣīr al-Dīn al-Ṭūsī. Although best known in the West as a poet, 'Umar Khayyam (1048-1131) is remembered in the modern Islamicate world as a mathematician. In keeping with the value for general proof developed by the end of the eleventh century, Khayyam was preoccupied with the creation of a general method

for solving cubic equations. He was the first to accomplish this in his *Treatise on Demonstrations of Problems of al-jabr [Algebra] and al-muqabala [Equations]* in which he used conic sections to solve whole categories of cubic equations.[161]

Nasir al-Din al-Tusi (1201-1274) was a Shi'ite Muslim who worked for the State of Assassins at the Alamut until it was conquered by the Mongols in 1256.[162] He was mentioned above for separating trigonometry from astronomy. Indeed, his contributions to mathematics included "the most comprehensive treatise on both spherical and plane trigonometry written in the Islamic world."[163] Al-Tusi also developed a mechanism, called the "Tusi Couple," in mathematical astronomy that solved the Ptolemaic problem of latitudinal motion. Later, the Tusi Couple appeared in Copernicus' famous heliocentric model, and it was also applied to the development of the steam engine, since his model can be used to translate a piston's linear motion into circular motion.[164]

Although I used al-Tusi, who lived predominantly in the 'Abbasid era but flourished briefly under the Mongols, to signal the end of my quantitative study of the 'Abbasid period, he did not signify the end of Islamic contributions to mathematics. Rather, significant mathematical achievements continued to come from the Islamicate world through the fifteenth century.[165] Ibn Khaldun's strong statement in favor of the soundness of geometry, quoted above, attests to mathematics' continued epistemological recognition into the fourteenth or fifteenth centuries at least.

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[161] For a translation of this text, see Omar Khayyam, *An Essay by the Uniquely Wise 'Abel Fath Omar Bin Al-Khayyam on Algebra and Equations:* Algebra wa Al-Muqabala, trans. Roshdi Khalil (Reading, UK: Garnet, 2008). For a briefer treatment of Khayyam's method and the significance of the book, see Katz, *A History of Mathematics*, 287-290.

[162] The Assassins were a subsect of Ismai'ilism, which was already a branch of Shi'ism. For more information about the Assassins and the Alamut, see the book *The Assassins: A Radical Sect in Islam* by Bernard Lewis.

[163] Katz, *A History of Mathematics*, 315.

[164] George Saliba, "Greek Astronomy and the Medieval Arabic Tradition," *American Scientist* 90 (2002): 366-367.

[165] Katz, *A History of Mathematics*, 267.

A core assumption underlying this thesis is the belief that mathematics (and other forms of knowledge) interacts with the cultural *milieu* in which it is performed at both the macro- and micro-levels. The study as a whole is considered macrohistory, since it investigates the changing place of math in Islamic intellectual history over several centuries and across the entire Islamicate world, but I found it necessary to support my conclusions with elements of microhistory. Closer study of individual scholars such as al-Kindi, ibn al-Haytham, and al-Ghazali not only textured the above history but provided crucial evidence for the claim that broad trends had real impact on the lives and thoughts of scholars. Because of the nature of the community in which the Islamicate scholastic elite acted, individual scholars also had the opportunity to affect the broad intellectual trends of their time; the men above were singled out precisely because their stature in that community ensured their lasting influence. Thus, it was evident that macro- and micro-level answers to the question I posed about the epistemological role of mathematics were necessarily intertwined. While traditionally macro- and microhistory are treated separately, in this context at least they are inseparable, and in fact, I would wish in future to support this macrohistory or others like it with even deeper microanalysis.

As it stands, the intertwined macro- and microhistory above has demonstrated that mathematics and Islam participated in a dynamic matrix of political, religious, social, and intellectual forces that contributed to how their relationship was defined. Islam's eventual recognition of mathematics as a producer of knowledge on par with revelation was by no means endemic to the religion, nor would mathematics necessarily attain equivalent standing in a very different *milieu*.

Appendix A: Quantitative Methodology

Data for the quantitative study were taken primarily from Boris Rosenfeld and Ekmeleddin Ihsanoğlu's *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th-19th c.)* (MAOS), which dramatically expands the 1900 survey *Mathematiker und Astronomen der Araber und ihre Werke* (MAA) by Heinrich Suter.[166]

Whereas Suter's bio-bibliography listed 500 individuals whose general dates were known, MAOS has 1423, given more or less in the order of their deaths. The study in the current paper began with the first intellectual whose life definitely intersected with the 'Abbasid period—Ja'far al-Sadiq (5), sixth of the twelve Imamiyya imams—and ended with Nasir al-Din al-Tusi (606), who is perhaps best known for working under the Mongols but did in fact live most of his life under the 'Abbasids. All entries were then coded by their religion and by whether the individual produced any mathematical works or was known to practice mathematics.

Religion was in some cases stated explicitly by Rosenfeld and Ihsanoğlu; in other cases, that information came from al-Nadim's bio-bibliographic *Index* (*Fihrist*). Many individuals were assumed to be Muslim because they knew Islamic inheritance or because they were listed as judges or jurists. While they were initially coded into Muslim, Christian, Jewish, Sabian, Zorotastrian, or other, the last five categories were taken together as "non-Muslims" for analysis.

An individual was determined to have produced mathematical works if he had at least one text to his name in MAOS that Rosenfeld and Ihsanoğlu had categorized as mathematics.[167]

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[166] For more information about the process of expanding Suter's work including a number of bio-bibliographic references used by Rosenfeld and Ihsanoğlu, see Boris Rosenfeld and Ekmeleddin Ihsanoğlu, *Mathematicians, Astronomers, and Other Scholars of Islamic Civilization and Their Works (7th-19th c.)*, 3-4.

[167] I trusted Rosenfeld and Ihsanoğlu to determine what texts constituted pure mathematics. While it can certainly be dangerous to try separating math and astronomy, as the authors of MAOS do, since trigonometry (firmly "math" to a modern mind) was so indispensable and inseparable from astronomy until Nasir al-Din al-Tusi pulled them apart in the mid-thirteenth century. However, I think there are certain natural, though by no means impermeable, lines between the subjects that Rosenfeld and Ihsanoğlu are careful to follow. Astrological and calendrical texts, preoccupied as they are with the interpretation of movements in the heavens for application on earth, are firmly

In some instances, no manuscript titles were known for listed individuals, but if Rosenfeld and Ihsanoğlu knew from their biographical sources that he was considered an "arithmetician," "geometer," or "mathematician," he was similarly coded as a practitioner of mathematics. This coding process was not intended to find *only* individuals who specialized in the study of pure mathematics, but rather to determine the extent to which mathematics was being performed by the Islamicate scholastic elite.[168]

In order to uncover patterns over time, I used the dates or time spans provided in *MAOS*—or, failing either of those, the general chronology of the list—to code individual scholars by time as well. I split the five hundred years of 'Abbasid rule into 25-year blocks (750, 775, etc.) and tallied the practitioners of mathematics who were active, to my best estimation, at each 25-year mark. I produced a time plot showing the fluctuations across time in the popularity of performing mathematics among scholars who could be identified as Muslim or non-Muslim (Table 2 and Figure 2).

While I had begun the coding process with 601 entries (604 people total), by the end I was left with 164 individuals, of whom 160 had practiced mathematics. Most of the rest were omitted for lacking a clearly determinable religion, but some were also eliminated for problematic dates. Of my remaining 120 mathematical practitioners, 97 were Muslim, but in order to compare Muslim and non-Muslim predilection on more even ground, they were considered proportionally through a chi-squared test of homogeneity.

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astronomical. Commentaries on Euclid, original texts on geometric figures, explanations of arithmetic, these are all distinctly mathematical. While I have neither the knowledge nor the resources to critique or to defend each of the compilers' classifications, I have found nothing objectionable in my own review of the titles they have categorized, and I assume the historians in question have done their work with due consideration. That consideration shows through in places where they elected *not* to make a distinction, when doing so would be inaccurate. In such cases, Rosenfeld and Ihsanoğlu wisely chose to categorize the text as both—usually both astronomical and mathematical—in which case I still counted it as performing mathematics.

[168] This scholastic elite represents only a small portion of the whole population living in the Islamicate world. However, most were given institutional or government positions by virtue of patronage, and they consequently would be the people with the opportunity and the interest to participate in an intellectual community.

Essentially, this hypothesis test was intended to reveal whether the proportions of Muslims and non-Muslims performing mathematics were similar across time in the Islamicate scholastic community. I took for my two populations the number of Muslims performing mathematics and the number of non-Muslims doing the same. The characteristic across which they were being compared was time.[169]

Table 2. Contingency Table for Chi-Squared Test for Homogeneity

Year	Muslims	Non-Muslims	Total Scholars Performing Math
750	3	0	3
775	3	0	3
800	2	0	2
825	8	0	8
850	10	1	11
875	9	2	11
900	15	4	19
925	11	6	17
950	8	5	13
975	11	6	17
1000	9	7	16
1025	9	3	12
1050	12	1	13
1075	9	0	9
1100	5	1	6
1125	10	0	10
1150	8	0	8
1175	9	1	10
1200	13	2	15
1225	4	1	5
1250	2	0	2
1275	0	1	1
Total Number of Instances at which Mathematics Performed	170	41	211

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[169] The foremost reason I chose to compare absolute numbers rather than proportions was the fact that individual scholars were often practicing mathematics across several time periods. As a result, individuals would be over-counted if each quantity was taken as a proportion of the total known population of Muslim (or non-Muslim) scholars.

To begin, the null hypothesis (H_0) claimed that the proportions of Muslim and non-Muslim scholars performing mathematics is the same at various times, and the alternative hypothesis (H_a) projected that the same proportions were unequal. We can note from Table 2 that the degrees of freedom for this test are 21, since $df = (number\ of\ rows - 1) \times (number\ of\ columns - 1) = 21 \times 1$. Electing a significance level of $\alpha = 0.1$, the x^2 critical value is 29.615, and $x^2 = \sum_{i=1}^{22} \frac{(Observed - Expected)^2}{Expected} = 32.646$. Since $x^2 = 32.646 > 29.615 = (x^2)^*$, we reject the null hypothesis. As a result, this test suggests a religious difference in the frequency of mathematical performance: The proportion of Muslims practicing mathematics during the 'Abbasid period was not similar to the proportion of non-Muslim intellectuals practicing mathematics.

Figure 2. Muslim and Non-Muslim Participating in Mathematics
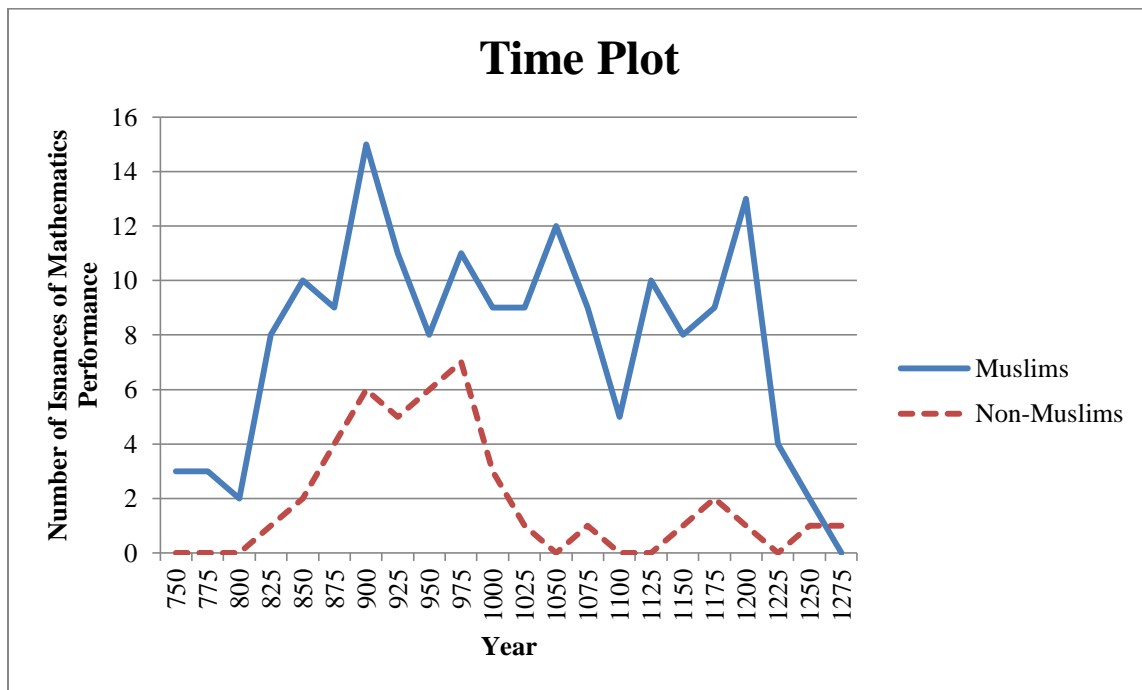
A graphical rendering of the same information provided additional information. Unfortunately, the sample sizes of non-Muslim scholars performing mathematics at most of the given times were too small to make hypothesis testing reliable; however, examining the graph

revealed trends. The steep slope of the solid blue line contrasts with the gradual slope of the dotted red line, suggesting that Islamic intellectuals took to mathematics much faster than non-Muslim scholars. While Islamic interest in math seems to have skyrocketed in the ninth century, reaching a pinnacle in 900, non-Muslims were more slowly attracted to it throughout both the ninth and tenth centuries. They did not reach the apex of their mathematical involvement until 975.[170] We also note that in the year 1100, a dip in mathematical performance occurred, but mathematics quickly rebounded to its normal levels. Aware that political and theological changes in the late eleventh century—culminating in 1100 in a widely-read tract by one of the most influential theological leaders of his time—challenged the idea that true knowledge could be created by virtue of human reason, math's brief decline in popularity c.1075 is not unexpected. Its subsequent capacity to rebound, however, was impressive. These patterns raised two questions that became central to this essay: First, why did mathematics begin to dominate the investigations of Muslim scholars over other forms of natural knowledge in the late ninth century? Second, why did this trend perpetuate to the end of the 'Abbasid period?

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[170] The almost complete disappearance of non-Muslim scholars after 1000 reflects the point at which Islam became a majority religion in most parts of the Islamicate world as more and more "People of the Book"—Christians, Jews, Zoroastrians, and Sabians, who were protected by the Qur'an from physical persecution—converted.

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