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Pascal and Fermat: Religion, Probability, and Other Mathematical Discoveries

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by

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Abstract

This final project primarily discusses how Blaise Pascal and Pierre de Fermat, two French seventeenth century mathematicians, founded the field of mathematical Probability and how this area continued to evolve after their contributions. Also included in this project is an analysis of how Pascal and Fermat were affected, or not, in their mathematical work by the widespread impact that the Catholic Church had on life in France during this time period. I further discuss two other central discoveries by these theorists: Pascal's Triangle and Fermat's Last Theorem. Lastly, the project analyzes how all of these aspects: the influence of the contemporary religion of the period on science and mathematics, Pascal's discoveries, and Fermat's different method of approach, impacted the trajectory of mathematical history.

Introduction

Blaise Pascal (1623 – 1662) and Pierre de Fermat (1601-1665) were two seventeenth century French mathematicians. During the end of the sixteenth century and throughout the seventeenth century, France was going through many transformations; of these the two that were most prevalent included the development of Catholic reform and the advancement of many new ideas, particularly in mathematics, science, and philosophy during the Scientific Revolution. Religion played an important role during this time period, and it affected almost everything from daily life to spiritual life in France during the time that Pascal and Fermat lived.

Blaise Pascal and Pierre de Fermat worked on their mathematical theories, despite religion being an often restrictive influence in France. These two were in communication via letters and founded the cornerstone of probability theory when they worked on what is known as *the problem of the points*. Pascal and Fermat were the founders of Probability Theory, however, there were many others who would shape the life of this field of mathematics. These two mathematicians managed to not only develop probability, but also established many other mathematical theories.

Separately, Pascal worked extensively in many fields of mathematics including the development of the still famous Pascal's Triangle. This Triangle has its cornerstone in probability, but it also arises in statistics, algebra, calculus, combinatorics, and more fields. Fermat, also separately, worked on many theorems, the most interesting and baffling is called Fermat's Last Theorem, which puzzled mathematicians for centuries after Fermat's death. Pascal's Triangle as well as the centuries' search for proof of Fermat's Last Theorem were seminal in mathematical development, and have gone on to be integral parts of many fields in today's mathematics.

Religion and the Scientific Revolution in France During the Seventeenth Century

The Catholic religion had gone through a huge upheaval after the Council of Trent, which took place in 1545-1563. This council began the transformation of thought in France within religion, culture, philosophy, academia, and more. Subsequent to this Council, the French Catholic Church tried to affect everything and anything that touched the lives of the people in France during the seventeenth century. To this end, the Church manipulated every day thoughts and feelings of French citizens so that they would see and hear Church teachings, or at least the message the Church wanted to portray, in most aspects of their everyday life. The Church thus began to deeply influence the schools, artwork, theater, literature, architecture, philosophers' thinking, and more.

During the 17^{th} Century, the Catholic Church as a whole was composed of at least three different factions. On one side was the French Catholic Church, on another, the Jesuits, and still another was the Jansenists. The Jesuits, or the Society of Jesus, were French Roman Catholic priests who thought that "salvation was to be obtained by a combination of one's own efforts and divine grace."[1] The Jansenists were a "sect that preached an extreme form of asceticism, sacrifice, and strict adherence to the Scriptures as the only path to salvation."[2] These three sects of religion impacted life in France greatly and in many different ways. Combined, these three religious schools used many different opportunities to try to control every-day life in France.

The most extensive influence the Church had during this time was their involvement in schools. Priests and nuns began to educate all French children, who would become adults and would then influence their own children, as explicitly planned. Henry Philips states that a "Christian education covers all the activities of the social space, and encourages a child in later life to be faithful…religion and civility are inseparable."[3] Students would learn and examine Scripture of the Church, and learn too the Church's definitions of right from wrong. Church and education became so entwined, that children "were to assemble at the school before attending divine office"[4] meaning that the children gathered at school before going to mass, on Sundays and during the school week, further entwining school and Church as one in their young minds. The Church was trying to entrench the views they wanted all French citizens to have. Jesuit and Jansenist schools had the same practices: they all knew that to truly achieve 'control' over their population was to make the Church and education one and the same.

During this time period, girls and boys would be taught separately, having different education in all other aspects except religion. It was a dramatically new practice for girls to be taught through a formal education, as beforehand, most girls were taught at home the skills they needed to become good housewives. However, the Church felt that it was important for girls to become formally educated because "they were the teachers in the home…the education of girls, and therefore of future wives and mothers, was for the Church the most effective way of extending its domain into domestic space."[5] By teaching girls, the Church could control what they were taught, and would undoubtedly emphasize Scripture and how to follow a life of the

Church, even after formal education ended. By schooling girls, the Church effectively was able to bring the importance of Church culture into the home. These were the girls who would one day either become nuns, furthering the cycle of religious thought, or mothers and housewives. By educating them in Scripture and all things holy, the Church was able to enter into many aspects of ordinary living during this time period. The girls learned Scripture, became mothers, and would help cement the relationship with the Church and their children, centering on religious attitudes and the philosophy that the Church felt that everyone should follow.

The Church also heavily influenced the written word, art, architecture, and new thought. After the Council of Trent, it was decreed that images in art and architecture had to adhere to the doctrines of faith. There couldn't be images anywhere that represented what the Church deemed to be 'falsehoods'. If an image was not exactly as described in the Scriptures, it would be destroyed or removed from view. An example of this is that "The Virgin could no longer be depicted as swooning before the cross and was to be depicted as standing, since no reference in the Gospels indicates any other position."[6] Here the Church wanted to make sure that all symbols, all artwork, etc… were viewed from Church's point of view, meaning that they closely followed Scripture. There could be no deviance from this – the Church's word was the arbiter of what people could see, and those who varied from it would get censured by the Church. There were people who argued with this initially, but most, after being indoctrinated in religious schools, went along with it. The written word was a bit less affected by the Church "given that books did not adorn buildings in the same way as images. It is interesting therefore to discover that many writers clearly desired not only to police themselves."[7] Policing the written word was much more difficult than policing public art and architecture as one had to have an actual copy of the book or pamphlet to see what was being written, and then if it wasn't something the Church wanted to be read, they'd then have to track down all copies and find ways to remove them. The Church definitely wanted to influence all books published during this time, but they did have a harder time of this than in some of the other areas of cultural or 'every-day life'. There were for example, many hidden printing presses that the Church was unable to control. Certainly in anything printed on the presses that they controlled, nothing that would go against Scripture was allowed, and thus there was considerable censorship.

Most scientists and mathematicians who developed their ideas during this time period "did not see themselves in opposition to Christian orthodoxy... [rather, they] accepted the new ideas which, they held, could be assimilated, if not by traditional teaching, at least by means of a reformulation of that teaching which did no harm to its fundamental principles."[8] This demonstrates that the new ideas that mathematicians and scientists developed needed to be framed in such a way so as not to directly contradict what the Church insisted upon as doctrine. In this way, mathematicians and scientists of this time period could explore and develop theories. In their own minds, they were not opposing the Church but going along with its teachings by adjusting the way the Church thought about the physical world, rather than about the spiritual one. Many, but certainly not all, mathematicians and scientists during this time period were reported to have thought of accommodation to, not outright rejection of, Church thought.[9] Therefore they believed they were applying the Church's teachings, and making new discoveries that might be argued to fit within existing doctrine.

It is interesting to note that the Church differed in its response to mathematics, which was seen as more abstract. For instance, Galileo Galilei (1564 – 1642), an Italian scientist, had an extremely tumultuous battle with the Church in regards to his findings and theories. Phillip states, in this regard that “Mathematics can also play a part in Christian apologetics as an argument in favour of providence: one can show that nature is a perfect order, mathematics constituting the science of that order.”[10] This seems to indicate that religious personae were more apt to feel unthreatened by mathematicians and mathematics because they can see that nature itself contains mathematical principles within it. Thus, it can be argued that these are principles that God built into the universe. On the other hand, Galileo had developed the theory that the planets revolved around the sun, instead of the Earth being the center of the universe. To the Church this was going against scripture: the earth and Man were viewed as being at the center of God’s creation. Mathematical ideas, however, were seen as commentary on a universal, God-given order that could help those studying Scripture.

In sum, the French Church did manage to influence most everyday situations that affected the people of France including their education, their worldview, their worship, the art and architecture to which they were exposed, and even the books they read. Certainly it was natural for children who were born after the Church established their formal schools to become heavily influenced by and then to spread its teachings. Their parents were already highly swayed by belief in Church Scripture, even though their indoctrination was not as thorough. Is it any wonder that people living in France during this time didn’t become involved in questioning these views, or become involved in learning about other viewpoints? They simply were not permitted exposure to other ways of thinking. Religion deeply influenced Blaise Pascal and Pierre de Fermat, as well, though not always in ways that one would assume given that they also lived during this time period.

The reason that Fermat's and Pascal's indoctrination into Church Doctrine was different than that of the typical French man or woman has to do with the following social trend. "Between 1600 and 1733 … the intellectual world of the educated elite changed more rapidly than at any time in previous history, and perhaps than at any time before the twentieth century…the only name we have for this great transformation is 'the Scientific Revolution.' "[11] The Scientific Revolution happened fast and all at once. There were many reasons for this, however "historians of science have often (and rightly) suggested that the key to the Scientific Revolution is 'the mathematization of nature'. "[12] Many new and different ways of thinking and of discovering the world were happening, in parallel with the Church's attempts at influencing the French populace. This seems paradoxical: however, the scientists who launched the discoveries that resulted in what was later called the "Scientific Revolution" actually considered themselves to be deeply religious.

As Wooton states: "Many of the scientists [or mathematicians] … were profoundly pious, but their religious faith was not what they had in common…but mathematics and, of course, a need for freedom of expression."[13] That is, that while scientists learned about the world from virtually the same, actively censored books, they shared a desire to use the language of mathematics, as it was being discovered, to decipher secrets of the natural world. They did not imagine in advance that these discoveries might be seen to threaten Church doctrine.

Johannes Kepler, a German mathematician and astronomer, once said, for example, that "if God was a mathematician (and who could doubt it), then one must expect to find a mathematical logic in the most unexpected places, for example in the organization of the solar system or in a snowflake."[14] Kepler discussed the plurality of worlds, as well as many other mathematicians/scientists of the time did. There were many mathematicians similar to Kepler, including Pascal, who were devout, who simply believed that God would want them to do it, as God created this world, and they were just exploring and figuring out how it worked

There were definitely mathematicians who feared the Church and what the Church would do if they published a piece that might seem to be going against their teachings. René Descartes, a French philosopher, mathematician, and scientist, for example, "was about to publish a series of treatises in natural science when he heard of the condemnation of Galileo and decided that that, since his philosophy supported Copernicanism, he dare not publish for fear of condemnation by the Catholic Church."[15] The Church, in regards to Galileo and Copernicanism (Heliocentrism which was that the Earth was not the center of the universe, but the sun was, with the Earth revolving around it) did condemn Galileo and Copernicus, insisting that Heliocentrism was not fact, but only a possibility. Church leaders urged Galileo to say that his finding was a hypothesis, and not a fact.[16] It was only when Galileo kept publishing about Heliocentrism,, after a decade of abstaining from publicizing his theory, that the Church banned his writings in 1633, and no other work by him could be published.

It seemed, that as long as a mathematician, or philosopher, or scientist did not openly and directly oppose the teachings of the Church, that the Church would let their work be published. However, if what was published was promulgated as fact, and did go against the Church's explicit teachings, there would be an Inquisition. If a scientist hypothesized something, or wrote about it as consistent with God's plans, such as saying God made the world and that this finding is a part of his creation, he Church would generally abstain from criticism or from finding the scientist to be heretical. It is interesting to note that the Scientific Revolution, which stemmed from establishing new facts and expanding knowledge, happened during the time when the Church was inarguably so involved in everyday life.

Pascal and Religion

Blaise Pascal (1623 - 1662), born in Clermont, France moved to Paris, after his mother died (some researchers think Pascal was three or four or seven years old when this move happened). Pascal was a frail child physically and so he was home-schooled. Because of his frailty, Étienne, his father, would not allow Pascal to study mathematics, as he didn't want studying such a complex field to tax his son[17]; but this only made Pascal more curious about it. Etienne, in fact, had many resources for study in his own library, and Pascal soon discovered on his own many of Euclid's theories about Geometry. Etienne finally gave his son a copy of *Euclid's Elements* after he realized that Pascal, at age 16, was already writing theorems for geometry! For example, at 16, Pascal had written a paper about the geometry of the cone that was admired by mathematicians much older than him[18]. After this, Étienne let Blaise have a free hand in studying, particularly in mathematics and science, realizing he was a prodigy.

As early as age 14, without Etienne's knowledge, Pascal had already begun to attend meetings conducted by Marin Mersenne, discussions about mathematics and science with well renowned mathematicians, scientists, and philosophers such as René Descartes and Thomas Hobbes along with many others. Mersenne also was in communication with Galileo, Fermat, and many others during this time. It is clear that Mersenne was a prime catalyst for change in the study of science and mathematics during the early Scientific Revolution time period. Influenced by his reading and by Mersnee's group of thinkers, Pascal also developed the first rudimentary calculator when he was in his teenage years. This calculator was mechanical and could only add, subtract, multiply, and divide numbers; but it was the first step in mathematical history in the direction of automatic machine calculation. Pascal also wanted to see if he could figure out how to make a calculator that would compute the square root of a number.[19] Unfortunately, it was very expensive and time-consuming to put together, so not many were made, and it was not mass-produced. One wonders what the advancement of mathematics would have been like if these rudimentary calculators could have been distributed more widely.

Pascal went through periods of his life where he researched and developed new mathematical ideas. He also went through phases of time, however, where he became a pious Jesuit, turning aside from math or science or knowledge about anything intellectual other than his belief and scripture. During the times Pascal explored mathematical propositions, he tended to put his religion aside, and vice versa. Thus, Pascal's life was very sectioned off, and religion and the study of mathematics did not intertwine for him at all. One could note that perhaps this is directly related to the fact that he was homeschooled, rather than schooled during his early years by the Church. Pascal, in fact, was not exposed to deeply religious influences until 1646, when he was 23 years old. His father fell on ice and broke his hip; and the nurses that took care of him were Jansenists.[20] Perhaps they awakened in him a yearning for a mother, so he was susceptible to their presence in his home, and to their teachings.

These nurses stayed after Pascal's father was healed, and taught Pascal the ways of Jansenists. Pascal, intrigued, discarded his thinking about mathematics and science at this point and delved deeply into the study of religion in general, and particularly of Jansenism. During this time period, Pascal thought and learned about aspects of this religion, he met with religious scholars and thought a good deal about Jansenism He did not write about religion, however, and it seemed that his conversion to religion was strictly an intellectual one, as opposed to a spiritual endeavor.

In 1650, at age 27, Pascal became very seriously ill; he had stomach problems and temporary paralysis[21], and his doctors told him that he needed to forget about the ways of Jansenism in order to get healthy. It was during this time period, that he would go back to mathematics and science and revolutionize the history of mathematics, specifically in the field of Probability.

Some years later, in November of 1654, very shortly after Pascal and Fermat finished writing to each other about the beginnings of Probability Theory, Pascal had another religious experience, which affected him greatly. Some sources claim that the new religious awareness resulted from a near- death carriage experience[22], while others suggest that was simply a mystical experience[23]. Sources do find that after it, he kept a piece of paper sewn in his jacket to remember the incident, and claiming “renunciation, total and sweet” of all his mathematical, scientific endeavors.[24] Throughout the rest of Pascal’s life, he divorced himself from the study of mathematics and science for good. When in 1660 Fermat wrote to Pascal concerning a problem he wanted to discuss of geometry, Pascal wrote back: “I can scarcely remember that there is such a thing as Geometry…I recognize Geometry to be so useless that I can find little difference between a man who is a geometrician and a clever craftsman.”[25] (It is believed that Pascal meant mathematicians in general here, not just geometricians.) Either way, it is very evident Pascal no longer pursued mathematics and didn’t find it necessary in his life, thinking it anti-thetical to the place that religion now held in his experience of life. In fact, Pascal eventually moved to the monastery of Port-Royal and studied religion even more and, this time, began to write about it. It was during this period that Pascal would craft his two most famous religious writings, including his most famous work, the *Pensées.*

Pascal wrote his first religious writings: *Provinciales* or Provincial Letters, which were eighteen letters (with a fragment of the a nineteenth found later) under the name Louis de Montalte. These letters were published between January 1956 and March 1657 in Paris. The letters addressed multiple subjects: some of them advocated Jansenism, some specifically supported Antoine Arnauld, others attacked Jesuits and casuistry. Casuistry "is the study of moral cases which provides the confessor with guidance on his direction of the penitent."[26] Arnauld wrote a book, *De la fréquente Communion* in 1643, condemning the Jesuits, and the outcry from this controversy meant that Arnauld was cast out of the college at Sorbonne in 1656, by Chancellor Séguier.[27] Ironically, Arnaulde was removed in part because Pascal brought the controversy over this thinker to life again in his letters. Pascal was driven 'underground' after these letters, and printers were asked not to print anymore of his letters; soon the printers were actually asked not to print anything without prior approval from the Church.[28]

The Provincial Letters discussed the fact that the Jesuits were too accommodating to someone who had sinned, that they just let him be absolved of his sins if he did penance, or they even sold or otherwise distributed papal indulgences. Pascal instead advocated for the Jansenists who followed Tradition, which said that a man should feel grief for his sins. "One of Pascal's underlying concerns in this whole dispute was that human reason had replaced the divine as authoritative."[29] At this point, Pascal had become a purist, who thought that the holy Scriptures were the texts to follow, that the divine and God were all-important, as opposed to the intellect. He felt that one should be guided by traditional belief and the Spirit, not by intellectually reasoning and deciding which parts of the Scriptures to follow and which parts to ignore. One can sense Pascal's thoughts were divorced fully from science and mathematics at this time. Pascal had completely compartmentalized his understanding of the world, and had begun simply living for the divine presence of God.

Blaise Pascal wrote his famous *Pensées* during this part of his lifetime. However, the *Pensées* were never meant to be seen, at least not in the form that is now known. The *Pensées* were "miscellaneous private jottings concerning God, religion, and many sorts of human behavior, analysed in support of Pascal's religious views. After Pascal's death, they were selected, arranged, edited, modified in the interest of intensifying their power of religion edification."[30] Pascal never intended them to be published, at least not in the form that is read and analyzed today. They were merely thoughts and musings of a man who wanted to explore and express his ideas about religion. This fact is important to note as one begins to read them. Further: take the *Pensées* with a grain of salt, as they were edited, and it may not be truly his words that one reads-especially because those who did publish his *Pensées* did so to enhance a particular variety of religious teachings.

In *Pensées* there is a small section titled: 'Mathematics/Intuition' which is especially interesting to read, as Pascal in between his bouts of religious piety was incredibly gifted at this kind of thought. Here, Pascal claimed that the mathematical mind is different from the intuitional mind. "Mathematicians who are only mathematicians therefore reason straightforwardly, providing only that everything is explained clearly in definitions and principles; otherwise they are unsound and intolerable."[31] Interestingly, Pascal is saying here, that for a pure mathematician, you do need to compartmentalize aspects of your life, and the mathematician in you has to adhere to certain rules and expectations that mathematics follows. It is interesting to read this, as in his correspondence with Fermat, as will be discussed later, the two had entirely different ways of thinking. One, Pascal, wrote as a pure mathematician, from a linear pursuit of principles, while the other, Fermat, was more intuitive. Fermat sensed that he just knew how math should be, and seemed to work backwards from there. It is interesting to note, that Pascal was a pure mathematician, whereas for Fermat, mathematics was a hobby, and not his full -time pursuit. Since the *Pensées* were written long after Pascal and Fermat's correspondence, one can wonder if this is the reason for Pascal's musings in this respect: possibly, Pascal didn't feel as if Fermat was a true mathematician, even though he helped solve *the problem of the points*. He could not be a true mathematician, in Pascal's view, because he was not a rigorously linear thinker, but relied too much on intuition- a very anti-Jansenist pursuit!

In another section of Pascal's *Pensées,* titled: 'A Letter to Further the Search for God,' Pascal noted how someone could have God in their lives. "…that God has established visible signs in the Church by which those who seek him sincerely should know him; and that he has nevertheless hidden them in such a way that he will only be perceived by those who seek him whole-heartedly."[32] Here Pascal was surely saying that only those who actively search, and search from within the Church, can actually find God and his teachings. That even though the Church is prevalent in everyday life in France during the time, someone needs to be actively searching for God to be able to see what and who God is; as well as having him be with you as you engage in other things in life. One wonders whether, for Pascal, this may have explained why Fermat never engaged in a deeply religious life and continued instead to explore mathematical reasoning: that Fermat never truly was exposed to the Church or to the kind of seeking that might occur there in order to have Pascal's experience of God. This will be discussed more later in the paper.

In the section titled: 'Foundations of Religion and Answers to Objections' in Pascal's *Pensées*; he wrote: "We understand nothing about God's work unless we take as the basis that he wanted to blind some and enlighten others."[33] One can take from this what one will; however, it seems possible that Pascal may have thought in the end that God picked certain people to fully become enlightened: that is, to experience spiritual life in a way that, for example, his old friend Fermat did not, but instead remained "blind". On the other hand, could he be saying also in retrospect that God had given him, Pascal, the ability to reason and to discover some facts about the world via understanding the language of mathematics, which was at the same time a language about God's work? With this being said, Pascal's work in mathematics could have been seen by him in the end as not going against God, but rather, *with* God's plan. Might this have helped Pascal in terms of feeling satisfied that he was, as a young man, working with the Church and with God; even if he wasn't particularly pious when he made most of his mathematical discoveries? Pascal "was even convinced that he could use his theories to justify a belief in God…religion was a game of infinite excitement and one worth playing."[34] This clearly indicates that it was definitely a possibility that for Pascal, his mathematical discoveries eventually were seen as going hand in hand with religion at least to some extent.

Keeping in mind that no one truly knows when Pascal wrote each musing of his *Pensées*, it is perhaps possible that he could have written some of them early on, while he was still working on his mathematical theories. That at first glance, his mathematical career seemed extremely compartmentalized from his religious life, and may have at times been so. However, it is clear that religion was important to Pascal throughout his life and mathematical career. As evidenced by his *Pensées*, it does seem as if, at some point, he felt he was going *with* God's plan as he developed his mathematical ideas, rather than against it – even if he wasn't completely and strictly religious during the times of his greatest mathematical achievements.

Fermat and Religion

Pierre de Fermat lived during the height of the Church's involvement in everyday life. Given that, how was it that Fermat managed to escape the Church's teachings and thoughts so completely? Fermat was born in August of 1601 in southwest France. His father was very wealthy, being a leather merchant, and perhaps as such, his father may have been exposed to wider ways of thinking than the typical Frenchman, which he may have passed along to his son. Details of Fermat's early schooling are unclear, especially before he left for college. Unless, like Pascal, who was tutored at home, Fermat most likely was taught at a local school in his hometown of Beaumont-de-Lomagne. Even if this were the case, his family's wealth might possibly have influenced him to be more free-thinking.

Fermat ended up serving in Parliament in Toulouse after college, and mathematics was more of a hobby that he explored in his free time rather than a central pursuit of his. He was very intuitively inclined, which most likely helped him to have the flashes of insight about mathematics that he did, even while thinking about it on a very part-time basis. Since he was in

Parliament, which was divided among the different sects of Catholicism at the time Fermat was involved, Fermat may have had conflicting thoughts about religion and how to be religious. It seems that he was not ever a religious man at all, though it is unclear whether at some point, he may have been conflicted about this. If we take Pascal's words in his *Pensées*, that one has to actively search for God in one's heart, one might see how Fermat could escape the dominance of Church culture in France. Fermat's adult life was in government, and in upholding the strict laws of France, so even though the Church had its hands in everyday life, the actual laws of the country were not very connected to religion. (They may not have contradicted Catholic doctrine, but they were not, on a day-to-day basis, so concerned with that doctrine.)

In fact, Fermat's position in Parliament was in "the Chamber of Petitions. If locals wanted to petition the King on any matter they first had to convince Fermat or one of his associates of the importance of their request"[35] . In fact, Fermat "had no political ambition, and did his best to avoid the rough and tumble of parliament. Instead he devoted all his spare energy to mathematics and, when not sentencing priests to be burnt at the stake, Fermat dedicated himself to his hobby."[36] From Singh's remarks it seems that Fermat was not even remotely religious: if part of his job involved condemning priests! However, that doesn't necessarily follow, if the priests were accused of heresy, which they may well have been. Or perhaps this exposure to priests who had "erred" made Fermat skeptical of the Church? His turn toward mathematics is perhaps made clearer when Singh states that "judges in seventeenth-century France were discouraged from socializing on the grounds that friends and acquaintances might one day be called before the court. Fraternizing with the locals would only lead to favoritism."[37] Fermat may have chosen to work at mathematical puzzles, then, as something he could do on his own, as he was denied the society of many people, except via letters about his extra-judicial pursuits.

In sum, no record has been discovered on what if anything Fermat felt personally towards the Church. It is clear that most of his free time was spent on studying and doing mathematics.

Pascal and Fermat: Probability and Beyond

Pascal and Fermat wrote letters to each other from July – September of 1654 discussing *the problem of the points*. Before discussing what Pascal and Fermat accomplished on this problem, it should be noted that this problem had a long history – and is important in discussing why Pascal and Fermat were the first ones to solve it. It has long been recorded that gambling has been a part of every major society in history, from the ancient Egyptians, ancient Greeks, ancient Romans, all the way up to Pascal and Fermat's time in seventeenth century France. However, "up to the time of the Renaissance, people perceived the future as little more than a matter of luck or the result of random variations"[38] and no one thought that the games of chance they played could be predicted or contain a specific number of possibilities. In 1494, Luca Paccioli became the first person to try his hand at predicting an event. Paccioli wrote *summa de arithmetic*, *geometria et proportianalità*, and proposed a problem: "A and B are playing a fair game of *balla*. They agree to continue until one has won six rounds. The game actually stops when A has five and B three. How should the stakes be divided?"[39] This problem, now known as "the problem of the points" started the beginning of thinking of risk analysis, fair division (which is another branch of mathematics that Pascal never even dreamt of but may possibly be attributed to the answer to this type of problem), and led to huge debate on how to be able divide money when a game gets interrupted.

In the sixteenth century, Girolamo Cardano wrote a mathematical treatise called *Liber de Ludo Aleae*, which explored games of chance, exemplified by dice-throwing. While never using the word probability, Cardano did calculate the chances of throwing particular numbers during random throws of dice. He calculated the probability of throwing a given number when rolling one die, as well as with two dice. Cardano used the word circuit to represent what, today, is known as the total number of possible outcomes. Cardano knew that these circuits would add up to more than just the eleven different types of outcomes of rolling two dice, and he was aware that in fact there were 36 different possibilities.[40] The mathematician wrote this treatise in 1525, and edited it in 1565, however it was never published until after his death in 1663.[41] The treatise was a major step forward toward the beginning of probability theory, but the central question that was posed by Paccoli was not answered. Mathematicians could now grasp the concept of different outcomes, but did not yet know how to approach the problem of declaring a winner if a game remained unfinished.

After Pascal's first period of religious immersion, in approximately 1650, he became a regular visitor to gambling places. It was here that Pascal met the Chevalier de Méré. The Chevalier had a hobby of figuring out the odds in the casinos. He mentioned the *problem of the points* to Pascal, which interested him a good deal. However, Pascal didn't think he had the resources to attempt to figure out this problem on his own. In 1654, Marin Mersenne put Pascal in touch with Fermat and thus started a correspondence that would revolutionize the subject of probability.

The complete known correspondence between Pascal and Fermat can be found in one of the appendices in *Games, Gods and Gambling*, by F.N. David. Snippets of this correspondence can be found in abridged versions in many other books about the history of probability and risk taking, but David's book contains the complete correspondence that has been found and translated into English.[42] It is the words from David's appendices that will be discussed here.

The work that Pascal and Fermat accomplished was only done via letters. They never met face to face before, during, and after their famous correspondence. The first of their letters have never been recovered, but due to Fermat's reply to Pascal, it is clear that Pascal introduced the idea of a simplified *problem of the points,* which then lead to the actual problem he wanted to discuss, which related to outcomes in games of chance that were interrupted. First, Pascal and Fermat tackled more basic challenges. One question that they considered was as follows: if a player threw a six-sided die eight times, attempting to throw a 6, but hadn't gotten one after 3 rolls of the die, and if this person then skipped his fourth throw, how much money should he make out of the stake (or pot, or ante) to which both players had originally contributed if they end the game there, after the third roll, but imagining they rolled all eight times?

The second problem that Pascal and Fermat tackled concerns two players playing a game of three points, in which each person has staked 32 pistoles (a common term for money). How should the sum of the pistols (64 all together) be divided, if the game had to be broken off before either player had achieved three points? They then also extrapolated this question to the case in which there might be three players instead of two. The two thinkers tried different ways to solve the problems. Fermat used pure algebra, listing all the different outcomes that could happen, while Pascal used a geometric premise, using the recursive method of reasoning.[43]

The first problem, concerning someone throwing a six-sided die eight times, was fairly easily solved by the two mathematicians. Fermat's method consisted of counting all possible outcomes and picking which ones would work in this particular example. This method is now known as the theoretical probability of what *could* happen on the next throw, from which is derived the probability of that *actually* happening. The mathematicians both agreed that after three unsuccessful attempts to roll a 6, the gambler should receive $\frac{125}{1296}$ of the total pot of money; as there were four rolls left (5th, 6th, 7th, and 8th roll after skipping the 4th) and so $6^4 = 1296$. Since the subject didn't get a 6 in the first three rolls, this could have happened in $5^3 = 125$ ways; thus the $\frac{125}{1296}$ of the pot. Fermat figured this out by using algebra on each consecutive roll, deducing what the pot amount should be. Pascal, however, figured this out recursively and with help from what is now known as Pascal's Triangle. He wanted to show Fermat a more universal way to calculate the answer to any type of similar problem.[44]

The *problem of points* was now discussed directly in the mathematicians' correspondence. Pascal summarized how the winnings should be distributed if two players staked 32 pistoles each (64 total) in a dice game in which the winner achieves three points first. He deduced how the pistoles should be divided if the two players had to suspend the game at any given point in its progress. If the first player has 2 points and the second player has 1 point, Pascal calculated that the first person should receive 48 pistoles and the second player, 16 pistoles. If the first player has 2 points and the second player has 0 points, Pascal claimed recursively that if they were to play another game, and the first player won that round that – he should be awarded all 64 pistoles. On the other hand, if the second player wins the second round, it would be the same situation as above: that player would be awarded 48 pistoles.

However, Pascal deduced, if the second round is never played (in which each of them is likely to win as the other), the first player should definitely keep the 48 pistoles plus one-half of the remaining stakes. In other words, he would receive his initial 48 points and then half of the remaining sixteen pistols, or eight more: $64 - 48 = 16; \frac{16}{2} = 8$. The second player would receive the remaining part of the stakes, or 8 pistoles.

Pascal continued to develop this line of reasoning. Should the first player have 1 point and the second player 0 points, and if they were to play another game, which the first gambler won, the situation would remain just the same as in the problem stated above. That is with the players divided at 2 points (first player) to 0 points (second player), they should subtract the 32 pistoles which the first player has already won from the overall stakes of 56 pistoles, which would equal 24 pistoles remaining in the stakes pot. This remainder would then be divided evenly. Therefore, the first player would get $32 + 12 = 44$ pistoles, and the second player would receive 20 pistoles.[45]

Pascal's theory is sound and comprehensive in deducing the ways in which player One might win, using the recursive method (that is, figuring out what could happen in future games and referencing it in the current situation). The *problem of the points* is most often viewed as the above-described situation, with two players who stop before finishing their agreed upon three points, using the kind of calculation that Pascal employed. However, in his correspondence, as seen through David's appendix, Pascal went further, explaining what would happen if the gamblers decided to play to more than just three points, but to any number of games and points.

In Pascal's letter addressing *the problem of the points*, he then considered the considerably more complicated example of 8 games, with the first one played to 5 points. For this deduction, Pascal used the device which is now termed Pascal's Triangle, see below in the section, Pascal's Triangle. Pascal used letters A, B, C, D, E, F, G, H (signifying the 8 possible games), and thus Row 8 of Pascal's Triangle, to help to analyze this question, and then calculated the combinations of the 8 letters to prove his point. As Pascal wrote:

> The total 128 pistoles equals
>
> 35, half the combinations of 4 letters
>
> + 56 combinations of 5 letters
>
> + 28 combinations of 6 letters
>
> + 8 combinations of 7 letters
>
> +1 combination of 8 letters

[One should note that Pascal found these 1, 8, 28, 56, as the first four terms in the 8^{th} row of Pascal's triangle, and the 35 as half of the fifth term, 70, of the 8^{th} row, and that 128 is what each of the two players would stake, totaling 256, which is the sum of the 8^{th} row of Pascal's Triangle]

> … Thus if I have won the first game out of 5, 35/128 of my opponent's stake is due to me: that is to say, if he has staked 128 pistoles, I take 35 and leave him the remainder, 93.[46]

Pascal didn't write out each item of the possible remaining stakes, as he was confident that Fermat could follow the logic presented in his letter. Pascal did use the 8^{th} row of the Triangle: $1, 8, 28, 56, 70, 56, 28, 8, 1$ (because that is the most possible games they could play), and with these numbers, which sum to 256, suggests that each player put in half of that, or 128 pistols each into the stakes pot. Since the overall game could be won in 5 games, or 1 more game than half; and since Pascal arbitrarily appointed player One to win the first game, he should be able to win at least the 128 pistoles he put into the pot.

Then, Pascal contended that Player One should earn one half of the 5th element in the row that denotes the Second Player's entry into the pot (70, as we start with element 0 in this triangle), which is 35. So out of the 128 pistoles that the Second Player put into the pot, he would earn 35 of those, meaning that Player One should receive (after playing and only wining one game out of the 8) his own stake, $128 + 35 = 163$ pistoles, while Player Two receives 93 pistoles.

In other letters between Pascal and Fermat, they went on to discuss what would happen if there were three players, instead of two. It seems that there were more letters that may have gotten lost, as Pascal did achieve the correct answer using his recursive formula along with his triangle, but he was trying to understand Fermat's method of counting every possible outcome as well. Likewise, Fermat tried to understand Pascal's method, but was unsuccessful, so continued with his method of counting each outcome possible and then figuring out which ones made sense. Pascal responded to a lost Fermat letter claiming "I must tell you that the solution of the problem of points for two players based on combinations [here he means counting all the possible outcomes, not combinations as known in the 21st century] is very accurate and true, but if there are more than two players it will not always be correct."[47]

Pascal continued to explain why, suggesting that if there were three players in the middle of a tournament, and Player One needs 1 game to win, Player Two needs 2 games to win, and

Player Three needs 2 games to win as well; the following outcome will happen playing at the most, three more games; then the 27 different possible outcomes are thus, being read downward[48]:

aaa	aaa	aaa	bbb	bbb	bbb	ccc	ccc	ccc
aaa	bbb	ccc	aaa	bbb	ccc	aaa	bbb	ccc
abc	abc	abc	abc	abc	abc	abc	abc	abc
111	111	111	111	1	1	111	1	1
	2		2	222	2		2	
		3			3	3	3	333

with a, b, c being the three different people; and the numbers after the horizontal line being who would win (player 1, 2, or 3). However, as Pascal pointed out that it only matters when one of the players gets the amount of games needed to win, unlike what Fermat had said, which is lost, that there are 19, 7, 7 possibilities for each player respectively to win. Pascal, correctly, noted that the game would stop once a player has actually won, so therefore the correct amount is 17, 5, 5; as reading it downwards only the player who got to the required number of games first would actually win; the lines that may have more than one number written under it, should only count for the player that would achieve it first, when looked at it from top to bottom. In reply, Fermat did end up agreeing with Pascal about the 17, 5, 5.[49]

Some mathematicians feel that Pascal made a greater contribution toward solving this problem than did Fermat, thus jump-starting the eventual science of probability. However, Fermat played an important role in that he and Pascal corresponded actively and tried to figure out each other’s methods. The process of communicating back and forth helped both thinkers to arrive at their eventual solutions. The idea of collaborating with others is essential in the field of mathematics, even today, in order to truly achieve better results. Further, both Pascal and Fermat came up together with the "realization that at most $(a + b - 1)$ more tosses will settle the game … [when playing] 2^{a+b-1} possible games.[50] Fermat's method of numerating every possible outcome forms the basis of probability, and most students learning in the 21st century start out by following exactly this course of reasoning. And it is true, that every probability question can be answered this way. However, as the numbers get larger and larger, so do the number of outcomes, so eventually it is not time efficient to use Fermat's method. Instead, we use a method that combines Pascal's recursive formula along with Pascal's Triangle, in addition to added methods that have been deduced since Pascal and Fermat's correspondence.

Fermat's letter in response to this one of Pascal's seems to have been lost, however Fermat wrote a letter soon after their correspondence about the two player *problem of the points* to Pierre de Carcavi, another mathematician during the 17th century, saying "I was overjoyed to have had the same thoughts as those of M.Pascal, for I greatly admire his genius and I believe him to be capable of solving any problem he attempts … If you combine your work [that Fermat helped him with] with his, everything will succeed…and thus be able to publish."[51] It seems clear, here, that Fermat not only agreed with Pascal's reasoning, but wanted to try to publish some of the work they were producing. Unfortunately, this never occurred [52] and it is to be noted how much could have changed if their work had been published at that time to a mass audience!

Instead, it was only through reading the correspondence of Fermat and Pascal and through the knowledge of some of their discussions, that Carcavi and Mersenne eventually advanced the study of probability. This is why, while Pascal and Fermat are considered the founders of probability theory, they did not discover or delineate all of the basic theorems. It was their effort, however, viewed through this correspondence, that provided the push for mathematicians to create this new subject within mathematics.

What Pascal and Fermat created together was a system with which to deduce what could happen, before it happens. “Pascal and Fermat held the key to a systematic method for calculating the probabilities of future events. Even though they did not turn it all the way, they inserted the key into the lock. The significance of their pioneering work for business management, for risk management, and, … for insurance was to be seized upon by others.”[53] Pascal and Fermat had no idea that what they had discussed would be the foundation of a separate mathematical subject called Probability. They also could not have foreseen that during the years that followed their famous correspondence, several thinkers would use their work to develop a variety of practical applications. Thus, probability theory would become foundational to the study of Statistics (along with the application of Pascal’s triangle), to the development of actuarial tables, and to further predictions of gaming, etc. I will now briefly consider some of the major, historically immediate applications of their work in the years that followed the Pascal-Fermat correspondence in order to demonstrate its importance.

John Graunt (1620-1674), who lived in London, and was a tradesman who used empirical (meaning experimental) probabilities, was most likely the first Englishman to actually use probabilities to help his trade. He created empirical probabilities relating to the mortality rates along with the fact that he started with plotting out who was christened over the course from 1592 - 1665[54]. Graunt knew his totals and percentages were not exact, as there wasn’t a nationwide death certificate practice during many of those years. Graunt took his tables and calculated the probability of living to a certain age. He had to take into consideration the years the Plague was raging, and he knew that his probabilities were not exact, but they were the first time someone had tried to keep track of who was alive, dead, and how long they might live. According to Graunt 's work "… gave impetus to the collection of vital statistics, to life-tables, to insurance; … that the empirical approach of the English to probability was not through the gaming table but through the raw material of experience."[55] Graunt's approach ultimately led to the idea of a nationwide census as well as to the keeping of records concerning mortality rates and the idea of an expected life age. This would then also end up helping car insurance and life insurance companies tabulate what they should charge for their services.

James Bernoulli (1654 – 1705), a Swiss mathematician, sometimes was known as Jacob, took Pascal and Fermat's correspondence further and developed further aspects of probability. His most famous work on this subject was the *Ars Conjectandi*, which was written in the later part of his life, but not published until 1713, after Bernoulli's death.[56] In *Ars Conjectandi*, Bernoulli generalizes the gambling problem to where "A has *m* units to stake, B has *n* and the chances of A:B winning a game are a:b. He showed that A's chance of winning a set of games by encompassing B's ruin is $\frac{a^n(a^m - b^m)}{a^{m+n} - b^{m+n}}$ and B's similar chance is $\frac{b^m(a^n - b^n)}{a^{m+n} - b^{m+n}}$."[57] Here, Bernoulli took what Pascal and Fermat had done on the gambling problem and generalized it to any game, as long as one knew the chances (aka probabilities) of winning. This was a big advance in probability as it could apply to many different types of games and scenarios. Bernoulli is also given credit for the development of the Binomial Distribution in statistics, which is often called the Bernoulli trials. More of this will be discussed in relationship to Pascal's Triangle in the next section.

Abraham de Moivre (1667 – 1754), a French born mathematician, moved to England sometime in the 1680's (researchers are unclear about the exact date). In 1718, de Moivre's first edition of *Doctrine of Chances* was published.[58] De Moivre was thought to have been a teacher, as this book was very well written and was directed for the masses to understand. The volume included the definition of probability, instructions about how to add probabilities, what expected value was, and about what were the probability of independent and dependent events, as well as offering instruction about conditional probabilities. De Moivre claimed that, when considering the chances of two people, A and B, playing a game of either success or failure, that the probability of A winning is: $\frac{(a+b)^n - b\hat{}n}{(a+b)^n}$ and that of B winning is: $\frac{b^n}{(a+b)^n}$ [59] . The number of times a winning event occurs is *n*, *a* is the probability that the player will win; while *b* is the probability that the player will fail.

David sumarizes de Moivre's work as follows: "The first edition of the *Doctrine of Chances,* is written by a man who was already superior to Montmort and the Bernoullis in his mathematical powers, and who, when he came to maturity, was to produce in his third edition the first modern book on probability theory."[60] What de Moivre did was to create an actual and deep resource about probability and its existing theories, but in addition he added his own new ideas that revolutionized the modern way of thinking in regards to this field.

In 1812 Pierre-Simon de Laplace (1749-1827), a French mathematician and astronomer, wrote his *Théorie analytique des probabilités*. This treatise had two parts. The first dealt with the philosophy of probabilities and with specific problems; the second part included statistical methods and applications including games of chance and tables of mortality.[61] This treatise pulled a considerable amount of information together, with de Laplace also adding his own mathematical questions to the historical ones that he included.

There have been more advances in the theory of probability and statistics since these men developed their contributions, but these men: Pascal, Fermat, Graunt, Bernoulli, de Moivre, and de Laplace were the ones who revolutionized this topic, and who developed it to the field that it largely is today.

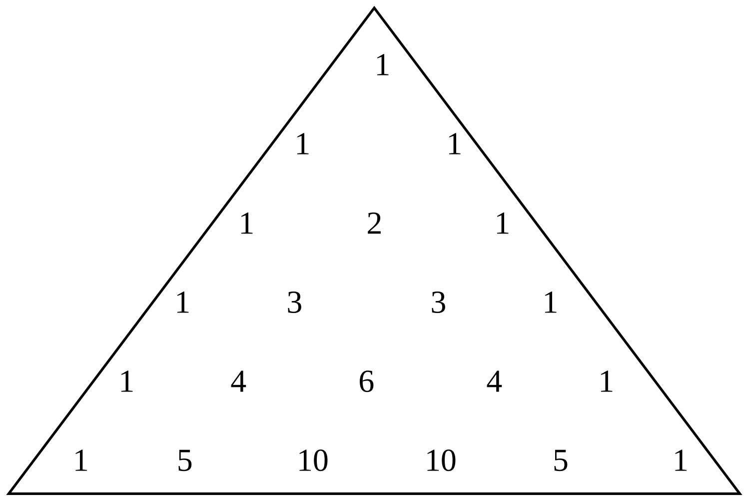
Pascal's Triangle

Along with being a co-founder of Probability, one of Pascal's other major mathematical accomplishments was the research and development of what is now known as Pascal's Triangle. Pascal's *Traité du triangle arithmétique*, was published in 1665, but he worked on it during 1663-1664. This triangle, which is also known as a Figurate Triangle, a Combinatorial Triangle, or a Binomial Triangle – depending on how a given mathematician was using it, has a long history. Arabic mathematicians knew about the Triangle in the 13th century, and Chinese mathematicians clearly used it in some form during the early 14th century. Mathematicians knew about some of the concepts or ideas that the Triangle helps to develop, such as triangular numbers, the Binomial Theorem, and combinatorics since Ancient Egypt, circa 300 BCE.

The first person to credit Pascal with the name of this Triangle was another French mathematician, Pierre Rémond de Montmort in 1708. His attribution was most likely the reason why Pascal is credited with the original and definitive work with this Triangle, since Montmort referred appreciatively in his writings to Pascal's *Traité du triangle arithmétique.*[62] *The Triate* was first mathematical work that analyzed the Triangle to the depth that Pascal did, as he pulled together many different connections. Other mathematicians before him had used the Triangle in reference. Below is an image of how Pascal viewed the Triangle: he named the top row, Row 0, or Column 0. Note that the numbers composing the triangle can continue forever.

1	1	1	1	1	1
1	2	3	4	5	
1	3	6	10		
1	4	10			
1	5				
1					

[Pascal's View]



[Today's Usual View]

Pascal's *Traité du triangle arithmétique* had two parts to it, a definition with corollaries and a problem in the first half; and in the second half had four sections: figurate numbers, combinations, dividing the stake in games of chance, and finding the powers of binomial expressions.[63] The definition part shows the triangle and how to construct each subsequent row (add the two numbers above it). The definition also shows the formula to find the figurative numbers: $f_k^l = f_k^{l-1} + f_{k-1}^l$, which Pascal called "triangulaires [and] pyramidaux,"[64] which mean the triangular numbers, and the pyramidal numbers. The definition also includes Pascal's first corollary: $f_0^1 = 1; where\ l = 2,3,4, \dots ; k = 1,2,3, \dots$ where the number is in the l^{th} column and the $(k+1)^{\text{th}}$ row.[65] Here Pascal thought that it is necessary to designate f_0^1, today, mathematicians just consider that part of the definition. The nineteen corollaries that Pascal included, are more formulas, to discover what we, today, take for granted. Some of which include what each row sums up to, what each column adds up to, the idea of symmetry, and others that all culminate in a formula that mathematician Cardano actually knew in the sixteenth century.

Today, Pascal's Triangle is used by students of mathematics to answer questions about combinations (${}_nC_r \ or \ \binom{n}{r}$), the binomial expansion, and probability. The easiest of these three to connect, is how the triangle relates to combinations. Starting with row 0 (or *n*), and going down; and starting with element 0 (or *r*) and going to the right; you can find any ${}_nC_r \ or \ \binom{n}{r}$, without having to use the more formal definition: ${}_nC_r = \frac{n!}{(n-r)!r!}$ or using a calculator. For instance, if you have ${}_6C_2$, by definition this would equal: $\frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6\bullet5\bullet4\bullet3\bullet2\bullet1}{4\bullet3\bullet2\bullet1\bullet2\bullet1} = \frac{6\bullet5\bullet\cancel{4\bullet3\bullet2\bullet1}}{\cancel{4\bullet3\bullet2\bullet1}\bullet2\bullet1} = \frac{6\bullet5}{2\bullet1} = \frac{30}{2} = 15$. However, by using the 6th row of Pascal's Triangle: $1,\ 6,\ 15,\ 20, 15,\ 6,\ 1$; and looking at element 2, remembering you start with element 0 on the left; we get the same answer: $15!$ As you can see, if the *n* and *r* gets very big, using the formal definition $\frac{n!}{(n-r)!r!}$, may be a bit time consuming, but if one has the Pascal's Triangle to look at, figuring out a combination problem takes no calculations at all. Since to write down Pascal's Triangle it involves only adding, instead of multiplying and dividing like the definition, one could say it's much easier to produce the triangle.

The second easiest way that Pascal's Triangle connects to other mathematics is through the binomial expansion, which Pascal discovered while in the midst of discussing *the problem of the points* with Fermat. The Binomial Theorem states that:

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r-1} ab^{n-1} + \binom{n}{r} b^n$$

As one can see, the Binomial Theorem directly takes into account about knowing combinations. Most students, particularly in Algebra I and Algebra II simply use this the 2nd or maybe 3rd power; and simply multiply the binomials together to get the same answer, although knowing the Binomial Theorem and using Pascal's Triangle, so you don't have to actually compute the coefficients of each term, will certainly be less time consuming.

For example: $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$, and $(a+b)^3 = (a+b)(a+b)(a+b) = (a^2 + 2ab + b^2)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$; after this it just simply is much easier to use the Binomial Theorem and utilizing Pascal's Triangle to know what the combinations are. So for example to expand: $(a+b)^9$, you would look at the 9th row of the Triangle: $1,\ 9,\ 36,\ 84,\ 126,\ 126,\ 84,\ 36,\ 9,\ 1$ to use for the coefficients; and the simply put, the exponents of the first term, *a*, goes from *n* (9) downwards; and the exponents of the second term, *b*, goes from 0 or nothing upwards to *n* (9). So therefore we get:

$$a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9$$

If one would have to multiply all of those binomials, it would take even the most serious mathematicians probably about five minutes or more to do, meaning the average student of math would likely take upwards of fifteen or twenty minutes to compute and combine like terms. However, with the binomial theorem and looking at the correct line of Pascal's Triangle, it took less than a minute.

As for probability, one can find the answer to many different questions using Pascal's Triangle. The important factor in using the Triangle is that either outcome which can happen needs to be equally likely; meaning either has a 50-50% chance of each happening. With real events, there are usually outside circumstances that become influencing variables (for example, the relative strength of the players), but at least with Pascal's Triangle, we can figure out the probability of an event happening without such outside influences.

Let's take an example that involves sports, as most people can relate to athletic games. What is the "probability that your [baseball] team will win the World Series after it has lost the first game?"[66] Some knowledge of the World Series is needed: The World Series is played until four out of seven games have been won. With this question, however, we know we will be considering how a team might win four games out of the remaining six, since the other team has already won once. This can happen in many different ways, and it would take some time to write out all the different means by which these two teams can proceed until four games have been won by one of them. So instead, we use Pascal's Triangle. Since there are six remaining games, there are $2^6 = 64$ different ways the outcomes can happen, and so we want to look at Row 6 $(1,\ 6,\ 15,\ 20,\ 15,\ 6,\ 1)$.

The 1 on the left means that there is only one way if one team wins the next four games, and the opposing team wins zero. The next number, 6, corresponds to the fact that there are six different outcomes that will have the team you're rooting for win the World Series with the other team winning one other game. The following number, 15, represents the fact that there are 15 possible ways that team you are rooting might win the World Series if the other team has won two other games, etc… However, since this is a real- life problem, and only the team that gets to four wins first claims the championship, the next numbers would correspond to the opposing team winning three other games, which would mean the team you're rooting for would lose the

World Series. At this point, then, one can stop counting. In sum the ways in which the team can win are as follows: : $1 + 6 + 15 = 22$ ways. So therefore there are $\frac{22}{64}$ ways your team can win the World Series; since there were $2^6 = 64$ possible outcomes that could happen. Therefore, there is a $\frac{22}{64} = 0.34375 = 34.375\%$ chance of the team you are rooting for will win the World Series if the opposing team has already won one game. This analysis assumes that either team is equally likely to win each game. In real-life there are many other factors involved, such as when one team in winning, it boosts their confidence which may increase their chances of winning; other factors involved include weather, home-field advantage, injuries, etc…

This row of Pascal's Triangle represents the pure mathematics of what could happen, though in real life, a team would not play all remaining six games, but stop after getting the 4 wins they need.[67] However "as Pascal remarked in his correspondence with Fermat, the mathematical laws must dominate the wishes of the players themselves, who are only abstractions of a general principle"[68] meaning that math is pure, and all possible outcomes that could happen should be taken into consideration, not just what could happen or when one should stop for one particular problem. Also not taken into consideration are the many variables a sports game can include- such as players' health, previous wins and losses, if they are home versus away. However, the pure math of this is important and will offer someone at least an idea of the probabilities involved. An example like this one, while not wholly based in pure mathematics, makes it easier for a non-mathematician to understand what is involved in trying to predict the outcome of a series of events.

Pascal's Triangle had long lasting effects on other fields of mathematics as well. John Wallis's (an English mathematician) work, based off of Pascal's Triangle in the later part of the seventeenth century, led directly to Isaac Newton's (an English mathematician (1643-1727)) generalized binomial theorem (that included fractional and negative exponents).[69] Wallis used Pascal's Triangle to figure out a table for inverse figurate numbers using intermediate values of Pascal's Triangle.[70] This work, plus the then developed idea of an anti-derivative lead Newton to his generalized binomial theorem, which was one of the advancements of the binomial theorem that Pascal knew. Pascal only knew the Binomial Theorem for only positive exponents. Furthermore, Newton's use of the generalized binomial theorem helped him discover what is now known as Calculus. This means that Pascal's work in probability had a direct relationship to the foundation of Calculus, a seemingly unrelated branch of mathematics!

Pascal's Triangle, furthermore, is useful, now, for a statistical distribution, called the binomial distribution, also known as the Bernoulli Trial. This distribution is calculated by: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0,1,2,3, \ldots, n$ where n is the number of trials, where p is the probability of success, and X is the number of successes we have; it is called the binomial distribution because either it can succeed or fail – thus two – thus a binomial answer [71] and is a key distribution in statistical analysis. The binomial distribution is used in many true/false polls, surveys, etc… and is used to figure out what the significance is of something happening for a null hypothesis test.

One can also use the binomial distribution as well as Pascal's Triangle to answer a question such as 'a coin has a 60% chance of coming up heads. If you flip it 10 times, what are the chances that exactly 8 of the flips will be heads?'. What you would do here would be plug it into the binomial distribution; where r = 8, n = 10, p = 0.60, and 1-p = 0.40 → $\binom{10}{8} \bullet 0.6^8 \bullet 0.4^2 = 0.120932352$, which can be approximated to 12%. This means that about 12% of the time, when someone flips a coin times; with the known chance it will be heads 60% of the time, you will get exactly 8 of the 10 flips to come up heads.

Pascal's Triangle has significant impact on the history of mathematics, not only in probability, but with statistics and even calculus. It is clear that Blaise Pascal did incredible and long-lasting work with his *Traité du triangle arithmétique*. He didn't know that his work would be as historically important as it came to be which goes to show that ideas should be published, even if one doesn't think anything will come of them.

Fermat's Last Theorem

Pierre de Fermat worked on many historic mathematic discoveries, however, other than his help in founding Probability, Fermat is most known for something called 'Fermat's Last Theorem'. Before describing Fermat's interest in this, it should be noted that his theorem was preceded by initial work conducted by Pythagoras (or someone in his Brotherhood)[72] in Ancient Egypt, sometime during the 6th century BCE. During Pythagoras's time, the following theorem, an equation for the relation among the sides of right-triangle, was developed: $x^2 + y^2 = z^2$, where x & y are the sides (or legs) of the triangle, and that z is the hypotenuse (longest side). It is interesting to note, that Pythagoras gets credit for this, as mathematicians in the West have named this formula the Pythagorean Theorem, however "it was actually used by the Chinese and

Babylonians one thousand years before. However, these cultures did not know that the theorem was true for every right-angled triangle."[73]

Pythagoras *was* able to convincingly use a proof to deduce that this equation was in fact true for every right triangle. Pythagoras was one of the first mathematicians to introduce a proof, of a theorem, in that he was able to demonstrate that logically no counterexample could exist that would disprove his theory. As such, the Pythagorean Theorem would endure the test of time, and many other mathematicians including Fermat would try to expand or develop it further.

Fermat, as an intuitive mathematician, didn't often use mathematical proofs to demonstrate the universality of his theorems; however, all but one eventually had been proven to be true with little difficulty. Fermat's Last Theorem, however, which ties into the Pythagorean Theorem, eluded proof by mathematicians for centuries. Fermat wrote in his copy of Diophantus's *Arithmetica*, in a margin, "it is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as the sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers."[74] Fermat meant that the Pythagorean Theorem $(x^2 + y^2 = z^2)$, would never hold true if one changed all the squares to a different orders of power. For instance, there isn't an $x^3 + y^3 = z^3$, or anything higher, so in other words $x^n + y^n \neq z^n, for\ n > 2$. As was often with Fermat, he did not leave any proof of this conjecture. Some researchers believe he did try a few examples, but that alone would have made it unlikely that he would have become so certain that his deduction was incontrovertibly true. Fermat did leave one other note on his copy of *Arithmetica:* "I have a truly marvelous demonstration of the proposition which this margin is too narrow to contain."[75] – indicating that he had, indeed, found proof of the theorem. It is very unfortunate that Fermat did not take the time to write the proof of this discovery somewhere else as it would puzzle mathematicians for centuries as to how he figured it out, and whether or not it could be true. But then, for Fermat, mathematics was a hobby. He did not hold himself to the practices of a full time scholar of mathematics within a university setting.

It is, in fact, pure chance that Fermat's discovery of this theorem became known in the wider mathematical community. Other than his brief correspondence with Pascal on *the problem of the points*, Fermat was not very engaged in the mathematical community of his time. However, Fermat's oldest son Clémen-Samuel, spent years collecting his father's notes after Fermat died in 1665, and in 1670 published Fermat's copy of *Arithmetica* complete with his father's notes – which ended up being 48 notes in all.[76] All of these notes left no proof of anything, and once they had been distributed to the wider mathematical community, scholars began to try to recreate his proofs: "There were just enough tantalizing glimpses of logic to leave mathematicians in no doubt that Fermat had proofs, but filling in the details was left as a challenge for them to take up."[77]

Fermat claimed he had proofs, but without them, mathematicians could not take his theories as theorems; then mathematics as a whole would be in jeopardy. "As centuries passed, all his other observations were proved one by one, but Fermat's Last Theorem stubbornly refused to give in. In fact, it is called the 'Last' Theorem because it remains the last one of the observations to be proven. Three centuries of effort failed to find an answer."[78]

The first mathematician to systematically attempt to solve Fermat's riddle was Leonard Euler. As Euler worked on the puzzle, he first tried to prove that $x^4 + y^4 \neq z^4$, thinking that if he could prove this, that proof could then be extrapolated to all of the n > 2. (Euler did this because he discovered elsewhere in Fermat's copy of *Arithmetica* that Fermat had done some work on this particular example of his theorem (where n = 4) and had deemed it a success.) Euler tried to prove the formula by contradiction, using a method of infinite descent. He was actually able to prove the theorem for the example in which n = 3, this was the first improvement in 100 years on proving Fermat's Last Theorem, it was not *the proof*, but a beginning step. Euler was not able to prove anything for n being higher than three, and definitely not for all n's higher.[79]

Even though it took about 100 years for someone to offer a successful proof of the theorem, all hope was not lost. In both the n = 3 and n = 4 cases, the case had been proven, and, as long as you could make the n be a multiple of 3 or 4, those cases were also proved. Also, mathematicians eventually realized that "one merely has to prove it for the prime values of *n*. All other cases are merely multiples of the prime case and would be proved implicitly." However prime numbers were indefinite, and even with that subset, it would take a long time to prove.[80]

Sophie Germain, born in France in 1776, would be the next to make headway of Fermat's Last Theorem. She tackled the problem in a different way, detailed in her letter to Gauss, a German mathematician. Germain focused on what would become known as Germain Primes, where p is prime as well as $2p + 1$ is also prime. However, Germain said that "there were probably no solutions."[81] In 1825, however, her method proved useful. At that time, Gustave Lejeune-Dirichlet and Adrien-Marie Legender extrapolated on her success. They both independently successively proved the case when n = 5, and both honored Germain in their triumph.

In 1847 Enrst Kummer, a German mathematician, sent a letter to the French Academy of Sciences, outlining the argument the current method of approaching the theorem using prime factorization was flawed. He wrote that “a complete proof of Fermat’s Last Theorem was beyond the current mathematical approaches”[82] and the French Academy of Sciences posted a letter to everyone claiming that eleven theories had been presented but none had completely solved Fermat’s Last Theorem.[83] From here, mathematicians were severely puzzled as they had more than a century of new mathematical discoveries to fall back on, as opposed to Fermat, and if they couldn’t solve it, who could? It is true that in the century between Fermat’s death and when the French Academy of Sciences proposed the competition that mathematics had taken a turn of complete upheaval in terms of huge amounts of new discoveries. If the current generation of mathematicians did not know enough about number theory, how could Fermat truly have known and thought he had a proof?

After World War II, with the success of Alan Turing’s ingenious computer, proving Fermat’s Last Theorem finally had some success. “With the arrival of the computer, awkward cases of Fermat’s Last Theorem could be dispatched with speed.”[84] In the 1980’s Samuel S. Wagstaff successively proved Fermat’s Last Theorem up to where n was 25,000; however, one still could not solve it for all n > 2. Finally, in 1986, some headway had been accomplished, “Fermat’s Last Theorem was now inextricably linked to the Taniyama-Shimura conjecture [made in 1955]. If somebody could prove that every elliptic equation is modular, then this would imply that Fermat’s equation had no solutions, and immediately prove Fermat’s Last Theorem.”[85]

In June of 1993, a mathematician, Andrew Wiles completed his third of three lectures that he titled “Modular Forms, Elliptic Curves, and Galois Representations”[86] No one had been given any advance notice other than the title about what his lectures may be about. Finally, after

Wiles put the last part of the proof up, and at this point everyone in the audience could see where he was going and what a momentous occasion it was, Wiles simply said " 'I think I'll stop here,' and then there was sustained applause."[87] However, that would not be the end of the puzzle that is Fermat's Last Theorem. For a theorem to be officially proven, a report, in writing, had to be submitted for authentication, which usually was in a journal of mathematics. Wiles published his in the journal *Inventiones Mathematicae* shortly after his series of lectures in June.[88] However, in September of 1993, the authenticators came across a fundamental flaw in Wiles's proof. "The error did not necessarily mean that Wiles's work was beyond salvation, but it did mean that he would have to strengthen his proof."[89] Wiles had made a flaw in extrapolating a method of mathematics that he had used in support for his theorem. Finally, in May of 1995, Wiles had strengthened his proof, and actually with the success of a smaller proof included in the longer Fermat's Last Theorem proof, managed to fix his major flaw. The smaller proof was proven by Andrew Wiles and Richard Taylor, who Wiles went to for help. These two proofs equaled a 130 pages and more description of the proof.[90]

Still, mathematicians are baffled by Fermat's Last Theorem, as Fermat could not have used the mechanics known to mathematicians of the twentieth century, in the seventeenth century. Did Fermat actually have a complete proof? Or did Fermat make a conjecture that could not actually be proven until many more mathematical discoveries had been found?

Analysis

Pascal and Fermat are two mathematicians whose theories have definitely withstood the test of time. Both of them revolutionized so many areas of mathematics, and further helped develop new areas of mathematics that both of them could never have dreamed of. Religion impacted both mathematicians, with Pascal in more overt ways, and Fermat much less so – he seemed to exist in a more protected sphere, as he came from a wealth family and worked in a government position that was relatively shielded from the reach of the Church. Further, as he engaged in mathematical theorizing as a hobby, he did not have to answer to a Church-dominated academy about his work. When discussing mathematics, the two men were not focused on religion at all. They were both simply trying to delve deeper and solve the problems that had historically come before them, which had some applicability to their current interests. They wanted to advance other work, and in doing so created new fields of math, either in direct response to their collaboration; or new fields that other mathematicians who used their beginnings continued to develop. Most of the mathematical discoveries the two did built on the work of the mathematicians who came before them, which they wanted to improve on or make more generalized. In this sense, it is fortunate that the Church of the time had allowed the work of the earlier thinkers to be preserved and used.

It is remarkable that although the Church was hugely involved in the culture of France during the 17^{th} century, these two mathematicians were able to compartmentalize their thoughts of mathematics. Both, Pascal and Fermat, were able to simply explore with the purity of mathematics, and neither of them worried what the Church might say. One could assume that this was because they weren’t going in direct contradiction to the Church’s teachings; they weren’t philosophizing about God and his world; they were simply portraying the mathematical universe that they were engrossed in. The Church itself gave their thoughts latitude as it did not perceive the work of pure mathematics to be a threat to its teaching. Rather, it saw the universal patterns demonstrated by the knowledge of mathematics as confirming a God of creation and order in some sense.

Since it is more than four centuries later, and many of the musings of each mathematician were not actually published by themselves, it will always be a puzzle to completely understand how they actually felt about math and religion. The best that researchers can do is hypothesize and infer based on clues left behind. These two mathematicians are regarded as great ones, and they definitely deserve that praise.

Now, in the twenty-first century, it is assumed that the majority of people ‘publish’ or post their musings/findings on the internet, and that they fully reflect their own voices. Without the building blocks provided by Pascal ad Fermat, however, it is doubtful that the computer would have been developed, creating avenues for sharing thoughts and discoveries of which these mathematicians would never have dreamed.

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